USER GUIDE FOR COMPROP2

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OVERVIEW OF COMPROP2

COMPROP2 is an interactive computer program for the calculation of the properties of various compressible flows. For engineering applications involving compressible flow analysis, it is inevitable that tedious tables and charts have to be used in order to calculate the compressible flow properties. However, there are restrictions when using those tables and charts, for example, the specific heat ratio which indicates the type of fluid used is one of them. Most of the tables and charts are constructed using a particular value for the specific heat ratio. For other values, tables may not be available and the original equations for a particular flow may need to be solved numerically in order to obtain the desired properties. Moreover, when a situation involving shock waves is encountered, such as an oblique shock or a conical shock, it is very difficult for the user to accurately read the properties from the charts if the desired Mach number is not shown and visual interpolation has to be used. In these situations it is very easy to make calculation mistakes. The software developed (**COMPROP2**) resolves the above-mentioned problems. The computer languages used are Fortran and Delphi. Fortran is used to build a dynamic link library, which handles all the numerical calculations. Delphi is used to construct an interactive user-friendly graphical interface for the user to have a convenient way to access the library. There are six modules in this computer program. The first five modules calculate the properties for: **Isentropic Flow**, **Normal Shock**, **Oblique Shock**, **Fanno Flow**, and **Rayleigh Flow**. The last module is for **Supersonic Airfoil Analysis**. For the first five modules, the user can input data and obtain output through a dialog box or from a graph, which is generated using the flow equations. For the supersonic airfoil analysis, a CAD environment is developed for the user to define the dimensions and shape of an airfoil. The software can then calculate the lift force, the drag force, and the pressure distribution of the airfoil according to the flow Mach number and the airfoil angle of attack.

COMPROP2 can be used for a variety of practical applications dealing with compressible flow. For example, it can be used to study the effect of area change, friction, and heat transfer on compressible flow. In most physical situations, more than one of these effects occur simultaneously; for example, flow in a rocket nozzle involves area change, friction and heat transfer. However, one of the effects is usually predominant; in the rocket nozzle, area change is the factor having the greatest influence on the flow. The frequent predominance of one factor provides a justification for separating the effects, including them one at a time, and studying the resultant property variations. Hence the different modules provided in **COMPROP2** can be used to study these predominant effects separately. Although a certain loss of generality is incurred by treating each of the effects individually, this procedure does simplify the equations of motion so that the result of each of the effects can be easily appreciated. Further, this simplification enables approximate solutions to be derived for a wide range of problems in compressible flow; such solutions are sufficiently accurate for many engineering applications. Attempts to include all effects simultaneously in the equations of motion lead to mathematical complexities that mask the physical situation. In many cases exact solutions to these generalized equations of motion are impossible.

ISENTROPIC FLOW MODULE

This module can be used to analyze compressible, isentropic flow through varying area channels, such as nozzles, diffusers, and turbine-blade passages. Friction and heat transfer are negligible for this isentropic flow; variation in properties are brought about by area change. One-dimensional, steady flow of an ideal gas is assumed in order to reduce the equations to a workable form.

To illustrate the use of the **Isentropic Flow module**, five example problems have been solved with the assistance of **COMPROP2**. Please refer to **COMPROP2** under "Help, Contents, Module" for the details on this particular module.

Example 1:

At a point in the flow over an F-15 high-performance fighter airplane, the pressure, temperature and Mach number are 1890 lb/ft^2, 450 °R, and 1.5, respectively. At this point calculate T_o, p_o, T^*, p^*, and the flow velocity with the assistance of **COMPROP2**.

Solution:

For $M_1 = 1.5$, the following information can be obtained from the **Isentropic Flow module**:

Isentropic Flow

Input

Mach No.	1.5	Rho0/Rho
P0/P		A/A$^\times$
T0/T		Prandtl-Meyer Angle

Pick from Graph

Output

Mach No.	1.500E+00
P0/P	3.671E+00
Rho0/Rho	2.532E+00
T0/T	1.450E+00
A/A$^\times$	1.176E+00
Prandtl-Meyer Angle	1.191E+01

Thus, $p_o = 3.671p = 3.671(1890) = \underline{\textbf{6938 lb/ft}^2}$

$T_o = 1.45T = 1.45(450) = \underline{\textbf{652.5 °R}}$

Isentropic Flow Module (Continued)

Example 1: continued

To evaluate T^*, p^*, use the **Isentropic Flow module** for M = 1.0

p* and T* can now be evaluated from:

$p^*=(p^*/p_o)(p_o/p)p =(1/1.893)(3.671)(1890) = $ **3665 lb/ft^2**

$T^*=(T^*/T_o)(T_o/T)T = (1/1.2)(1.45)(450) = $ **543.8 °R**

The flow velocity can be calculated from: V = Ma

where $\qquad a = (\gamma RT)^{1/2} = [(1.4)(1716)(450)]^{1/2} = 1040\text{ft/s}$

Thus $\qquad$ V = Ma = (1.5)(1040) = **1560 ft/s**

Isentropic Flow Module (Continued)

Example 2:

Consider the isentropic subsonic-supersonic flow through a convergent-divergent nozzle. The reservoir pressure and temperature are 10 atm and 300 K, respectively. There are two locations in the nozzle where $A/A^* = 6$: one in the convergent section and the other in the divergent section. At each location, calculate M, p, T, and u with the assistance of **COMPROP2**.

Solution:

In the *convergent* section, with $A/A^* = 6$, from the **Isentropic Flow module**

The subsonic and supersonic solutions are obtained.

	Subsonic	Supersonic
M	0.097	3.368
$p = (p/p_o)p_o$	9.94 atm	0.1584 atm
$T = (T/T_o)T_o$	299.4 K	91.77 K
$a = (\gamma RT)^{1/2}$	346.8 m/s	192.0 m/s
$V = Ma$	33.6 m/s	646.7 m/s

Isentropic Flow Module (Continued)

Example 3:

A supersonic wind tunnel is designed to produce Mach 2.5 flow in the test section with standard sea level conditions. Calculate (with the assistance of **COMPROP2**) the exit area ratio and reservoir conditions necessary to achieve these design conditions.

Solution:

Using the **Isentropic Flow module** for M = 2.5 shown below, the area ratio is

$A_e/A^* = $ 2.673

The condition of the reservoir p_o, T_o can be calculated from the above results and the standard sea level conditions ($p_e = 1$ atm and $T_e = 288$ K) as shown below:

$p_o = (p_o/p_e)p_e = 17.09(1) = $ **17.09 atm**

$T_o = (T_o/T_e)T_e = 2.25(288) = $ **648 K**

Isentropic Flow Module (Continued)

Example 4:

Consider a rocket engine burning hydrogen and oxygen, the combustion chamber temperature and pressure are 3517 K and 25 atm, respectively. The molecular weight of the chemically reacting gas in the combustion chamber is 16 and $\gamma = 1.22$. The pressure at the exit of the convergent divergent rocket nozzle is 1.174×10^{-2} atm. The area of the throat is 0.4 m^2. Assuming a calorically perfect gas and isentropic flow, calculate (a) the exit Mach number, (b) the exit velocity, (c) the mass flow through the nozzle, and (d) the area of the exit. Use **COMPROP2** for these calculations.

Solution:

$\gamma = 1.22$ is not typically available in the compressible flow tables provided in textbooks. They usually provide values for $\gamma = 1.4$. Thus to solve this problem, one needs to use the governing equations directly. However, for **COMPROP2** this is not an issue. The software is designed to handle different values of γ. Simply change the default value of 1.4 for γ to 1.22 in the **Isentropic Flow module**. Using the **Isentropic Flow module** with $\gamma = 1.22$ and $p_o/p_e = 25/1.174\times10^{-2} = 2129.47$ we get:

(a) $M_e = $ **5.21**

Isentropic Flow Module (Continued)

Example 4: continued

(b) $T_e = (T_e/T_o)T_e = (1/3.983)3517 = 883$ K

$a_e = (\gamma RT_e)^{1/2} = [(1.22)(519.6)(883)]^{1/2} = 748.2$ m/s

Thus $V_e = M_e a_e = 5.21(748.2) = \underline{\textbf{3898 m/s}}$

(c) and (d) The results for parts (c) and (d) can be evaluated directly since the value of (A_e/A^*) is given in the output.

Since $A^* = 0.4$ m^2, from the above output, we have

$A_e = (A_e/A^*)A^* = 121(0.4) = \underline{\textbf{48.4 m}^2}$

$\rho_e = p_e/(RT_e) = (1.174\times10^{-2})(1.01\times10^5)/[(519.6)(883)] = 2.58\times10^{-3}$ kg/m^3

$\dot{m} = \rho_e A_e V_e = (0.00258)(3898)(48.4) = \underline{\textbf{487.5 kg/s}}$
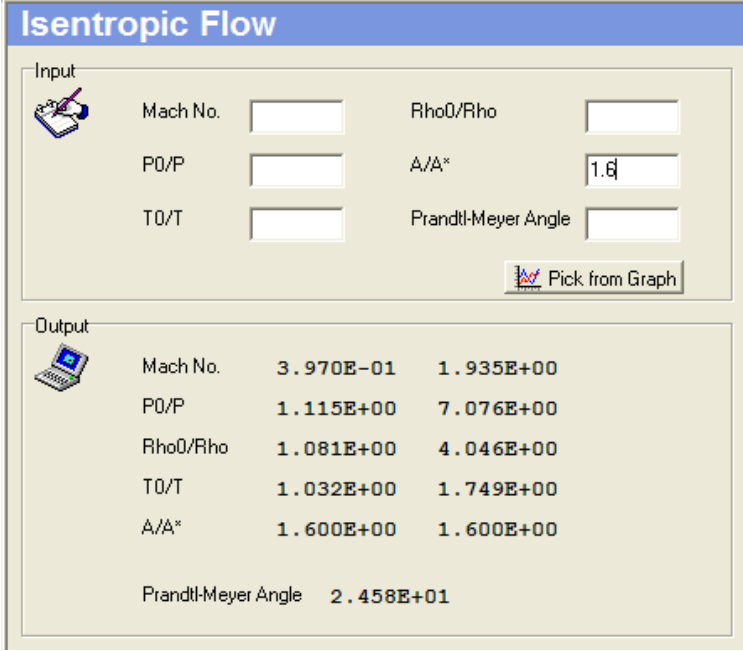
Isentropic Flow Module

Example 5 :

Consider a convergent-divergent duct with an exit to throat area ratio of 1.6. Calculate the exit-to-reservoir pressure ratio required to achieve sonic flow at the throat, but subsonic flow everywhere else. Use **COMPROP2** for these calculations.

Solution:

Since M =1 at the throat, $A_e = A^*$, and $A_e/A_t = A_e/A^* = 1.6$. Using $A_e/A^* = 1.6$ as an input to the **Isentropic Flow module**, we have

Thus, $p_e/p_o = 1/1.115 = $ **0.9**

NORMAL SHOCK MODULE

The shock process represents an abrupt change in fluid properties, in which finite variations in pressure, temperature, and density occur over a shock thickness comparable to the mean free path of the gas molecules involved. The supersonic flow adjusts to the presence of the body by means of such shock waves, whereas subsonic flow can adjust by gradual changes in flow properties. Shocks may also occur in the flow of a compressible medium through nozzles or ducts and thus may have a decisive effect on these flows. An understanding of the shock process and its ramifications is essential to the study of compressible flow.

This module deals with a plane shock normal to the flow direction. This case represents the simplest example of a shock, in that changes in flow properties occur only in the direction of flow; thus, it can be treated with the equations of one-dimensional gas dynamics. The next module will cover the oblique shock wave, positioned at an angle to the flow direction.

To illustrate the use of the **Normal Shock module**, three example problems have been solved with the assistance of **COMPROP2**. Please refer to **COMPROP2** under "Help, Contents, Module" for the details on this particular module.

Example 1:

A normal shock wave is standing in the test section of a supersonic wind tunnel. Upstream of the wave, $M_1 = 3$, $p_1 = 0.5$ atm and $T_1 = 200$ K. Find M_2, p_2, T_2 and u_2 downstream of the wave with the assistance of **COMPROP2**.

Solution:

Using the **Normal Shock module**, for $M_1 = 3$ as the input, we have

Normal Shock Module (Continued)

Example 1: continued

```
Normal Shock
 ┌Input───────────────────────────────────────────────┐
 │  [icon]   M1      [3      ]        P2/P1   [       ] │
 │                                                     │
 │           M2      [       ]        T2/T1   [       ] │
 │                                                     │
 │           P02/P01 [       ]        P02/P1  [       ] │
 │                                                     │
 │                                       [~ Pick from Graph] │
 └─────────────────────────────────────────────────────┘
 ┌Output──────────────────────────────────────────────┐
 │  [icon]   M1        3.000E+00     P02/P1  1.206E+01  │
 │           P2/P1     1.033E+01     M2      4.752E-01  │
 │           Rho2/Rho1 3.857E+00                        │
 │           T2/T1     2.679E+00                        │
 │           P02/P01   3.283E-01                        │
 └─────────────────────────────────────────────────────┘
```

$p_2 = (p_2/p_1)p_1 = (10.33)(0.5) = \underline{\textbf{5.165 atm}}$

$T_2 = (T_2/T_1)T_1 = 2.679(200) = \underline{\textbf{535.8 K}}$

$a_2 = (\gamma R T_2)^{1/2} = [(1.4)(287)(535.8)]^{1/2} = \underline{\textbf{464 m/s}}$

$V_2 = M_2 a_2 = (0.4752)(464) = \underline{\textbf{220 m/s}}$

Normal Shock Module (Continued)

Example 2:

Consider two flows, one of helium and one of air, at the same Mach number of 5. Denoting the strength of a normal shock by the pressure ratio across the shock, p_2/p_1, which gas will result in the stronger shock? For a monatomic gas such as helium, $\gamma = 1.67$, and for a diatomic gas such as air, $\gamma = 1.4$. Use **COMPROP2** for this problem.

Solution:

Using the **Normal Shock module** with $\gamma = 1.4$ and $M_1 = 5$ we get:

Specific heat ratio (Gamma) = 1.400

Normal Shock

Input

M1	5	P2/P1	
M2		T2/T1	
P02/P01		P02/P1	

Pick from Graph

Output

M1	5.000E+00	P02/P1	3.265E+01
P2/P1	2.900E+01	M2	4.152E-01
Rho2/Rho1	5.000E+00		
T2/T1	5.800E+00		
P02/P01	6.172E-02		

Thus $(p_2/p_1) = \underline{\textbf{29}}$ **(air)**

For helium ($\gamma = 1.67$), the compressible flow tables provided in typical textbooks can not be used since the tables are based on $\gamma = 1.4$. Thus to solve this problem, the governing equation for pressure ratio across a normal shock should be used. However, for **COMPROP2** this is not an issue. The software is designed to handle different values of γ. Simply change the default value of 1.4 for γ to 1.67 in the **Normal Shock module**. Using the **Normal Shock module** with $\gamma = 1.67$ and $M_1 = 5$ we get:

Normal Shock Module (Continued)

Example 2: continued

Thus $(p_2/p_1) =$ __31__ **(helium)**

Comparing the pressure ratio across the normal shock for air and helium, we conclude that for equal upstream Mach numbers, the **shock strength is greater in helium compared to air.**

Normal Shock Module (Continued)
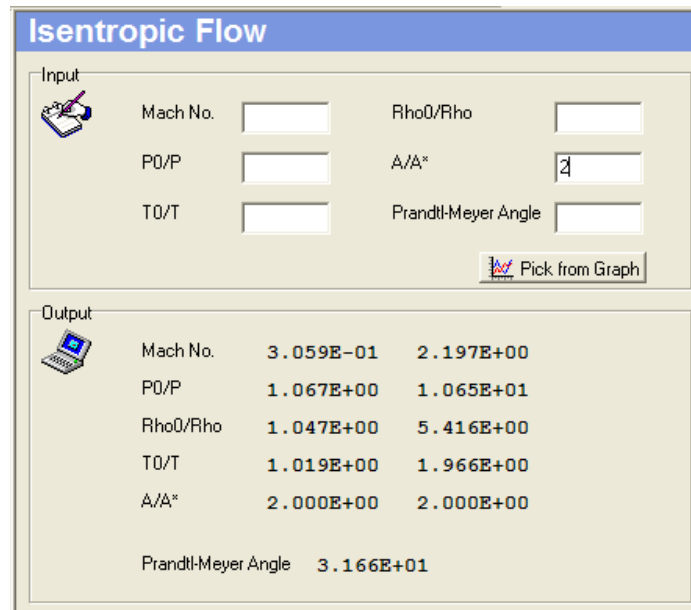
Example 3:

Consider a convergent-divergent nozzle with an exit to throat area ratio of 3. A normal shock wave is inside the divergent portion at a location where the local area ratio is $A/A_t = 2$. Calculate the exit-to-reservoir pressure ratio with the assistance of **COMPROP2**.

Note: The subjects of isentropic flow and normal shock waves were discussed in the prevoius two modules. Many compressible flow systems involve a combination of these two flows. It is instructive at this time to consider one such application to gain an appreciation of some of the interactions that may occur in engineering systems. The particular devise that will be discussed as described in the problem statement given above is the convergent-divergent nozzel.

Solution:

For this case, we have an isentropic subsonic-supersonic expansion through the part of the nozzle upstream of the normal shock. This part of the problem will be solved using the **Isentropic Flow module**. Let the subscripts 1 and 2 denote conditions immediately upstream and downstream of the shock, respectively. The local Mach number M_1 just ahead of the shock is obtained from the **Isentropic Flow module** for $A_1/A_1^* = 2.0$, namely $M_1 = 2.2$ as shown below.

Normal Shock Module (Continued)

Example 3: continued

For $M_1 = 2.2$, use the **Normal Shock module** to obtain $M_2 = 0.547$ and $p_{o2}/p_{o1} = 0.6281$ as shown below.

Normal Shock

Input

M1	2.2	P2/P1
M2		T2/T1
P02/P01		P02/P1

Pick from Graph

Output

M1	2.200E+00		P02/P1	6.716E+00
P2/P1	5.480E+00		M2	5.471E-01
Rho2/Rho1	2.951E+00			
T2/T1	1.857E+00			
P02/P01	6.281E-01			

For $M_2 = 0.547$ from the **Isentropic Flow module**, we can find $A_2/A_2^* = 1.26$ as shown below. Note that A^* is different for 1 and 2 (it changes across the shock wave), but $A_1 = A_2$ (the normal shock is assumed to be infinitely thin).

Isentropic Flow

Input

Mach No.	0.547	Rho0/Rho
P0/P		A/A*
T0/T		Prandtl-Meyer Angle

Pick from Graph

Output

Mach No.	5.470E-01
P0/P	1.226E+00
Rho0/Rho	1.156E+00
T0/T	1.060E+00
A/A*	1.259E+00
Prandtl-Meyer Angle	–
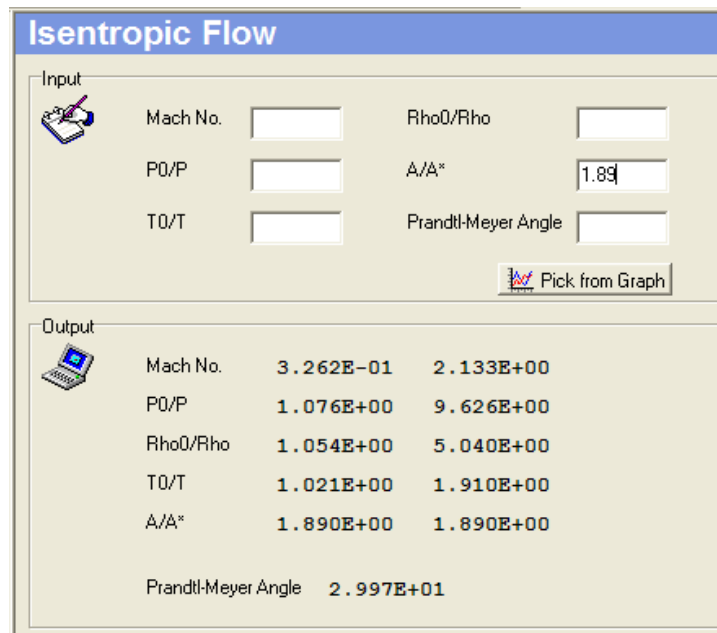
Normal Shock Module (Continued)

<u>Example 3</u>: continued

Proceeding with calculation, we have

$A_e/A_2^* = (A_e/A_2)(A_2/A_2^*) = (A_e/A_t)(A_t/A_2)(A_2/A_2^*) = (3)(1/2)(1.26) = 1.89$

The flow is *subsonic* behind the normal shock wave, and hence is subsonic throughout the remainder of the divergent portion downstream of the shock. For $A_e/A_2^* = 1.89$, from the **Isentropic Flow module**, we have

From the above results we have, $M_e = 0.32$ and $p_{oe}/p_e = 1.076$. Since $p_o = p_{o1}$ and $p_{oe} = p_{o2}$, we have

$p_e/p_o = (p_e/p_{oe})(p_{oe}/p_{o2})(p_{o2}/p_{o1})(p_{o1}/p_o) = (1/1.076)(1)(0.6281)(1) = \underline{\textbf{0.584}}$

Note: This problem required the use of the **Isentropic Flow module** and the **Normal Shock module**.

OBLIQUE SHOCK MODULE

   The normal shock wave, a compression shock normal to the flow direction, was discussed in the previous module. However, in a wide variety of physical situations, a compression shock wave occurs which is inclined at an angle to the flow. Such a wave is called an oblique shock. An oblique shock wave, either straight or curved, can occur in such varied examples as supersonic flow over a thin airfoil or in supersonic flow through an overexpanded nozzle.

   The analysis of the multidimensional oblique shock wave represents a departure from the one-dimensional flow covered in the previous module, yet in many ways the method of handling the oblique shock parallels that of handling the normal shock. Even though inclined to the flow direction, the oblique shock still represents a sudden, almost discontinuous change in fluid properties, with the shock process itself being adiabatic. This module focuses on the two-dimensional straight oblique shock wave, a type that might occur during the presence of a wedge in a supersonic stream or during a supersonic compression in a corner.

   To illustrate the use of the **Oblique Shock module**, six example problems have been solved with the assistance of **COMPROP2**. Please refer to **COMPROP2** under "Help, Contents, Module" for the details on this particular module.

Example 1:

A uniform supersonic stream with $M_1 = 3.0$, $p_1 = 1$ atm, and $T_1 = 288$ K encounters a compression corner (see figure) which deflects the stream by an angle $\delta = 20^o$. Calculate the shock wave angle, and p_2, T_2, M_2, p_{o2}, and T_{o2} behind the shock wave with the assistance of **COMPROP2**.
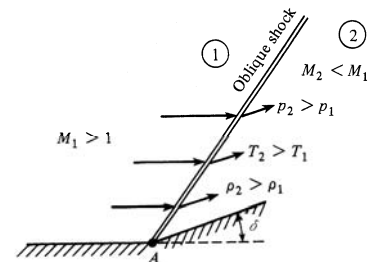
Solution:

Using the **Oblique Shock module**, for $M_1 = 3.0$ and $\delta = 20^o$, as shown below, we have

Shock wave angle = __37.8 ^{o}__

$p_2 = (p_2/p_1)p_1 = 3.771(1) = $ __3.771 atm__

$T_2 = (T_2/T_1)T_1 = 1.56(288) = $ __449.3 K__

$M_2 = $ __1.994__

Oblique Shock Module (Continued)

<u>**Example 1**</u>**: continued**

From the **Isentropic Flow module**, for $M_1 = 3.0$, we get: $p_{o1}/p_1 = 36.73$ and $T_{o1}/T_1 = 2.8$

Hence, from the output of these two modules, we get

$p_{o2} = (p_{o2}/p_{o1})(p_{o1}/p_1)p_1 = (0.7960)(36.73)(1) = $ <u>**29.24 atm**</u>

$T_{o2} = T_{o1} = (T_{o1}/T_1)T_1 = (2.8)(288) = $ <u>**806.4 K**</u>

Oblique Shock Module (Continued)

Example 2:

In Example 1, the deflection angle is increased to $\delta = 30^{\text{o}}$. Calculate the pressure and Mach number behind the wave, and compare these results with those of Example 1. Use **COMPROP2** for the calculations.

Solution:

From the **Oblique Shock module**, for $M_1 = 3$ and $\delta = 30^{\text{o}}$, we have

Thus $\qquad p_2 = (p_2/p_1)p_1 = (6.356)(1) = \underline{\textbf{6.356 atm}}$

$\qquad\qquad M_2 = \underline{\textbf{1.41}}$

Note: Compare these results to Example 1. When δ is increased from 20° to 30°, the shock wave becomes *stronger*, as evidenced by the increased pressure behind the shock (6.356 atm compared to 3.771 atm). The Mach number behind the shock is reduced (1.41 compared to 1.994). Also, as δ is increased, shock wave angle also increases (52° compared to 37.8^{o}).

Oblique Shock Module (Continued)

<u>**Example 3:**</u>

In Example 1, the free-stream Mach number is increased to 5. Calculate the pressure and Mach number behind the wave, and compare these results with those of Example 1. Use **COMPROP2** for the calculations.

<u>**Solution:**</u>

From the **Oblique Shock module**, for $M_1 = 5$ and $\delta = 20^o$, we have

Thus
$$p_2 = (p_2/p_1)p_1 = (7.037)(1) = \underline{\textbf{7.037 atm}}$$

$$M_2 = \underline{\textbf{3.02}}$$

Note: Compare these results to Example 1. When M_1 is increased from 3 to 5, the shock wave becomes *stronger*, as evidenced by the increased pressure behind the shock (7.037 atm compared to 3.771 atm). The Mach number behind the shock is increased (3.02 compared to 1.994). Also, as M_1 is increased, shock wave angle is decreased (29.8^o compared to 37.8^o).

Oblique Shock Module (Continued)

Example 4:

Consider a Mach 4 flow over a compression corner with a deflection angle of 32°. Calculate the oblique shock wave angle for the weak shock case using (a) Using Oblique Shock Wave Charts available in most textbooks and (b) the β-δ-M equation which can be found most textbooks. Compare the results from the two sets of calculations.

Solution:

We will solve this problem using **COMPROP2**.

(a) In **COMPROP2**, we can use the "Pick from Graph" function in the **Oblique Shock module** instead of using Oblique Shock Wave Charts.
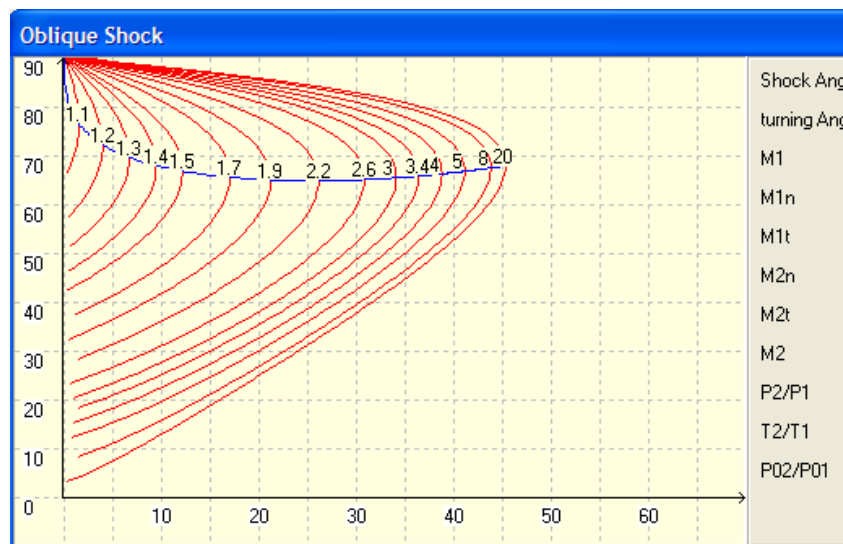
We can directly pick up the desired value of the oblique shock wave angle for the weak shock case for $\delta = 32^o$ and $M_1 = 4$ from the figure given above. The result is:

$$\beta = \underline{\mathbf{48.26^o}}$$

For the conditions selected in the above figure an output can also be generated by **COMPROP2**. To do this, left-click on the mouse and the generated output is shown below:

Oblique Shock Module (Continued)

Example 4: continued

Oblique Shock

┌─ Input ───┐
│ (1.) M1 and shock angle   [   ] [   ] │
│ (2.) Shock and Turning angle   [   ] [   ] │
│ (3.) Turning angle and M1   [32   ] [4   ] │
│   [📈 Pick from Graph] │
└───┘

┌─ Output ──┐
│ Max turning angle = **3.877E+01**   [Strong/Weak] │
│ Shock angle for this max turning angle = **6.606E+01** │
│   │
│ Shock angle   **4.826E+01**   M2t   **1.633E+00** │
│ Turning angle **3.200E+01**   M2   **1.701E+00** │
│ M1   **4.000E+00**   P2/P1 **1.023E+01** │
│ M1n   **2.985E+00**   T2/T1 **2.661E+00** │
│ M1t   **2.663E+00**   P02/P01 **3.327E-01** │
│ M2n   **4.761E-01**   *(Weak Oblique Shock)* │
└───┘

(b) For this part, using **COMPROP2** there is no need to use the β-δ-M equation. The user can simply use the interactive part of the **Oblique Shock module** (third option in the Input menu shown above) with the values of Turning Angle $(\delta) = 32^o$ and $M_1 = 4$. The result would be identical to Part (a), $\beta = \underline{\mathbf{48.26^o}}$.

Note: This problem clearly demonstrates the convenience rendered by **COMPROP2** in solving oblique shock problems.

Oblique Shock Module (Continued)

Example 5:

A uniform supersonic stream with M_1 = 1.5, p_1 = 1700 lb/ft², and T_1 = 460 °R encounters an expansion corner (see figure) which deflects the stream by an angle δ = 20°. Calculate M_2, p_2, T_2, p_{o2}, T_{o2}, and the angles the forward and rearward Mach lines make with respect to the upstream flow direction. Use **COMPROP2** for these calculations.
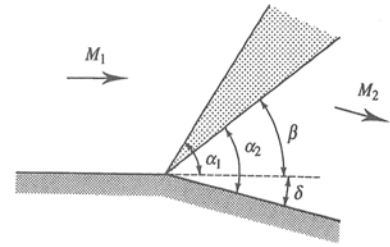
Note: This problem deals with **Expansion Wave (Prandtl-Meyer Flow)**. When a supersonic compression takes place at a concave corner, the flow is "turned into itself" and an oblique shock occurs at the corner. When supersonic flow passes over a convex corner, the flow is "turned away from itself" and an expansion wave is formed. An expansion wave emanating from a sharp corner is called *Prandtl-Meyer expansion wave*. The analysis of Prandtl-Meyer flow requires the relation between the Prandtle-Meyer angle and the Mach number. In most textbbooks this relation is either listed as a separate table or is included in the isentropic flow tables. In **COMPROP2**, the **Prandtl-Meyer Angle** is part of the **Isentropic Flow module**. It is instructive at this time to consider two examples of Prandtl-Meyer flow to gain an appreciation of some of the interactions that may occur in engineering systems.

Solution:

From the **Isentropic Flow module**, for M_1 = 1.5, the Prandtl –Meyer angle (θ_1) is 11.91°, as shown below:
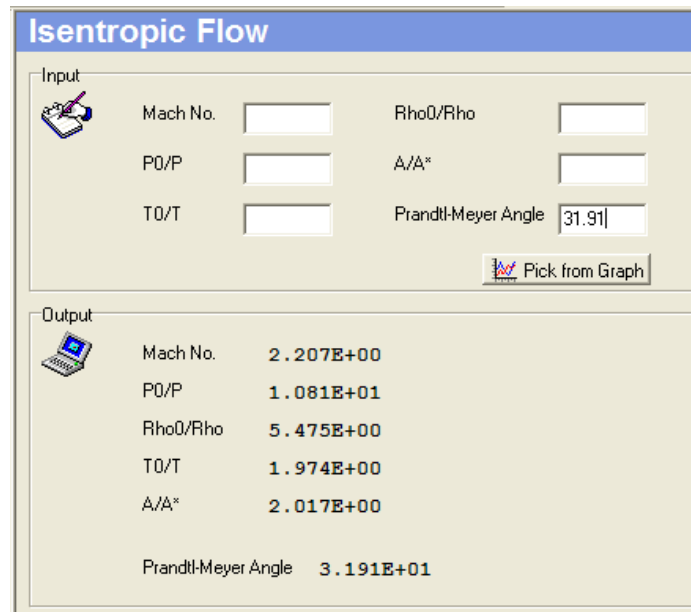
Oblique Shock Module (Continued)

Example 5 : continued

So $\quad \theta_2 = \theta_1 + \delta = 11.91 + 20 = 31.91^\circ$

From the **Isentropic Flow module**, for $\theta_2 = 31.91^\circ$, we have

Therefore

$M_2 = \underline{\mathbf{2.207}}$

$p_2 = (p_2/p_{o2})(p_{o2}/p_{o1})(p_{o1}/p_1)p_1 = (1/10.81)(1)(3.671)(1700) = \underline{\mathbf{577.3\ lb/ft^2}}$

$T_2 = (T_2/T_{o2})(T_{o2}/T_{o1})(T_{o1}/T_1)T_1 = (1/1.974)(1)(1.45)(460) = \underline{\mathbf{337.9\ ^oR}}$

$p_{o2} = p_{o1} = (p_{o1}/p_1)p_1 = (3.671)(1700) = \underline{\mathbf{6241\ lb/ft^2}}$

$T_{o2} = T_{o1} = (T_{o1}/T_1)T_1 = (1.45)(460) = \underline{\mathbf{667\ ^oR}}$

The Mach angle (α), can be obtained from the following equation, $\alpha = \arcsin(1/M)$. Thus, $\alpha_1 = 41.81^\circ$ and $\alpha_2 = 26.95^\circ$. Referring to the figure for the problem, we see that:

Angle of forward Mach line $= \alpha_1 = \underline{\mathbf{41.81^o}}$

Angle of rearward Mach line $= \beta = \alpha_2 - \delta = 26.95 - 20 = \underline{\mathbf{6.95^o}}$

Oblique Shock Module (Continued)

Example 6:

Consider an infinitely thin flat plate at a 5° angle of attack in a Mach 2.6 free stream (see figure). Calculate the lift and drag coefficients using **COMPROP2**. Note in the figure, "c" is the span.
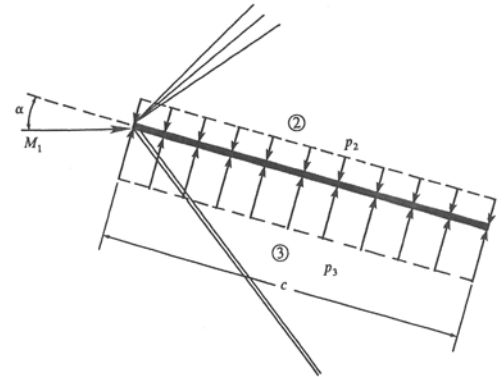
Solution:

From the **Isentropic Flow module**, for $M_1 = 2.6$, the Prandtl-Meyer (θ_1) is 41.41^o, as shown below:

So $\theta_2 = \theta_1 + \delta = 41.41 + 5 = 46.41^o$

From the **Isentropic Flow module,** for $\theta_2 = 46.41^o$, we obtain $p_{o2}/p_2 = 28.5$ as shown below.

Thus,

$p_2/p_1 = (p_2/p_{o2})(p_{o2}/p_{o1})(p_{o1}/p_1) = (1/28.5)(1)(19.95) = 0.7$

Oblique Shock Module (Continued)

Example 6: continued

Isentropic Flow

Input

Mach No. [　　　]   Rho0/Rho [　　　]

P0/P [　　　]   A/A* [　　　]

T0/T [　　　]   Prandtl-Meyer Angle [46.41]

Pick from Graph

Output

Mach No.   2.832E+00

P0/P   2.850E+01

Rho0/Rho   1.094E+01

T0/T   2.604E+00

A/A*   3.609E+00

Prandtl-Meyer Angle   4.641E+01

From the **Oblique Shock module**, for $M_1 = 2.6$ and $\alpha = \delta = 5^o$, we have

Oblique Shock

Input

(1.) M1 and shock angle   [　　　] [　　　]

(2.) Shock and Turning angle   [　　　] [　　　]

(3.) Turning angle and M1   [5] [2.6]

Pick from Graph

Output

Max turning angle = 3.081E+01   Strong/Weak

Shock angle for this max turning angle = 6.487E+01

Shock angle   2.641E+01   M2t   2.219E+00

Turning angle   5.000E+00   M2   2.384E+00

M1   2.600E+00   P2/P1   1.394E+00

M1n   1.157E+00   T2/T1   1.101E+00

M1t   2.329E+00   P02/P01   9.963E-01

M2n   8.704E-01   *(Weak Oblique Shock)*

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Oblique Shock Module (Continued)

Example 6 : continued

From the **Normal Shock module**, for $M_{1n} = 1.16$, we obtain $p_3/p_1 = 1.403$ as shown below.

The lift per unit span L' is: $\qquad L' = (p_3 - p_2)\, c\, \cos\alpha$

The drag per unit span D' is: $\qquad D' = (p_3 - p_2)\, c\, \sin\alpha$

The free-stream dynamic pressure is defined as $q_1 = (\gamma/2)\, p_1\, M_1^2$

The lift coefficient is then calculated from

$C_L = L'/(q_1 c) = 2/(\gamma M_1^2)[(p_3/p_1) - (p_2/p_1)]\cos\alpha \ = 2/(1.4)(2.6)^2(1.403 - 0.7)\cos 5^o = \underline{\textbf{0.148}}$

The drag coefficient is then calculated from

$C_D = D'/(q_1 c) = 2/(\gamma M_1^2)[(p_3/p_1) - (p_2/p_1)]\sin 2/(1.4)(2.6)^2(1.403 - 0.7)\sin 5^o = \underline{\textbf{0.013}}$

FANNO FLOW MODULE

In the Isentropic Flow module, compressible flow in ducts was analyzed for the case in which changes in flow properties were brought about solely by area change. In a real flow situation, however, frictional forces are present and may have a decisive effect on the resultant flow characteristics. The first part of this module is concerned with compressible flow with friction in constant-area, insulated ducts, which eliminate the effects of area change and heat addition. In a practical sense, these restrictions limit the applicability of the resultant analysis; however, certain problems such as flow in short ducts can be handled and, furthermore, an insight is provided into the general effects of friction on a compressible flow. The second part of this module deals with flow with friction in constant-area ducts, in which the fluid temperature is assumed constant. The latter case approximates the flow of a gas through long, uninsulated pipeline. Thus, these two cases cover a wide range of frictional flows and are consequently of great significance.
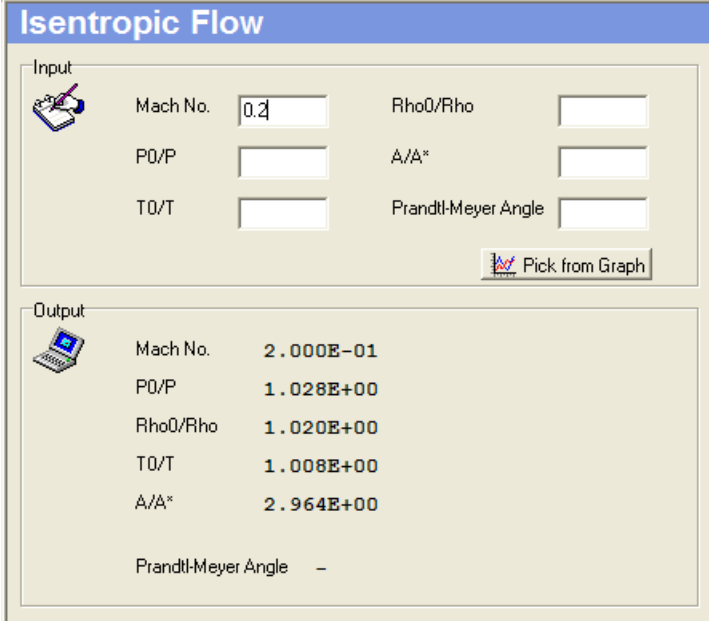
To illustrate the use of the **Fanno Flow module**, an example problem has been solved with the assistance of **COMPROP2**. Please refer to **COMPROP2** under "Help, Contents, Module" for the details on this particular module.

Example 1:

Consider the flow of air through a pipe of inside diameter = 0.15 m and length = 30 m. The inlet flow conditions are $M_1 = 0.3$, $p_1 = 1$ atm, and $T_1 = 273K$. Assuming f = const. = 0.005, calculate the flow conditions at the exit, M_2, p_2, T_2, and p_{o2} using **COMPROP2**.

Solution:

From the **Isentropic Flow module**, for $M_1 = 0.3$ as an input, we obtain $p_{o1}/p_1 = 1.064$ as shown below:

Hence,

$p_{o1} = (p_{o1}/p_1)p_1 = 1.064(1\text{atm}) = 1.064$ atm

Fanno Flow Module (Continued)

Example 1: continued

From the **Fanno Flow module**, for $M_1 = 0.3$ and $4f(L_1^*/D) = 5.299$, we have

Fanno Flow Module (Continued)

Example 1: continued

Hence,

$$\frac{4\bar{f}L_2^*}{D} = \frac{4\bar{f}L_1^*}{D} - \frac{4fL}{D} = 5.299 - \frac{(4)(0.005)(30)}{0.15} = 1.2993$$

From the **Fanno Flow module**, $4f(L_2^*/D) = 1.2993$, we have

Hence,

$M_2 = \underline{\textbf{0.474}}$

$p_2 = (p_2/p^*)(p^*/p_1)p_1 = 2.259(1/3.619)(1atm) = \underline{\textbf{0.624 atm}}$

$T_2 = (T_2/T^*)(T^*/T_1)T_1 = 1.148(1/1.179)(273) = \underline{\textbf{265.8 K}}$

$p_{o2} = (p_{o2}/p_o^*)(p_o^*/p_{o1})p_{o1} = 1.392(1/2.035)(1.064) = \underline{\textbf{0.728 atm}}$

RAYLEIGH FLOW MODULE

We have discussed in Isentropic Flow and Fanno Flow modules the effects on a gas flow of area change and friction. For these cases, flows were assumed to be adiabatic. In this module, the effect of heat addition or loss on a one-dimensional frictionless gas flow in a constant-area duct will be investigated. Flows with heat transfer occur in a variety of situations, for example, combustion chambers, in which the heat addition is supplied internally by a chemical reaction, or heat exchangers, in which heat flow occurs across the system boundaries.

To illustrate the use of the **Rayleigh Flow module**, an example problem has been solved with the assistance of **COMPROP2**. Please refer to **COMPROP2** under "Help, Contents, Module" for the details on this particular module.

Example 1:

Air enters a constant area duct at $M_1 = 0.2$, $p_1 = 1$ atm, and $T_1 = 273$ K. Inside the duct, the heat added per unit mass is $q = 1.0 \times 10^6$ J/kg. Calculate the flow properties M_2, p_2, T_2, ρ_2, T_{o2}, and p_{o2} at the exit of the duct with the assistance of the **COMPROP2** program.

Solution:

From the **Isentropic Flow module**, for $M_1 = 0.2$, we have

Rayleigh Flow Module (Continued)

<u>Example 1</u>: continued

Hence,

$T_{o1} = 1.008 T_1 = 1.008(273) = 275.2$ K
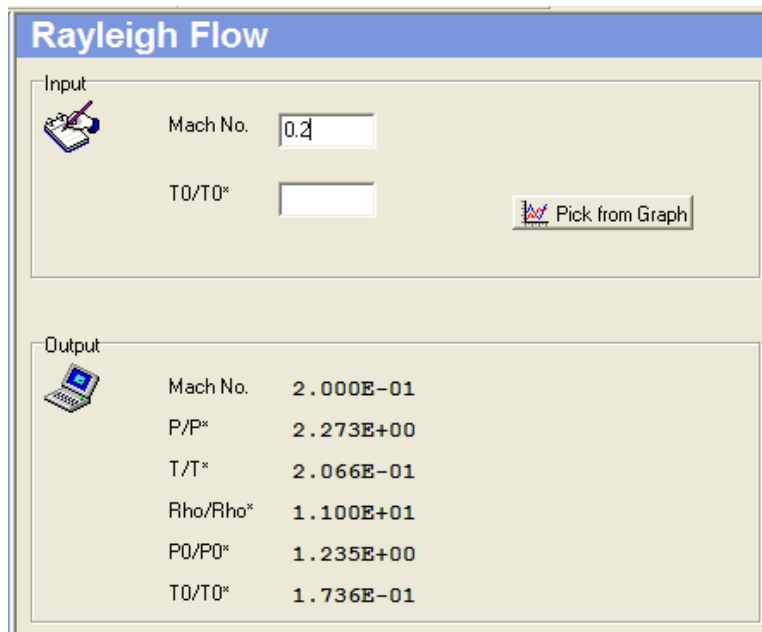
$p_{o1} = 1.028 p_1 = 1.028(1 atm) = 1.028$ atm

The specific heat can be evaluated from $c_p = \gamma R/(\gamma - 1) = (1.4)(287)/0.4 = 1005$ J/kg·K

Now, using the equation for the exit temperature (T_{o2}), we calculate

$T_{o2} = q/c_p + T_{o1} = (1.0 \times 10^6/1005) + 275.2 = \underline{\mathbf{1270\ K}}$

From the **Rayleigh Flow model**, for $M_1 = 0.2$, we have
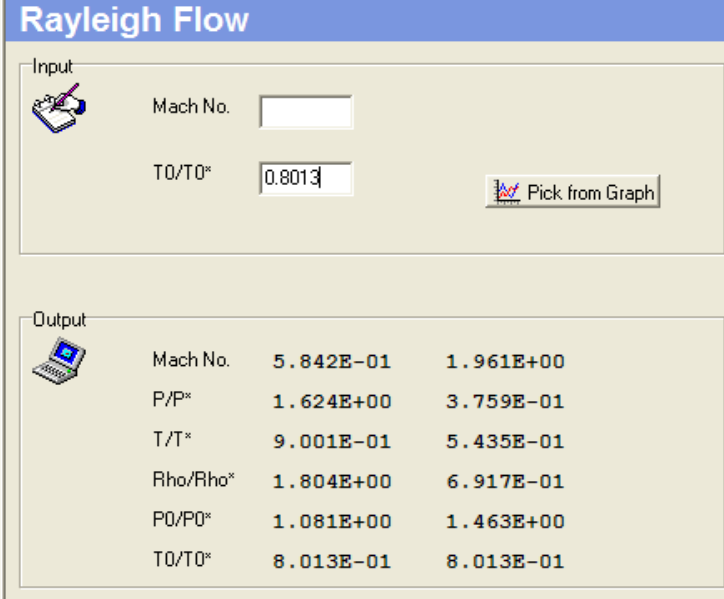
Hence,

$T_{o2}/T_o^* = (T_{o2}/T_{o1})(T_{o1}/T_o^*) = (1270/275.2)(0.1736) = 0.8013$

Rayleigh Flow Module (Continued)

<u>Example 1</u>: continued

From the **Rayleigh Flow module**, for $T_{o2}/T_o^* = 0.8013$, we have $M_2 = $ **<u>0.58</u>,** as shown below
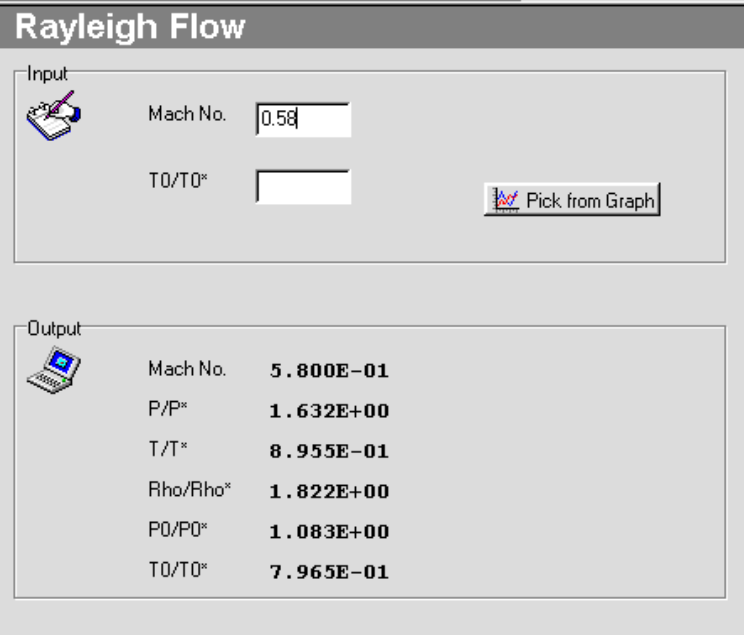
From the **Rayleigh Flow module**, for $M_2 = 0.58$**,** we have

Rayleigh Flow Module (Continued)

Example 1: continued

Hence,

$T_2 = (T_2/T^*)(T^*/T_1)T_1 = (0.8955)(1/0.2066)(273) = \underline{\textbf{1183 K}}$

$p_2 = (p_2/p^*)(p^*/p_1)p_1 = (1.632)(1/2.273)(1) = \underline{\textbf{0.718 atm}}$

$p_{o2} = (p_{o2}/p_o^*)(p_o^*/p_{o1})p_{o1} = (1.083)(1/1.235)(1.028) = \underline{\textbf{0.902 atm}}$

Since 1 atm = 1.01×10^5 N/m^2,

$\rho_2 = p_2/RT_2 = (0.718)(1.01 \times 10^5)/[(278)(1183)] = \underline{\textbf{0.214 kg/m}^{\textbf{3}}}$
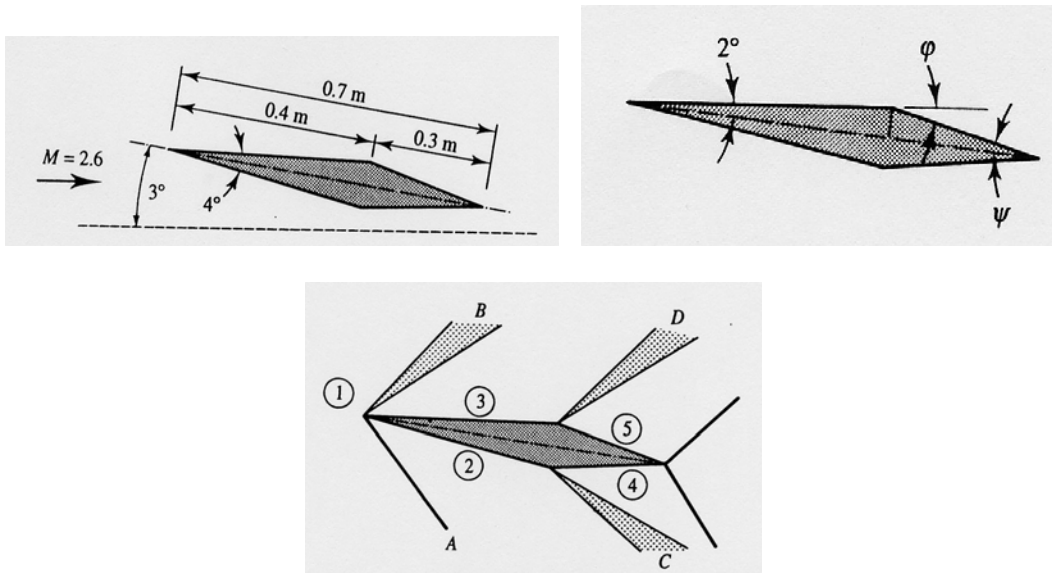
AIRFOIL MODULE

The design of an airfoil should be such as to provide a lift force normal to the undisturbed flow accompanied by low drag force in the direction of the undisturbed flow. The shape of a wing section to be used in low-speed, incompressible flow is the well-known teardrop or streamlined profile. In supersonic flow, however, the design must be completely modified owing to the occurrence of shocks. The high pressures after the shock wave produce excessive drag forces on the airfoil. To minimize wave drag, or drag due to the presence of shocks, the supersonic airfoil must have a pointed nose and also be as thin as possible. The ideal case is a flat plate airfoil, possessing zero thickness.

To illustrate the use of the **Airfoil module** in the analysis and design of supersonic airfoils, an example problem has been solved with the assistance of **COMPROP2**. Please refer to **COMPROP2** under "Help, Contents, Module" for the details on this particular module.

Example 1 (Supersonic Airfoil Analysis):

Consider the supersonic airfoil shown below and calculate the lift and drag forces by two methods: (a) hand calculation with the assistance of different **COMPROP2** modules and (b) use of the **Airfoil module** alone in the **COMPROP2** program.

Given:

Where: Angle of attack = 3°
                    Mach no. ahead of airfoil = 2.6
                    Pressure ahead of airfoil = 40 kPa

Airfoil Module (Continued)

Example 1 (Supersonic Airfoil Analysis): continued

The angle of turn for various waves is summarized below

Type of wave	Angle of turn
Shock Wave (A, from 1 to 2)	5°
Expansion Wave (B, from1 to 3)	1°
Expansion Wave (C, from 2 to 4)	4.67°
Expansion Wave (D, from 3 to 5)	4.67°

Solution:

Part (a): For oblique shock wave A, $M_1 = 2.6$ and the turning angle = 5°, from the **Oblique Shock module**, we have

Oblique Shock

Input

(1.) M1 and shock angle

(2.) Shock and Turning angle

(3.) Turning angle and M1   `5`   `2.6`

Pick from Graph

Output

Max turning angle = 3.081E+01   Strong/Weak
Shock angle for this max turning angle = 6.487E+01

Shock angle	2.641E+01	M2t	2.219E+00
Turning angle	5.000E+00	M2	2.384E+00
M1	2.600E+00	P2/P1	1.394E+00
M1n	1.157E+00	T2/T1	1.101E+00
M1t	2.329E+00	P02/P01	9.963E-01
M2n	8.704E-01	*(Weak Oblique Shock)*	

For expansion wave B, from the **Isentropic Flow module**, for $M_1 = 2.6$, we have

Airfoil Module (Continued)

Example 1 (Supersonic Airfoil Analysis): continued

Since the flow is turned through 1° by the expansion wave, the new angle (Prandtl-Meyer angle) is now 42.41^o, using the **Isentropic Flow module** again, we have
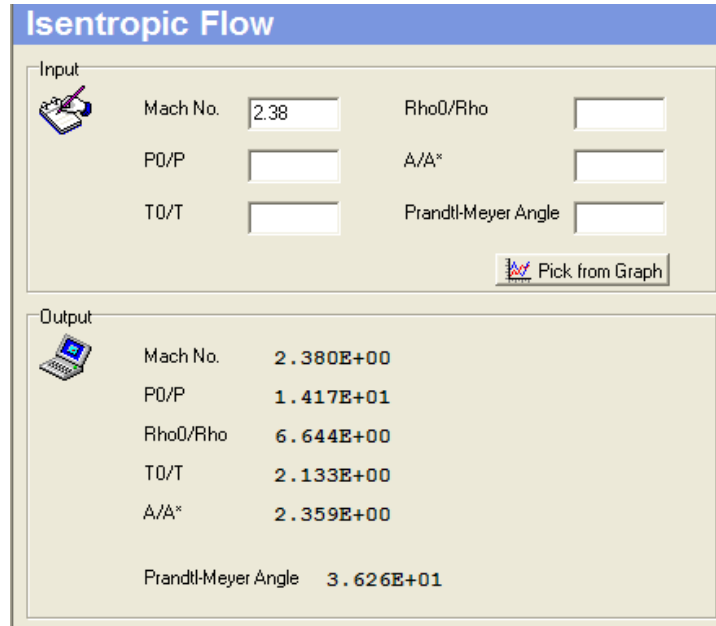
Hence, $p_3 = (p_3/p_{o3})(p_{o1}/p_1)p_1 = (1/21.38)(19.95)(40) = 37.32$ kPa

Airfoil Module (Continued)
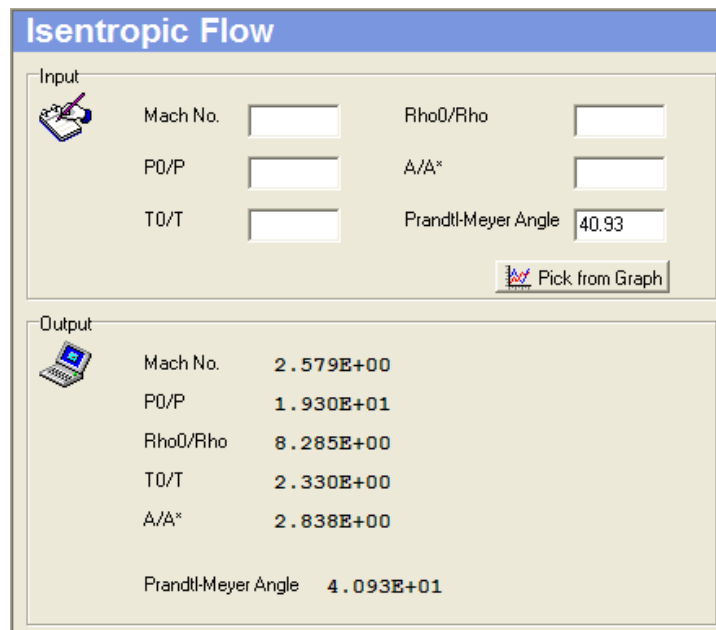
Example 1 (Supersonic Airfoil Analysis): continued

Now consider the expansion wave C with Mach number ahead ($M_2 = 2.38$, obtained from the **Oblique Shock module**), from the **Isentropic Flow module** we have

Isentropic Flow

Input

Mach No.	2.38	Rho0/Rho	
P0/P		A/A*	
T0/T		Prandtl-Meyer Angle	

Pick from Graph

Output

Mach No.	2.380E+00
P0/P	1.417E+01
Rho0/Rho	6.644E+00
T0/T	2.133E+00
A/A*	2.359E+00
Prandtl-Meyer Angle	3.626E+01

Since the flow is turned by the expansion wave through 4.67^o, the new angle (Prandtl-Meyer angle) is now $36.26^o + 4.67^o = 40.93^o$, with this angle as the new input to the **Isentropic Flow module**, we have:

Isentropic Flow

Input

Mach No.		Rho0/Rho	
P0/P		A/A*	
T0/T		Prandtl-Meyer Angle	40.93

Pick from Graph

Output

Mach No.	2.579E+00
P0/P	1.930E+01
Rho0/Rho	8.285E+00
T0/T	2.330E+00
A/A*	2.838E+00
Prandtl-Meyer Angle	4.093E+01

Airfoil Module (Continued)

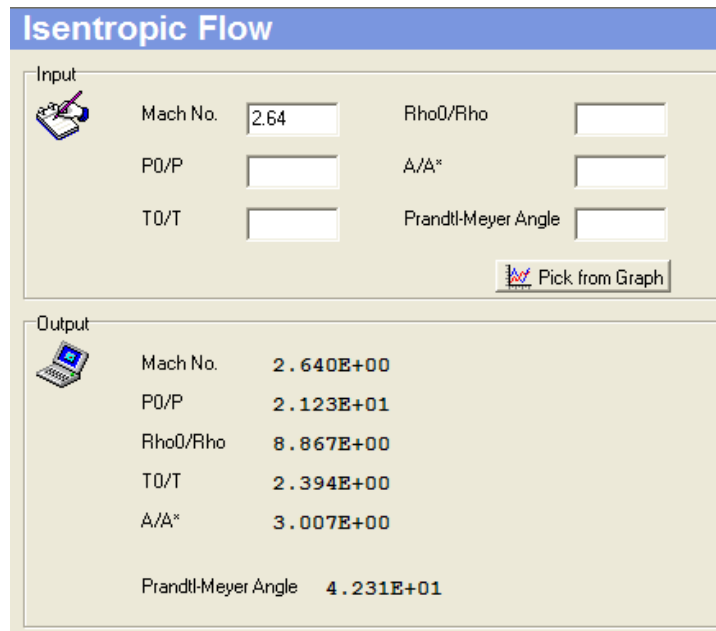## Example 1 (Supersonic Airfoil Analysis): continued

Hence,

$p_4 = (p_4/p_{o4})(p_{o2}/p_2)p_2 = (14.17)(1/19.3)(56.1) = 41.19$ kPa

[**Note**: p_2 is from $p_2/p_1 = 1.403$ (the **Normal Shock module**), $p_2 = (1.403)(40) = 56.1$ kPa]

For expansion wave D, with $M_3 = 2.64$, from **Isentropic Flow module** we have

Isentropic Flow

Input

Mach No. [2.64]   Rho0/Rho [   ]

P0/P [   ]   A/A* [   ]

T0/T [   ]   Prandtl-Meyer Angle [   ]

Pick from Graph

Output

Mach No.	2.640E+00
P0/P	2.123E+01
Rho0/Rho	8.867E+00
T0/T	2.394E+00
A/A*	3.007E+00
Prandtl-Meyer Angle	4.231E+01

Now the expansion angle (Prandtl-Meyer angle) should be $42.31^o + 4.67^o = 46.98^o$ and this angle is now the new input to the **Isentropic Flow module**.

Airfoil Module (Continued)

Example 1 (Supersonic Airfoil Analysis): continued

Then, $p_5 = (p_5/p_{o5})(p_{o3}/p_3)p_3 = (1/29.73)(21.23)(37.27) = 26.61\text{kPa}$

Therefore, $p_2 = 56.1$ kPa, $p_3 = 37.32$ kPa, $p_4 = 41.19$ kPa, and $p_5 = 26.61$ kPa.

For the areas,

$$A_2 = A_3 = 0.4/\cos 2° = 0.4 \text{ m}^2$$
$$A_4 = A_5 = 0.3/\cos 2.67° = 0.3 \text{ m}^2$$

The lift force per meter span is then

Lift Force $= 56.1 \times 0.4 \times \cos 5° - 37.32 \times 0.4 \times \cos 1° + 41.19 \times 0.3 \times \cos 0.33°$
$\qquad - 26.61 \times 0.3 \times \cos 5.67° = \textbf{\underline{11.84 kN/m span}}$

The drag force per meter span is then

Drag Force $= 56.1 \times 0.4 \times \sin 5° - 37.32 \times 0.4 \times \sin 1° + 41.19 \times 0.3 \times \sin 0.33°$
$\qquad - 26.61 \times 0.3 \times \sin 5.67° = \textbf{\underline{0.978 kN/m span}}$

Airfoil Module (Continued)

### Example 1 (Supersonic Airfoil Analysis): continued

Part (b): Now, let's consider the usage of the **Airfoil module** to calculate the lift and drag forces for the problem that was solved in Part (a). Select the module by clicking the icon and input the airfoil chord length which is 0.7m, the Mach number ahead of the airfoil which is 2.6, the pressure ahead of the airfoil which is 40 kPa, and finally the angle of attack which is 3°. Since the length of the airfoil is 0.7m, with the other specifications, user can easily draw the airfoil in the CAD environment provided by the module.
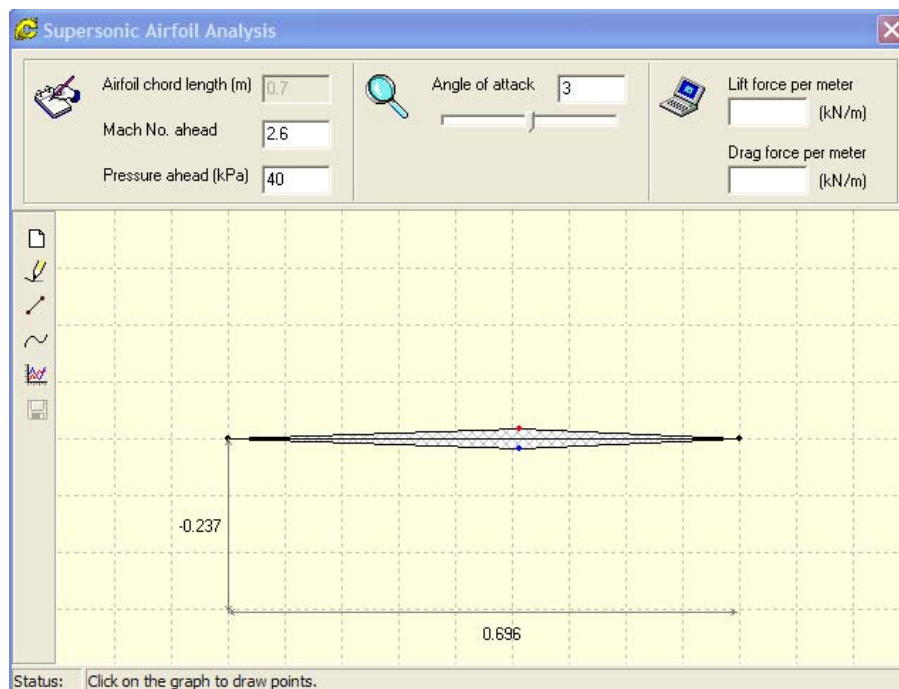
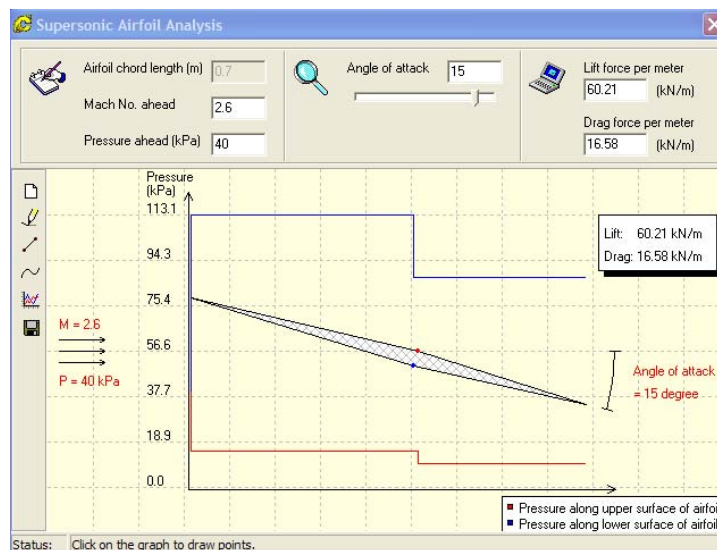Click the calculation icon, then you can get the lift and the drag force!

Airfoil Module (Continued)

<u>Example 1 (Supersonic Airfoil Analysis)</u>: continued
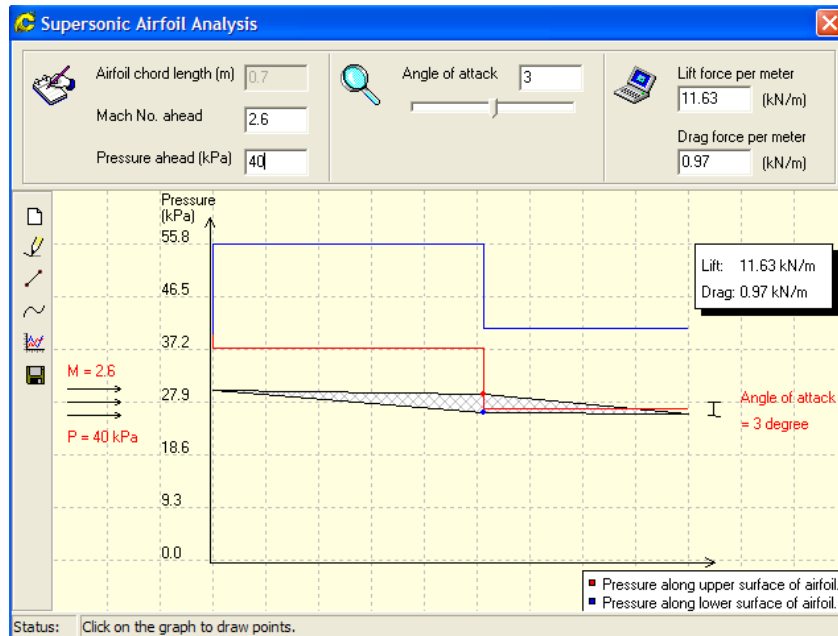
The **lift force** from the output is **11.63 kN/m span** and the **drag force** is **0.97 kN/m span**. Comparing to Part (a) of the example problem both results are almost identical.

The user can also conduct parametric studies for design purposes by varying the angle of attack, the pressure ahead of the airfoil, or even the shape of the airfoil. For example, the angle of attack is now changed to 15°.

Airfoil Module (Continued)

Example 1 (Supersonic Airfoil Analysis): continued

The lift force changes from 11.65 to 60.21 kN/m. If the user wants to change the shape of the foil, all one needs to do is to drag the red and blue dots. This is shown below.
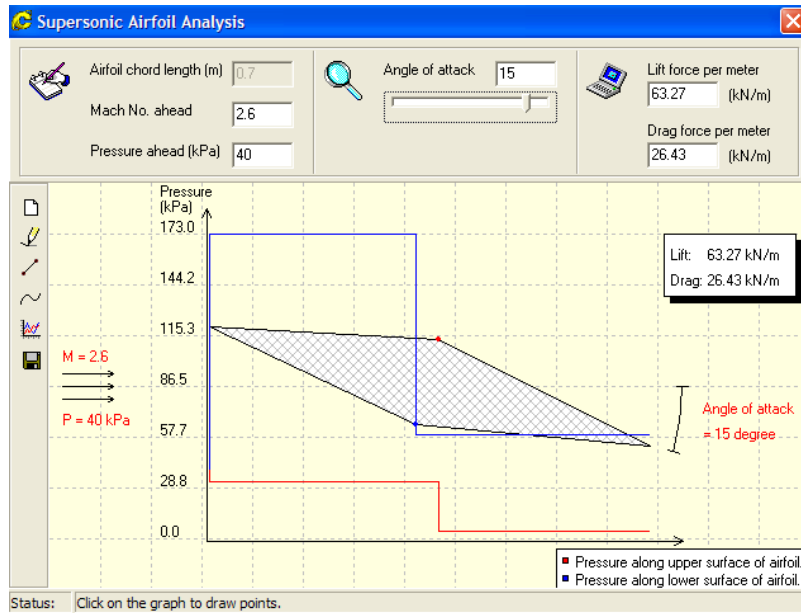
In this case, it can be observed that the lift force is slightly larger (63.27 kN/m compared to 60.21 kN/m); however, the drag force has increased significantly (26.43 kN/m compared to 16.58 kN/m).

Note: This example problem demonstrated how the different modules in **COMPROP2** could be used for analysis/design of the **Supersonic Airfoil**. It was also demonstrated that the special module called **Airfoil module** is much easier to use for this type of problem. In addition, this module offers tremendous opportunity for in depth analysis and design of supersonic airfoils.