Index

acceleration, 14, 76, 355
    centripetal, 27
tangential, 27
algebraic geometry, vii
analytic, 44
angle
    at a corner, 21
    on a regular surface, 170
angle excess, 337
angle of parallelism, 344
angular velocity, 3, 27
arc length, 16, 169, 355
area formula, 172
area-preserving maps, 183
asymptote of Dupin indicatrix, 217
asymptotic curve, 218, 278
asymptotic direction, 218

Bertrand mate, 89
bijection, 48
bitangent, 69
Bolyai, 339
Bonnet’s Formula, 283
bounded, 48

cardioid, 9, 32, 51
catenary, 40, 46
catenoid, 220, 243
Cauchy-Binet Theorem, 359
center of curvature, 37
Central projection, 150
Christoffel symbols
    of the first kind, 252
    of the second kind, 252
circle, 2, 6, 72
Clairaut relation, 317
class $C^r$, 14
Classification Theorem of Surfaces, 297
Codazzi equations, 259
colatitude, 126, 142, 181
compact, 172, 300
compatibility condition of a system of PDEs, 269
concave set, 64
cone, 134, 261, 320
conformal map, 182
conic surface, 126
conics, vii
conjugate directions, 221, 255
contact, order of, 33, 34, 91
continuity, 4
contraction, 388
convex set, 64
coordinate functions, 5, 71
coordinate lines, 130, 215
coordinate neighborhood, 141
coordinate patch, 129
coordinate system on a regular surface, 141
coordinates
    geodesic, 323
    geodesic polar, 329, 331
    normal, 331
    polar, 310
corner, 21, 290
critical point, 140
critical value, 18, 140
Crofton’s Theorem, 112
cross product, 77, 396
curvature
  first, 356
  Gaussian, 222, 226, 288, 292, 300, 334, 404
  geodesic, 275, 288, 292, 300
  mean, 222
  normal, 210, 275
  of a plane curve, 26
  of a space curve, 82
  principal, 212
  second, 357
  total, 107
  vector, 275
curve
  closed, 47, 103
  convex, 65
  diameter of, 51
  knotted, 115
  parallel, 32, 340
  parametric, 5, 71
  planar, 72, 88, 280
  regular, 18, 80
  simple, 48, 103
  trefoil, 60
cusp, 75, 290
cycloid, 7
curtate, 8, 128
prolate, 8
cylinder, 167, 232, 312
Darboux frame, 275
defect, 344
degree, 56
developable surface, 232
diameter, 106
diffeomorphism, 152
differentiable, 138
  vector function, 76
differential, 138, 139, 379
direction, 17
directrix, 232
discrete set, 289
distance
  Euclidean formula, 4
  on a surface, 306
Dupin indicatrix, 216
degree, 296
Einstein summation convention, 386
Elements, vii, 273, 339
ellipse, 6, 32, 50
ellipsoid, 126, 212
elliptic geometry, 335
Enneper’s surface, 208, 220, 244
envelope, 243
epicycloid, 11
Euclid, 273
Euclid’s postulates, vii
Euler characteristic, 297, 300
Euler’s curvature formula, 216, 276
evolute, 37
exponential map, 329
exterior of a closed curve, 49
external angle, 290
extremum, 24
face, 296
Fary-Milnor Theorem, 116
Fenchel’s Theorem, 108
first fundamental form, 165–176
flat torus, 368, 371
Four-Vertex Theorem, 68
frame, standard, 13
Frenet frame, 80, 358
function
  differentiable, 76
  in polar coordinates, 8
function graph, 6, 126, 158, 224
Fundamental Theorem
  of Curves, 362
of Plane Curves, 42  
of Space Curves, 97  
of Surface Theory, 268  

Gauss map, 192  
  differential of, 196  
Gauss's equations, 248, 264  
Gauss's formula, 252  
Gauss's Linking Formula, 121  
Gauss-Bonnet Theorem, 288, 292, 300  
Gaussian curvature, see curvature,  
  Gaussian  
geodesic, 305, 307, 341  
geodesic circle, 331  
geodesic coordinates, see  
  coordinates, geodesic  
geodesic curvature, see curvature,  
  geodesic  
geodesic torsion, 282, 320  
global property, viii  
Gram-Schmidt orthonormalization, 73  
Green's Theorem, 50  
helicoid, 243  
  helix, 74, 85, 107  
homomorphic, 140, 365  
homotopy, 115  
Horn's Lemma, 109  
hyperboloid, 75  
hypocycloid, 12  
ideal point, 344  
indicatrix  
  binormal, 107  
  principal, 107  
  tangent, 107  
inflexion point, 40  
intrinsic geometry, 175  
intrinsic properties, 247, 260  
invariant  
geometric, 25  
topological, 297, 303  
  under reparametrization, 18  
involute, 38  
isometry, 40, 42, 261  
isoperimetric inequality, 61  
Jacobi's Theorem, 112, 305  
Jacobian, 157  
Klein bottle, 372  
knot, 115  
  trefoil, 116  
Kronecker delta, 251, 379, 394  
  generalized, 394  
Lagrange multipliers, 211  
latitude, 181  
  latitude on a surface of revolution, 314  
lemniscate, 60  
length, 4  
lifting, 55  
limaçon, 60, 70  
limit, 4  
  line, 1, 72, 107  
  of curvature, 214  
  of stricture, 234, 242  
line element, 175  
line integral, 22  
link, 119  
link diagram, 120  
Liouville surface, 267, 320  
Liouville's Formula, 283  
Lissajous figure, 7, 31  
Lobachevsky, 339  
local properties, 1  
local property, viii  
locus, 5, 10, 72  
longitude, 181  
Lorentz tensor, 400, 401  
Lorentz transformation, 400  
loxodrome, 132, 170, 177
mean curvature, see curvature, mean
Mean Value Theorem, 227
measure, 112
meridian, 181
on a surface of revolution, 130
meridian on a surface of revolution, 314
metric tensor, 174, 366
minimal surface, 231, 239
Minkowski spacetime, 400
Möbius strip, 160
moment of inertia, 402
monkey saddle, 203, 229
moving trihedron, 91
natural equation, 43
natural equations, 362
non-Euclidean geometry, 340
nonorientable, 160
normal curvature, see curvature, normal
normal plane, 91
normal variation, 179, 229, 239
order of contact, 34
orientable surface, 159, 370
orientation
diagram, 164
of a curve, 30
of a surface, 163
orthogonal
coordinate system, 381
orthogonal transformation, 399
orthonormal basis, 13, 30, 81, 249
osculating
$k$-plane, 361
circle, 35
helix, 96
paraboloid, 201
plane, 90
sphere, 92
parabola, 38
parabolic, 201
parallel
coordinate lines, 178
surface, 244
parallel curve, see curve, parallel
parallels on a surface of revolution, 130
parameter, 1, 19
parametric equations, 5, 71
parametrization, 10, 129
by arc length, 19
orthogonal, 170, 323
path-connected, 264
pedal curve, 33
permutation, 393
even, 393
odd, 393
sign of, 393
pitch, 86, 89, 107
planar, 201
plane, 125
Plateau’s problem, 242
Playfair Axiom, 339
Poincaré disk, 348
Poincaré upper half-plane, 176, 256, 319, 345
point
critical, 18, 80
elliptic, 201, 207, 217
hyperbolic, 201, 207, 217
parabolic, 207, 217
planar, 207, 217
regular, 18, 80
umbilical, 214
positive definite, 172
positively oriented, 163
principal directions, 212, 249
proper orthogonal, 399
pseudosphere, 225, 229, 257, 350, 352
quadratic form, 199
radius of curvature, 27
rational normal curve, 362
rectifying plane, 92, 106
region, simple, 286, 323
regular, 356
regular value, 140
regularity condition, 140, 241, 365
reparametrization
  negatively oriented, 18, 76, 157
  positively oriented, 18, 76, 157
  regular, 18, 76
Riemann sum, 16
rotation index, 53
ruled surface, 231, 232
ruling, 232
scalar, 382
scaling factors, 381
secant surface, 132, 228
second derivative test, 203, 209, 225
second fundamental form, 198–207
self-adjoint operator, 199
self-intersection, 24
sensed-parallel, 343
simply connected, 285
singular point of a surface, 140
slices, 130
smoothness condition, 14
space cardioid, 75, 86, 107
special orthogonal, 399
Spectral Theorem, 212
speed, 16, 76, 355
sphere, 126, 168, 200, 223, 312
spherical geometry, 335
spiral
  exponential, 24
  linear, 15, 24
stereographic projection, 149
Stokes’ Theorem, 104
surface
  parametrized, 129
  regular, 133, 140, 365
surface of revolution, 127, 147, 177, 219, 229, 257, 314
tangent line, 77
tangent plane, 135, 136
tangent space, 135, 155
tangent vector, 80, 133
tangential indicatrix, 53
tangential surface, 158, 177, 228, 242
Tchebysheff net, 267
tensor, 386
  addition, 387
  antisymmetric, 392
  moment of inertia, 402
  multiplication, 387
  numerical, 394
  proper, 399
  symmetric, 391
Theorem of Beltrami-Enneper, 230
Theorem of Turning Tangents, 288, 292
Theorema Egregium, 260
topology, vii
torsion, 83
torus, 88, 130, 177, 202, 257, 283, 320
  flat, 369
torus knot, 88, 283
total angle function, 55
trace, 222
tractrix, 225
trajectory, 27
translation surface, 221, 258
transposition, 393
transversal intersection, 35
transverse intersection, 159
triangulation, 296
triple-vector product, 235, 278
tube, 158, 177, 228, 257
twisted cubic, 73
ultraparallel, 344
umbilical point, 264
unit normal to a surface, 157

vector, 1
  acceleration, 14
  affine, 385
  binormal, 81
  contravariant, 384
  covariant, 385
  direction, 2
  first normal, 357
  position, 13
  principal normal, 80
  unit normal, 26
  unit tangent, 18, 19, 80
  velocity, 14
vector field, 385
velocity, 14, 76, 355
vertex
  of a curve, 30
  of a triangulation, 296
Weingarten equations, 207
winding number, 57