Example 1.3

Calculate the Mean Thermal Conductivity, Thermal Resistance, and Surface Temperature of a Silicon Chip with Temperature-Dependent Conductivity.

Input:

Q=Heat dissipation, W. $Q := 50$ $A_k := 0.5\cdot 0.5$

T_2=Temperature at chip base, °C. $T_2 := 100$

t=Chip thickness, in.

A_k=Chip cross-sectional area for conduction. $t := 0.02$

Thermal Conductivity:

T_1=Chip temperature guess for first iteration. $T_1 := 150$

k_m=Mean thermal conductivity for pure silicon, W/in.°C.

$$k_m(T) := \frac{3429}{T - T_2}\left[\left(\frac{T_2 + 273.16}{300}\right)^{\frac{-1}{3}} - \left(\frac{T + 273.16}{300}\right)^{\frac{-1}{3}}\right]$$

First Iteration:

$k := k_m(T_1)$ $k = 2.618$

$R := \dfrac{t}{k\cdot A_k}$ $R = 0.031$

$T_1 := T_2 + Q\cdot R$ $T_1 = 101.528$

Second Iteration:

$k := k_m(T_1)$ $k = 2.84$

$R := \dfrac{t}{k\cdot A_k}$ $R = 0.028$

$T_1 := T_2 + Q\cdot R$ $T_1 = 101.408$

Third Iteration:

$k := k_m(T_1)$ $k = 2.841$

$R := \dfrac{t}{k\cdot A_k}$ $R = 0.028$

$T_1 := T_2 + Q\cdot R$ $T_1 = 101.408$

Example 1.5

Estimate the Average Temperature of a 9 in. x 4 in. Chassis Panel.

Input:

Q =Heat dissipation, W.
ΔT=Desired surface temperature rise.
A_s=Plate total surface area.
h_c=Convective heat transfer coefficient.

$Q := 7$ $A := 2 \cdot 9 \cdot 4$

$h := 0.004$

Required Forumlae:

$$\Delta T(Q_c, h_c, A_c) := \frac{1}{h_c \cdot A_c} \cdot Q_c \qquad R(h_c, A_c) := \frac{1}{h_c \cdot A_c}$$

Solution:

$\Delta T := \Delta T(Q, h, A)$ $R := R(h, A)$

$\Delta T = 24.306$ $R = 3.472$

Example 1.7

Estimate the Heat Transferred to Ambient by Radiation from a 10 in. x 12 in. Chassis Panel.

Input:

ΔT=Given temperature rise.
A_s=Plate total surface area.
ε =Emissivity.
T_A = Ambient temperature.

$A := 10 \cdot 12$ $\varepsilon := 0.8$

$\Delta T := 10$ $T_A := 20$

Required Forumlae:

$$h_r(T_A) := 1.463 \cdot 10^{-10} (T_A + 273.16)^3$$

$$h := h_r(T_A)$$

$$Q(A_s, \Delta T_s, \varepsilon_s) := \varepsilon_s \cdot h \cdot A_s \cdot \Delta T_s$$

Solution:

$$h = 3.686 \times 10^{-3}$$

$Q_r := Q(A, \Delta T, \varepsilon)$ $Q_r = 3.539$

Illustrative Example 1.8

Power Transistor with Heat Sink on Printed Circuit Board.

Heat Sink Input:

A_{sT} = Heat sink top surface area.
A_{sB} = Heat sink bottom surface area.
ε_s = Heat sink emissivity.
T_A = Ambient temperature.
ΔT_s = First heat temperature rise estimate.

$A_{sT} := 1.5 \cdot 0.75$ $\quad A_{sB} := (1.5 - 0.5) \cdot 0.75$

$\varepsilon_s := 0.8$

$T_A := 50$ $\quad \Delta T_s := 100$

Chip to Sink Input:

Ignore any spreading. Use straightforward conduct through lid edges as major resistance.

$t_{lid} := 0.02$ $\quad l_{lid} := 0.1$ $\quad k_{lid} := 0.4$ $\quad P_{lid} := 2 \cdot (0.5 + 0.5)$

Chip to PCB Input:

$t_{flag} := 0.02$ $\quad A_{flag} := 0.25 \cdot 0.25$ $\quad k_{flag} := 0.4$ $\quad Q_{Chip} := 6$

Transistor/PCB Spreading Input:

Use ΔT_s for all ΔT.

$W_{PCB} := 1.5$ $\quad L_{PCB} := 1.5$ $\quad t_{Cu} := 0.0014$ $\quad k_{Cu} := 10$

$\Delta x := 0.5$ $\quad \Delta y := 0.75$

Heat Sink Calculations:

$$h_{rs} := 1.463 \cdot 10^{-10} \cdot \left(T_A + 273.16\right)^3 \qquad h_{rs} = 4.937 \times 10^{-3}$$

Only heat top radiates to ambient

$$R_{rs} := \frac{1}{\varepsilon_s \cdot A_{sT} \cdot h_{rs}} \qquad R_{rs} = 225.04$$

First estimate of convection resistance -

$$h_{cs} := 0.0018 \cdot \left[\frac{\Delta T_s}{\dfrac{1.5 \cdot 0.75}{2 \cdot (1 \cdot 0.75)}} \right]^{0.33} \qquad h_{cs} = 9.047 \times 10^{-3}$$

$$R_{cs} := \frac{1}{h_{cs} \cdot \left(A_{sT} + A_{sB}\right)} \qquad R_{cs} = 58.952$$

Chip to Sink Calculations:

$$R_{chiptosink} := \frac{l_{lid}}{k_{lid} \cdot (t_{lid} + P_{lid})}$$

$$\boxed{R_{chiptosink} = 0.124}$$

Chip to PCB Calculations:

$$R_{chiptoPCB} := \frac{t_{flag}}{k_{flag} \cdot A_{flag}}$$

$$\boxed{R_{chiptoPCB} = 0.8}$$

Transistor/PCB Spreading Calculations:

Prepare input to use in Ellison's spreading formulae.

$$h_{cTopPCB} := 0.0018 \cdot \left[\frac{\Delta T_s}{\frac{W_{PCB} \cdot L_{PCB}}{2 \cdot (W_{PCB} + L_{PCB})}} \right]^{0.33}$$

$$h_{cTopPCB} = 0.011$$

$$h_{cBotPCB} := 0.5 \cdot h_{cTopPCB}$$

$$h_{cBotPCB} = 5.686 \times 10^{-3}$$

Use same radiation h for PCB as used for heat sink

$$h_{rPCB} := h_{rs}$$

Use only PCB copper as planar conductor, ignore conduction resistance through PCB.

$$Biot\tau_{Top} := \frac{(h_{cTopPCB} + h_{rPCB}) \cdot t_{Cu}}{k_{Cu}} \qquad Biot\tau_{Bot} := \frac{(h_{cBotPCB} + h_{rPCB}) \cdot t_{Cu}}{k_{Cu}}$$

$$Biot\tau_{Top} = 2.283 \times 10^{-6} \qquad Biot\tau_{Bot} = 1.487 \times 10^{-6}$$

$$\alpha := \frac{0.5}{1.5} \qquad \beta := \frac{0.75}{1.5} \qquad \tau := \frac{t_{Cu}}{W_{PCB}} \qquad \alpha = 0.333 \qquad \beta = 0.5 \qquad \tau = 9.333 \times 10^{-4}$$

Using average spreading for Newtonian cooling from bottom only, must extrapolate graphs.

$$\psi_{Sp} := 20 \qquad R_{Sp} := \frac{\psi_{Sp}}{k_{Cu} \cdot \sqrt{\Delta x \cdot \Delta y}} \qquad R_{Sp} = 3.266$$

Mathcad ave psi for one-sided cooling program gives

$$\psi_{Sp} := 30 \qquad R_{Sp} := \frac{\psi_{Sp}}{k_{Cu} \cdot \sqrt{\Delta x \cdot \Delta y}} \qquad \boxed{R_{Sp} = 4.899}$$

$$R_U := \frac{t_{Cu}}{k_{Cu} \cdot W_{PCB} \cdot L_{PCB}} + \frac{1}{\left(\frac{h_{cTopPCB} + h_{cBotPCB}}{2} + 2 \cdot h_{rPCB}\right) \cdot \left(W_{PCB} \cdot L_{PCB}\right)} \qquad R_U = 24.15$$

$$R_{TransPCB} := R_{Sp} + R_U \qquad \boxed{R_{TransPCB} = 29.049}$$

Mathcad max psi for one-sided cooling program gives

$$\psi_{Sp} := 40 \qquad R_{Sp} := \frac{\psi_{Sp}}{k_{Cu} \cdot \sqrt{\Delta x \cdot \Delta y}} \qquad \boxed{R_{Sp} = 6.532}$$

$$R_U := \frac{t_{Cu}}{k_{Cu} \cdot W_{PCB} \cdot L_{PCB}} + \frac{1}{\left(\frac{h_{cTopPCB} + h_{cBotPCB}}{2} + 2 \cdot h_{rPCB}\right) \cdot \left(W_{PCB} \cdot L_{PCB}\right)} \qquad R_U = 24.15$$

$$R_{TransPCB} := R_{Sp} + R_U \qquad R_{TransPCB} = 30.682$$

Using max spreading for Newtonian cooling from both sides gives

$$\psi_{Sp} := 40 \qquad R_{Sp} := \frac{\psi_{Sp}}{k_{Cu} \cdot \sqrt{\Delta x \cdot \Delta y}} \qquad R_{Sp} = 6.532$$

$$R_{2U} := \frac{t_{Cu}}{k_{Cu} \cdot W_{PCB} \cdot L_{PCB}} + \frac{1}{\left(h_{cBotPCB} + h_{rPCB}\right) \cdot \left(W_{PCB} \cdot L_{PCB}\right)} \qquad R_{2U} = 41.836$$

$$R_{1U} := \frac{1}{\left(h_{cTopPCB} + h_{rPCB}\right) \cdot W_{PCB} \cdot L_{PCB}} \qquad R_{1U} = 27.251$$

$$R_U := \frac{R_{1U} \cdot R_{2U}}{R_{1U} + R_{2U}} \qquad R_U = 16.502$$

$$R_{TransPCB} := R_{Sp} + R_U \qquad \boxed{R_{TransPCB} = 23.034}$$

TAMS results for two layers, two-sided cooling
is about equal for average and max resistance.
TAMS gives nearly identical result compared to the
Mathcad 2-sided cooling, spreading result.

$$\boxed{R_{TransPCB} := 26}$$

Add All Resistances and Calculate Temperature Rise:

$$R_{Sink} := \frac{R_{rs} \cdot R_{cs}}{R_{rs} + R_{cs}} \qquad R_{Sink} = 46.714$$

$$R_T := R_{Sink} + R_{chiptosink} \qquad R_T = 46.838$$

$$R_{chiptoPCB} = 0.8 \qquad R_{TransPCB} = 26$$

$$R_B := R_{chiptoPCB} + R_{TransPCB} \qquad R_B = 26.8$$

$$R := \frac{R_T \cdot R_B}{R_T + R_B} \qquad R = 17.046$$

Note that the chip to sink resistance is very small compared to R top and chip to PCB resistance is very small compared to R bottom so chip temperature is not much different that sink and PCB temperatures. This means $\Delta T_{ChiptoAmb}$ can be used for ΔT Sink or PCB to Amb.

$$\Delta T_{ChiptoAmb} := R \cdot Q_{Chip} \qquad \boxed{\Delta T_{ChiptoAmb} = 102.278}$$

$$Q_T := \frac{\Delta T_{ChiptoAmb}}{R_T} \qquad \boxed{Q_T = 2.184}$$

$$Q_B := \frac{\Delta T_{ChiptoAmb}}{R_B} \qquad \boxed{Q_B = 3.816}$$

Forced Air Cooled Box From Notes

Created September 27, 2006

Input Data:

Inlet: $W_I := 5.0$ $H_I := 1.0$ $f_I := 0.45$

Box: $H_B := 4.5$ $W_B := 8.0$

Circuit Boards: $S_B := 1.0$ $L_{Card} := 11.0$

Components: $L := 0.5$ $B := 0.5$ $S := 0.5$ $H := S_B$ $N := 5$ 5 across.

Power Supply: $f_{In\_PS} := 0.45$ $W_{PS} := 2.0$ $f_{PS} := 0.381$

Fan: $f_{Fan} := 0.45$ $d_{Fan} := 3.0$ $f_B := \dfrac{H_B \cdot S_B - N \cdot L \cdot H}{S_B \cdot H_B}$

Resistance Calculations:

Inlet: $A_{Inlet\_perf} := W_I \cdot H_I \cdot f_I$ $R_{Inlet\_perf} := \dfrac{1.5 \cdot 10^{-3}}{A_{Inlet\_perf}^{2}}$ $A_{Inlet\_perf} = 2.25$

Expansion from Inlet: $R_{Inlet\_perf} = 2.963 \times 10^{-4}$

$A_{1Inlet\_Exp} := W_I \cdot H_I$ $A_{2Inlet\_Exp} := H_B \cdot W_B$ $A_{1Inlet\_Exp} = 5$ $A_{2Inlet\_Exp} = 36$

$$R_{Inlet\_expan} := 1.29 \cdot 10^{-3} \cdot \left[\frac{1}{A_{1Inlet\_Exp}} \cdot \left(1 - \frac{A_{1Inlet\_Exp}}{A_{2Inlet\_Exp}}\right)\right]^2$$ $R_{Inlet\_expan} = 3.8262 \times 10^{-5}$

Circuit Boards Taken One At a Time:

Contraction: $A_{1BC} := H_B \cdot S_B$ $A_{2BC} := H_B \cdot S_B \cdot f_B$ $R_{Cont} := \dfrac{0.5 \cdot 10^{-3}}{A_{2BC}^{2}} \cdot \left[1 - \left(\dfrac{A_{2BC}}{A_{1BC}}\right)\right]^{\frac{3}{4}}$

Card: $f_B = 0.4444$ Use $f_B := 0.5$ $R_{Cont} = 8.0437 \times 10^{-5}$

$$R_{Card} := \frac{5.18 \cdot (1) \cdot L_{Card} \cdot 10^{-4}}{(H_B \cdot S_B)^2}$$ $R_{Card} = 2.8138 \times 10^{-4}$

Card Expansion: $R_{Expan} := 1.29 \cdot 10^{-3} \cdot \left[\frac{1}{H_B \cdot S_B \cdot f_B} \cdot (1 - f_B) \right]^2$

$\boxed{R_{Expan} = 6.3704 \times 10^{-5}}$

One Board: $R_{Channel} := R_{Cont} + R_{Card} + R_{Expan}$ $\boxed{R_{Channel} = 4.2552 \times 10^{-4}}$

Card Cage: $R_{Cardcage} := \left(\frac{\sqrt{R_{Channel}}}{6} \right)^2$ $\boxed{R_{Cardcage} = 1.182 \times 10^{-5}}$

Power Supply: $R_{PS\_inlet} := \frac{1.5 \cdot 10^{-3}}{(W_{PS} \cdot H_B \cdot f_{In\_PS})^2}$ $R_{PS\_exit} := R_{PS\_inlet}$

$\boxed{R_{PS\_inlet} = 9.1449 \times 10^{-5}}$

$A_{1PS\_C} := W_{PS} \cdot H_B$ $A_{2PS\_C} := f_{PS} \cdot W_{PS} \cdot H_B$ $A_{1PS\_E} := A_{2PS\_C}$

$$R_{PS\_internal} := 9 \cdot \left[\frac{0.5 \cdot 10^{-3}}{A_{2PS\_C}{}^2} \cdot (1 - f_{PS})^{\frac{3}{4}} + 1.29 \cdot 10^{-3} \cdot \left[\frac{1}{A_{1PS\_E}} \cdot (f_{PS}) \right]^2 \right]$$

$\boxed{R_{PS\_internal} = 4.1042 \times 10^{-4}}$

$R_{PS} := R_{PS\_inlet} + R_{PS\_internal} + R_{PS\_exit}$ $\boxed{R_{PS} = 5.9331 \times 10^{-4}}$

Fan: $R_{Exit\_perf} := \frac{1.5 \cdot 10^{-3}}{\left[\pi \cdot \left(\frac{d_{Fan}}{2} \right)^2 \cdot f_{Fan} \right]^2}$ $\boxed{R_{Exit\_perf} = 1.4825 \times 10^{-4}}$

System: Box Internals: $R_{Enc\_internal} := \left(\frac{\sqrt{R_{Cardcage}} \cdot \sqrt{R_{PS}}}{\sqrt{R_{Cardcage}} + \sqrt{R_{PS}}} \right)^2$

$\boxed{R_{Enc\_internal} = 9.0769 \times 10^{-6}}$

$R_{Sys} := R_{Inlet\_perf} + R_{Inlet\_expan} + R_{Enc\_internal} + R_{Exit\_perf}$ $\boxed{R_{Sys} = 4.9189 \times 10^{-4}}$

Calculate Necessary Plotting Data:

Since this is an intermediate fan case, both h_{vd} and h_{vi} are required in addition to H_L:

$$h_{vd}(G) := \frac{1.29 \cdot 10^{-3} \cdot G^2}{\left[\pi \cdot \left(\frac{d_{Fan}}{2}\right)^2\right]^2} \qquad h_{vi}(G) := \frac{1.29 \cdot 10^{-3} \cdot G^2}{\left(W_B \cdot H_B\right)^2} \qquad H_L(G) := R_{Sys} \cdot G^2$$

$$\Delta h_v(G) := h_{vd}(G) - h_{vi}(G)$$

$$x := 43$$

$$H_{vd} := h_{vd}(x) \qquad H_{vi} := h_{vi}(x) \qquad \Delta H_v := \Delta h_v(x) \qquad HL := H_L(x)$$

$$H_{vd} = 0.0477 \qquad H_{vi} = 1.8404 \times 10^{-3} \qquad \Delta H_v = 0.0459 \qquad HL = 0.9095$$

G	hvd	hvi	Δhv	HL
0	0	0	0	0
1	$2.58*10^{-5}$	$9.95*10^{-7}$	$2.48*10^{-5}$	$4.92*10^{-4}$
5	$6.46*10^{-4}$	$2.49*10^{-5}$	$6.21*10^{-4}$	0.0123
10	$2.58*10^{-3}$	$9.95*10^{-5}$	$2.48*10^{-3}$	0.049
15	$5.81*10^{-3}$	$2.24*10^{-4}$	$5.59*10^{-3}$	0.111
20	0.010	$3.98*10^{-4}$	$9.93*10^{-3}$	0.20
25	0.016	$6.22*10^{-4}$	0.0155	0.31
30	0.0232	$8.96*10^{-4}$	0.023	0.442
35	0.0316	$1.22*10^{-3}$	0.030	0.602
40	0.041	$1.59*10^{-3}$	0.040	0.7864
43	0.0477	$1.84*10^{-3}$	0.0459	0.907
45	0.0523	$2.016*10^{-3}$	0.050	0.9952

Using fan curve in Notes, the fan and [HL + (hvd-hvi)] curves intersect at 21 CFM. $G_0 := 21$

Card Cage Results: $$G_{Cardcage} := G_0 \cdot \sqrt{\frac{R_{Enc\_internal}}{R_{Cardcage}}}$$ $G_{Cardcage} = 18.4026$

$$G_{Channel} := \frac{G_{Cardcage}}{6}$$ $G_{Channel} = 3.0671$

Power Supply: $$G_{PS} := G_0 - G_{Cardcage}$$ $G_{PS} = 2.5974$

Try Teerstra for PCB Using Rather Arbitrary Component Dimensions:

$L := 0.5$ $B := 0.5$ $S := 0.5$

$$H := S_B \qquad V := \frac{G_{Channel}}{\frac{H_B \cdot S_B}{144}} \qquad V = 98.147 \qquad Re_{2H} := 2 \cdot H \cdot \frac{V}{5 \cdot 0.023}$$ $Re_{2H} = 1.7069 \times 10^3$

$$\gamma := 1 + \left(\frac{B}{H}\right)\left(\frac{H}{L}\right) \cdot \left(\frac{L}{L+S}\right) \qquad \zeta := 1 - \left(\frac{B}{H}\right) \cdot \left(\frac{L}{L+S}\right) \qquad \chi := \left(\frac{B}{H}\right) + \left(1 - \frac{B}{H}\right)\left[1 + \left(\frac{2 \cdot B}{H}\right) \cdot \left(\frac{H}{L}\right) \cdot \left(\frac{L}{L+S}\right)\right]$$

$\zeta = 0.75$ $\gamma = 1.5$ $\chi = 1.5$

$$\xi := \frac{B}{H} + \left(1 - \frac{B}{H}\right) \cdot \left(\frac{L}{L+S}\right)$$ $\xi = 0.75$

$$A_{Bar} := \frac{\gamma^2}{\zeta^3 \chi} \qquad B_{Bar} := \frac{\gamma^{\frac{5}{4}}}{\zeta^3 \cdot \xi}$$ $A_{Bar} = 3.5556$ $B_{Bar} = 5.2465$

$$f_{2H} := \left[\left(\frac{96 \cdot A_{Bar}}{Re_{2H}}\right)^3 + \left(\frac{0.347 \cdot B_{Bar}}{Re_{2H}^{\frac{1}{4}}}\right)^3\right]^{\frac{1}{3}}$$ $f_{2H} = 0.3132$

$$R_{Card} := \frac{1.29 \cdot 10^{-3}}{(H_B \cdot S_B)^2}\left(\frac{L_{Card}}{2 \cdot H}\right) \cdot f_{2H}$$ $R_{Card} = 1.0973 \times 10^{-4}$

The single card resistance using the McLean resistance was 4.04×10^{-4}. We should perform at least one more iteration to correct the results.

$$R_{Channel} := R_{Cont} + R_{Card} + R_{Expan} \qquad R_{Channel} = 2.5387 \times 10^{-4}$$

$$R_{Cardcage} := \left(\frac{\sqrt{R_{Channel}}}{6}\right)^2 \qquad \boxed{R_{Cardcage} = 7.052 \times 10^{-6}}$$

$$R_{Enc\_internal} := \left(\frac{\sqrt{R_{Cardcage}} \cdot \sqrt{R_{PS}}}{\sqrt{R_{Cardcage}} + \sqrt{R_{PS}}}\right)^2 \qquad \boxed{R_{Enc\_internal} = 5.7336 \times 10^{-6}}$$

$$R_{Sys} := R_{Inlet\_perf} + R_{Inlet\_expan} + R_{Enc\_internal} + R_{Exit\_perf} \qquad \boxed{R_{Sys} = 4.8854 \times 10^{-4}}$$

This new R_{Sys} is not much different that the first R_{Sys}=4.92x10^{-4} so we won't calculate a new G_0.

$$G_{Cardcage} := G_0 \cdot \sqrt{\frac{R_{Enc\_internal}}{R_{Cardcage}}} \qquad \boxed{G_{Cardcage} = 18.9356}$$

$$G_{Channel} := \frac{G_{Cardcage}}{6} \qquad \boxed{G_{Channel} = 3.1559}$$

$$G_{PS} := G_0 - G_{Cardcage} \qquad \boxed{G_{PS} = 2.0644}$$

22 Watt Cards $$\Delta T_{C\_22W} := \frac{1.76 \cdot (22)}{G_{Channel}} \qquad \boxed{\Delta T_{C\_22W} = 12.2689}$$

11 Watt Card $$\Delta T_{C\_11W} := \frac{1.76 \cdot (11)}{G_{Channel}} \qquad \boxed{\Delta T_{C\_11W} = 6.1345}$$

33 Watt Card $$\Delta T_{C\_33W} := \frac{1.76 \cdot (33)}{G_{Channel}} \qquad \boxed{\Delta T_{C\_33W} = 18.4034}$$

$$\Delta T_{PS} := \frac{1.76 \cdot 68}{G_{PS}} \qquad \boxed{\Delta T_{PS} = 57.9735}$$

Fan Inlet: $$\Delta T_{Fan\_Inlet} := \frac{1.76 \cdot (4 \cdot 22 + 11 + 33 + 68)}{G_0} \qquad \boxed{\Delta T_{Fan\_Inlet} = 16.7619}$$

Example 4.4a Experimental Airflow Resistance, Pressure Loss Heatsink, L= 12 in. Using Yovanovich Correlations

Input Heat Sink Geometry (Inches):

$W := 8.02$ $H := 1.0$ $L := 12$ $N_f := 25$ $t_f := 0.1$

Input Heat Sink Total (for two sinks each 4.01 in. wide) Volumetric Flow Rate (ft.^{3}/min.): $G := 25$

Calculate Some Values:

$$N_c := N_f - 1 \qquad S := \frac{W - N_f \cdot t_f}{N_f - 1} \qquad D_H := \frac{4 \cdot S \cdot H}{2 \cdot H + 2S} \qquad D_H = 0.374 \qquad S = 0.23$$

$$A_{c\_Total} := (N_f - 1) S \cdot H \qquad \sigma := \frac{A_{c\_Total}}{W \cdot H} \qquad A_c := S \cdot H \qquad \sigma = 0.688$$

$$G_{Channel} := \frac{G}{N_f - 1} \qquad V_f := \frac{G_{Channel}}{\frac{A_c}{144}} \qquad V_f = 652.174 \qquad G_{Channel} = 1.042$$

f_{app} from Yovanovich:

$$Re_{DH} := \frac{V_f \cdot D_H}{5(0.023)} \qquad Re_{DH} = 2.121 \times 10^3$$

$$Re_{RtA} := \frac{V_f \cdot \sqrt{A_c}}{5 \cdot (0.023)} \qquad Re_{RtA} = 2.72 \times 10^3$$

$$\varepsilon := \frac{S}{H} \qquad g := \frac{1}{1.086957^{1-\varepsilon} \cdot \left(\sqrt{\varepsilon} - \varepsilon^{\frac{3}{2}}\right) + \varepsilon} \qquad \varepsilon = 0.23 \qquad g = 1.603 \qquad zPlus := \frac{L}{\sqrt{A_c} \cdot Re_{RtA}}$$

$$f_{Turb} := \frac{0.079}{Re_{DH}^{0.25}} \qquad f_{Turb} = 0.012$$

$$zPlus = 9.2 \times 10^{-3}$$

$$f_{app} := \frac{1}{Re_{RtA}} \cdot \left[\left(\frac{3.44}{\sqrt{zPlus}}\right)^2 + (8 \cdot \pi \cdot g)^2\right]^{\frac{1}{2}} \qquad f_{app} = 0.02$$

If Re_{DH}<2000, Set f=f_{app}. If Re_{DH}>10,000, Set f=$f_{turb.}$: $f := f_{app}$

Get K_c, K_e Based on Re_{DH}, σ From Text Graphs: $K_c := 0.31$ $K_e := 0.05$

Calculate Airflow Resistance and Pressure Loss:

$$R_{af} := \frac{1.29 \cdot 10^{-3}}{N_c^{\,2} \cdot A_c^{\,2}} \cdot \left(K_c + K_e + 4 \cdot f \cdot \frac{L}{\sqrt{A_c}}\right) \qquad H_L := R_{af} \cdot G^2 \qquad R_{af} = 9.928 \times 10^{-5} \qquad H_L = 0.062$$

Example 4.4b Experimental Airflow Resistance, Pressure Loss Heatsink, Length =12 in. Using Handbook of Heat Transfer Correlations

Input Heat Sink Geometry (Inches):

$W := 8.02$ $H := 1.0$ $L := 12$ $N_f := 25$ $t_f := 0.1$

Input Heat Sink Total (for two sinks each 4.01 in. wide) Volumetric Flow Rate (ft.^{3}/min.): $G := 25$

Calculate Some Values:

$$N_c := N_f - 1 \qquad S := \frac{W - N_f \cdot t_f}{N_f - 1} \qquad D_H := \frac{4 \cdot S \cdot H}{2 \cdot H + 2S} \qquad D_H = 0.374 \qquad S = 0.23$$

$$A_{c\_Total} := (N_f - 1) S \cdot H \qquad \sigma := \frac{A_{c\_Total}}{W \cdot H} \qquad A_c := S \cdot H \qquad \sigma = 0.688$$

$$G_{Channel} := \frac{G}{N_f - 1} \qquad V_f := \frac{G_{Channel}}{\frac{A_c}{144}} \qquad V_f = 652.174 \qquad G_{Channel} = 1.042$$

f from Table or Curve:

$$Re_{DH} := \frac{V_f \cdot D_H}{5(0.023)} \qquad Re_{DH} = 2.121 \times 10^3$$

$$x := \frac{L}{D_H \cdot Re_{DH}} \qquad y := 24.2 \qquad f_{Lam} := \frac{y}{Re_{DH}} \qquad x = 0.015 \qquad f_{Lam} = 0.011$$

$$f_{Turb} := \frac{0.079}{Re_{DH}^{0.25}} \qquad f_{Turb} = 0.012$$

If Re_{DH}<2000, Set f=f_{app}. If Re_{DH}>10,000, Set f=$f_{turb.}$: $f := f_{Lam}$

Get K_c, K_e Based on Re_{DH}, σ From Text Graphs: $K_c := 0.31$ $K_e := 0.05$

Calculate Airflow Resistance and Pressure Loss:

$$R_{af} := \frac{1.29 \cdot 10^{-3}}{N_c^2 \cdot A_c^2} \cdot \left(K_c + K_e + 4 \cdot f \cdot \frac{L}{D_H}\right) \qquad R_{af} = 7.724 \times 10^{-5}$$

$$H_L := R_{af} \cdot G^2 \qquad H_L = 0.048$$

Example 4.6 Cylindrical Pin Fins (Forced Air) by Khan

W. A. Khan, J. R. Culham, and M. M. Yovanovich, "Modeling of Cylindrical Pin-Fin Heat Sinks for Electronic Packaging, 21st IEEE Semi-Therm Symposium, 2005.

Input, Data, Symbolics Mostly Following Original Paper:

Sample Problem Data from Paper Using SI Units:

Footprint (*m*):--------------------------------------- $W := \frac{25.4}{1000}$ $L := \frac{25.4}{1000}$

Heat Source (m^2):--------------------------------

Pin Diameter Thickness (*m*):------------------- $D := \frac{2}{1000}$

Baseplate Thickness (*m*):---------------------- $t_b := \frac{2}{1000}$

Overall Height of Heat Sink (*m*):-------------- $H_T := \frac{12}{1000}$

Pin Height (*mm*):-------------------------------------- $H := H_T - t_b$ $H = 0.01$

Number of Pins (In-Line) N_T, N_L:---------------- $N_T := 7$ $N_L := 7$

Number of Pins (Staggered) N_T, N_L:------------ $N_T := 8$ $N_L := 7$

Approach Velocity (*m/s*):--------------------------- $V_a := 3$

Thermal Conductivity of Solid (*W/m-K*):------- $k := 180$

Thermal Conductivity of Air (*W/m-K*):---------- $k_f := 0.026$

Kinematic Viscosity of Air (m^2/s):-------------- $\nu := 1.58 \cdot 10^{-5}$

Density of Air (kg/m^3):----------------------------- $\rho := 1.1614$

Prandtl Number of Air:------------------------------- $Pr := 0.71$

Heat Load (*W*):-- $Q := 50$

Ambient Temperature (^oC):----------------------- $T_a := 27$

Calculate Variables and Results:

$$P_L := \frac{L}{N_L} \qquad p_L := \frac{P_L}{D} \qquad P_T := \frac{W}{N_T} \qquad p_T := \frac{P_T}{D} \qquad P_D := \sqrt{{P_L}^2 + \left(\frac{P_T}{2}\right)^2} \qquad p_D := \frac{P_D}{D}$$

$$V_{Max} := \max\left(\frac{p_T}{p_T - 1} \cdot V_a, \frac{p_T}{p_D - 1} \cdot V_a\right) \qquad V_{Max} = 6.684$$

$$Re_D := \frac{D \cdot V_{Max}}{\nu} \qquad Re_D = 846.103$$

Select from first or second of two following lines for in-line or staggered pins, respectively:

$$C_1 := \left(0.2 + \exp\left(-0.55 \cdot p_L\right)\right) \cdot {p_T}^{0.285} \cdot {p_L}^{0.212}$$

$$C_1 := \frac{0.61 \cdot {p_T}^{0.091} \cdot {p_L}^{0.053}}{1 - 2 \cdot \exp\left(-1.09 \cdot p_L\right)}$$

$$h_b := \frac{0.75 \cdot k_f}{D} \cdot \sqrt{\frac{p_T - 1}{N_L \cdot p_L \cdot p_T}} \cdot {Re_D}^{\frac{1}{2}} \cdot Pr^{\frac{1}{3}} \qquad h_b = 47.563$$

$$h_{fin} := \frac{C_1 \cdot k_f}{D} \cdot {Re_D}^{\frac{1}{2}} \cdot Pr^{\frac{1}{3}} \qquad h_{fin} = 257.935$$

$$A_b := L \cdot W - N_T \cdot N_L \cdot \frac{\pi \cdot D^2}{4} \qquad A_b = 4.912 \times 10^{-4}$$

$$A_{fin} := \pi \cdot D \cdot H \qquad A_{fin} = 6.283 \times 10^{-5}$$

$$m := \sqrt{4 \cdot \frac{h_{fin}}{k \cdot D}} \qquad \eta_{fin} := \frac{\tanh(m \cdot H)}{m \cdot H} \qquad \eta_{fin} = 0.914$$

$$R_{fin} := \frac{1}{h_{fin} \cdot A_{fin} \cdot \eta_{fin}} \qquad R_m := \frac{t_b}{k \cdot L \cdot W} \qquad R_{fin} = 67.488$$

$$R_m = 0.017$$

$$R_b := \frac{1}{h_b \cdot \left(L \cdot W - N_T \cdot N_L \cdot \frac{\pi}{4} \cdot D^2 \right)} \qquad R_b = 42.801$$

$$R_{th} := \frac{1}{\left(\frac{N_T \cdot N_L}{R_{fin}} \right) + \frac{1}{R_b}} + R_m \qquad \boxed{R_{th} = 1.352}$$

Calculate Pressure Variables and Results:

$$\sigma := \frac{p_T - 1}{p_T} \qquad K_c := -0.0311 \cdot \sigma^2 - 0.3722 \cdot \sigma + 1.0676 \qquad \sigma = 0.449$$

$$K_e := 0.9301 \cdot \sigma^2 - 2.5746 \cdot \sigma + 0.973 \qquad K_c = 0.894$$

Select from first or second of two following lines for in-line or staggered pins, respectively:

$$K_e = 4.829 \times 10^{-3}$$

$$K_1 := 1.009 \cdot \left(\frac{p_T - 1}{p_L - 1} \right)^{\frac{1.09}{Re_D^{0.0553}}} \qquad f := K_1 \cdot \left[0.233 + \frac{45.78}{(p_T - 1)^{1.1} \cdot Re_D} \right]$$

$$K_1 := 1.175 \cdot \left(\frac{p_L}{p_T \cdot Re_D^{0.3124}} \right) + 0.5 \cdot Re_D^{0.0807} \qquad f := K_1 \cdot \frac{\frac{378.6}{S_T^{\frac{13.1}{p_T}}}}{Re_D^{\frac{0.68}{p_T^{1.29}}}}$$

$$K_1 = 1.009$$

$$f = 0.304$$

$$\Delta P := (K_c + K_e + f \cdot N_L) \cdot \frac{\rho \cdot V_{Max}^2}{2} \qquad \boxed{\Delta P = 78.453}$$

Important Note: The above friction factor definition via the ΔP formula is different that for plate fins.

Cylindrical Pin Fins (Forced Air) by Khan et. al.,
But Mixed Units Used.

W. A. Khan, J. R. Culham, and M. M. Yovanovich, "Modeling of Cylindrical Pin-Fin Heat Sinks for Electronic Packaging, 21st IEEE Semi-Therm Symposium, 2005.

Input, Data, Symbolics Mostly Following Original Paper:

Sample Problem Data from Paper Using SI Units:

Footprint (*in.*):-------------------------------------- $W := 1.0$ $L := 1.0$

Heat Source (*in.*^{2}):--------------------------------

Pin Diameter Thickness (*in.*):------------------ $D := 0.07874$

Baseplate Thickness (*in.*):---------------------- $t_b := 0.0787$

Overall Height of Heat Sink (*m*):------------- $H_T := 0.472441$

Pin Height (*in.*):-------------------------------------- $H := H_T - t_b$ $H = 0.394$

Number of Pins (In-Line) N_T, N_L:---------------- $N_T := 7$ $N_L := 7$

Number of Pins (Staggered) N_T, N_L:------------ $N_T := 8$ $N_L := 7$

Approach Velocity (*ft./min.*):-------------------------- $V_a := 590.55118$

Thermal Conductivity of Solid (*W/in.-K*):------- $k := 4.572$

Thermal Conductivity of Air (*W/in.-K*):---------- $k_f := 0.0006604$

Kinematic Viscosity of Air (in.^{2}/*s*):-------------- $\nu := 0.0245$

Prandtl Number of Air:------------------------------- $Pr := 0.71$

Heat Load (*W*):-- $Q := 50$

Ambient Temperature (^oC):----------------------- $T_a := 27$

Calculate Thermal Variables and Results:

$$P_L := \frac{L}{N_L} \qquad p_L := \frac{P_L}{D} \qquad P_T := \frac{W}{N_T} \qquad p_T := \frac{P_T}{D} \qquad P_D := \sqrt{P_L^{\,2} + \left(\frac{P_T}{2}\right)^2} \qquad p_D := \frac{P_D}{D}$$

$$P_D = 0.16$$

$$V_{Max} := \max\left(\frac{p_T}{p_T - 1}\cdot V_a, \frac{p_T}{p_D - 1}\cdot V_a\right) \qquad V_{Max} = 1.316 \times 10^3$$

$$p_D = 2.028$$

$$Re_D := \frac{D \cdot V_{Max}}{5\nu} \qquad \boxed{Re_D = 845.755}$$

Select from first or second of two following lines for in-line or staggered pins, respectively:

$$\boxed{C_1 := \left(0.2 + \exp\left(-0.55 \cdot p_L\right)\right) \cdot p_T^{\,0.285} \cdot p_L^{\,0.212}}$$

$$\boxed{C_1 := \frac{0.61 \cdot p_T^{\,0.091} \cdot p_L^{\,0.053}}{1 - 2 \cdot \exp\left(-1.09 \cdot p_L\right)}}$$

$$h_b := \frac{0.75 \cdot k_f}{D} \cdot \sqrt{\frac{p_T - 1}{N_L \cdot p_L \cdot p_T}} \cdot Re_D^{\frac{1}{2}} \cdot Pr^{\frac{1}{3}} \qquad h_b = 0.031$$

$$h_{fin} := \frac{C_1 \cdot k_f}{D} \cdot Re_D^{\frac{1}{2}} \cdot Pr^{\frac{1}{3}} \qquad h_{fin} = 0.166$$

$$A_b := L \cdot W - N_T \cdot N_L \cdot \frac{\pi \cdot D^2}{4} \qquad A_b = 0.761$$

$$A_{fin} := \pi \cdot D \cdot H \qquad A_{fin} = 0.097$$

$$m := \sqrt{4\cdot\frac{h_{fin}}{k\cdot D}} \qquad \eta_{fin} := \frac{\tanh(m\cdot H)}{m\cdot H} \qquad \eta_{fin} = 0.914$$

$$R_{fin} := \frac{1}{h_{fin}\cdot A_{fin}\cdot\eta_{fin}} \qquad R_m := \frac{t_b}{k\cdot L\cdot W} \qquad R_{fin} = 67.495$$

$$R_m = 0.017$$

$$R_b := \frac{1}{h_b\cdot\left(L\cdot W - N_T\cdot N_L\cdot\frac{\pi}{4}\cdot D^2\right)} \qquad R_b = 42.81$$

$$R_{th} := \frac{1}{\left(\frac{N_T\cdot N_L}{R_{fin}}\right)+\frac{1}{R_b}} + R_m \qquad \boxed{R_{th} = 1.352}$$

Calculate Pressure Variables and Results:

$$\sigma := \frac{p_T - 1}{p_T} \qquad K_c := -0.0311\cdot\sigma^2 - 0.3722\cdot\sigma + 1.0676 \qquad \boxed{\sigma = 0.449}$$

$$K_e := 0.9301\cdot\sigma^2 - 2.5746\cdot\sigma + 0.973 \qquad \boxed{K_c = 0.894}$$

Select from first or second of two following lines for in-line or staggered pins, respectively:

$$\boxed{K_e = 4.827\times 10^{-3}}$$

$$K_1 := 1.009\cdot\left(\frac{p_T - 1}{p_L - 1}\right)^{\frac{1.09}{Re_D^{0.0553}}} \qquad f := K_1\cdot\left[0.233 + \frac{45.78}{(p_T - 1)^{1.1}\cdot Re_D}\right]$$

$$K_1 := 1.175\cdot\left(\frac{p_L}{p_T\cdot Re_D^{0.3124}}\right) + 0.5\cdot Re_D^{0.0807} \qquad f := K_1\cdot\frac{378.6}{p_T^{\frac{13.1}{p_T}}}\cdot\frac{1}{Re_D^{\frac{0.68}{p_T^{1.29}}}}$$

$$\boxed{K_1 = 1.009} \qquad \boxed{f = 0.304}$$

Important Note: The friction factor definition via the Δh formula is different that for plate fins.
$\Delta h=(K_c+K_e+f_{Pins}N_L)h_{v\text{-}Pins}$ where = one velocity head in Pin array.

Use $h_{v\text{-}Pins}$=$1.2910^{-3}G^2/(WH\sigma)^2$, then $\Delta h = \Delta h$[in. H_2O]. This $G/()^2$ is equivalent to V_{Max}

$$G := V_a\cdot\frac{W\cdot(H)}{144} \qquad G = 1.615 \qquad \Delta h := (K_c + K_e + f\cdot N_L)1.29\cdot 10^{-3}\cdot\frac{G^2}{(W\cdot H\cdot\sigma)^2} \qquad \Delta h = 0.326$$

Δh=0.326 in. H2O converted to Pa by dividing by 4.019×10^{-3} is 81.1 and is slightly different than the SI calc. because my air density (built into the 1.29×10^{-3}) is a little different that Khan's.

Also:

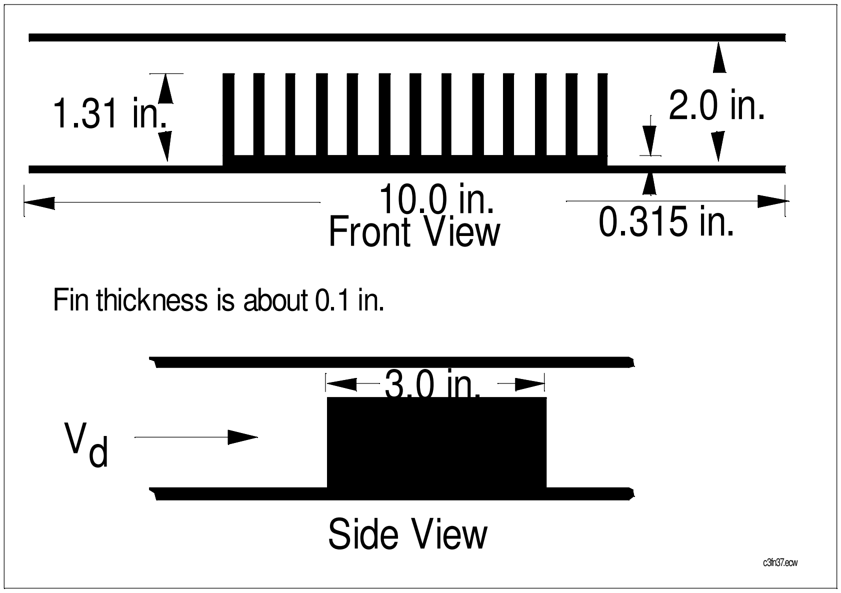
$$R := \left(K_c + K_e + f \cdot N_L\right) 1.29 \cdot 10^{-3} \cdot \frac{1}{(W \cdot H \cdot \sigma)^2}$$

$$R = 0.125$$

$$\Delta h := R \cdot G^2 \qquad \Delta h = 0.326$$

Example 4.8 Aavid Thermalloy Heat Sink Part Number 62705
Bernoulli's Equation Based on SI Units and Using Muzychka & Yovanovich Correlations

Airflow By-Pass and Thermal Resistance Calculator.
This method may be applied to any heat sink in a card channel by changing the input values.



Some Physical Constants (ρ[kg/m³]):

$\rho := 1.18 \qquad Pr := 0.72$

Input Heat Sink Geometry (Inches):

$W := 4.0 \quad H := 0.995 \quad L := 3 \quad N_f := 13 \quad t_f := 0.1 \quad H_T := 1.31 \quad t_b := 0.315$

Input Duct Geometry (Inches):

Input Duct Volumetric Flow Rate (ft.³/min.):

$W_d := 10.0 \quad H_d := 2.0 \quad H_T := t_b + H \quad G := 50$

Calculate Some Values:

$N_p := N_f - 1 \qquad S := \dfrac{W - N_f \cdot t_f}{N_f - 1} \qquad D_H := \dfrac{4 \cdot S \cdot H}{2 \cdot H + S} \qquad D_H = 0.404 \qquad S = 0.225$

$A_d := W_d \cdot H_d \qquad V_d := \dfrac{G}{\dfrac{A_d}{144}} \qquad V_d = 360$

$A_f := (N_f - 1) \cdot S \cdot H \qquad A_{hs} := W \cdot H_T - A_f \qquad \sigma := \dfrac{A_f}{A_f + A_{hs}} \qquad A_c := S \cdot H \qquad \sigma = 0.513$

$A_f = 2.687 \qquad A_{hs} = 2.554$

Airflow Calculation:

1st Iteration - Use a Guess for V_f. $V_f := 248$

After 1st Iteration - Use End Result.

Calculate Reynold's No. and Ratio r:

$$Re_{DH} := \frac{V_f \cdot D_H}{5 \cdot (0.023)} \qquad Re_{DH} = 871.858$$

fapp from Yovanovich:

$$A := S \cdot H \qquad Re_{RtA} := \frac{V_f \cdot \sqrt{A}}{5 \cdot (0.023)} \qquad Re_{RtA} = 1.02 \times 10^{3} \qquad A = 0.224$$

$$\varepsilon := \frac{S}{H} \qquad g := \frac{1}{1.086957^{1-\varepsilon} \cdot \left(\sqrt{\varepsilon} - \varepsilon^{\frac{3}{2}}\right) + \varepsilon} \qquad zPlus := \frac{L}{\sqrt{A} \cdot Re_{RtA}}$$

$$zPlus = 6.214 \times 10^{-3}$$

$$\varepsilon = 0.226 \qquad g = 1.616$$

$$f_{app} := \frac{1}{Re_{RtA}} \cdot \left[\left(\frac{3.44}{\sqrt{zPlus}}\right)^2 + (8 \cdot \pi \cdot g)^2\right]^{\frac{1}{2}} \qquad f_{app} = 0.058$$

If Re_{DH}<2000, Set f=f_{app}. If Re_{DH}>10,000, Set f=$f_{turb.}$: $f := f_{app}$

Get K_c, K_e Based on Re_{DH}, σ From Text Graphs: $K_c := 0.55$ $K_e := 0.13$

Calculate Airflow Resistance and Pressure Loss:

$$R_{af} := \frac{1.29 \cdot 10^{-3}}{N_p^{\,2} \cdot A_c^{\,2}} \cdot \left(K_c + K_e + 4 \cdot f_{app} \cdot \frac{L}{\sqrt{A}}\right) \qquad R_{af} = 3.864 \times 10^{-4}$$

$$G_f := \frac{A_f}{144} \cdot V_f \qquad \Delta p_{fH20} := R_{af} \cdot G_f^{\,2} \qquad \Delta p_{fH20} = 8.272 \times 10^{-3}$$

$$G_f = 4.627$$

Convert pressure from in.H_20 to Pa:

$$\Delta p_{SI} := \frac{\Delta p_{fH20}}{4.019 \cdot 10^{-3}}$$

Calculate "Constants" for Quadratic Equation:

$$A_b := A_d - A_f - A_{hs} \qquad \boxed{A_b = 14.76} \qquad \boxed{\Delta p_{SI} = 2.058}$$

Va (duct or approach) = Vd (approach) but in M/s: $C_V := 12 \cdot \frac{2.54}{100 \cdot 60}$ $\quad C_V = 5.08 \times 10^{-3}$

$$V_a := V_d \cdot C_V \qquad \boxed{V_a = 1.829}$$

$$a := 1 - \left(\frac{A_f}{A_b}\right)^2 \qquad b := 2 \cdot \frac{A_d}{A_b} \cdot \frac{A_f}{A_b} \cdot V_a \qquad c := \frac{2 \cdot \Delta p_{SI}}{\rho} - \left(\frac{A_d}{A_b}\right)^2 \cdot V_a^{\,2}$$

$$\boxed{a = 0.967} \quad \boxed{b = 0.902} \quad \boxed{c = -2.652}$$

Vsol is V in fins but in M/s: $V_{sol} := \frac{-b + \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$ $\quad V_{sol} = 1.254$

Convert to ft/min: $V_f := \frac{V_{sol}}{C_V}$ $\quad G_f := V_f \cdot \frac{A_f}{144}$ $\quad \boxed{G_f = 4.606}$ $\quad \boxed{V_f = 246.887}$

Assume Laminar flow. For V_d=360, a V_f of 200 ft./min. was used to start the calculation which gave Re_{DH}=703.

The iteration sequence beginning with V_d=360, the first velocity V_f=200, was:

V_f Used	V_f Calc
200	300
220	280
250	244
240	257
245	251

and after the revised f, K_c, K_e, ----> V_f = 247 ft./min. and Re_{DH}=868.

Heat Transfer Calculation Using Calculated V_f:

Input Fin Channel Velocity V_f: $V_f := 247$

Some Physical Constants: **ρ recast in gm/in.³ Cp in J/(gm*C) -**

$\rho := 0.02 \quad C_p := 1.0 \quad k := 6.5 \cdot 10^{-4} \quad Pr := 0.72$

Calculate Some Values:

$A_s := 2 \cdot (N_f - 1) \cdot L \cdot H + W \cdot L \quad \boxed{A_s = 83.64} \quad Re_{RtA} := \dfrac{V_f \cdot \sqrt{A}}{5 \cdot (0.023)} \quad zPlus := \dfrac{L}{\sqrt{A} \cdot Re_{RtA}}$

Mass flow rate times specific heat , mdot*Cp, put in a convenient function form:

$mdotCp(V) := \rho \cdot 12^3 \cdot V \cdot \dfrac{1}{60} \cdot A_f \cdot \dfrac{1}{12^2} \cdot C_p \qquad Re_{DH} := \dfrac{V_f \cdot D_H}{5 \cdot 0.023}$

Calculate Laminar h From Yovanovich Using Average h, Isothermal Wall, Symmetric Heating:

$C_1 := 1.5 \quad C_2 := 0.409 \quad C_3 := 3.01 \quad \gamma := 0.1 \quad zStar := \dfrac{zPlus}{Pr}$

$$Nu := \left[\left[C_1 \cdot C_2 \cdot \left(\frac{f_{app} \cdot Re_{RtA}}{zStar} \right)^{\frac{1}{3}} \right]^5 + \left[C_3 \cdot \left(f_{app} \cdot \frac{Re_{RtA}}{8 \cdot \sqrt{\pi} \cdot \varepsilon^{\gamma}} \right) \right]^5 \right]^{\frac{1}{5}} \qquad \boxed{Nu = 15.463}$$

$$\boxed{h_{Laminar} := \frac{k}{\sqrt{A}} \cdot Nu} \qquad \boxed{h_{Laminar} = 0.021}$$

Turbulent Flow Heat Transfer Coefficient:

$$h_{Turb}(Re) := \left[1 + 1.68 \cdot \left(\frac{D_H}{L} \right)^{0.58} \right] \cdot 0.023 \cdot \frac{k}{D_H} \cdot Re^{0.8} \qquad h_{Turbulent} := h_{Turb}(Re_{DH})$$

$$\boxed{h_{Turbulent} = 0.013}$$

Calculate Fin Efficiency Using Primitives R_{Prim_k}, $R_{Prim\_c\_Lam}$:

$$R_k := \frac{H}{5 \cdot L \cdot t_f} \qquad R_{Prim\_c\_Lam} := \frac{1}{h_{Laminar} \cdot 2 \cdot H \cdot L}$$

$$\eta_{Laminar} := \sqrt{\frac{R_{Prim\_c\_Lam}}{R_k}} \cdot \tanh\left(\sqrt{\frac{R_k}{R_{Prim\_c\_Lam}}}\right)$$

$$\boxed{\eta_{Laminar} = 0.973}$$

$$R_{Prim\_c\_Turb} := \frac{1}{h_{Turbulent} \cdot 2 \cdot H \cdot L} \qquad \eta_{Turbulent} := \sqrt{\frac{R_{Prim\_c\_Turb}}{R_k}} \cdot \tanh\left(\sqrt{\frac{R_k}{R_{Prim\_c\_Turb}}}\right)$$

$$\boxed{\eta_{Turbulent} = 0.984}$$

Calculate R_C for Laminar and Turbulent Flow:

$$R_{C\_Laminar} := \frac{1}{\eta_{Laminar} \cdot h_{Laminar} \cdot A_s} \qquad R_{C\_Turbulent} := \frac{1}{\eta_{Turbulent} \cdot h_{Turbulent} \cdot A_s}$$

$$\boxed{R_{C\_Laminar} = 0.579} \qquad \boxed{R_{C\_Turbulent} = 0.96}$$

Calculate R_I for Laminar and Turbulent Flow:

$$mdotCpCalc := mdotCp\left(V_f\right)$$

$$\boxed{mdotCpCalc = 2.654}$$

$$\beta_{Laminar} := \frac{1}{mdotCpCalc \cdot R_{C\_Laminar}}$$

$$\boxed{\beta_{Laminar} = 0.651}$$

$$R_{I\_Laminar} := \beta_{Laminar} \cdot \frac{R_{C\_Laminar}}{1 - e^{-\beta_{Laminar}}}$$

$$\boxed{R_{I\_Laminar} = 0.787}$$

$$\beta_{Turbulent} := \frac{1}{mdotCpCalc \cdot R_{C\_Turbulent}}$$

$$\boxed{\beta_{Turbulent} = 0.392}$$

$$R_{I\_Turbulent} := \beta_{Turbulent} \cdot \frac{R_{C\_Turbulent}}{1 - e^{-\beta_{Turbulent}}}$$

$$\boxed{R_{I\_Turbulent} = 1.161}$$

Heat Transfer Analysis Using Method of Jonsson and Mosfegh:

Check Some Ratios for Model Validity:

$$r_H := \frac{H}{H_d} \qquad r_W := \frac{W}{W_d} \qquad \boxed{r_H = 0.497} \qquad \boxed{r_W = 0.4}$$

Model is in range of geometry limits.

Problem Geometry Remains the Same But There Are Some Different Definitions:

$$D_H := \frac{2 \cdot W_d \cdot H_d}{W_d + H_d} \qquad \boxed{D_H = 3.333}$$

Calculate and Check Reynold's Number:

Velocity based on bypass area + fin channel area.

$$Re_{DH} := \frac{\left(\frac{G}{A_b + A_f}\right) \cdot 144 \cdot D_H}{5 \cdot (0.023)} \qquad \boxed{Re_{DH} = 1.196 \times 10^4}$$

Which is in region >2000 and <16500.

Get Constants from Jonsson and Mosfegh Table:

$C_1 := 88.28 \quad m_1 := 0.6029 \quad m_2 := -0.1098 \quad m_3 := -0.5623 \quad m_4 := 0.08713 \quad m_5 := 0.4139$

Calculate Results:

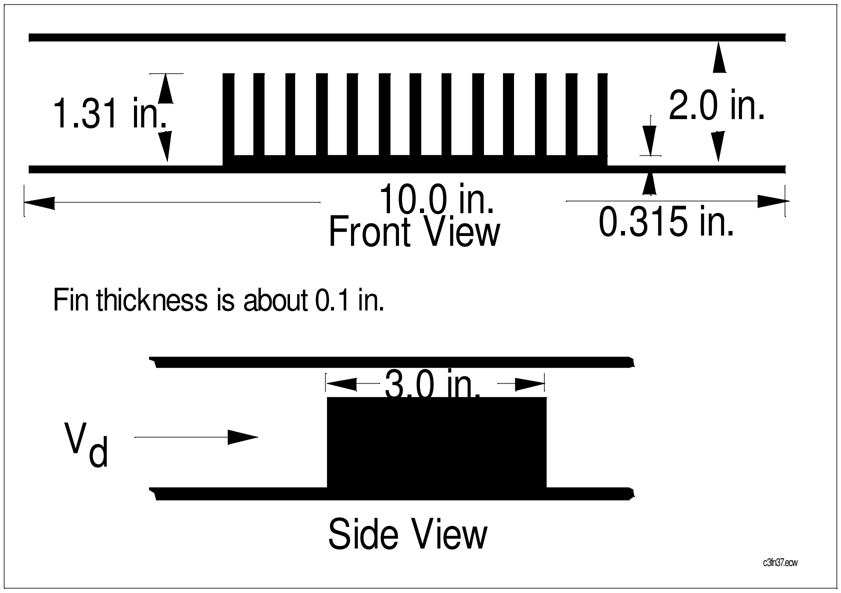
$$Nu_L := C_1 \cdot \left(\frac{Re_{DH}}{1000}\right)^{m_1} \cdot \left(\frac{W_d}{W}\right)^{m_2} \cdot \left(\frac{H_d}{H}\right)^{m_3} \cdot \left(\frac{S}{H}\right)^{m_4} \cdot \left(\frac{t_f}{H}\right)^{m_5} \qquad \boxed{Nu_L = 81.702}$$

$$k_{Air} := 6.5 \cdot 10^{-4} \qquad h := \frac{k_{Air}}{L} \cdot Nu_L \qquad \boxed{h = 0.018}$$

$$A_{Sink} := 2 \cdot N_f \cdot L \cdot H + W \cdot L \qquad R_{Sink} := \frac{1}{h \cdot A_{Sink}} \qquad \boxed{R_{Sink} = 0.63}$$

Application Example 4.9: Aavid Thermalloy Heat Sink Part Number 62705
Bernoulli's Equation Based on SI Units

Airflow By-Pass and Thermal Resistance Calculator.
This method may be applied to any heat sink in a card channel by changing the input values.



Some Physical Constants (ρ[kg/m³]):

$\rho := 1.18$

Input Heat Sink Geometry (Inches):

$W := 4.0 \quad H := 0.995 \quad L := 3 \quad N_f := 13 \quad t_f := 0.1 \quad H_T := 1.31$

Input Duct Geometry (Inches):

$W_d := 10.0 \quad H_d := 2.0$

Input Duct Volumetric Flow Rate (ft.³/min.):

$G := 50$

Calculate Some Values:

$N_p := N_f - 1 \quad S := \frac{W - N_f \cdot t_f}{N_f - 1} \quad D_H := \frac{4 \cdot S \cdot H}{2 \cdot H + S} \quad D_H = 0.404 \quad S = 0.225$

$A_d := W_d \cdot H_d \quad V_d := \frac{G}{\frac{A_d}{144}} \quad V_d = 360$

$A_f := (N_f - 1) \cdot S \cdot H \quad A_{hs} := W \cdot H_T - A_f \quad \sigma := \frac{A_f}{A_f + A_{hs}} \quad A_c := S \cdot H \quad \sigma = 0.513$

Airflow Calculation:

1st Iteration - Use a Guess for V_f. $V_f := 275$

After 1st Iteration - Use End Result.

Calculate Reynold's No. and Ratio r:

$$Re_{DH} := \frac{V_f \cdot D_H}{5 \cdot (0.023)} \qquad r := \frac{L}{D_H \cdot Re_{DH}} \qquad Re_{DH} = 966.778 \qquad r = 7.675 \times 10^{-3}$$

Get fRe for Laminar Based on Ratio r From Text Graph: $fRe := 25.6$

This r results in fRe=: $f_{lam} := \dfrac{fRe}{Re_{DH}}$ $\quad f_{turb} := \dfrac{0.079}{Re_{DH}^{0.25}}$

$$f_{lam} = 0.026$$

$$f_{turb} = 0.014$$

If Re_{DH}<2000, Set f=f_{lam}. If Re_{DH}>10,000, Set f=f_{turb}.: $f := f_{lam}$

Get K_c, K_e Based on Re_{DH}, σ From Text Graphs: $K_c := 0.56$ $\quad K_e := 0.12$

Calculate Airflow Resistance and Pressure Loss:

$$R_{af} := \frac{1.29 \cdot 10^{-3}}{N_p^{\,2} \cdot A_c^{\,2}} \cdot \left(K_c + K_e + 4 \cdot f \cdot \frac{L}{D_H}\right) \qquad R_{af} = 2.62 \times 10^{-4}$$

$$G_f := \frac{A_f}{144} \cdot V_f \qquad \Delta p_{fH20} := R_{af} \cdot G_f^{\,2} \qquad \Delta p_{fH20} = 6.897 \times 10^{-3} \qquad G_f = 5.13$$

Convert pressure from in.$H_2$0 to Pa: $\quad \Delta p_{SI} := \dfrac{\Delta p_{fH20}}{4.019 \cdot 10^{-3}} \qquad \Delta p_{SI} = 1.716$

Calculate "Constants" for Quadratic Equation:

$$A_b := A_d - A_f - A_{hs} \qquad A_b = 14.76$$

Va (duct or approach) = Vd (approach) but in M/s: $\quad C_V := 12 \cdot \dfrac{2.54}{100 \cdot 60} \qquad C_V = 5.08 \times 10^{-3}$

$$V_a := V_d \cdot C_V \qquad V_a = 1.829$$

$$a := 1 - \left(\frac{A_f}{A_b}\right)^2 \qquad b := 2\cdot\frac{A_d}{A_b}\cdot\frac{A_f}{A_b}\cdot V_a \qquad c := \frac{2\cdot\Delta p_{SI}}{\rho} - \left(\frac{A_d}{A_b}\right)^2\cdot V_a^{\,2}$$

$a = 0.967$ $\quad b = 0.902$ $\quad c = -3.232$

Vsol is V in fins but in M/s: $V_{sol} := \frac{-b + \sqrt{b^2 - 4\cdot a\cdot c}}{2\cdot a}$ $\qquad V_{sol} = 1.42$

Convert to ft/min: $V_f := \frac{V_{sol}}{C_V}$ $\qquad G_f := V_f\cdot\frac{A_f}{144}$ $\qquad G_f = 5.217$ $\qquad V_f = 279.613$

Assume Laminar flow. For V_d=360, a V_f of 400 ft./min. was used to start the calculation which gave Re_{DH}=1406.

The iteration sequence beginning with V_d=360, the first velocity V_f=400, was:

V_f Used	V_f Calc
400	126
300	254
250	299
270	284
277	278

and after the revised f, K_c, K_e, ----> V_f = 280 ft./min. and Re_{DH}=985.

Heat Transfer Calculation Using Calculated V_f:

Input Fin Channel Velocity V_f: $V_f := 278$

Some Physical Constants: **ρ recast in gm/in.^{3} Cp in J/(gm*C) -**

$\rho := 0.02 \qquad C_p := 1.0 \qquad k := 6.5\cdot 10^{-4} \qquad Pr := 0.72$

Calculate Some Values:

$A_s := 2\cdot(N_f - 1)\cdot L\cdot H + W\cdot L$ $\qquad A_s = 83.64$

Mass flow rate times specific heat , mdot*Cp, put in a convenient function form:

$$mdotCp(V) := \rho\cdot 12^3\cdot V\cdot\frac{1}{60}\cdot A_f\cdot\frac{1}{12^2}\cdot C_p$$

Other values: $r_{Aspect\_ratio} := \frac{H}{S}$ $r_{Aspect\_ratio} = 4.422$

Get $Nu_{Rect.}$ Fully Dev. From Text: $Nu_{Circ.} := 3.66$ $Nu_{Rect.} := 4.44$

$$r_{Nu} := \frac{Nu_{Rect.}}{Nu_{Circ.}}$$ $r_{Nu} = 1.213$

Re-Calculate Re_{DH} for V_f: $\text{ReynoldsNumber}(V) := \frac{V \cdot D_H}{5 \cdot 0.023}$

$Re := \text{ReynoldsNumber}(V_f)$ $Re = 977.325$

Laminar Flow Heat Transfer Coefficient:

$$h_{Lam}(Re) := r_{Nu} \cdot \left(\frac{k}{D_H}\right) \cdot \left[3.66 + \frac{0.104 \cdot \frac{Re \cdot Pr}{\frac{L}{D_H}}}{1 + 0.016 \cdot \left(\frac{Re \cdot Pr}{\frac{L}{D_H}}\right)^{0.8}}\right]$$

$h_{Laminar} := h_{Lam}(Re)$

$h_{Laminar} = 0.019$

Turbulent Flow Heat Transfer Coefficient:

$$h_{Turb}(Re) := \left[1 + 1.68 \cdot \left(\frac{D_H}{L}\right)^{0.58}\right] \cdot 0.023 \cdot \frac{k}{D_H} \cdot Re^{0.8}$$

$h_{Turbulent} := h_{Turb}(Re)$

$h_{Turbulent} = 0.014$

Calculate Fin Efficiency Using Primitives R_{Prim_k}, $R_{Prim\_c\_Lam}$:

$$R_k := \frac{H}{5 \cdot L \cdot t_f} \qquad R_{Prim\_c\_Lam} := \frac{1}{h_{Laminar} \cdot 2 \cdot H \cdot L}$$

$$\eta_{Laminar} := \sqrt{\frac{R_{Prim\_c\_Lam}}{R_k}} \cdot \tanh\left(\sqrt{\frac{R_k}{R_{Prim\_c\_Lam}}}\right)$$

$\eta_{Laminar} = 0.976$

$$R_{Prim\_c\_Turb} := \frac{1}{h_{Turbulent} \cdot 2 \cdot H \cdot L} \qquad \eta_{Turbulent} := \sqrt{\frac{R_{Prim\_c\_Turb}}{R_k}} \cdot \tanh\left(\sqrt{\frac{R_k}{R_{Prim\_c\_Turb}}}\right)$$

$\eta_{Turbulent} = 0.982$

Calculate R_C for Laminar and Turbulent Flow:

$$R_{C\_Laminar} := \frac{1}{\eta_{Laminar} \cdot h_{Laminar} \cdot A_s} \qquad R_{C\_Turbulent} := \frac{1}{\eta_{Turbulent} \cdot h_{Turbulent} \cdot A_s}$$

$$R_{C\_Laminar} = 0.642 \qquad R_{C\_Turbulent} = 0.875$$

Calculate R_I for Laminar and Turbulent Flow:

$$mdotCpCalc := mdotCp\left(V_f\right)$$

$$mdotCpCalc = 2.987$$

$$\beta_{Laminar} := \frac{1}{mdotCpCalc \cdot R_{C\_Laminar}} \qquad \beta_{Laminar} = 0.521$$

$$R_{I\_Laminar} := \beta_{Laminar} \cdot \frac{R_{C\_Laminar}}{1 - e^{-\beta_{Laminar}}} \qquad R_{I\_Laminar} = 0.824$$

$$\beta_{Turbulent} := \frac{1}{mdotCpCalc \cdot R_{C\_Turbulent}} \qquad \beta_{Turbulent} = 0.382$$

$$R_{I\_Turbulent} := \beta_{Turbulent} \cdot \frac{R_{C\_Turbulent}}{1 - e^{-\beta_{Turbulent}}} \qquad R_{I\_Turbulent} = 1.053$$

Heat Transfer Analysis Using Method of Jonsson and Mosfegh:

Check Some Ratios for Model Validity:

$$r_H := \frac{H}{H_d} \qquad r_W := \frac{W}{W_d} \qquad r_H = 0.497 \qquad r_W = 0.4$$

Model is in range of geometry limits.

Problem Geometry Remains the Same But There Are Some Different Definitions:

$$D_H := \frac{2 \cdot W_d \cdot H_d}{W_d + H_d} \qquad D_H = 3.333$$

Calculate and Check Reynold's Number:

Velocity based on bypass area + fin channel area.

$$Re_{DH} := \frac{\left(\frac{G}{A_b + A_f}\right) \cdot 144 \cdot D_H}{5 \cdot (0.023)} \qquad Re_{DH} = 1.196 \times 10^4$$

Which is in region >2000 and <16500.

Get Constants from Jonsson and Mosfegh Table:

$C_1 := 88.28 \quad m_1 := 0.6029 \quad m_2 := -0.1098 \quad m_3 := -0.5623 \quad m_4 := 0.08713 \quad m_5 := 0.4139$

Calculate Results:

$$Nu_L := C_1 \cdot \left(\frac{Re_{DH}}{1000}\right)^{m_1} \cdot \left(\frac{W_d}{W}\right)^{m_2} \cdot \left(\frac{H_d}{H}\right)^{m_3} \cdot \left(\frac{S}{H}\right)^{m_4} \cdot \left(\frac{t_f}{H}\right)^{m_5} \qquad Nu_L = 81.702$$

$$k_{Air} := 6.5 \cdot 10^{-4} \qquad h := \frac{k_{Air}}{L} \cdot Nu_L \qquad h = 0.018$$

$$A_{Sink} := 2 \cdot N_f \cdot L \cdot H + W \cdot L \qquad R_{Sink} := \frac{1}{h \cdot A_{Sink}} \qquad R_{Sink} = 0.63$$

Example 4.11 Application of Jonsson and Mosfegh to Khan's Pin Fin Problem

Input Heat Sink Geometry (Inches):

In-line Circular Pins

$N_L := 7$ $\quad N_T := 7$ $\quad W := 1$ $\quad H := 0.394$ $\quad t_f := 0.07874$ $\quad L := 1$ $\quad G_{Max} := 1.615$

$$S := \frac{W - N_T \cdot t_f}{N_T - 1} \qquad S = 0.075$$

We are using the same CFM that we used in the Khan version of the problem.

Calculate Values:

$$W_d := W + S \qquad H_d := 0.394 \qquad r_W := \frac{W}{W_d} \qquad r_H := \frac{H}{H_d} \qquad r_W = 0.93 \qquad r_H = 1$$

$$A_b := W_d \cdot H_d - H \cdot W \qquad A_f := N_T \cdot H \cdot S \qquad W_d = 1.075 \qquad A_b = 0.029$$

$$A_f = 0.206$$

$$Re_{Max}(G) := \frac{\left(\dfrac{G}{\dfrac{A_b + A_f}{144}}\right) \cdot \left(\dfrac{2 \cdot W_d \cdot H_d}{W_d + H_d}\right)}{5 \cdot 0.023} \qquad MaxRe := Re_{Max}(G_{Max}) \qquad MaxRe = 4.946 \times 10^3$$

$$D_H := \frac{2 \cdot W_d \cdot H_d}{W_d + H_d} \qquad Re(G) := \frac{\left(\dfrac{G}{\dfrac{A_b + A_f}{144}}\right) \cdot \left(\dfrac{2 \cdot W_d \cdot H_d}{W_d + H_d}\right)}{5 \cdot 0.023}$$

$C_2 := 5.375$ $\quad n_1 := -0.1759$ $\quad n_2 := -0.7161$ $\quad n_3 := -0.8230$ $\quad n_4 := -0.5401$ $\quad n_5 := 0.5990$

$$\Delta h(G) := \frac{1.29 \cdot 10^{-3} \cdot G^2}{\left(A_b + A_f\right)^2} \cdot C_2 \cdot \left[\frac{\left(\dfrac{G}{\dfrac{A_b + A_f}{144}}\right) \cdot \left(\dfrac{2 \cdot W_d \cdot H_d}{W_d + H_d}\right) \cdot 10^{-3}}{5 \cdot 0.023}\right]^{n_1} \ldots$$
$$+ \left(\frac{W_d}{W}\right)^{n_2} \cdot \left(\frac{H_d}{H}\right)^{n_3} \cdot \left(\frac{S}{H}\right)^{n_4} \cdot \left(\frac{t_f}{H}\right)^{n_5} \cdot \frac{2.05}{W} \cdot \frac{L}{2.05}$$

$$\Delta hKhan := \Delta h\left(G_{Max}\right) \qquad R := \frac{\Delta hKhan}{{G_{Max}}^2}$$

$$\boxed{\Delta hKhan = 1.134} \qquad \boxed{R = 0.435}$$

Application Example 6.6

Calculate the thermal resistance and temperature rise for a winged heatsink.

Input Heat Sink Geometry (Inches):

$W := 1.0$ $H := 0.75$ $L := 2$ $Q := 5$ $V := 100$ $\nu := 0.029$

$k_{Air} := 6.92 \cdot 10^{-4}$

Calculate Area:

$A_s := 4 \cdot (H \cdot L) + W \cdot L$ $A_s = 8$

Calculate Heat Transfer Coefficient Using Text Equation (6.14) and Figure 6-7:

$h_c := 0.00109 \cdot \sqrt{\frac{V}{L}}$ $h_c = 7.707 \times 10^{-3}$ $f := 1.38$ $h_L := f \cdot h_c$ $h_L = 0.011$

Calculate Heat Transfer Coefficient Using Text Equation (6.15):

$Re_L := V \cdot \frac{L}{5\nu}$ $h_L := 0.374 \cdot \left(\frac{k_{Air}}{L}\right) \cdot Re_L^{0.607}$ $Re_L = 1.379 \times 10^3$ $h_L = 0.01$

As expected, the two h_Ls agree to within about ten percent.

Calculate Heat Sink Thermal Resistance and Temperature Rise Above Local Ambient:

$R_c := \frac{1}{h_L \cdot A_s}$ $\Delta T := R_c \cdot Q$ $R_c = 12$ $\Delta T = 60.001$

Calculate Thermal Resistance and Temperature Rise Without Heat Sink, i.e. Convection Resistance from Case Top:

$R_c := \frac{1}{h_L \cdot W \cdot L}$ $\Delta T := R_c \cdot Q$ $R_c = 48.001$ $\Delta T = 240.003$

Application Example 7.7: Double Sided Heat Sink Pressure and Thermal Conductance Using Yovanovich Correlations

Some Physical Constants (ρ[kg/m^3]):

$Pr := 0.71 \qquad \nu := 0.029 \qquad k := 6.6 \cdot 10^{-4}$

Input Heat Sink Geometry (Inches):

$W := 8.0 \qquad H := 1.0 \qquad L := 12 \qquad N_f := 25 \qquad t_f := 0.1 \qquad H_T := 1.25$

Input Duct Volumetric Flow Rate (ft.^{3}/min.), Heat:

$G := 26 \qquad Q := 62.8$

Calculate Some Values:

$$N_p := N_f - 1 \qquad S := \frac{W - N_f \cdot t_f}{N_f - 1} \qquad D_H := \frac{4 \cdot S \cdot H}{2 \cdot H + 2S} \qquad D_H = 0.373 \qquad S = 0.229$$

$$A_f := (N_f - 1) \cdot S \cdot H \qquad A_{hs} := W \cdot H_T - A_f \qquad \sigma := \frac{A_f}{A_f + A_{hs}} \qquad A_c := S \cdot H \qquad \sigma = 0.55$$

$A_f = 5.5$

Airflow Calculation:

$$V_f := \frac{G}{\frac{A_f}{144}} \qquad V_f = 680.727$$

Calculate Reynold's No. and Ratio r:

$$Re_{DH} := \frac{V_f \cdot D_H}{5 \cdot \nu} \qquad Re_{DH} = 1.751 \times 10^3$$

fapp from Yovanovich:

$$A := S \cdot H \qquad Re_{RtA} := \frac{V_f \cdot \sqrt{A}}{5 \cdot \nu} \qquad Re_{RtA} = 2.247 \times 10^3$$

$$\varepsilon := \frac{S}{H} \qquad g := \frac{1}{1.086957^{1-\varepsilon} \cdot \left(\sqrt{\varepsilon} - \varepsilon^{\frac{3}{2}}\right) + \varepsilon} \qquad zPlus := \frac{L}{\sqrt{A} \cdot Re_{RtA}}$$

$\varepsilon = 0.229$

$g = 1.606 \qquad zPlus = 0.011$

$$f_{app} := \frac{1}{Re_{RtA}} \cdot \left[\left(\frac{3.44}{\sqrt{zPlus}}\right)^2 + (8 \cdot \pi \cdot g)^2\right]^{\frac{1}{2}}$$

$f_{app} = 0.023$

If Re_{DH}<2000, Set f=f_{app}. If Re_{DH}>10,000, Set f=$f_{turb.}$: $f := f_{app}$

Get K_c, K_e Based on Re_{DH}, σ From Text Graphs: $K_c := 0.46$ $K_e := 0.1$

Calculate Airflow Resistance and Pressure Loss:

$$R_{af} := \frac{1.29 \cdot 10^{-3}}{N_p^{\;2} \cdot A_c^{\;2}} \cdot \left(K_c + K_e + 4 \cdot f_{app} \cdot \frac{L}{D_H}\right)$$

$R_{af} = 1.506 \times 10^{-4}$

$$G_f := \frac{A_f}{144} \cdot V_f \qquad \Delta p_{fH20} := R_{af} \cdot G_f^{\;2}$$

$\Delta p_{fH20} = 0.102$

Heat Transfer Calculation Using Muzychka and Yovanovich with Calculated V_f:

Input Fin Channel Velocity V_f: Use V_f calculated from G input in beginning.

Some Physical Constants: ρ recast in gm/in.^{3} Cp in J/(gm*C) -

Calculate Some Values:

$$A_s := 2(N_f - 1) \cdot L \cdot H + (N_f - 1) \cdot S \cdot L$$

$A_s = 642$

Mass flow rate times specific heat , mdot*Cp, put in a convenient function form:

$$mdotCp(V) := \frac{V \cdot A_f}{262}$$

Calculate Laminar h From Yovanovich Using Average h, Isothermal Wall, Symmetric Heating:

$$C_1 := 1.5 \qquad C_2 := 0.409 \qquad C_3 := 3.01 \qquad \gamma := 0.1 \qquad zStar := \frac{zPlus}{Pr}$$

$$Nu := \left[\left[C_1 \cdot C_2 \cdot \left(\frac{f_{app} \cdot Re_{RtA}}{zStar}\right)^{\frac{1}{3}}\right]^5 + \left[C_3 \cdot \left(f_{app} \cdot \frac{Re_{RtA}}{8 \cdot \sqrt{\pi} \cdot \varepsilon^{\gamma}}\right)\right]^5\right]^{\frac{1}{5}}$$ $\boxed{Nu = 13.205}$

$$h_{Laminar} := \frac{k}{\sqrt{A}} \cdot Nu$$ $\boxed{h_{Laminar} = 0.018}$

Calculate Fin Efficiency Using Primitives R_{Prim_k}, $R_{Prim\_c\_Lam}$:

$$R_k := \frac{H}{5 \cdot L \cdot t_f} \qquad R_{Prim\_c\_Lam} := \frac{1}{h_{Laminar} \cdot 2 \cdot H \cdot L}$$ $R_{Prim\_c\_Lam} = 2.289$

$$\eta_{Laminar} := \sqrt{\frac{R_{Prim\_c\_Lam}}{R_k}} \cdot \tanh\left(\sqrt{\frac{R_k}{R_{Prim\_c\_Lam}}}\right)$$ $\boxed{\eta_{Laminar} = 0.976}$

Calculate R_C for Laminar and Turbulent Flow:

$$R_{C\_Laminar} := \frac{1}{\eta_{Laminar} \cdot h_{Laminar} \cdot A_s}$$ $\boxed{R_{C\_Laminar} = 0.088}$

Calculate R_I for Laminar and Turbulent Flow:

$$mdotCpCalc := mdotCp(V_f)$$

$\boxed{mdotCpCalc = 14.29}$

$$\beta_{Laminar} := \frac{1}{mdotCpCalc \cdot R_{C\_Laminar}}$$ $\beta_{Laminar} = 0.799$

$$R_{I\_Laminar} := \beta_{Laminar} \cdot \frac{R_{C\_Laminar}}{1 - e^{-\beta_{Laminar}}}$$ $\boxed{R_{I\_Laminar} = 0.127}$

$$C_{I\_Laminar} := \frac{1}{R_{I\_Laminar}}$$ $\boxed{C_{I\_Laminar} = 7.86}$

$$\Delta T_{Laminar} := R_{I\_Laminar} \cdot Q$$ $\boxed{\Delta T_{Laminar} = 7.989}$

Heat Transfer Calculated Using Kays:

Calculate Some Values:

$$A_s := 2(N_f - 1)\cdot L\cdot H + (N_f - 1)\cdot S\cdot L \qquad A_s = 642$$

Mass flow rate times specific heat , mdot*Cp, put in a convenient function form:

$$mdotCp(V) := \frac{V\cdot A_f}{262}$$

Calculate Laminar h From Kays Using Average h, Isothermal Wall, Symmetric Heating:

Get Nu Rectangular Duct, Circular Duct for Fully Developed Flow: $Nu_{Rec} := 4.7$ $\quad Nu_{Cir} := 3.66$

$$r_{Nu} := \frac{Nu_{Rec}}{Nu_{Cir}} \qquad Nu := \left[3.66 + 0.104\cdot\left[\frac{\frac{Re_{DH}\cdot Pr}{\frac{L}{D_H}}}{1 + 0.016\cdot\left(\frac{Re_{DH}\cdot Pr}{\frac{L}{D_H}}\right)^{0.8}}\right]\right]\cdot r_{Nu}$$

$$Nu = 8.675$$

$$h_{Laminar} := \frac{k}{D_H}\cdot Nu \qquad h_{Laminar} = 0.01535$$

Turbulent Flow Heat Transfer Coefficient:

$$r := \frac{L}{D_H} \qquad r = 32.182$$

$$h_{Turb}(Re) := 1 + \left(\frac{24}{Re^{0.23}}\right)^{2.08\cdot 10^{-6}\cdot Re - 0.815}$$ For 2 < r = L/D < 20

$$h_{Turb}(Re) := \left(1 + \frac{6\cdot D_H}{L}\right)\cdot 0.023\cdot\frac{k}{D_H}\cdot Re^{0.8}\cdot Pr^{0.4}$$ For 20 < r = L/D < 60

$$h_{Turbulent} := h_{Turb}(Re_{DH}) \qquad Nu_T := \frac{D_H}{k}\cdot h_{Turbulent} \qquad Nu_T = 9.354 \qquad h_{Turbulent} = 0.017$$

Calculate Fin Efficiency Using Primitives R_{Prim_k}, $R_{Prim\_c\_Lam}$:

$$R_k := \frac{H}{5\cdot L\cdot t_f} \qquad R_{Prim\_c\_Lam} := \frac{1}{h_{Laminar}\cdot 2\cdot H\cdot L} \qquad R_{Prim\_c\_Lam} = 2.714$$

$$\eta_{Laminar} := \sqrt{\frac{R_{Prim\_c\_Lam}}{R_k}} \cdot \tanh\left(\sqrt{\frac{R_k}{R_{Prim\_c\_Lam}}}\right)$$

$$\boxed{\eta_{Laminar} = 0.98}$$

$$R_{Prim\_c\_Turb} := \frac{1}{h_{Turbulent} \cdot 2 \cdot H \cdot L} \qquad \eta_{Turbulent} := \sqrt{\frac{R_{Prim\_c\_Turb}}{R_k}} \cdot \tanh\left(\sqrt{\frac{R_k}{R_{Prim\_c\_Turb}}}\right)$$

$$\boxed{\eta_{Turbulent} = 0.978}$$

Calculate R_C for Laminar and Turbulent Flow:

$$R_{C\_Laminar} := \frac{1}{\eta_{Laminar} \cdot h_{Laminar} \cdot A_s} \qquad R_{C\_Turbulent} := \frac{1}{\eta_{Turbulent} \cdot h_{Turbulent} \cdot A_s}$$

$$\boxed{R_{C\_Laminar} = 0.104} \qquad \boxed{R_{C\_Turbulent} = 0.096}$$

Calculate R_I for Laminar and Turbulent Flow:

$$mdotCpCalc := mdotCp(V_f)$$

$$\boxed{mdotCpCalc = 14.29}$$

$$\beta_{Laminar} := \frac{1}{mdotCpCalc \cdot R_{C\_Laminar}}$$

$$\beta_{Laminar} = 0.676$$

$$R_{I\_Laminar} := \beta_{Laminar} \cdot \frac{R_{C\_Laminar}}{1 - e^{-\beta_{Laminar}}}$$

$$\boxed{R_{I\_Laminar} = 0.142}$$

$$\beta_{Turbulent} := \frac{1}{mdotCpCalc \cdot R_{C\_Turbulent}}$$

$$\beta_{Turbulent} = 0.728$$

$$R_{I\_Turbulent} := \beta_{Turbulent} \cdot \frac{R_{C\_Turbulent}}{1 - e^{-\beta_{Turbulent}}}$$

$$R_{I\_Turbulent} = 0.135$$

$$C_{I\_Laminar} := \frac{1}{R_{I\_Laminar}} \qquad C_{I\_Turbulent} := \frac{1}{R_{I\_Turbulent}}$$

$$\boxed{C_{I\_Laminar} = 7.022}$$

$$\boxed{C_{I\_Turbulent} = 7.389}$$

$$\Delta T_{Laminar} := R_{I\_Laminar} \cdot Q$$

$$\boxed{\Delta T_{Laminar} = 8.944}$$

$$\Delta T_{Turbulent} := R_{I\_Turbulent} \cdot Q$$

$$\boxed{\Delta T_{Turbulent} = 8.499}$$

Application Example 7.9 Aavid Thermalloy Heat Sink Part Number 62705 Bernoulli's Equation Based on SI Units and Using Muzychka & Yovanovich Correlations; Adjusted Kays and Crawford; Jonsson & Moshfegh

Some Physical Constants (ρ[kg/m^3]):

$Pr := 0.71 \qquad \nu := 0.029 \qquad k := 6.6 \cdot 10^{-4}$

Input Heat Sink Geometry (Inches):

$W := 4.0 \quad H := 0.995 \quad L := 3 \quad N_f := 13 \quad t_f := 0.1 \quad H_T := 1.31 \quad t_b := 0.315$

$H_d := 2.0 \quad W_d := 10 \qquad H_T := t_b + H \qquad V_f := 248 \quad G := 50$

Calculate Some Values:

$$N_p := N_f - 1 \qquad S := \frac{W - N_f \cdot t_f}{N_f - 1} \qquad D_H := \frac{4 \cdot S \cdot H}{2 \cdot H + S} \qquad D_H = 0.404 \qquad S = 0.225$$

$$A_{c\_Total} := (N_f - 1) \cdot S \cdot H \qquad A_{hs} := W \cdot H_T - A_{c\_Total} \qquad \sigma := \frac{A_{c\_Total}}{A_{c\_Total} + A_{hs}} \qquad A_c := S \cdot H \qquad \sigma = 0.513$$

$$A_{c\_Total} = 2.687 \qquad A_{hs} = 2.554$$

$$V_a := V_f \cdot \frac{A_{c\_Total}}{W \cdot H} \qquad V_a = 167.4$$

Note: This Va is calculated assuming that the heatsink is ducted so that there is no bypass. This is how we presume a vendor would test the heat sink, i.e. the vendor would duct the sink and measure the approach V prior to entering the sink.

f_{app} Calculation:

Calculate Reynold's No. and Ratio r:

$$Re_{DH} := \frac{V_f \cdot D_H}{5 \cdot \nu} \qquad Re_{DH} = 691.473$$

fapp from Yovanovich:

$$A := S \cdot H \qquad Re_{RtA} := \frac{V_f \cdot \sqrt{A}}{5 \cdot \nu} \qquad Re_{RtA} = 809.257$$

$$\varepsilon := \frac{S}{H} \qquad g := \frac{1}{1.086957^{1-\varepsilon} \cdot \left(\sqrt{\varepsilon} - \varepsilon^{\frac{3}{2}}\right) + \varepsilon} \qquad zPlus := \frac{L}{\sqrt{A} \cdot Re_{RtA}}$$

$$f_{app} := \frac{1}{Re_{RtA}} \cdot \left[\left(\frac{3.44}{\sqrt{zPlus}}\right)^2 + (8 \cdot \pi \cdot g)^2\right]^{\frac{1}{2}}$$ $\boxed{f_{app} = 0.069}$

Heat Transfer Calculation Using Muzychka and Yovanovich with Calculated V_f:

Calculate Some Values:

$$A_s := 2 \cdot (N_f) \cdot L \cdot H + (N_f - 1) \cdot S \cdot L$$ $\boxed{A_s = 85.71}$

Mass flow rate times specific heat , mdot*Cp, put in a convenient function form:

$$mdotCp(V) := \frac{V \cdot A_{c\_Total}}{262}$$

Calculate Laminar h From Yovanovich Using Average h, Isothermal Wall, Symmetric Heating:

$C_1 := 1.5$ $C_2 := 0.409$ $C_3 := 3.01$ $\gamma := 0.1$ $zStar := \frac{zPlus}{Pr}$

$$Nu := \left[\left[C_1 \cdot C_2 \cdot \left(\frac{f_{app} \cdot Re_{RtA}}{zStar}\right)^{\frac{1}{3}}\right]^5 + \left[C_3 \cdot \left(f_{app} \cdot \frac{Re_{RtA}}{8 \cdot \sqrt{\pi} \cdot \varepsilon^{\gamma}}\right)\right]^5\right]^{\frac{1}{5}}$$ $\boxed{Nu = 14.496}$

$$h_{Laminar} := \frac{k}{\sqrt{A}} \cdot Nu$$ $\boxed{h_{Laminar} = 0.02022}$

Calculate Fin Efficiency Using Primitives R_{Prim_k}, $R_{Prim\_c\_Lam}$:

$R_k := \frac{H}{5 \cdot L \cdot t_f}$ $R_{Prim\_c\_Lam} := \frac{1}{h_{Laminar} \cdot 2 \cdot H \cdot L}$

$$\eta_{Laminar} := \sqrt{\frac{R_{Prim\_c\_Lam}}{R_k}} \cdot \tanh\left(\sqrt{\frac{R_k}{R_{Prim\_c\_Lam}}}\right)$$ $\boxed{\eta_{Laminar} = 0.974}$

Calculate R_C for Laminar and Turbulent Flow:

$$R_{C\_Laminar} := \frac{1}{\eta_{Laminar} \cdot h_{Laminar} \cdot A_s}$$

$$\boxed{R_{C\_Laminar} = 0.592}$$

Calculate R_I for Laminar and Turbulent Flow:

$$mdotCpCalc := mdotCp(V_f)$$

$$\boxed{mdotCpCalc = 2.543}$$

$$\beta_{Laminar} := \frac{1}{mdotCpCalc \cdot R_{C\_Laminar}}$$

$$\beta_{Laminar} = 0.664$$

$$R_{I\_Laminar} := \beta_{Laminar} \cdot \frac{R_{C\_Laminar}}{1 - e^{-\beta_{Laminar}}}$$

$$\boxed{R_{I\_Laminar} = 0.811}$$

Turbulent Flow Heat Transfer Coefficient:

$$h_{Turb}(Re) := \left[1 + \frac{24}{Re_{DH}^{\,0.23}} \cdot \left(\frac{L}{D_H}\right)^{2.08 \cdot 10^{-6} \cdot (Re) - 0.815}\right] \cdot 0.023 \cdot \frac{k}{D_H} \cdot Re^{0.8}$$

$$h_{Turbulent} := h_{Turb}(Re_{DH})$$

$$Nu_T := \frac{D_H}{k} \cdot h_{Turbulent}$$

$$\boxed{Nu_T = 8.793}$$

$$\boxed{h_{Turbulent} = 0.01435}$$

$$R_{Prim\_c\_Turb} := \frac{1}{h_{Turbulent} \cdot 2 \cdot H \cdot L}$$

$$\eta_{Turbulent} := \sqrt{\frac{R_{Prim\_c\_Turb}}{R_k}} \cdot \tanh\left(\sqrt{\frac{R_k}{R_{Prim\_c\_Turb}}}\right)$$

$$\boxed{\eta_{Turbulent} = 0.981}$$

$$R_{C\_Turbulent} := \frac{1}{\eta_{Turbulent} \cdot h_{Turbulent} \cdot A_s}$$

$$\boxed{R_{C\_Turbulent} = 0.828}$$

$$\beta_{Turbulent} := \frac{1}{mdotCpCalc \cdot R_{C\_Turbulent}}$$

$$\boxed{\beta_{Turbulent} = 0.475}$$

$$R_{I\_Turbulent} := \beta_{Turbulent} \cdot \frac{R_{C\_Turbulent}}{1 - e^{-\beta_{Turbulent}}}$$

$$\boxed{R_{I\_Turbulent} = 1.04}$$

$$\beta_{Turbulent} := \frac{1}{mdotCpCalc \cdot R_{C\_Turbulent}}$$

$$\beta_{Turbulent} = 0.475$$

$$R_{I\_Turbulent} := \beta_{Turbulent} \cdot \frac{R_{C\_Turbulent}}{1 - e^{-\beta_{Turbulent}}}$$

$$R_{I\_Turbulent} = 1.04$$

Heat Transfer Calculated Using Kays:

$V_f := 275$

$$Re_{DH} := \frac{V_f \cdot D_H}{5 \cdot \nu}$$

$$V_a := V_f \cdot \frac{A_{c\_Total}}{W \cdot H}$$

$V_a = 185.625$

Mass flow rate times specific heat , mdot*Cp, put in a convenient function form:

$$mdotCp(V) := \frac{V \cdot A_{c\_Total}}{262}$$

$Re_{DH} = 766.755$

Calculate Laminar h From Kays Using Average h, Isothermal Wall, Symmetric Heating:

Get Nu Rectangular Duct, Circular Duct for Fully Developed Flow: $Nu_{Rec} := 4.7$ $\quad Nu_{Cir} := 3.66$

$$r_{Nu} := \frac{Nu_{Rec}}{Nu_{Cir}}$$

$$Nu := \left[3.66 + 0.104 \cdot \left[\frac{\dfrac{Re_{DH} \cdot Pr}{\dfrac{L}{D_H}}}{1 + 0.016 \cdot \left(\dfrac{Re_{DH} \cdot Pr}{\dfrac{L}{D_H}}\right)^{0.8}}\right]\right] \cdot r_{Nu}$$

$Nu = 11.244$

$$h_{Laminar} := \frac{k}{D_H} \cdot Nu$$

$h_{Laminar} = 0.01836$

Calculate Fin Efficiency Using Primitives R_{Prim_k}, $R_{Prim\_c\_Lam}$:

$$R_k := \frac{H}{5 \cdot L \cdot t_f} \qquad R_{Prim\_c\_Lam} := \frac{1}{h_{Laminar} \cdot 2 \cdot H \cdot L}$$

$$\eta_{Laminar} := \sqrt{\frac{R_{Prim\_c\_Lam}}{R_k}} \cdot \tanh\left(\sqrt{\frac{R_k}{R_{Prim\_c\_Lam}}}\right)$$

$\eta_{Laminar} = 0.976$

$$R_{Prim\_c\_Turb} := \frac{1}{h_{Turbulent} \cdot 2 \cdot H \cdot L} \qquad \eta_{Turbulent} := \sqrt{\frac{R_{Prim\_c\_Turb}}{R_k}} \cdot \tanh\left(\sqrt{\frac{R_k}{R_{Prim\_c\_Turb}}}\right)$$

$\eta_{Turbulent} = 0.981$

$$\beta_{Turbulent} := \frac{1}{mdotCpCalc \cdot R_{C\_Turbulent}}$$

$\beta_{Turbulent} = 0.475$

Calculate R_C for Laminar and Turbulent Flow:

$$R_{C\_Laminar} := \frac{1}{\eta_{Laminar} \cdot h_{Laminar} \cdot A_s} \qquad R_{C\_Turbulent} := \frac{1}{\eta_{Turbulent} \cdot h_{Turbulent} \cdot A_s}$$

$R_{C\_Laminar} = 0.651$ $\quad R_{C\_Turbulent} = 0.828$

Calculate R_I for Laminar Flow:

$$mdotCpCalc := mdotCp\left(V_f\right)$$

$mdotCpCalc = 2.82$

$$\beta_{Laminar} := \frac{1}{mdotCpCalc \cdot R_{C\_Laminar}}$$

$\beta_{Laminar} = 0.545$

$$R_{I\_Laminar} := \beta_{Laminar} \cdot \frac{R_{C\_Laminar}}{1 - e^{-\beta_{Laminar}}}$$

$R_{I\_Laminar} = 0.844$

Heat Transfer Analysis Using Method of Jonsson and Mosfegh:

Check Some Ratios for Model Validity:

$$r_H := \frac{H}{H_d} \qquad r_W := \frac{W}{W_d}$$

$r_H = 0.497$ $\quad r_W = 0.4$

Model is in range of geometry limits

Problem Geometry Remains the Same But There Are Some Different Definitions:

$$D_H := \frac{2 \cdot W_d \cdot H_d}{W_d + H_d}$$

$D_H = 3.333$

Calculate and Check Reynold's Number:
Velocity based on bypass area + fin channel area.

$$A_b := W_d \cdot H_d - W \cdot H_T \qquad A_f := A_{c\_Total}$$

$A_b = 14.76$ $\quad H_d = 2$ $\quad W_d = 10$ $\quad W = 4$

$$Re_{DH} := \frac{\left(\frac{G}{A_b + A_f}\right) \cdot 144 \cdot D_H}{5 \cdot 0.029}$$

$Re_{DH} = 9.487 \times 10^3$

Which is in region >2000 and <16500.

Get Constants from Jonsson and Mosfegh Table:

$C_1 := 88.28 \quad m_1 := 0.6029 \quad m_2 := -0.1098 \quad m_3 := -0.5623 \quad m_4 := 0.08713 \quad m_5 := 0.4139$

Calculate Results:

$$Nu_L := C_1 \cdot \left(\frac{Re_{DH}}{1000}\right)^{m_1} \cdot \left(\frac{W_d}{W}\right)^{m_2} \cdot \left(\frac{H_d}{H}\right)^{m_3} \cdot \left(\frac{S}{H}\right)^{m_4} \cdot \left(\frac{t_f}{H}\right)^{m_5}$$

$\boxed{Nu_L = 71.045}$

$k_{Air} := 6.5 \cdot 10^{-4} \qquad h := \frac{k_{Air}}{L} \cdot Nu_L$

$\boxed{h = 0.015}$

$A_{Sink} := 2 \cdot N_f \cdot L \cdot H + W \cdot L \qquad R_{Sink} := \frac{1}{h \cdot A_{Sink}}$

$\boxed{R_{Sink} = 0.725}$

Example 7.11 Cylindrical Pin Fins (Forced Air) by Khan: This Example Also Used in Example 4.6 for Pressure Loss.

W. A. Khan, J. R. Culham, and M. M. Yovanovich, "Modeling of Cylindrical Pin-Fin Heat Sinks for Electronic Packaging, 21st IEEE Semi-Therm Symposium, 2005.

Input, Data, Symbolics Mostly Following Original Paper:

Sample Problem Data from Paper Using SI Units:

Footprint (*m*):-- $W := \frac{25.4}{1000}$ $L := \frac{25.4}{1000}$

Heat Source (*m*^{2}):--------------------------------

Pin Diameter Thickness (*m*):------------------- $D := \frac{2}{1000}$

Baseplate Thickness (*m*):---------------------- $t_b := \frac{2}{1000}$

Overall Height of Heat Sink (*m*):-------------- $H_T := \frac{12}{1000}$

Pin Height (*mm*):-------------------------------------- $H := H_T - t_b$ $H = 0.01$

Number of Pins (In-Line) N_T, N_L:---------------- $N_T := 7$ $N_L := 7$

Number of Pins (Staggered) N_T, N_L:------------ $N_T := 8$ $N_L := 7$

Approach Velocity (*m/s*):--------------------------- $V_a := 3$

Thermal Conductivity of Solid (*W/m-K*):------- $k := 180$

Thermal Conductivity of Air (*W/m-K*):---------- $k_f := 0.026$

Kinematic Viscosity of Air (*m*^{2}*/s*):-------------- $\nu := 1.58 \cdot 10^{-5}$

Density of Air (*kg/m*^{3}):----------------------------- $\rho := 1.1614$

Prandtl Number of Air:------------------------------ $Pr := 0.71$

Heat Load (*W*):-- $Q := 50$

Ambient Temperature (^oC):----------------------- $T_a := 27$

Calculate Variables and Results:

$$P_L := \frac{L}{N_L} \qquad p_L := \frac{P_L}{D} \qquad P_T := \frac{W}{N_T} \qquad p_T := \frac{P_T}{D} \qquad P_D := \sqrt{P_L^{\,2} + \left(\frac{P_T}{2}\right)^2} \qquad p_D := \frac{P_D}{D}$$

$$V_{Max} := \max\left(\frac{p_T}{p_T - 1}\cdot V_a, \frac{p_T}{p_D - 1}\cdot V_a\right) \qquad V_{Max} = 6.684$$

$$Re_D := \frac{D\cdot V_{Max}}{\nu} \qquad \boxed{Re_D = 846.103}$$

Select from first or second of two following lines for in-line or staggered pins, respectively:

$$\boxed{C_1 := \left(0.2 + \exp\left(-0.55\cdot p_L\right)\right)\cdot p_T^{\,0.285}\cdot p_L^{\,0.212}}$$

$$\boxed{C_1 := \frac{0.61\cdot p_T^{\,0.091}\cdot p_L^{\,0.053}}{1 - 2\cdot \exp\left(-1.09\cdot p_L\right)}}$$

$$h_b := \frac{0.75\cdot k_f}{D}\cdot\sqrt{\frac{p_T - 1}{N_L\cdot p_L\cdot p_T}}\cdot Re_D^{\frac{1}{2}}\cdot Pr^{\frac{1}{3}} \qquad h_b = 47.563$$

$$h_{fin} := \frac{C_1\cdot k_f}{D}\cdot Re_D^{\frac{1}{2}}\cdot Pr^{\frac{1}{3}} \qquad h_{fin} = 257.935$$

$$A_b := L\cdot W - N_T\cdot N_L\cdot\frac{\pi\cdot D^2}{4} \qquad A_b = 4.912\times 10^{-4}$$

$$A_{fin} := \pi\cdot D\cdot H \qquad A_{fin} = 6.283\times 10^{-5}$$

$$m := \sqrt{4\cdot\frac{h_{fin}}{k\cdot D}} \qquad \eta_{fin} := \frac{\tanh(m\cdot H)}{m\cdot H} \qquad \eta_{fin} = 0.914$$

$$R_{fin} := \frac{1}{h_{fin} \cdot A_{fin} \cdot \eta_{fin}} \qquad R_m := \frac{t_b}{k \cdot L \cdot W} \qquad R_{fin} = 67.488$$

$$R_m = 0.017$$

$$R_b := \frac{1}{h_b \cdot \left(L \cdot W - N_T \cdot N_L \cdot \frac{\pi}{4} \cdot D^2\right)} \qquad R_b = 42.801$$

$$R_{th} := \frac{1}{\left(\frac{N_T \cdot N_L}{R_{fin}}\right) + \frac{1}{R_b}} + R_m \qquad \boxed{R_{th} = 1.352}$$

Calculate Pressure Variables and Results:

$$\sigma := \frac{p_T - 1}{p_T} \qquad K_c := -0.0311 \cdot \sigma^2 - 0.3722 \cdot \sigma + 1.0676 \qquad \sigma = 0.449$$

$$K_e := 0.9301 \cdot \sigma^2 - 2.5746 \cdot \sigma + 0.973 \qquad K_c = 0.894$$

Select from first or second of two following lines for in-line or staggered pins, respectively:

$$K_e = 4.829 \times 10^{-3}$$

$$K_1 := 1.009 \cdot \left(\frac{p_T - 1}{p_L - 1}\right)^{\frac{1.09}{Re_D^{0.0553}}} \qquad f := K_1 \cdot \left[0.233 + \frac{45.78}{(p_T - 1)^{1.1} \cdot Re_D}\right]$$

$$K_1 := 1.175 \cdot \left(\frac{p_L}{p_T \cdot Re_D^{0.3124}}\right) + 0.5 \cdot Re_D^{0.0807} \qquad f := K_1 \cdot \frac{S_T^{\frac{378.6}{\frac{13.1}{p_T}}}}{Re_D^{\frac{0.68}{p_T^{1.29}}}}$$

$$K_1 = 1.009$$

$$f = 0.304$$

$$\Delta P := \left(K_c + K_e + f \cdot N_L\right) \cdot \frac{\rho \cdot V_{Max}^2}{2} \qquad \boxed{\Delta P = 78.453}$$

Important Note: The above friction factor definition via the ΔP formula is different that for plate fins.

Cylindrical Pin Fins (Forced Air) by Khan et. al., But Mixed Units Used.

W. A. Khan, J. R. Culham, and M. M. Yovanovich, "Modeling of Cylindrical Pin-Fin Heat Sinks for Electronic Packaging, 21st IEEE Semi-Therm Symposium, 2005.

Input, Data, Symbolics Mostly Following Original Paper:

Sample Problem Data from Paper Using SI Units:

Footprint (*in.*):-------------------------------------- $W := 1.0$ $L := 1.0$

Heat Source (*in.*^{2}):--------------------------------

Pin Diameter Thickness (*in.*):------------------ $D := 0.07874$

Baseplate Thickness (*in.*):---------------------- $t_b := 0.0787$

Overall Height of Heat Sink (*m*):------------- $H_T := 0.472441$

Pin Height (*in.*):-------------------------------------- $H := H_T - t_b$ $H = 0.394$

Number of Pins (In-Line) N_T, N_L:---------------- $N_T := 7$ $N_L := 7$

Number of Pins (Staggered) N_T, N_L:------------ $N_T := 8$ $N_L := 7$

Approach Velocity (*ft./min.*):------------------------- $V_a := 590.55118$

Thermal Conductivity of Solid (*W/in.-K*):------- $k := 4.572$

Thermal Conductivity of Air (*W/in.-K*):---------- $k_f := 0.0006604$

Kinematic Viscosity of Air (in.^{2}/*s*):-------------- $\nu := 0.0245$

Prandtl Number of Air:------------------------------- $Pr := 0.71$

Heat Load (*W*):-- $Q := 50$

Ambient Temperature (^oC):----------------------- $T_a := 27$

Calculate Thermal Variables and Results:

$$P_L := \frac{L}{N_L} \qquad p_L := \frac{P_L}{D} \qquad P_T := \frac{W}{N_T} \qquad p_T := \frac{P_T}{D} \qquad P_D := \sqrt{P_L^{\,2} + \left(\frac{P_T}{2}\right)^2} \qquad p_D := \frac{P_D}{D}$$

$$P_D = 0.16$$

$$V_{Max} := \max\left(\frac{p_T}{p_T - 1}\cdot V_a, \frac{p_T}{p_D - 1}\cdot V_a\right) \qquad V_{Max} = 1.316 \times 10^3$$

$$p_D = 2.028$$

$$Re_D := \frac{D \cdot V_{Max}}{5\nu} \qquad \boxed{Re_D = 845.755}$$

Select from first or second of two following lines for in-line or staggered pins, respectively:

$$\boxed{C_1 := \left(0.2 + \exp\left(-0.55 \cdot p_L\right)\right) \cdot p_T^{\,0.285} \cdot p_L^{\,0.212}}$$

$$\boxed{C_1 := \frac{0.61 \cdot p_T^{\,0.091} \cdot p_L^{\,0.053}}{1 - 2 \cdot \exp\left(-1.09 \cdot p_L\right)}}$$

$$h_b := \frac{0.75 \cdot k_f}{D} \cdot \sqrt{\frac{p_T - 1}{N_L \cdot p_L \cdot p_T}} \cdot Re_D^{\frac{1}{2}} \cdot Pr^{\frac{1}{3}} \qquad h_b = 0.031$$

$$h_{fin} := \frac{C_1 \cdot k_f}{D} \cdot Re_D^{\frac{1}{2}} \cdot Pr^{\frac{1}{3}} \qquad h_{fin} = 0.166$$

$$A_b := L \cdot W - N_T \cdot N_L \cdot \frac{\pi \cdot D^2}{4} \qquad A_b = 0.761$$

$$A_{fin} := \pi \cdot D \cdot H \qquad A_{fin} = 0.097$$

$$m := \sqrt{4 \cdot \frac{h_{fin}}{k \cdot D}} \qquad \eta_{fin} := \frac{\tanh(m \cdot H)}{m \cdot H} \qquad \eta_{fin} = 0.914$$

$$R_{fin} := \frac{1}{h_{fin} \cdot A_{fin} \cdot \eta_{fin}} \qquad R_m := \frac{t_b}{k \cdot L \cdot W} \qquad R_{fin} = 67.495$$

$$R_m = 0.017$$

$$R_b := \frac{1}{h_b \cdot \left(L \cdot W - N_T \cdot N_L \cdot \frac{\pi}{4} \cdot D^2\right)} \qquad R_b = 42.81$$

$$R_{th} := \frac{1}{\left(\frac{N_T \cdot N_L}{R_{fin}}\right) + \frac{1}{R_b}} + R_m \qquad \boxed{R_{th} = 1.352}$$

Calculate Pressure Variables and Results:

$$\sigma := \frac{p_T - 1}{p_T} \qquad K_c := -0.0311 \cdot \sigma^2 - 0.3722 \cdot \sigma + 1.0676 \qquad \boxed{\sigma = 0.449}$$

$$K_e := 0.9301 \cdot \sigma^2 - 2.5746 \cdot \sigma + 0.973 \qquad \boxed{K_c = 0.894}$$

Select from first or second of two following lines for in-line or staggered pins, respectively:

$$\boxed{K_e = 4.827 \times 10^{-3}}$$

$$K_1 := 1.009 \cdot \left(\frac{p_T - 1}{p_L - 1}\right)^{\frac{1.09}{Re_D^{0.0553}}} \qquad f := K_1 \cdot \left[0.233 + \frac{45.78}{(p_T - 1)^{1.1} \cdot Re_D}\right]$$

$$K_1 := 1.175 \cdot \left(\frac{p_L}{p_T \cdot Re_D^{0.3124}}\right) + 0.5 \cdot Re_D^{0.0807} \qquad f := K_1 \cdot \frac{p_T^{\frac{378.6}{p_T^{\frac{13.1}{p_T}}}}}{Re_D^{\frac{0.68}{p_T^{1.29}}}}$$

$$\boxed{K_1 = 1.009}$$

$$\boxed{f = 0.304}$$

Important Note: The friction factor definition via the Δh formula is different that for plate fins.
$\Delta h=(K_c+K_e+f_{Pins}N_L)h_{v\text{-}Pins}$ where = one velocity head in Pin array.

Use $h_{v\text{-}Pins}=1.2910^{-3}G^2/(WH\sigma)^2$, then $\Delta h = \Delta h$[in. H_2O]. This $G/()^2$ is equivalent to V_{Max}

$$G := V_a \cdot \frac{W \cdot (H)}{144} \qquad G = 1.615 \qquad \Delta h := \left(K_c + K_e + f \cdot N_L\right)1.29 \cdot 10^{-3} \cdot \frac{G^2}{(W \cdot H \cdot \sigma)^2} \qquad \Delta h = 0.326$$

Δh=0.326 in. H2O converted to Pa by dividing by 4.019×10^{-3} is 81.1 and is slightly different than the SI calc. because my air density (built into the 1.29×10^{-3}) is a little different that Khan's.

Also:

$$R := \left(K_c + K_e + f \cdot N_L\right) 1.29 \cdot 10^{-3} \cdot \frac{1}{(W \cdot H \cdot \sigma)^2}$$

$$R = 0.125$$

$$\Delta h := R \cdot G^2 \qquad \Delta h = 0.326$$

Application Example 8.4: Vertical Flat Plate

Input Plate Geometry (Inches):

$W := 9.0$ $H := 6.0$ $Q := 8$

Calculate Results:

$$A_S := 2 \cdot H \cdot W \qquad \Delta T := \left(\frac{Q}{\frac{0.0024}{H^{0.25}} \cdot A_S} \right)^{\frac{1}{1.25}}$$

$\Delta T = 22.243$

$$Q_{Check} := 0.0024 \cdot \left(\frac{\Delta T}{H} \right)^{0.25} \cdot A_S \cdot \Delta T$$

$Q_{Check} = 8$

$$h := 0.0024 \cdot \left(\frac{\Delta T}{H} \right)^{0.25}$$

$h = 3.33 \times 10^{-3}$

Application Example 8.5: Vertical Flat Plate with Radiation, ε=0.8

Input Plate Geometry (Inches):

$W := 9.0$ $\quad H := 6.0$ $\quad Q := 8$ $\quad \varepsilon := 0.8$ $\quad T_A := 30$

Calculate Results:

$$A_S := 2 \cdot H \cdot W$$

Use First ΔT Guess of 40, Then Update from It. Table:

$$\Delta T := 12.1$$

$$h_c := 0.0024 \cdot \left(\frac{\Delta T}{H}\right)^{0.25} \qquad h_r := 1.463 \cdot 10^{-10} \cdot \left(T_A + 273.16\right)^3$$

Calculate New ΔT:

$$h_r = 4.076 \times 10^{-3}$$

$$\Delta T := \frac{Q}{\left(h_c + \varepsilon \cdot h_r\right) \cdot A_S} \qquad h_c = 2.86 \times 10^{-3} \qquad \Delta T = 12.102$$

Tabulated Iteration Results:

Iteration	ΔT	hc	ΔT
1	40	0.00386	10.41
2	20	0.00324	11.39
3	11	0.00279	12.24
4	12	0.00285	12.11
5	12.1	0.00285	12.10

$$T_S := \Delta T + 20 + 273.16$$

Check on Accuracy of Approximate h_r=0.0041:

$$h_r := 3.657 \cdot 10^{-11} \cdot \left[T_S^{\,3} + T_S^{\,2} \cdot \left(T_A + 273.16\right) + T_S \cdot \left(T_A + 273.16\right)^2 + \left(T_A + 273.16\right)^3\right] \qquad h_r = 4.118 \times 10^{-3}$$

Application Example 8.7: Array of Convecting PCBs.

Symmetric Isothermal.

Input Data:

$W := 10$ $L := 10$ $Q := 25$ $T_I := 30$

First Use Forumlae and Graphs to Estimate b_{opt}:

Guess ΔT=25 C. $\Delta T := 25$

From Fig. 8.10 for Isothermal we get $r = b_{opt}/L^{1/4} = 0.2$.

$r := 0.2$ $b_{opt} := r \cdot L^{0.25}$ $b_{opt} = 0.356$

Guess ΔT=25 C. $\Delta T := 25$ $q := \frac{Q}{2} \cdot \frac{1}{L \cdot W}$ $q = 0.125$

From Fig. 8.11 for Isoflux we get $r = b_{opt}/L^{1/5} = 0.125$.

$r := 0.125$ $b_{opt} := r \cdot L^{0.2}$ $b_{opt} = 0.198$

Create Formulae:

$$\gamma div\beta(T_W) := \left[5.46 \cdot 10^5 \cdot \exp\left[-9.2817 \cdot 10^{-3} \cdot \left(\frac{T_W - T_I}{2} + T_I\right)\right]\right]$$

$$\beta := \frac{1}{273.16 + T_I} \qquad k_{Air}(T_W) := 6.0583 \cdot 10^{-4} + 1.6906 \cdot 10^{-6} \cdot \left(\frac{T_W - T_I}{2} + T_I\right)$$

$$Ra(T_W, b) := \gamma div\beta(T_W) \cdot \beta \cdot \frac{b^4}{L} \cdot (T_W - T_I)$$

$$Nu(T_W, b) := \frac{1}{\left[\left(\frac{24}{Ra(T_W, b)}\right)^2 + \frac{1}{\left(0.59 \cdot Ra(T_W, b)^{0.25}\right)^2}\right]^{0.5}}$$

$$\Delta T(T_W, b) := \frac{1}{\frac{k_{Air}(T_W)}{b} \cdot Nu(T_W, b) \cdot L \cdot W} \cdot \frac{Q}{2}$$

$$r_{opt}(T_W) := \frac{54^{\frac{1}{4}}}{\left[\left[5.46\cdot 10^{5}\cdot \exp\left[-9.2817\cdot 10^{-3}\cdot\left(\frac{T_W - T_I}{2} + T_I\right)\right]\right]\cdot(T_W - T_I)\cdot\frac{1}{273.16 + T_I}\right]^{\frac{1}{4}}}$$

Calculate Optimum b and T_W for b_{opt}:

First Estimate Optimum b, Guess T_W = 40:

$\boxed{r_{opt} = b_{opt}/L^{1/4}}$ $\quad$ $T_W := 74.7$

$b_{opt} := r_{opt}(T_W)\cdot L^{0.25}$ $\quad$ $b_{opt} = 0.323$

$\Delta T_{New} := \Delta T(T_W, b_{opt})$ $\quad \Delta T_{New} = 44.627$ $\quad T_W := \Delta T_{New} + T_I$ $\quad T_W = 74.627$

Calculate T_W for Various Values of b by Iterating T_W:

$b := 0.5$ $\quad$ $T_W := 68.55$

$$\Delta T(T_W) := \frac{1}{\frac{k_{Air}(T_W)}{b}\cdot Nu(T_W, b)\cdot L\cdot W}\cdot\frac{Q}{2}$$

$T_W := \Delta T(T_W) + T_I$ $\quad$ $T_W = 68.582$

Symmetric Isoflux for Nu(L).

Create Formulae:

$$\gamma div\beta(T_W) := \left[5.46\cdot 10^{5}\cdot \exp\left[-9.2817\cdot 10^{-3}\cdot\left(\frac{T_W - T_I}{2} + T_I\right)\right]\right]$$

$$\beta := \frac{1}{273.16 + T_I} \qquad k_{Air}(T_W) := 6.0583\cdot 10^{-4} + 1.6906\cdot 10^{-6}\cdot\left(\frac{T_W - T_I}{2} + T_I\right)$$

$$RaIsoF(T_W, b, q) := \gamma div\beta(T_W)\cdot\beta\cdot\frac{b^5}{L}\cdot\frac{q}{k_{Air}(T_W)}$$

$$\mathrm{NuIsoF}(T_W, b, q) := \frac{1}{\left(\frac{48}{\mathrm{RaIsoF}(T_W, b, q)} + \frac{2.51}{\mathrm{RaIsoF}(T_W, b, q)^{0.4}}\right)^{0.5}}$$

$$\Delta T(T_W, b, q) := \frac{1}{\frac{k_{Air}(T_W)}{b} \cdot \mathrm{NuIsoF}(T_W, b, q) \cdot L \cdot W} \cdot \frac{Q}{2}$$

$$r_{optIsoF}(T_W, q) := \frac{6.9^{\frac{1}{5}}}{\left[\left[5.46 \cdot 10^5 \cdot \exp\left[-9.2817 \cdot 10^{-3} \cdot \left(\frac{T_W - T_I}{2} + T_I\right)\right]\right] \cdot \frac{\frac{q}{k_{Air}(T_W)}}{273.16 + T_I}\right]^{\frac{1}{5}}}$$

Calculate Optimum b and T_W for b_{opt}:

$$q := \frac{\frac{Q}{2}}{L \cdot W} \qquad \boxed{q = 0.125}$$

First Estimate Optimum b, Guess T_W = 50:

$\boxed{r_{optIsoF} = b_{opt}/L^{1/5}}$ $\qquad$ $\boxed{T_W := 135}$

$$b_{opt} := r_{optIsoF}(T_W, q) \cdot L^{\frac{1}{5}} \qquad \boxed{b_{opt} = 0.218}$$

$$\Delta T_{New} := \Delta T(T_W, b_{opt}, q) \quad \Delta T_{New} = 104.112 \quad T_W := \Delta T_{New} + T_I \quad \boxed{T_W = 134.112}$$

Calculate T_W for Various Values of b, Same Q, by Iterating T_W:

$b := 0.5$ $\qquad$ $\boxed{T_W := 76.7}$

$$\Delta T(T_W, b, q) := \frac{1}{\frac{k_{Air}(T_W)}{b} \cdot \mathrm{NuIsoF}(T_W, b, q) \cdot L \cdot W} \cdot \frac{Q}{2} \qquad T_W := \Delta T(T_W, b, q) + T_I$$

$$\boxed{T_W = 76.672}$$

Symmetric Isoflux for Nu(L/2).

Create Formulae:

$$\gamma div\beta(T_W) := \left[5.46\cdot 10^5\cdot \exp\left[-9.2817\cdot 10^{-3}\cdot\left(\frac{T_W - T_I}{2} + T_I\right)\right]\right]$$

$$\beta := \frac{1}{273.16 + T_I} \qquad k_{Air}(T_W) := 6.0583\cdot 10^{-4} + 1.6906\cdot 10^{-6}\cdot\left(\frac{T_W - T_I}{2} + T_I\right)$$

$$RaIsoF(T_W, b, q) := \gamma div\beta(T_W)\cdot\beta\cdot\frac{b^5}{L}\cdot\frac{q}{k_{Air}(T_W)}$$

$$NuIsoF(T_W, b, q) := \frac{1}{\left(\frac{12}{RaIsoF(T_W, b, q)} + \frac{1.85}{RaIsoF(T_W, b, q)^{0.4}}\right)^{0.5}}$$

$$\Delta T(T_W, b, q) := \frac{1}{\frac{k_{Air}(T_W)}{b}\cdot NuIsoF(T_W, b, q)\cdot L\cdot W}\cdot\frac{Q}{2}$$

$$r_{optIsoF}(T_W, q) := \frac{6.9^{\frac{1}{5}}}{\left[\left[5.46\cdot 10^5\cdot \exp\left[-9.2817\cdot 10^{-3}\cdot\left(\frac{T_W - T_I}{2} + T_I\right)\right]\right]\cdot\frac{\frac{q}{k_{Air}(T_W)}}{273.16 + T_I}\right]^{\frac{1}{5}}}$$

Calculate Optimum b and T_W for b_{opt}:

$$q := \frac{\frac{Q}{2}}{L\cdot W} \qquad \boxed{q = 0.125}$$

First Estimate Optimum b, Guess T_W = 50:

$\boxed{r_{optIsoF} = b_{opt}/L^{1/5}}$ $\qquad T_W := 88.9$

$$b_{opt} := r_{optIsoF}(T_W, q)\cdot L^{\frac{1}{5}} \qquad \boxed{b_{opt} = 0.207}$$

$\Delta T_{New} := \Delta T(T_W, b_{opt}, q)$ $\Delta T_{New} = 58.865$ $T_W := \Delta T_{New} + T_I$ $\boxed{T_W = 88.865}$

Calculate T_W for Various Values of b, Same Q, by Iterating T_W:

$b := 0.5$ $\boxed{T_W := 66}$

$$\Delta T(T_W, b, q) := \frac{1}{\frac{k_{Air}(T_W)}{b} \cdot NuIsoF(T_W, b, q) \cdot L \cdot W} \cdot \frac{Q}{2}$$

$T_W := \Delta T(T_W, b, q) + T_I$

$\boxed{T_W = 65.988}$

Calculation Heat Sink Using Van de Pol & Tierney - Simplified h_H

$T_A := 20$ $\Delta T := 50$ $t_f := 0.06$ $W := 1.86$ $L := 1.0$ $H := 5$ $N_f := 6$ $k_{Al} := 5$

$$S := \frac{(W - N_f \cdot t_f)}{N_f - 1}$$

$t_b := 0.63$ $S = 0.3$

$A_E := 2 \cdot H \cdot (L)$ $A_I := \left[2 \cdot (N_f - 1) \cdot L + W\right] \cdot H$

$A_I = 59.3$ $A_E = 10$

$\frac{L}{S} = 3.333$ $\frac{H}{S} = 16.667$

$hRatio := 0.76$

$$h_H := 0.0024 \cdot \left(\frac{\Delta T}{H}\right)^{0.25} \qquad h_c := hRatio \cdot h_H$$

$h_H = 4.268 \times 10^{-3}$ $h_c = 3.244 \times 10^{-3}$

$$R_k := \frac{L}{k_{Al} \cdot H \cdot t_f} \qquad R_c := \frac{1}{2 \cdot h_c \cdot L \cdot H}$$

$R_k = 0.667$ $R_c = 30.83$

$$\eta := \sqrt{\frac{R_c}{R_k}} \cdot \tanh\left(\sqrt{\frac{R_k}{R_c}}\right)$$

$\eta = 0.993$

$C_I := \eta \cdot h_c \cdot A_I$ $C_E := \eta \cdot h_H \cdot A_E$ $C := C_I + C_E$

$C_I = 0.191$ $C_E = 0.0424$ $C = 0.233$

$Q := C \cdot \Delta T$ $R := \frac{1}{C}$

$Q = 11.667$ $R = 4.286$

Calculation Heat Sink Using Van de Pol & Tierney - Most Exact h_c, h_H

$T_A := 20$ $\quad \Delta T := 50$ $\quad t_f := 0.15$ $\quad S := 0.35$ $\quad L := 2.62$ $\quad H := 6$ $\quad N_f := 9$ $\quad k_{Al} := 5$

$W := N_f \cdot t_f + (N_f - 1) \cdot S \qquad W = 4.15 \qquad t_b := 0.63$

$$T_M := \frac{2 \cdot T_A + \Delta T}{2} \quad T_S := T_A + \Delta T \quad z := \frac{H}{S} \quad x := \frac{L}{S} \qquad z = 17.143 \quad x = 7.486$$

$$C_1 := 5.454 \cdot 10^5 \cdot \exp\left(-9.254 \cdot 10^{-3} \cdot T_S\right) \qquad \beta := \frac{1}{T_M + 273.16} \qquad V := -11.8$$

$$GrPr_H := C_1 \cdot \beta \cdot \Delta T \cdot H^3 \qquad r := \frac{2x \cdot S}{2 \cdot x + 1} \qquad GrPr := C_1 \cdot \beta \cdot \Delta T \cdot r^3$$

$$PSI(a) := \frac{24 \cdot \left(1 - 0.483 \cdot \exp\left(\frac{-0.17}{a}\right)\right)}{\left[\left(1 + \frac{a}{2}\right) \cdot \left[1 + (1 - \exp(-0.83 \cdot a)) \cdot \left(9.14 \cdot \sqrt{a} \cdot \exp(V \cdot S) - 0.61\right)\right]\right]^3}$$

$$\psi := PSI\left(\frac{1}{x}\right) \qquad RaChan := \left(\frac{r}{H}\right) \cdot GrPr \qquad \psi = 20.475 \quad RaChan = 86.599$$

$$Nur := \frac{RaChan}{\psi} \cdot \left[1 - \exp\left[-\psi \cdot \left(\frac{0.5}{RaChan}\right)^{\frac{3}{4}}\right]\right] \qquad Nu_H := 0.595 \cdot GrPr_H^{\frac{1}{4}}$$

$$hRatio := \frac{H}{r} \frac{Nur}{Nu_H} \qquad Nur = 1.475 \quad Nu_H = 33.194 \quad hRatio = 0.813$$

$$k_{Air} := 6.0583 \cdot 10^{-4} + 1.6906 \cdot 10^{-6} \cdot T_M \qquad h_H := \frac{k_{Air}}{H} \cdot Nu_H \qquad h_c := hRatio \cdot h_H$$

$k_{Air} = 6.819 \times 10^{-4}$ $h_H = 3.773 \times 10^{-3}$ $h_c = 3.066 \times 10^{-3}$

$$R_k := \frac{L}{k_{Al} \cdot H \cdot t_f} \qquad R_c := \frac{1}{2 \cdot h_c \cdot L \cdot H} \qquad R_k = 0.582 \qquad R_c = 10.375$$

$$\eta := \sqrt{\frac{R_c}{R_k}} \cdot \tanh\left(\sqrt{\frac{R_k}{R_c}}\right) \qquad A_I := \left[2 \cdot \left(N_f - 1\right) \cdot L + W\right] \cdot H \qquad \eta = 0.982$$

$$A_E := 2 \cdot H \cdot \left(L + t_b\right) + W \cdot H + 2 \cdot t_b \cdot (W + H) + 2 \cdot N_f \cdot \left(t_f \cdot L\right)$$

$A_I = 276.42$ $A_E = 83.763$

$$C_I := \eta \cdot h_c \cdot A_I \quad C_E := \eta \cdot h_H \cdot A_E \quad C := C_I + C_E \qquad C_I = 0.832 \quad C_E = 0.31 \quad C = 1.142$$

$$Q := C \cdot \Delta T \qquad R := \frac{1}{C} \qquad Q = 57.108 \qquad R = 0.876$$

Calculation Heat Sink Using Van de Pol & Tierney - Simplified h_H

$$h_H := 0.0024 \cdot \left(\frac{\Delta T}{H}\right)^{0.25} \qquad h_c := hRatio \cdot h_H$$

$h_H = 4.078 \times 10^{-3}$ $h_c = 3.314 \times 10^{-3}$

$$R_k := \frac{L}{k_{Al} \cdot H \cdot t_f} \qquad R_c := \frac{1}{2 \cdot h_c \cdot L \cdot H} \qquad R_k = 0.582 \qquad R_c = 9.598$$

$$\eta := \sqrt{\frac{R_c}{R_k}} \cdot \tanh\left(\sqrt{\frac{R_k}{R_c}}\right) \qquad \eta = 0.98$$

$$C_I := \eta \cdot h_c \cdot A_I \quad C_E := \eta \cdot h_H \cdot A_E \quad C := C_I + C_E \qquad C_I = 0.898 \quad C_E = 0.335 \quad C = 1.233$$

$$Q := C \cdot \Delta T \qquad R := \frac{1}{C} \qquad Q = 61.637 \qquad R = 0.811$$

Calculation Heat Sink Using Van de Pol & Tierney - Small Device h_H

$$h_H := 0.0022 \cdot \left(\frac{\Delta T}{H}\right)^{0.35} \qquad h_c := hRatio \cdot h_H$$

$$\boxed{h_H = 4.621 \times 10^{-3}} \qquad \boxed{h_c = 3.755 \times 10^{-3}}$$

$$R_k := \frac{L}{k_{Al} \cdot H \cdot t_f} \qquad R_c := \frac{1}{2 \cdot h_c \cdot L \cdot H} \qquad R_k = 0.582 \qquad R_c = 8.47$$

$$\eta := \sqrt{\frac{R_c}{R_k}} \cdot \tanh\left(\sqrt{\frac{R_k}{R_c}}\right) \qquad \boxed{\eta = 0.978}$$

$$C_I := \eta \cdot h_c \cdot A_I \quad C_E := \eta \cdot h_H \cdot A_E \quad C := C_I + C_E \quad \boxed{C_I = 1.015} \boxed{C_E = 0.378} \boxed{C = 1.393}$$

$$Q := C \cdot \Delta T \qquad R := \frac{1}{C} \qquad \boxed{Q = 69.662} \qquad \boxed{R = 0.718}$$

Application Example 10.3 Radiation Shape Factors for Parallel Circuit Boards - Board to Board Effects

Input Starting Values: $LL := 10.0$ $SS := 1$ $WW := 10$

x=L/S, y=W/S: $x := \dfrac{LL}{SS}$ $y := \dfrac{WW}{SS}$ $x = 10$ $y = 10$

Full Text Formula:

$$FPar(y) := \left(\frac{2}{\pi \cdot x \cdot y}\right) \cdot \left[\ln\left[\sqrt{\frac{(1+x^2)\cdot(1+y^2)}{1+x^2+y^2}}\right] + \left(y \cdot \sqrt{1+x^2} \cdot \operatorname{atan}\left(\frac{y}{\sqrt{1+x^2}}\right)\right) \ldots \atop + \left(x \cdot \sqrt{1+y^2} \cdot \operatorname{atan}\left(\frac{x}{\sqrt{1+y^2}}\right)\right) - y \cdot \operatorname{atan}(y) - x \cdot \operatorname{atan}(x) \right]$$

Single Value Result for y=20: $F := FPar(y)$ $F = 0.82699$

Single Value Result for y=Infinity: $FInf := FPar(2000)$ $FInf = 0.905$

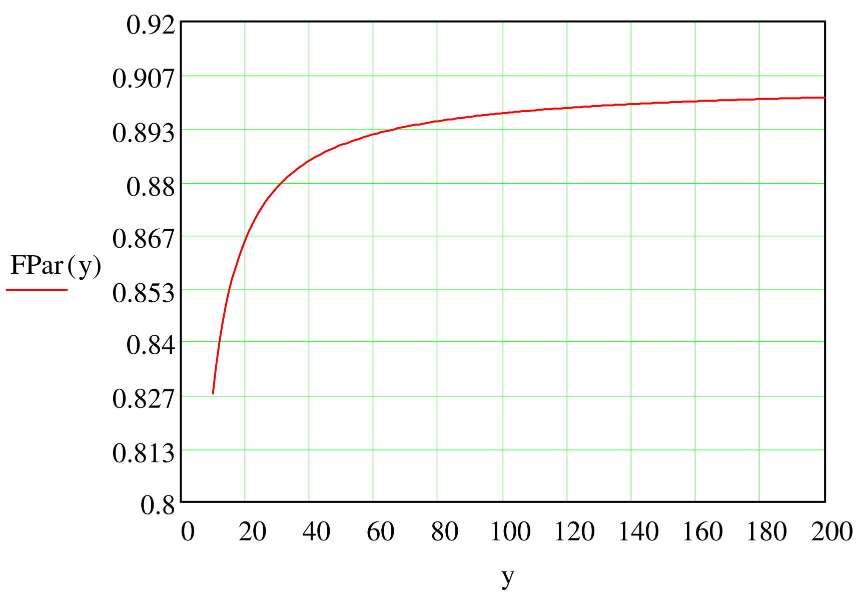
Crossed String Formula:

$L_1 := LL$ $L_2 := LL$ $L_5 := SS$ $L_6 := SS$ $L_3 := \sqrt{{L_5}^2 + LL^2}$

$L_4 := L_3$ $FCS := \dfrac{(L_3 + L_4) - (L_5 + L_6)}{2 \cdot L_1}$ $FCS = 0.905$

Graphical Results:

$y := 10, 11 .. 200$

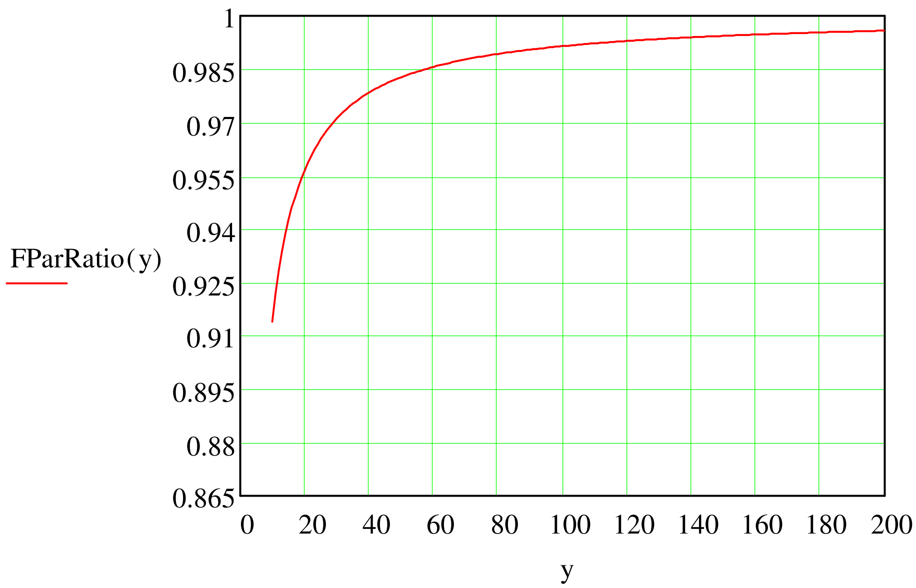


Fractional Formula:

$$FParRatio(y) := \left(\frac{2}{\pi \cdot x \cdot y \cdot FCS}\right) \cdot \left[\ln\left[\sqrt{\frac{\left(1+x^2\right)\cdot\left(1+y^2\right)}{1+x^2+y^2}}\right] + \left(y\cdot\sqrt{1+x^2}\cdot \mathrm{atan}\left(\frac{y}{\sqrt{1+x^2}}\right)\right) \ldots \right.$$
$$\left. + \left(x\cdot\sqrt{1+y^2}\cdot \mathrm{atan}\left(\frac{x}{\sqrt{1+y^2}}\right)\right) - y\cdot \mathrm{atan}(y) - x\cdot \mathrm{atan}(x) \right]$$

Graphical Results:

$y := 10, 11 .. 200$



Save Plot Data for Input to Grapher:

$i := 10, 11 .. 200$ $\quad FM_i := FParRatio(i)$ $\quad x_i := i$

E:\temp\X.dat

x

E:\temp\FM.dat

FM

Application Example 10.3 Radiation Shape Factors for Parallel Circuit Boards - End Effects for Long Direction (W)

Can Only Get Single Value Result Because Both xP, yP Are Functions of W:

$x=L_2/W$, $y=L_1/W$: $\quad xP := \frac{SS}{WW} \qquad yP := \frac{LL}{WW} \qquad xP = 0.1 \qquad yP = 1\times 10^{0}$

$$z := xP^2 + yP^2$$

$$FPerP := \left(\frac{1}{4\cdot\pi\cdot yP}\right)\cdot\left[\ln\left[\left[\frac{\left(1+xP^2\right)\cdot\left(1+yP^2\right)}{1+z}\right]\cdot\left[\frac{yP^2\cdot(1+z)}{\left(1+yP^2\right)\cdot z}\right]^{yP^2}\cdot\left[\frac{xP^2\cdot(1+z)}{\left(1+xP^2\right)\cdot z}\right]^{xP^2}\right]\right] \ldots$$
$$+\left(\frac{1}{\pi\cdot yP}\right)\cdot\left(yP\cdot\operatorname{atan}\left(\frac{1}{yP}\right)+xP\cdot\operatorname{atan}\left(\frac{1}{xP}\right)-\sqrt{z}\cdot\operatorname{atan}\left(\frac{1}{\sqrt{z}}\right)\right)$$

$FTotal := 4\cdot FPerP + F \qquad \boxed{FPerP = 0.043251} \qquad \boxed{FTotal = 1}$

Application Example 10.6 for Sealed and Vented Box with External Radiation

Input Airflow Resistance Data:

$W_I := 5.0$ $H_I := 1.0$ $W_E := 5.0$ $H_E := 1.0$

$L := 5.0$ $D_{PCB} := 5.0$ $b := 0.75$ $a := 0.25$ $l := 1.0$ $f_I := 0.35$ $f_E := 0.35$

Calculate Total Airflow Resistance:

$C_{Inlet} := 1.9$ $A_I := f_I \cdot W_I \cdot H_I$ $R_{Inlet} := \dfrac{C_{Inlet} \cdot 10^{-3}}{A_I^2}$ $A_I = 1.75$ $R_{Inlet} = 6.204 \times 10^{-4}$

$A_{1ExpantoCards} := W_I \cdot H_I$ $A_{2ExpantoCards} := D_{PCB} \cdot 5 \cdot (b + a)$

$$R_{ExpantoCards} := 1.29 \cdot 10^{-3} \cdot \left[\frac{1}{A_{1ExpantoCards}} \cdot \left(1 - \frac{A_{1ExpantoCards}}{A_{2ExpantoCards}}\right)\right]^2$$

$A_{1ExpantoCards} = 5$ $A_{2ExpantoCards} = 25$ $R_{ExpantoCards} = 3.302 \times 10^{-5}$

Calculate free area ratio of card cage

$A_1 := D_{PCB} \cdot 5(b + a)$ $A_2 := D_{PCB} \cdot 5(b + a) - 4 \cdot 5 \cdot (l \cdot a)$ $A_{fCards} := \dfrac{A_2}{A_1}$

$A_1 = 25$ $A_2 = 20$ $A_{fCards} = 0.8$

$$R_{ConttoCards} := \frac{0.5 \cdot 10^{-3}}{A_2^2} \cdot \left(1 - \frac{A_2}{A_1}\right)^{\frac{3}{4}}$$

$R_{ConttoCards} = 3.738 \times 10^{-7}$

$$R_{CardCage} := \frac{3.08 \cdot (1) \cdot L \cdot 10^{-4}}{\left[D_{PCB} \cdot 5(b + a)\right]^2}$$

$R_{CardCage} = 2.464 \times 10^{-6}$

$A_{1ExpanfromCards} := D_{PCB} \cdot 5(b + a) - 4 \cdot 5 \cdot (l \cdot a)$ $A_{1ExpanfromCards} = 20$

$A_{2ExpanfromCards} := D_{PCB} \cdot 5(b + a)$ $A_{2ExpanfromCards} = 25$

$$R_{ExpanfromCards} := 1.29 \cdot 10^{-3} \cdot \left[\frac{1}{A_{1ExpanfromCards}} \cdot \left(1 - \frac{A_{1ExpanfromCards}}{A_{2ExpanfromCards}}\right)\right]^2$$

$R_{ExpanfromCards} = 1.29 \times 10^{-7}$

$A_{1ConttoExit} := D_{PCB} \cdot 5(b + a)$ $\quad$ $A_{2ConttoExit} := W_E \cdot H_E$

$A_{1ConttoExit} = 25$ $\quad$ $A_{2ConttoExit} = 5$

$$R_{ConttoExit} := \frac{0.5 \cdot 10^{-3}}{A_{2ConttoExit}^{\,2}} \cdot \left[1 - \left(\frac{A_{2ConttoExit}}{A_{1ConttoExit}}\right)\right]^{\frac{3}{4}} \qquad R_{ConttoExit} = 1.692 \times 10^{-5}$$

$C_{Exit} := 1.9$ $\quad$ $A_E := f_E \cdot W_E \cdot H_E$ $\quad$ $R_{Exit} := \dfrac{C_{Exit} \cdot 10^{-3}}{A_E^{\,2}}$ $\quad$ $A_E = 1.75$ $\quad$ $R_{Exit} = 6.204 \times 10^{-4}$

$$R_{AF} := R_{Inlet} + R_{ExpantoCards} + R_{ConttoCards} + R_{CardCage} + R_{ExpanfromCards} + R_{ConttoExit} + R_{Exit}$$

$R_{AF} = 1.294 \times 10^{-3}$

Enter Box Dimensions, Dissipation Height (inches):

$W := 7$ $\quad$ $H := 7$ $\quad$ $D := 7$ $\quad$ $d_H := 5$ $\quad$ $T_A := 20$ $\quad$ $\varepsilon := 0.8$ $\quad$ $\sigma := 3.657 \cdot 10^{-11}$

Enter Total Box Dissipation (W):

$Q_{Box} := 10$

Calculate Area Values (in.^2):

$A_{Left} := H \cdot D$ $\quad$ $A_{Right} := A_{Left}$ $\quad$ $A_{Front} := W \cdot H$ $\quad$ $A_{Back} := A_{Front}$

$A_{Top} := W \cdot D$ $\quad$ $A_{Bottom} := A_{Top}$

$A_{Left} = 49$ $\quad$ $A_{Right} = 49$ $\quad$ $A_{Front} = 49$ $\quad$ $A_{Back} = 49$ $\quad$ $A_{Top} = 49$ $\quad$ $A_{Bottom} = 49$

Values at Beginning of Iteration:

$\Delta TWA := 4.7$ $\quad$ $\Delta TAir\_TA := 14.32$ $\quad$ $\Delta TAirW := \Delta TAir\_TA - \Delta TWA$ $\quad$ $\Delta TAirW = 9.62$

Set Up Equation for Qd:

Sealed Box Uses Qd=0, Vented Iterates

$Q_d := 5$

Use Qd=1*10^-20 for Sealed Box. Qd=5 for First Iteration of Vented Box.
Disable After First Iteration.
This Line Enabled by "Right Click Drop Down Menu"

Set Up Formulae for Airdraft (CFM) and Thermal-Fluid Resistance:

$$G_{Constant} := 1.53 \cdot 10^{-2} \cdot \left(\frac{d_H}{R_{AF}}\right)^{\frac{1}{3}} \qquad G_{Constant} = 0.24$$

$G := G_{Constant} \cdot Q_d^{\frac{1}{3}}$ $\quad$ $G = \blacksquare$ $\quad$ **Disable After First Iteration.**

Vented Box Q_d. Enable These Two Lines After First Iteration. After First Iteration Get G Calculated From Previous Iteration. Activated by "Right Click Drop Down Menu"

$G := 0.34$ $\qquad Q_d := \frac{\Delta TAir\_TA \cdot G}{1.76}$ $\qquad Q_d = 2.766$

$$G := G_{Constant} \cdot Q_d^{\frac{1}{3}} \qquad G = 0.337$$

$$R_f := \frac{1.76}{G} \qquad R_f = 5.222$$

Set Up and Calculate External Convection Resistance:

$$RCE\_Const := \frac{1}{\left[\left[A_{Top} \cdot 0.0022 \left[\frac{1}{\frac{A_{Top}}{2 \cdot (W+D)}}\right]^{0.25}\right] + A_{Bottom} \cdot 0.0011 \cdot \left[\frac{1}{\frac{A_{Bottom}}{2 \cdot (W+D)}}\right]^{0.25}\right] \ldots + \left[2 \cdot A_{Left} \cdot 0.0024 \cdot \left(\frac{1}{H}\right)^{0.25}\right] + 2 \cdot A_{Front} \cdot 0.0024 \cdot \left(\frac{1}{H}\right)^{0.25}}$$

$RCE\_Const = 2.327$

$$RCE := \frac{RCE\_Const}{\Delta TWA^{0.25}} \qquad RCE = 1.58$$

Set Up and Calculate External Radiation Resistance:

$$A_{Total} := A_{Front} + A_{Back} + A_{Left} + A_{Right} + A_{Top} + A_{Bottom}$$

$$R_r := \frac{1}{\varepsilon \cdot A_{Total} \cdot \sigma \cdot \left[(\Delta TWA + T_A + 273.16)^3 + (\Delta TWA + T_A + 273.16)^2 \cdot (T_A + 273.16) \ldots + (\Delta TWA + T_A + 273.16) \cdot (T_A + 273.16)^2 + (T_A + 273.16)^3\right]}$$

$R_r = 1.126$

Total External Resistance:

$RE := \frac{RCE \cdot R_r}{RCE + R_r}$ $\qquad RE = 0.658$ $\qquad \Delta TWA := RE \cdot (Q_{Box} - Q_d)$ $\qquad \Delta TWA = 4.757$

Set Up and Calculate Internal Convection Resistance:

$$RCI\_Const := \frac{1}{\left[\left[A_{Top} \cdot 0.0022 \left[\frac{1}{\frac{A_{Top}}{2 \cdot (W+D)}}\right]^{0.25}\right] + A_{Bottom} \cdot 0.0011 \cdot \left[\frac{1}{\frac{A_{Bottom}}{2 \cdot (W+D)}}\right]^{0.25}\right] \ldots + \left[2 \cdot A_{Left} \cdot 0.0024 \cdot \left(\frac{1}{H}\right)^{0.25}\right] + 2 \cdot A_{Front} \cdot 0.0024 \cdot \left(\frac{1}{H}\right)^{0.25}}$$

$RCI\_Const = 2.327$ $\qquad RCI := \frac{RCI\_Const}{\Delta TAirW^{0.25}}$ $\qquad RCI = 1.321$

Total of Total External Resistance (Conv and Rad) and Internal Convection Resistance:

$Rx := RCI + RE$ $\qquad Rx = 1.979$

Calculate Total System Thermal Resistance And Total Air Temperature Rise:

$R_{Total} := \frac{Rx \cdot R_f}{Rx + R_f}$ $\qquad R_{Total} = 1.435$

$TAir\_TA := R_{Total} \cdot Q_{Box}$ $\qquad TAir\_TA = 14.35$

Convection Calculation Heat Sink Using Van de Pol & Tierney and Fin Radiation

$T_A := 20$ $t_b := 0.63$ $t_f := 0.15$ $S := 0.35$ $L := 2.62$ $H := 8$ $N_f := 9$ $k_{Al} := 5$

$W := N_f \cdot t_f + (N_f - 1) \cdot S$ $W = 4.15$ $\frac{L}{S} = 7.486$ $\frac{H}{S} = 22.857$

$A_I := [2 \cdot (N_f - 1) \cdot L + W] \cdot H$ $A_E := 2 \cdot H \cdot L$ $A_I = 368.56$ $A_E = 41.92$

Convection Calculation for ΔT=10:

$\Delta T := 10$ $hRatio := 0.57$ $T_S := T_A + \Delta T$ $T_S = 30$

$$h_H := 0.0024 \cdot \left(\frac{\Delta T}{H}\right)^{0.25} \qquad h_c := hRatio \cdot h_H$$

$h_H = 2.538 \times 10^{-3}$ $h_c = 1.446 \times 10^{-3}$

Radiation Calculation for ΔT=10:

$\varepsilon := 0.8$ $SF := 0.1$ **for H = 8.0 in. and** $\frac{L}{S} = 7.486$ $\frac{H}{L} = 3.053$

$$h_r := 3.657 \cdot 10^{-11} \cdot \left[\left[(T_A + \Delta T + 273.16)^3 + (T_A + \Delta T + 273.16)^2 \cdot (T_A + 273.16) \ldots \atop + (T_A + \Delta T + 273.16) \cdot (T_A + 273.16)^2 + (T_A + 273.16)^3\right]\right]$$

$$h_r = 3.878 \times 10^{-3}$$

Fin Efficiency Calculation for ΔT=10:

$$R_k := \frac{L}{k_{Al} \cdot H \cdot t_f} \qquad R_c := \frac{1}{2 \cdot (h_c + SF \cdot h_r) \cdot H \cdot L} \qquad R_k = 0.437 \qquad R_c = 13.005$$

$$\eta := \sqrt{\frac{R_c}{R_k}} \cdot \tanh\left(\sqrt{\frac{R_k}{R_c}}\right)$$ $\eta = 0.989$

Heat Calculation for ΔT=10:

$$C_I := \eta \cdot (h_c + SF \cdot h_r) \cdot A_I \quad C_E := \eta \cdot (h_H + \varepsilon \cdot h_r) \cdot A_E \quad C_E := \eta \cdot h_H \cdot A_E \quad C := C_I + C_E$$

$C_I = 0.669$ $C_E = 0.105$ $C = 0.774$

$$Q := C \cdot \Delta T \qquad R := \frac{1}{C}$$ $Q = 7.738$ $R = 1.292$

Convection Calculation for ΔT=25:

$\Delta T := 25$ $hRatio := 0.71$ $T_S := T_A + \Delta T$ $T_S = 45$

$$h_H := 0.0024 \cdot \left(\frac{\Delta T}{H}\right)^{0.25} \qquad h_c := hRatio \cdot h_H$$

$h_H = 3.191 \times 10^{-3}$ $h_c = 2.266 \times 10^{-3}$

Radiation Calculation for ΔT=25:

$\varepsilon := 0.8$ $SF := 0.1$ **for H = 8.0 in. and** $\frac{L}{S} = 7.486$ $\frac{H}{L} = 3.053$

$$h_r := 3.657 \cdot 10^{-11} \cdot \left[\left[(T_A + \Delta T + 273.16)^3 + (T_A + \Delta T + 273.16)^2 \cdot (T_A + 273.16) \ldots \atop + (T_A + \Delta T + 273.16) \cdot (T_A + 273.16)^2 + (T_A + 273.16)^3\right]\right]$$

$$h_r = 4.184 \times 10^{-3}$$

Fin Efficiency Calculation for ΔT=25:

$$R_k := \frac{L}{k_{Al} \cdot H \cdot t_f} \qquad R_c := \frac{1}{2 \cdot (h_c + SF \cdot h_r) \cdot H \cdot L} \qquad R_k = 0.437 \qquad R_c = 8.888$$

$$\eta := \sqrt{\frac{R_c}{R_k}} \cdot \tanh\left(\sqrt{\frac{R_k}{R_c}}\right)$$

$\boxed{\eta = 0.984}$

Heat Calculation for ΔT=25:

$$C_I := \eta \cdot \left(h_c + SF \cdot h_r\right) \cdot A_I \quad C_E := \eta \cdot \left(h_H + \varepsilon \cdot h_r\right) \cdot A_E \quad C_E := \eta \cdot h_H \cdot A_E \quad C := C_I + C_E$$

$\boxed{C_I = 0.973}$ $\boxed{C_E = 0.132}$ $\boxed{C = 1.105}$

$$Q := C \cdot \Delta T \qquad R := \frac{1}{C}$$

$\boxed{Q = 27.624}$ $\boxed{R = 0.905}$

Convection Calculation for ΔT=50:

$\boxed{\Delta T := 50}$ $\boxed{hRatio := 0.77}$ $T_S := T_A + \Delta T$ $\boxed{T_S = 70}$

$$h_H := 0.0024 \cdot \left(\frac{\Delta T}{H}\right)^{0.25} \qquad h_c := hRatio \cdot h_H$$

$\boxed{h_H = 3.795 \times 10^{-3}}$ $\boxed{h_c = 2.922 \times 10^{-3}}$

Radiation Calculation for ΔT=50:

$\boxed{\varepsilon := 0.8}$ $\boxed{SF := 0.1}$ **for H = 8.0 in. and** $\frac{L}{S} = 7.486$ $\frac{H}{L} = 3.053$

$$h_r := 3.657 \cdot 10^{-11} \cdot \left[\left[\left(T_A + \Delta T + 273.16\right)^3 + \left(T_A + \Delta T + 273.16\right)^2 \cdot \left(T_A + 273.16\right) \ldots \\ + \left(T_A + \Delta T + 273.16\right) \cdot \left(T_A + 273.16\right)^2 + \left(T_A + 273.16\right)^3\right]\right]$$

$$h_r = 4.74 \times 10^{-3}$$

Fin Efficiency Calculation for ΔT=50:

$$R_k := \frac{L}{k_{Al} \cdot H \cdot t_f} \qquad R_c := \frac{1}{2 \cdot (h_c + SF \cdot h_r) \cdot H \cdot L} \qquad R_k = 0.437 \qquad R_c = 7.025$$

$$\eta := \sqrt{\frac{R_c}{R_k}} \cdot \tanh\left(\sqrt{\frac{R_k}{R_c}}\right) \qquad \boxed{\eta = 0.98}$$

Heat Calculation for ΔT=50:

$$C_I := \eta \cdot (h_c + SF \cdot h_r) \cdot A_I \qquad C_E := \eta \cdot (h_H + \varepsilon \cdot h_r) \cdot A_E \qquad C_E := \eta \cdot h_H \cdot A_E \qquad C := C_I + C_E$$

$$\boxed{C_I = 1.226} \quad \boxed{C_E = 0.156} \quad \boxed{C = 1.382}$$

$$Q := C \cdot \Delta T \qquad R := \frac{1}{C} \qquad \boxed{Q = 69.108} \qquad \boxed{R = 0.724}$$

Convection Calculation for ΔT=100:

$$\boxed{\Delta T := 100} \quad \boxed{hRatio := 0.79} \quad T_S := T_A + \Delta T \quad \boxed{T_S = 120}$$

$$h_H := 0.0024 \cdot \left(\frac{\Delta T}{H}\right)^{0.25} \qquad h_c := hRatio \cdot h_H$$

$$\boxed{h_H = 4.513 \times 10^{-3}} \qquad \boxed{h_c = 3.565 \times 10^{-3}}$$

Radiation Calculation for ΔT=100:

$$\boxed{\varepsilon := 0.8} \quad \boxed{SF := 0.1}$$

for H = 8.0 in. and $\frac{L}{S} = 7.486 \qquad \frac{H}{L} = 3.053$

$$h_r := 3.657 \cdot 10^{-11} \cdot \left[\left[(T_A + \Delta T + 273.16)^3 + (T_A + \Delta T + 273.16)^2 \cdot (T_A + 273.16) \ldots \right.\right.$$
$$\left.\left. + (T_A + \Delta T + 273.16) \cdot (T_A + 273.16)^2 + (T_A + 273.16)^3\right]\right]$$

$$h_r = 6.037 \times 10^{-3}$$

Fin Efficiency Calculation for ΔT=100:

$$R_k := \frac{L}{k_{Al} \cdot H \cdot t_f} \qquad R_c := \frac{1}{2 \cdot (h_c + SF \cdot h_r) \cdot H \cdot L} \qquad R_k = 0.437 \qquad R_c = 5.722$$

$$\eta := \sqrt{\frac{R_c}{R_k}} \cdot \tanh\left(\sqrt{\frac{R_k}{R_c}}\right) \qquad \boxed{\eta = 0.975}$$

Heat Calculation for ΔT=50:

$$C_I := \eta \cdot (h_c + SF \cdot h_r) \cdot A_I \qquad C_E := \eta \cdot (h_H + \varepsilon \cdot h_r) \cdot A_E \qquad C_E := \eta \cdot h_H \cdot A_E \qquad C := C_I + C_E$$

$$\boxed{C_I = 1.498} \quad \boxed{C_E = 0.185} \quad \boxed{C = 1.683}$$

$$Q := C \cdot \Delta T \qquad R := \frac{1}{C} \qquad \boxed{Q = 168.3} \qquad \boxed{R = 0.594}$$

Convection Calculation Heat Sink Using Van de Pol & Tierney and Fin Radiation

$T_A := 20$ $t_b := 0.63$ $t_f := 0.15$ $W := 4.15$ $L := 2.62$ $H := 8$ $\varepsilon := 0.8$ $k_{Al} := 5$

$\Delta T := 50$ $N_f := 14$

$$S := \frac{W - N_f \cdot t_f}{N_f - 1} \qquad S = 0.1577 \qquad A_I := \left[2\cdot\left(N_f - 1\right)\cdot L + W\right]\cdot H \qquad A_E := 2\cdot H\cdot L$$

$A_I = 578.16$ $A_E = 41.92$

$\frac{L}{S} = 16.615$ $\frac{H}{S} = 50.732$ $hRatio := 0.999$

$\frac{H}{L} = 3.053$ $SF := 0.47$

Convection Calculation for ΔT=50:

$$T_S := T_A + \Delta T \qquad T_S = 70$$

$$h_H := 0.0024\cdot\left(\frac{\Delta T}{H}\right)^{0.25} \qquad h_c := hRatio\cdot h_H \qquad h_H = 3.795\times 10^{-3} \qquad h_c = 3.791\times 10^{-3}$$

Radiation Calculation for ΔT=50:

for H = 8.0 in. and $\frac{L}{S} = 16.615$ $\frac{H}{L} = 3.053$

$$h_r := 3.657\cdot 10^{-11}\cdot\left[\left[\left(T_A + \Delta T + 273.16\right)^3 + \left(T_A + \Delta T + 273.16\right)^2\cdot\left(T_A + 273.16\right) \ldots \right.\right.$$
$$\left.\left. + \left(T_A + \Delta T + 273.16\right)\cdot\left(T_A + 273.16\right)^2 + \left(T_A + 273.16\right)^3\right]\right]$$

$$h_r = 4.74\times 10^{-3}$$

Fin Efficiency Calculation for ΔT=50:

$$R_k := \frac{L}{k_{Al} \cdot H \cdot t_f} \qquad R_c := \frac{1}{2 \cdot \left(h_c + SF \cdot h_r\right) \cdot H \cdot L} \qquad R_k = 0.437 \qquad R_c = 3.963$$

$$\eta := \sqrt{\frac{R_c}{R_k}} \cdot \tanh\left(\sqrt{\frac{R_k}{R_c}}\right) \qquad \boxed{\eta = 0.965}$$

Heat Calculation for ΔT=50:

$$C_I := \eta \cdot \left(h_c + SF \cdot h_r\right) \cdot A_I \quad C_E := \eta \cdot \left(h_H + \varepsilon \cdot h_r\right) \cdot A_E \quad C_E := \eta \cdot h_H \cdot A_E \quad C := C_I + C_E$$

$$\boxed{C_I = 3.357} \quad \boxed{C_E = 0.153} \quad \boxed{C = 3.511}$$

$$Q := C \cdot \Delta T \qquad R := \frac{1}{C} \qquad \boxed{Q = 175.546} \qquad \boxed{R = 0.285}$$

$$h_r = 4.74 \times 10^{-3}$$

Fin Efficiency Calculation for ΔT=100:

$$R_k := \frac{L}{k_{Al} \cdot H \cdot t_f} \qquad R_c := \frac{1}{2 \cdot \left(h_c + SF \cdot h_r\right) \cdot H \cdot L} \qquad R_k = 0.437 \qquad R_c = 3.963$$

$$\eta := \sqrt{\frac{R_c}{R_k}} \cdot \tanh\left(\sqrt{\frac{R_k}{R_c}}\right) \qquad \boxed{\eta = 0.965}$$

Heat Calculation for ΔT=50:

$$C_I := \eta \cdot \left(h_c + SF \cdot h_r\right) \cdot A_I \quad C_E := \eta \cdot \left(h_H + \varepsilon \cdot h_r\right) \cdot A_E \quad C_E := \eta \cdot h_H \cdot A_E \quad C := C_I + C_E$$

$$\boxed{C_I = 3.357} \quad \boxed{C_E = 0.153} \quad \boxed{C = 3.511}$$

$$Q := C \cdot \Delta T \qquad R := \frac{1}{C} \qquad \boxed{Q = 175.546} \qquad \boxed{R = 0.285}$$

Application Example 11.6: Metal Cooled PCB with Forced Air Cooling

$V := 300$ $L := 6$ $W := 4$ $t := 0.0625$ $k := 5.0$ $T_0 := 40$ $f := 1.54$

$Q := 8.33$ $\Delta T_{L\_A} := 20 + 40$

$h := 0.00109 \cdot \sqrt{\frac{V}{W}} \cdot f$ $h = 0.0145$

$R_s := \frac{1}{2 \cdot h \cdot W \cdot L}$ $R_k := \frac{L}{k \cdot W \cdot t}$ $R_s = 1.433$ $R_k = 4.8$

First Iteration:

$\frac{\frac{T_0}{Q}}{R_s} = 3.351$ $\frac{R_k}{R_s} = 3.349$ $R := \frac{\left(\frac{\frac{T_0}{Q}}{R_s} - 1\right) \cdot R_s}{\cosh\left(\sqrt{\frac{R_k}{R_s}}\right)} + R_s$ $\frac{R}{R_s} = 1.735$ $R = 2.487$ $Q := \frac{\Delta T_{L\_A}}{R}$

$Q = 24.129$

Calculate a New R and Q:

$\frac{\frac{T_0}{Q}}{R_s} = 1.157$ $\frac{R_k}{R_s} = 3.349$ $R := \frac{\left(\frac{\frac{T_0}{Q}}{R_s} - 1\right) \cdot R_s}{\cosh\left(\sqrt{\frac{R_k}{R_s}}\right)} + R_s$ $\frac{R}{R_s} = 1.049$ $R = 1.503$ $Q := \frac{\Delta T_{L\_A}}{R}$

$Q = 39.91$

Calculate a New R and Q:

$\frac{\frac{T_0}{Q}}{R_s} = 0.699$ $\frac{R_k}{R_s} = 3.349$ $R := \frac{\left(\frac{\frac{T_0}{Q}}{R_s} - 1\right) \cdot R_s}{\cosh\left(\sqrt{\frac{R_k}{R_s}}\right)} + R_s$ $\frac{R}{R_s} = 0.906$ $R = 1.298$ $Q := \frac{\Delta T_{L\_A}}{R}$

$Q = 46.212$

Calculate a New R and Q:

$\frac{\frac{T_0}{Q}}{R_s} = 0.604$ $\frac{R_k}{R_s} = 3.349$ $R := \frac{\left(\frac{\frac{T_0}{Q}}{R_s} - 1\right) \cdot R_s}{\cosh\left(\sqrt{\frac{R_k}{R_s}}\right)} + R_s$ $\frac{R}{R_s} = 0.876$ $R = 1.256$ $Q := \frac{\Delta T_{L\_A}}{R}$

$Q = 47.785$

Calculate a New R and Q:

$$\frac{\frac{T_0}{Q}}{R_s} = 0.584 \quad \frac{R_k}{R_s} = 3.349 \quad R := \frac{\left(\frac{\frac{T_0}{Q}}{R_s} - 1\right)\cdot R_s}{\cosh\left(\sqrt{\frac{R_k}{R_s}}\right)} + R_s \quad \frac{R}{R_s} = 0.87 \quad R = 1.247 \quad Q := \frac{\Delta T_{L\_A}}{R}$$

$$\boxed{Q = 48.127}$$

Calculate a New R and Q:

$$\frac{\frac{T_0}{Q}}{R_s} = 0.58 \quad \frac{R_k}{R_s} = 3.349 \quad R := \frac{\left(\frac{\frac{T_0}{Q}}{R_s} - 1\right)\cdot R_s}{\cosh\left(\sqrt{\frac{R_k}{R_s}}\right)} + R_s \quad \frac{R}{R_s} = 0.869 \quad R = 1.245 \quad Q := \frac{\Delta T_{L\_A}}{R}$$

$$\boxed{Q = 48.199}$$

Calculate Q_0 Into T_0:

$$Q_0 := \frac{Q \tanh\left(\sqrt{\frac{R_k}{R_s}}\right)}{\sqrt{\frac{R_k}{R_s}}}\cdot\left(\frac{\frac{T_0}{Q}}{R_s} - 1\right)$$

$$\boxed{Q_0 = -10.529}$$

Application Example 11.12: Calculation of PCB Conductivities for PCB

Using New ks and Isotropic for each layer:

First Layer is Mix of Cu and EG
Second Layer is EG
Third Layer is Cu
Fourth Layer is EG

$k_{Cu} := 10$ $\quad k_{EG} := 0.0140$ $\quad t_1 := 0.0014$ $\quad t_2 := 0.03$ $\quad t_3 := 0.0014$ $\quad t_4 := 0.03$

First make calculations using series and parallel method for <u>One Layer Orthogonal</u>:

Layer 1 In-Plane and Through Plane:

$f_{1\_Cu} := 0.5$ $\quad f_{1\_EG} := 0.5$

$k_1 := f_{1\_EG} \cdot k_{EG} + f_{1\_Cu} \cdot k_{Cu}$ $\quad k_1 = 5.007$

Layer 2: $k_2 := k_{EG}$ **Layer 3:** $k_3 := k_{Cu}$ **Layer 4:** $k_4 := k_{EG}$

Entire PCB:

$t := t_1 + t_2 + t_3 + t_4$ $\quad f_1 := \frac{t_1}{t}$ $\quad f_2 := \frac{t_2}{t}$ $\quad f_3 := \frac{t_3}{t}$ $\quad f_4 := \frac{t_4}{t}$

$f_1 = 0.022$ $\quad f_2 = 0.478$ $\quad f_3 = 0.022$ $\quad f_4 = 0.478$ $\quad t = 0.063$

$k_P := k_1 \cdot f_1 + k_2 \cdot f_2 + k_3 \cdot f_3 + k_4 \cdot f_4$ $\quad f_1 = 0.022$ $\quad f_2 = 0.478$ $\quad f_3 = 0.022$ $\quad f_4 = 0.478$ $\quad k_P = 0.348$

Check: $\quad f := f_1 + f_2 + f_3 + f_4$ $\quad f = 1$

$$k_N := \frac{1}{\frac{f_1}{k_1} + \frac{f_2}{k_2} + \frac{f_3}{k_3} + \frac{f_4}{k_4}} \qquad k_N = 0.015$$

Make calculations using Azar Method:

$k_{Cu} := 9.779$ $\quad k_{EG} := 0.0140$

$$k_P(Z_{Cu}, Z) := 0.0203 + 8.89 \cdot \left(\frac{Z_{Cu}}{Z}\right) \qquad k_N(Z_{Cu}, Z) := \frac{1}{66.53 \cdot \left(1 - \frac{Z_{Cu}}{Z}\right) + 0.10 \cdot \left(\frac{Z_{Cu}}{Z}\right)}$$

$kP := k_P(0.0014 + 0.0014 \cdot 0.5, 0.0628)$ $\quad kN := k_N(0.0014 + 0.0014 \cdot 0.5, 0.0628)$ $\quad kP = 0.318$ $\quad kN = 0.016$

Redo Series and Parallel Method, Dividing PCB Into Top, Bottom Halves k_P for each layer, k_N from top to bottom surface, i.e. Two Layer Othogonal:

Top Half, In-Plane: The result for layer 1 may be used here.

$t_{1\_Top} := t_1 \qquad t_{2\_Top} := t_2 \qquad t_{Top} := t_{1\_Top} + t_{2\_Top}$

$k_{1\_Top} := k_1 \qquad k_{2\_Top} := k_2 \qquad f_{1\_Top} := \dfrac{t_{1\_Top}}{t_{Top}} \qquad f_{2\_Top} := \dfrac{t_{2\_Top}}{t_{Top}}$

$k_{P\_Top} := k_{1\_Top} \cdot f_{1\_Top} + k_{2\_Top} \cdot f_{2\_Top} \qquad k_{1\_Top} = 5.007 \quad k_{2\_Top} = 0.014$

$\boxed{t_{Top} = 0.031} \qquad \boxed{f_{1\_Top} = 0.045} \qquad \boxed{f_{2\_Top} = 0.955} \qquad \boxed{k_{P\_Top} = 0.237}$

Bottom Half, In-Plane:

$t_{1\_Bot} := t_3 \qquad t_{2\_Bot} := t_4 \qquad t_{Bot} := t_{1\_Bot} + t_{2\_Bot}$

$k_{1\_Bot} := k_3 \qquad k_{2\_Bot} := k_4 \qquad f_{1\_Bot} := \dfrac{t_{1\_Bot}}{t_{Bot}} \qquad f_{2\_Bot} := \dfrac{t_{2\_Bot}}{t_{Bot}}$

$k_{P\_Bot} := k_{1\_Bot} \cdot f_{1\_Bot} + k_{2\_Bot} \cdot f_{2\_Bot} \qquad k_{1\_Bot} = 10 \quad k_3 = 10 \quad k_{2\_Bot} = 0.014$

$\boxed{t_{Bot} = 0.031} \qquad \boxed{f_{1\_Bot} = 0.045} \qquad \boxed{f_{2\_Bot} = 0.955} \qquad \boxed{k_{P\_Bot} = 0.459}$

For thermal network applications with the two layers of nodes placed at the top and bottom surfaces, the k_N for two separate top and bottom halves should be the k_N calculated for the entire PCB.

Application Example 11.14: TO-220 Power Transistor on Heat Sink

$k_1 := 4.0$ $k_2 := 4.0$ $H := 1.71 \cdot 10^5$ $k_g := 6.7 \cdot 10^{-4}$

$M_0 := 1.469 \cdot 10^{-5}$ $T_{g0} := 273.16 + 50$ $T_g := 273.16 + 30$ $P_g := 1$ $P_{g0} := P_g$

First row of Table 11.3:

$$k_M := \frac{2 \cdot k_1 \cdot k_2}{k_1 + k_2} \qquad k_M = 4 \qquad R_1 := \sqrt{2 \cdot \left(120 \cdot 10^{-6}\right)^2} \qquad R_2 := \sqrt{2 \cdot \left(65 \cdot 10^{-6}\right)^2} \qquad R_3 := \sqrt{2 \cdot \left(10 \cdot 10^{-6}\right)^2}$$

$$R_1 = 1.697 \times 10^{-4} \qquad k_M = 4$$

$$hc1_{100} := 9.22 \cdot k_M \cdot R_1^{-0.598} \cdot \left(\frac{100}{H}\right)^{0.95} \qquad hc1_{100} = 5.625 \qquad rc1_{100} := \frac{1}{hc1_{100}} \qquad rc1_{100} = 0.178$$

$$hg1_{100} := \frac{k_g}{\left[1.53 \cdot R_1 \cdot \left(\frac{100}{H}\right)^{-0.097} + M_0 \cdot \left(\frac{T_g}{T_{g0}}\right) \cdot \left(\frac{P_{g0}}{P_g}\right)\right]} \qquad hg1_{100} = 1.222 \qquad \frac{1}{hg1_{100}} = 0.818$$

$$\frac{1}{hg1_{100} + hc1_{100}} = 0.146$$

Create Plots:

$$i := 1, 2 .. 500$$

$$p_i := i$$

$$rg1_i := \frac{1.53 \cdot R_1 \cdot \left(\frac{i}{H}\right)^{-0.097} + M_0 \cdot \left(\frac{T_g}{T_{g0}}\right) \cdot \left(\frac{P_{g0}}{P_g}\right)}{k_g} \qquad rc1_i := \frac{1}{9.22 \cdot k_M \cdot R_1^{-0.598} \cdot \left(\frac{i}{H}\right)^{0.95}}$$

$$r1_i := \frac{1}{\frac{1}{rg1_i} + \frac{1}{rc1_i}}$$

$$rg2_i := \frac{1.53 \cdot R_2 \cdot \left(\frac{i}{H}\right)^{-0.097} + M_0 \cdot \left(\frac{T_g}{T_{g0}}\right) \cdot \left(\frac{P_{g0}}{P_g}\right)}{k_g} \qquad rc2_i := \frac{1}{9.22 \cdot k_M \cdot R_2^{-0.598} \cdot \left(\frac{i}{H}\right)^{0.95}}$$

$$r2_i := \frac{1}{\frac{1}{rg2_i} + \frac{1}{rc2_i}}$$

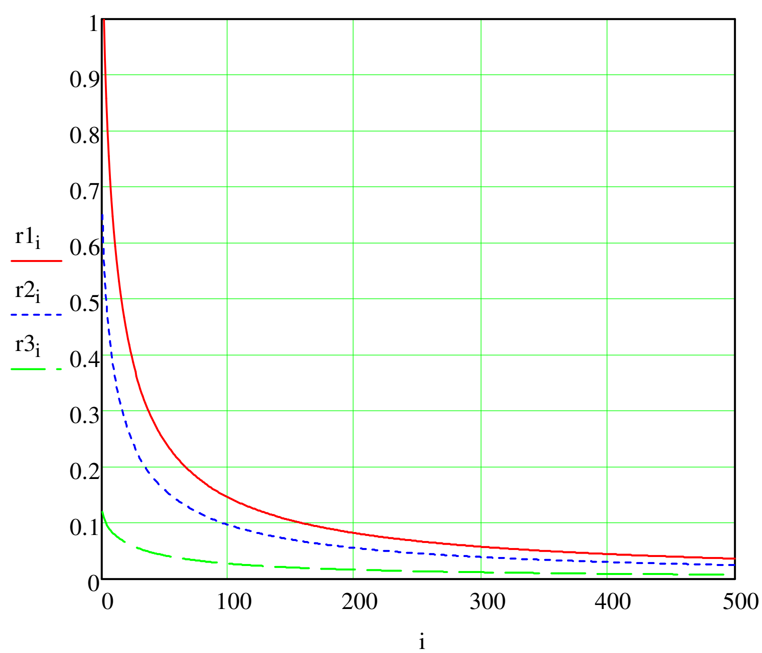
$$rg3_i := \frac{1.53 \cdot R_3 \cdot \left(\frac{i}{H}\right)^{-0.097} + M_0 \cdot \left(\frac{T_g}{T_{g0}}\right) \cdot \left(\frac{P_{g0}}{P_g}\right)}{k_g} \qquad rc3_i := \frac{1}{9.22 \cdot k_M \cdot R_3^{-0.598} \cdot \left(\frac{i}{H}\right)^{0.95}}$$

$$r3_i := \frac{1}{\frac{1}{rg3_i} + \frac{1}{rc3_i}}$$

$rc1_{100} = 0.178$ $\quad rg1_{100} = 0.818$ $\quad r1_{100} = 0.146$

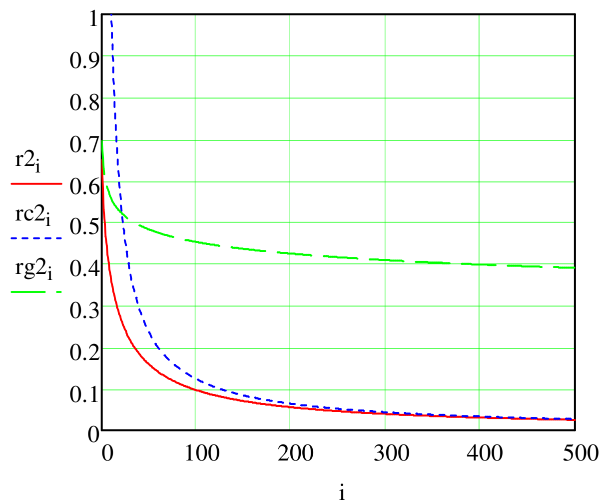
$rc2_{100} = 0.123$ $\quad rg2_{100} = 0.453$ $\quad r2_{100} = 0.097$

$rc3_{100} = 0.04$ $\quad rg3_{100} = 0.087$ $\quad r3_{100} = 0.028$



pa.dat InterfaceR1.dat InterfaceR2.dat InterfaceR3.dat

p r1 r2 r3



Application 12.13: Heat Sink with Two Convecting Sides, One Finned, One Unfinned.

$N_f := 14$ $L := 1.0$ $t_b := 0.2$ $a := 4.0$ $b := 4.0$ $k := 5.0$ $h_2 := 0.01$

$T_{A2} := 30$ $T_{A1} := 30$ $\Delta x := 0.4$ $\Delta y := 0.4$ $Q := 20$

Calculate Finned Side h_e:

$$A_f := 2 \cdot (N_f - 1) \cdot L \cdot b + a \cdot b \qquad R_{Sink\_Fins} := \frac{1}{h_2 \cdot A_f} \qquad h_e := \frac{1}{R_{Sink\_Fins} \cdot a \cdot b}$$

$A_f = 120$ $R_{Sink\_Fins} = 0.83333$ $h_e = 0.075$

Calculate Finned Side Spreading Parameters:

$$\rho := \frac{a}{b} \quad \alpha := \frac{\Delta x}{a} \quad \beta := \frac{\Delta y}{a} \quad \tau := \frac{t_b}{a} \quad Biot\tau_2 := \frac{h_e \cdot t_b}{k}$$

$\rho = 1$ $\alpha = 0.1$ $\beta = 0.1$ $Biot\tau_2 = 0.003$

$\tau = 0.05$

Get Dimensionless Spreading Resistance for Various Values of h_1:

$h_1 := 0.0$ $Biot\tau_1 := \frac{h_1 \cdot t_b}{k}$ $Biot\tau_1 = 0 \times 10^{0}$ $\psi_{Sp1} := 0.8031$ from Figure 12-13

$h_1 := 0.005$ $Biot\tau_1 := \frac{h_1 \cdot t_b}{k}$ $Biot\tau_1 = 2 \times 10^{-4}$ $\psi_{Sp2} := 0.8025$ calculated

$h_1 := 0.01$ $Biot\tau_1 := \frac{h_1 \cdot t_b}{k}$ $Biot\tau_1 = 4 \times 10^{-4}$ $\psi_{Sp3} := 0.8018$ calculated

$h_1 := 0.075$ $Biot\tau_1 := \frac{h_1 \cdot t_b}{k}$ $Biot\tau_1 = 0.003$ $\Psi_{Sp4} := 0.7935$ calculated

Calculate Spreading Resistance for Various Values of h_1:

$$R_{Spread}(Psi) := \frac{Psi}{k \cdot \sqrt{\Delta x \cdot \Delta y}}$$

$R_{Sp1} := R_{Spread}(\psi_{Sp1})$ $R_{Sp1} = 0.40155$

$R_{Sp2} := R_{Spread}(\psi_{Sp2})$ $R_{Sp2} = 0.40125$

$R_{Sp3} := R_{Spread}(\psi_{Sp3})$ $R_{Sp3} = 0.4009$

$R_{Sp4} := R_{Spread}(\Psi_{Sp4})$ $R_{Sp4} = 0.39675$

Calculate Uniform Source Resistance Term for Elevated ΔT_{A1}=30:

$$R_U(h_1, T_{A1}) := \frac{1 + h_1 \cdot a \cdot b \cdot T_{A1}}{k \cdot a \cdot b} \cdot \frac{t_b + \frac{k}{h_e}}{1 + \frac{h_1 \cdot t_b}{k} + \frac{h_1}{h_e}}$$

$R_{U1a} := R_U(0.0, 0.0)$ $\quad \boxed{R_{U1a} = 0.83583}$

$R_{U1b} := R_U(0.0, 30)$ $\quad \boxed{R_{U1b} = 0.83583}$

$R_{U2a} := R_U(0.005, 0)$ $\quad \boxed{R_{U2a} = 0.78345}$

$R_{U2b} := R_U(0.005, 30)$ $\quad \boxed{R_{U2b} = 2.66372}$

$R_{U3a} := R_U(0.01, 0)$ $\quad \boxed{R_{U3a} = 0.73724}$

$R_{U3b} := R_U(0.01, 30)$ $\quad \boxed{R_{U3b} = 4.27599}$

$R_{U4a} := R_U(0.075, 0)$ $\quad \boxed{R_{U4a} = 0.41729}$

$R_{U4b} := R_U(0.075, 30)$ $\quad \boxed{R_{U4b} = 15.43976}$

Calculate the Total Resistance from Source to Ambient T_{A2}. Since the four R_{Sp} are nearly identical, use only one value.

$\boxed{R_{Sp} := 0.401}$ $\quad \Delta T(R_U) := (R_{Sp} + R_U) \cdot Q$

$\boxed{\Delta T(R_{U1a}) = 24.73667}$

$\boxed{\Delta T(R_{U1b}) = 24.73667}$

$\boxed{\Delta T(R_{U2a}) = 23.68894}$

$\boxed{\Delta T(R_{U2b}) = 61.29439}$

$\boxed{\Delta T(R_{U3a}) = 22.7648}$

$\boxed{\Delta T(R_{U3b}) = 93.53982}$

$\boxed{\Delta T(R_{U4a}) = 16.36581}$

$\boxed{\Delta T(R_{U4b}) = 316.81514}$

Gordon Ellison's Calculation of a Single Value of Average Thermal Spreading Resistance ψ_{AveSp}

$\alpha := 0.001 \quad \beta := 0.001 \quad \rho := 1 \quad Bi\tau := 10^{20} \quad \tau := 1$

$Lmax := 3000 \quad Mmax := 3000$

Recommended	
$\alpha = \beta$	Lmax=Mmax
0.5	50
0.25	75
0.1	100
0.05	150
0.025	175
0.01	300
0.005	400
0.0025	700
0.001	1000

$l := 1 .. Lmax$

$$Lterm_l := \left(\frac{1}{l^3}\right)\cdot\left[\sin(l\cdot\pi\cdot\alpha)^2\cdot\left(\frac{1+\frac{Bi\tau}{2\cdot l\cdot\pi\cdot\tau}\cdot\tanh(2\cdot l\cdot\pi\cdot\tau)}{\frac{Bi\tau}{2\cdot l\cdot\pi\cdot\tau}+\tanh(2\cdot l\cdot\pi\cdot\tau)}\right)\right]$$

$m := 1 .. Mmax$

$$Mterm_m := \frac{1}{m^3}\cdot\sin(m\cdot\pi\cdot\beta\cdot\rho)^2\cdot\left(\frac{1+\frac{Bi\tau}{2\cdot m\cdot\pi\cdot\tau\cdot\rho}\cdot\tanh(2\cdot m\cdot\pi\cdot\tau\cdot\rho)}{\frac{Bi\tau}{2\cdot m\cdot\pi\cdot\tau\cdot\rho}+\tanh(2\cdot m\cdot\pi\cdot\tau\cdot\rho)}\right)$$

$l := 1 .. Lmax$

$m := 1 .. Mmax$

$$LMterm_{l,m} := \frac{1}{l^2\cdot m^2}\cdot\sin(l\cdot\pi\cdot\alpha)^2\cdot\sin(m\cdot\pi\cdot\beta\cdot\rho)^2\cdot\left[\frac{1+\frac{Bi\tau}{2\cdot\pi\cdot\tau\cdot\sqrt{l^2+m^2\cdot\rho^2}}\cdot\tanh\left(2\cdot\pi\cdot\sqrt{l^2+m^2\cdot\rho^2}\cdot\tau\right)}{2\cdot\pi\cdot\sqrt{l^2+m^2\cdot\rho^2}\cdot\left(\frac{Bi\tau}{2\cdot\pi\cdot\tau\cdot\sqrt{l^2+m^2\cdot\rho^2}}+\tanh\left(2\cdot\pi\cdot\sqrt{l^2+m^2\cdot\rho^2}\cdot\tau\right)\right)}\right]$$

$$Ltotal := \frac{\rho}{\pi^3\cdot\alpha}\cdot\sqrt{\frac{\beta}{\alpha}}\cdot\sum_{l=1}^{Lmax} Lterm_l \qquad Mtotal := \left(\frac{1}{\rho^2}\right)\cdot\left(\frac{1}{\pi^3}\right)\cdot\frac{1}{\beta}\cdot\sqrt{\frac{\alpha}{\beta}}\cdot\sum_{m=1}^{Mmax} Mterm_m \qquad LMtotal := \frac{4}{\pi^4\cdot\rho\cdot\alpha\cdot\beta}\cdot\left(\frac{1}{\sqrt{\alpha\cdot\beta}}\right)\cdot\sum_{l=1}^{Lmax}\sum_{m=1}^{Mmax} LMterm_{l,m}$$

$\psi_{AveSp} := Ltotal + Mtotal + LMtotal \quad \psi_{AveSp} = 0.471 \quad \psi_{UCond} := \rho\cdot\sqrt{\alpha\cdot\beta}\cdot\tau \quad \psi := \psi_{AveSp} + \psi_{UCond} \quad \psi = 0.4718727056 \quad \psi_{AveSp} = 0.471 \quad \psi_{UCond} = 1\times 10^{-}$

Calculation and Display of Convergence Plot of Thermal Spreading Resistance Ψ_{Sp}

$$LMsubtotal_1 := \frac{4}{\pi^4 \cdot \rho \cdot \alpha \cdot \beta} \cdot \left(\frac{1}{\sqrt{\alpha \cdot \beta}}\right) \cdot LMterm_{1,1} \qquad Lsubtotal_1 := \frac{\rho}{\pi^3 \cdot \alpha} \cdot \sqrt{\frac{\beta}{\alpha}} \cdot Lterm_1 \qquad Msubtotal_1 := \left(\frac{1}{\rho^2}\right) \cdot \left(\frac{1}{\pi^3}\right) \cdot \frac{1}{\beta} \cdot \left(\sqrt{\frac{\alpha}{\beta}}\right) \cdot Mterm_1$$

$j := 2 .. Lmax$

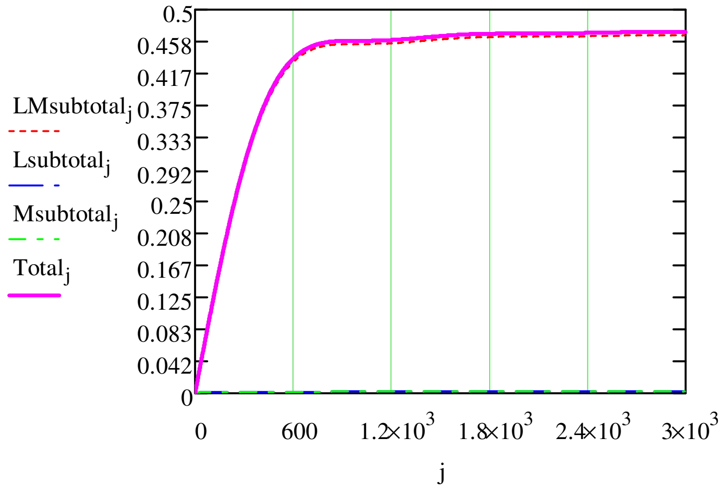
$$Total_1 := LMsubtotal_1 + Lsubtotal_1 + Msubtotal_1$$

$$LMsubtotal_j := LMsubtotal_{j-1} + \frac{4}{\pi^4 \cdot \rho \cdot \alpha \cdot \beta} \cdot \left(\frac{1}{\sqrt{\alpha \cdot \beta}}\right) \cdot \sum_{l=1}^{j} \sum_{m=j}^{j} LMterm_{l,m} + \frac{4}{\pi^4 \cdot \rho \cdot \alpha \cdot \beta} \cdot \left(\frac{1}{\sqrt{\alpha \cdot \beta}}\right) \cdot \sum_{l=j}^{j} \sum_{m=1}^{j-1} LMterm_{l,m}$$

$$Lsubtotal_j := \frac{\rho}{\pi^3 \cdot \alpha} \cdot \sqrt{\frac{\beta}{\alpha}} \cdot Lterm_j + Lsubtotal_{j-1} \qquad Msubtotal_j := \frac{1}{\pi^3 \cdot \rho^2 \cdot \beta} \cdot \sqrt{\frac{\alpha}{\beta}} \cdot Mterm_j + Msubtotal_{j-1}$$

$$Total_j := LMsubtotal_j + Lsubtotal_j + Msubtotal_j$$

$j := 1 .. Lmax$



Warning: Mathcad 8 gives the correct plot results that agree with ΨSp calculated at bottom of page 1. Mathcad 2000i does not seem to give the correct plot results. Mathcad 2000iC seems to give the correct results.

The following explains the unusual summation method used for the convergence plot by way of an example (LMAX=MMAX):

Sum[(l=1 to 3, m=1 to 3), xlm] = {x11 + x12 + x21 + x22} + {x13 + x23 +x33} + {x31 + x32}

= {Sum[l=1 to 2, m=1 to 2), xlm]} + {Sum[(l=1 to 3, m=3 to 3), xlm]} + {Sum[l=3 to 3, m=1 to 3-1), xlm]}

Thus to get a single plot point, the first {} is the previous plot point and the second {} plus third {} are the additions required to get this point.

Then in general

Sum[(l=1 to j, m=1 to j), xlm] = Sum[(l=1 to j-1, m=1 to j-1), xlm] + Sum[(l=1 to j, m=j to j), xlm] + Sum[(l=j to j, m=1 to j-1), xlm]

The Calculation of the Spreading Resistance from a Centered Circular Source on a Circular Disk

Source: Lee, S. et. al., MathCad Worksheet by Ellison.

Input, A Data Title Block:

Source radius/disk radius: $\varepsilon := 0.01$

Dimensionless thickness (t/b): $\tau := 0.01$

Biot * τ: $\text{Biotx}\tau := 1 \cdot 10^{10}$

Intermediate Variable Calculation(s):

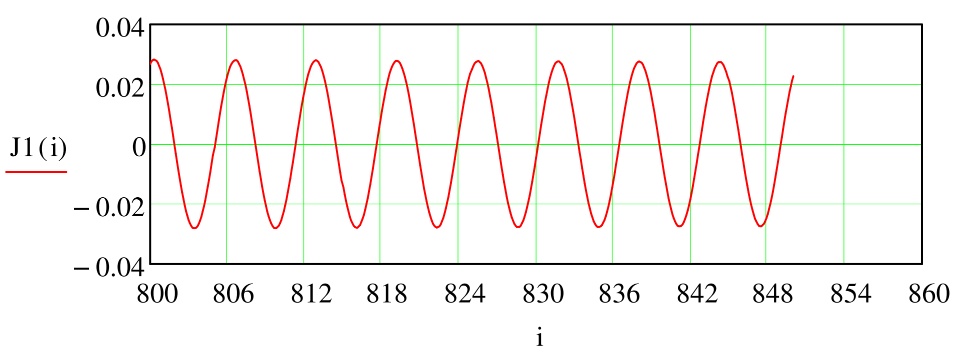
$$Bi := \frac{\text{Biotx}\tau}{\tau} \qquad x_1 := Bi \qquad x_2 := \tau$$

Calculate Eigenvalues λ_n That Satisfies nth Root of $J1(\lambda_n)=0$ and Manually Insert Into Column Vector:

$$ORIGIN := 1 \qquad f(x) := J1(x) \qquad x := 802$$

$$\text{root}(f(x), x) = 801.89106$$

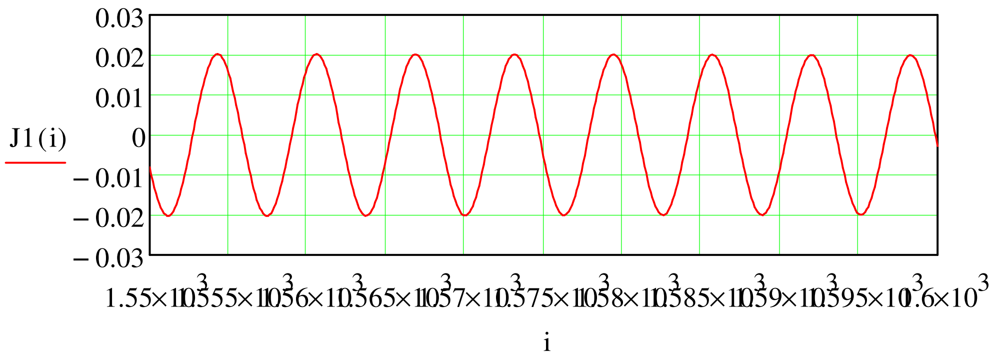
$$i := 800, 800.1 .. 850$$



$$i := 1550, 1550.1 .. 1600$$

$\lambda :=$

	1
1	3.83185
2	7.01558
3	10.17347
4	13.32369
5	16.47465
6	19.61672
7	22.76018
8	25.90301
9	29.04682
10	[illegible]



10	32.19096
11	35.33221
12	38.47401
13	41.61781
14	44.75231
15	47.90072
16	51.04354
17	54.18663
18	...

Calculate Maximum Thermal Spreading Resistance Ψ_{Max}:

Set Maximum Number of Series Terms:

$$Max := 500$$

Calculate Intermediate Φ Functions:

$$j := 1, 2 .. Max$$

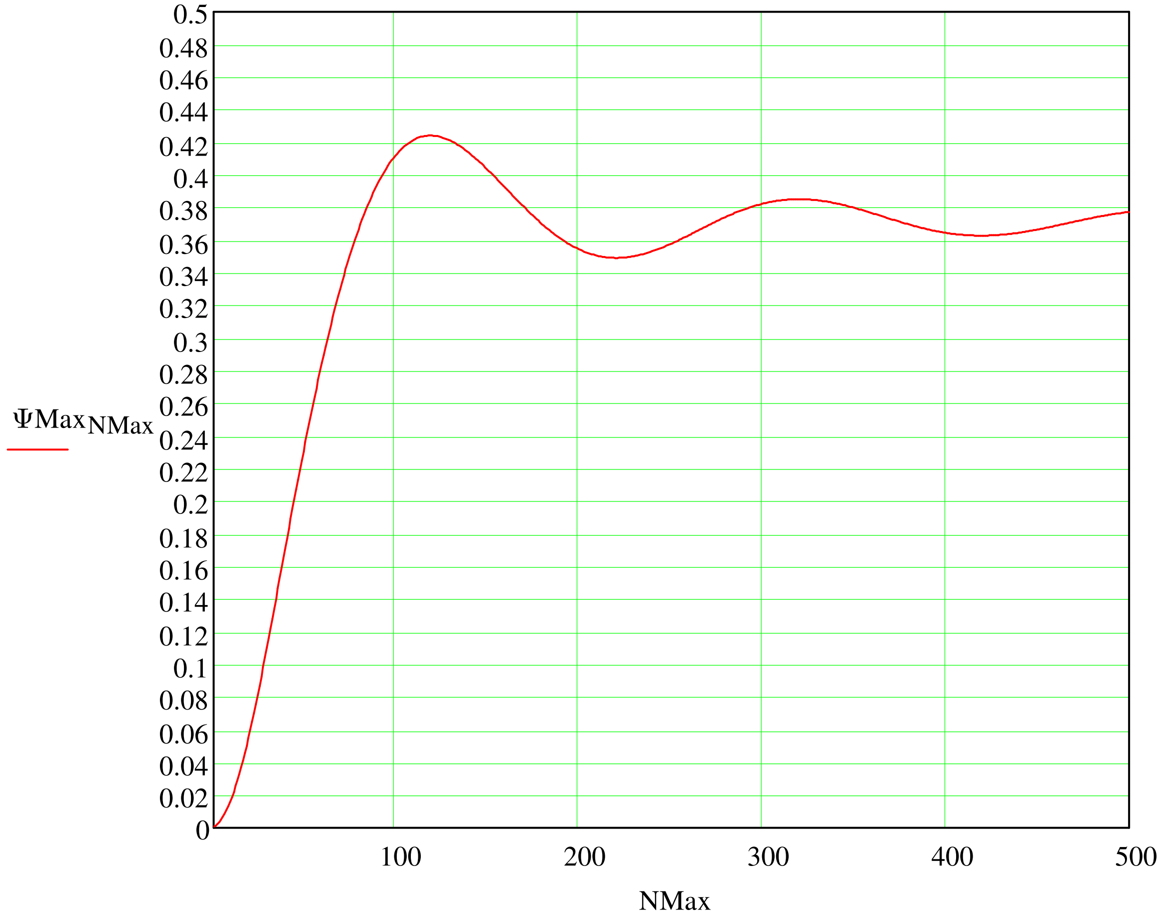
$$\Phi_j := \left(\frac{\tanh(\lambda_j \cdot \tau) + \frac{\lambda_j}{Bi}}{1 + \frac{\lambda_j}{Bi} \cdot \tanh(\lambda_j \cdot \tau)} \right)$$

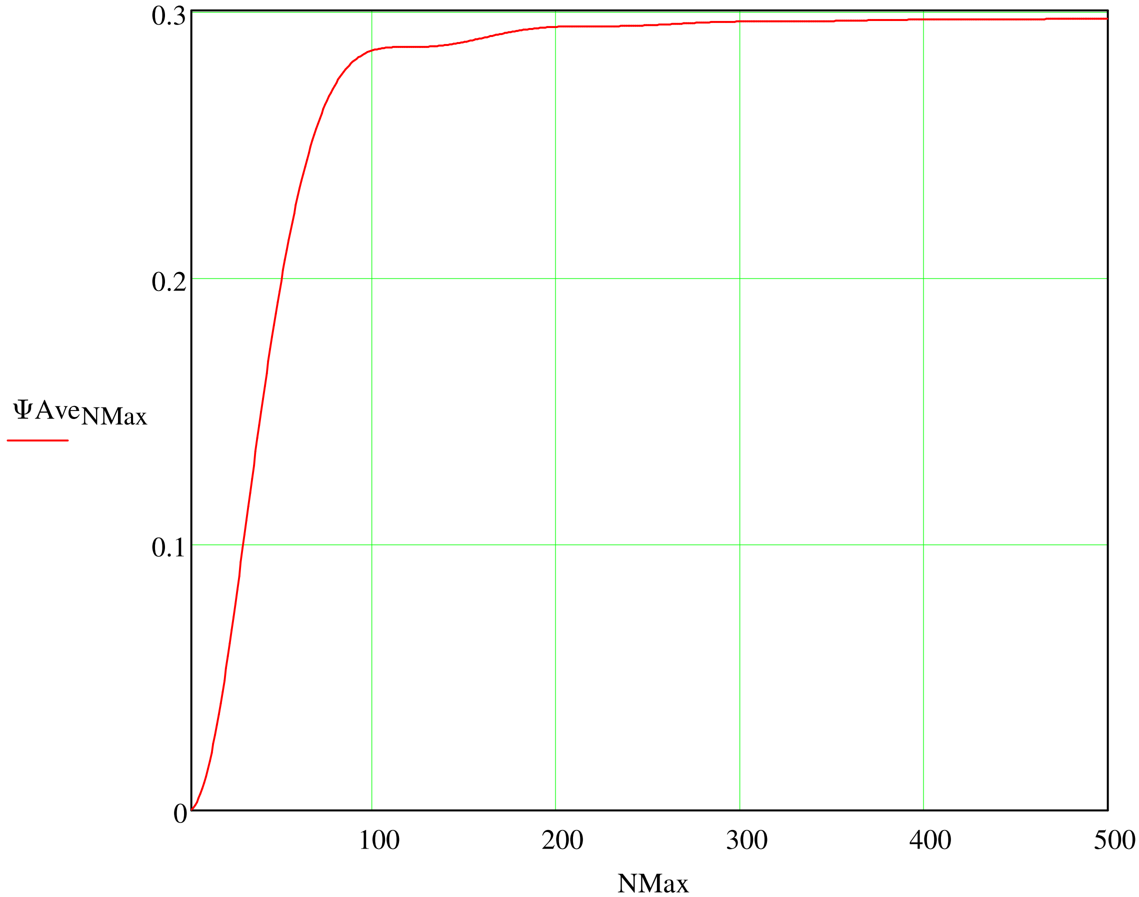
Calculate and Plot Each Series:

$$NMax := 1, 2 .. Max$$

$$\Psi Max_{NMax} := \left(\frac{2}{\sqrt{\pi}} \right) \cdot \sum_{n=1}^{NMax} \left[\Phi_n \cdot \frac{J1(\lambda_n \cdot \varepsilon)}{(\lambda_n)^2 \cdot J0(\lambda_n)^2} \right]$$

$$\Psi Ave_{NMax} := \left(\frac{4}{\varepsilon \sqrt{\pi}} \right) \cdot \sum_{n=1}^{NMax} \left[\Phi_n \cdot \frac{J1(\lambda_n \cdot \varepsilon)^2}{(\lambda_n)^3 \cdot J0(\lambda_n)^2} \right]$$


0.5
0.48
0.46
0.44
0.42
0.4
0.38
0.36
0.34
0.32
0.3
0.28
0.26
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
ΨMax_NMax
100
200
300
400
500
NMax


0.3
0.2
0.1
0
100
200
300
400
500
ΨAve_NMax
NMax

Gordon Ellison's Calculation of a Single Value of Thermal Spreading Resistance ψ_{Sp}

$\alpha := 0.25 \quad \beta := 0.25 \quad \rho := 1.0 \quad Bi\tau := 2\cdot 10^{-5} \quad \tau := 2.5\cdot 10^{-3}$

$Lmax := 25 \qquad Mmax := Lmax$

$l := 1 .. Lmax$

$$Lterm_l := \left(\frac{1}{l^2}\right)\cdot\left[\sin(l\cdot\pi\cdot\alpha)\cdot\left(\frac{1 + \frac{Bi\tau}{2\cdot l\cdot\pi\cdot\tau}\cdot\tanh(2\cdot l\cdot\pi\cdot\tau)}{\frac{Bi\tau}{2\cdot l\cdot\pi\cdot\tau} + \tanh(2\cdot l\cdot\pi\cdot\tau)}\right)\right]$$

$m := 1 .. Mmax$

$$Mterm_m := \frac{1}{m^2}\cdot\sin(m\cdot\pi\cdot\beta\cdot\rho)\cdot\left(\frac{1 + \frac{Bi\tau}{2\cdot m\cdot\pi\cdot\tau\cdot\rho}\cdot\tanh(2\cdot m\cdot\pi\cdot\tau\cdot\rho)}{\frac{Bi\tau}{2\cdot m\cdot\pi\cdot\tau\cdot\rho} + \tanh(2\cdot m\cdot\pi\cdot\tau\cdot\rho)}\right)$$

Recommended

$\alpha = \beta$	Lmax=Mmax
0.5	50
0.25	75
0.1	100
0.05	150
0.025	175
0.01	300
0.005	700
0.0025	1500
0.001	3000"

Warning: This version of Mathcad (2001i) reports an internal error for Lmax=3000. The workable lower limit is not exactly known. This same error does not occur with Mathcad 8. It has been reported to Mathsoft. It is possible that the error is also due to memory limitations. It is possible that the difficulty is that of a memory limitation.

$l := 1 .. Lmax$

$m := 1 .. Mmax$

$$LMterm_{l,m} := \frac{1}{l\cdot m}\cdot\sin(l\cdot\pi\cdot\alpha)\cdot\sin(m\cdot\pi\cdot\beta\cdot\rho)\cdot\left[\frac{1 + \frac{Bi\tau}{2\cdot\pi\cdot\tau\cdot\sqrt{l^2 + m^2\cdot\rho^2}}\cdot\tanh\left(2\cdot\pi\cdot\sqrt{l^2 + m^2\cdot\rho^2}\cdot\tau\right)}{2\cdot\pi\cdot\sqrt{l^2 + m^2\cdot\rho^2}\cdot\left(\frac{Bi\tau}{2\cdot\pi\cdot\tau\cdot\sqrt{l^2 + m^2\cdot\rho^2}} + \tanh\left(2\cdot\pi\cdot\sqrt{l^2 + m^2\cdot\rho^2}\cdot\tau\right)\right)}\right]$$

$$Ltotal := \frac{\rho}{\pi^2}\cdot\sqrt{\frac{\beta}{\alpha}}\cdot\sum_{l=1}^{Lmax} Lterm_l \qquad Mtotal := \left(\frac{1}{\rho}\right)\cdot\left(\frac{1}{\pi^2}\right)\cdot\sqrt{\frac{\alpha}{\beta}}\cdot\sum_{m=1}^{Mmax} Mterm_m \qquad LMtotal := \frac{4}{\pi^2}\cdot\left(\frac{1}{\sqrt{\alpha\cdot\beta}}\right)\cdot\sum_{l=1}^{Lmax}\sum_{m=1}^{Mmax} LMterm_{l,m}$$

$\psi_{Sp} := Ltotal + Mtotal + LMtotal \qquad \psi_{Sp} = 17.42954$

Calculation and Display of Convergence Plot of Thermal Spreading Resistance Ψ_{Sp}

$$LMsubtotal_1 := \frac{4}{\pi^2}\cdot\left(\frac{1}{\sqrt{\alpha\cdot\beta}}\right)\cdot LMterm_{1,1} \quad Lsubtotal_1 := \frac{\rho}{\pi^2}\cdot\sqrt{\frac{\beta}{\alpha}}\cdot Lterm_1 \quad Msubtotal_1 := \left(\frac{1}{\rho}\right)\cdot\left(\frac{1}{\pi^2}\right)\cdot\left(\sqrt{\frac{\alpha}{\beta}}\right)\cdot Mterm_1$$

$j := 2..Lmax$

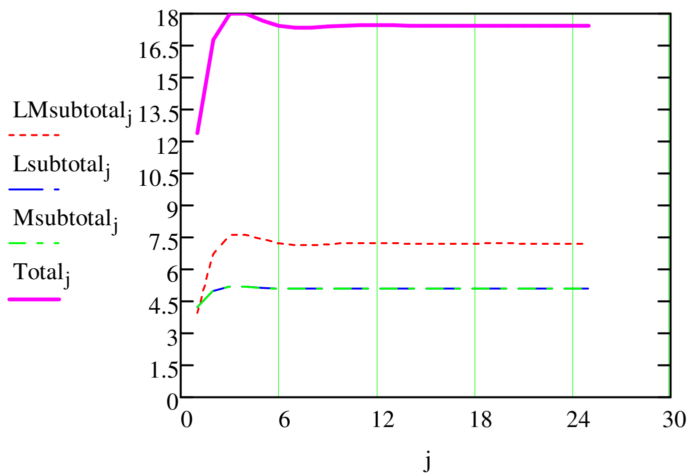
$$Total_1 := LMsubtotal_1 + Lsubtotal_1 + Msubtotal_1$$

$$LMsubtotal_j := LMsubtotal_{j-1} + \frac{4}{\pi^2}\cdot\left(\frac{1}{\sqrt{\alpha\cdot\beta}}\right)\cdot\sum_{l=1}^{j}\sum_{m=j}^{j} LMterm_{l,m} + \frac{4}{\pi^2}\cdot\left(\frac{1}{\sqrt{\alpha\cdot\beta}}\right)\cdot\sum_{l=j}^{j}\sum_{m=1}^{j-1} LMterm_{l,m}$$

$$Lsubtotal_j := \frac{\rho}{\pi^2}\cdot\sqrt{\frac{\beta}{\alpha}}\cdot Lterm_j + Lsubtotal_{j-1} \qquad Msubtotal_j := \frac{1}{\pi^2}\cdot\sqrt{\frac{\alpha}{\beta}}\cdot\frac{1}{\rho}\cdot Mterm_j + Msubtotal_{j-1}$$

$$Total_j := LMsubtotal_j + Lsubtotal_j + Msubtotal_j$$

$j := 1..Lmax$



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The following explains the unusual summation method used for the convergence plot by way of an example (LMAX=MMAX):

Sum[(l=1 to 3, m=1 to 3), xlm] = {x11 + x12 + x21 + x22} + {x13 + x23 +x33} + {x31 + x32}

= {Sum[l=1 to 2, m=1 to 2), xlm]} + {Sum[(l=1 to 3, m=3 to 3), xlm]} + {Sum[l=3 to 3, m=1 to 3-1), xlm]}

Thus to get a single plot point, the first {} is the previous plot point and the second {} plus third {} are the additions required to get this point.

Then in general

Sum[(l=1 to j, m=1 to j), xlm] = Sum[(l=1 to j-1, m=1 to j-1), xlm] + Sum[(l=1 to j, m=j to j), xlm] +
Sum[(l=j to j, m=1 to j-1), xlm]

Some Results for h=2.5x10^11:

α	β	ψSp	ψTotal	RTotal	TAMS(k=1)
0.05	0.05	0.427	0.429	8.58	8.6
0.05	0.10	0.361	0.364	5.15	5.2
0.05	1.00	0.115	0.1237	0.553	0.55

Gordon Ellison's Calculation of a Single Value of Thermal Spreading Resistance ψ_{Sp} for Two Sided Cooling

$\alpha := 0.1$ $\quad\beta := 0.1$ $\quad\rho := 1.0$ $\quad Bi2\tau := 0.0017$ $\quad Bi1\tau := 4\cdot 10^{-4}$ $\quad\tau := 0.05$

$Lmax := 300$ $\qquad Mmax := Lmax$

$l := 1..Lmax$

$$Lterm_l := \left(\frac{1}{l^2}\right)\cdot\left[\sin(l\cdot\pi\cdot\alpha)\cdot\left[\frac{\frac{2\cdot l\cdot\pi\cdot\tau}{Bi2\tau}+\tanh(2\cdot l\cdot\pi\cdot\tau)}{\left(1+\frac{Bi1\tau}{Bi2\tau}\right)+\left(\frac{Bi1\tau}{2\cdot l\cdot\pi\cdot\tau}+\frac{2\cdot l\cdot\pi\cdot\tau}{Bi2\tau}\right)\cdot\tanh(2\cdot l\cdot\pi\cdot\tau)}\right]\right]$$

$m := 1..Mmax$

$$Mterm_m := \frac{1}{m^2}\cdot\sin(m\cdot\pi\cdot\beta\cdot\rho)\cdot\left[\frac{\frac{2\cdot m\cdot\pi\cdot\tau\cdot\rho}{Bi2\tau}+\tanh(2\cdot m\cdot\pi\cdot\tau\cdot\rho)}{\left(1+\frac{Bi1\tau}{Bi2\tau}\right)+\left(\frac{2\cdot m\cdot\pi\cdot\tau\cdot\rho}{Bi2\tau}+\frac{Bi1\tau}{2\cdot m\cdot\pi\cdot\tau\cdot\rho}\right)\cdot\tanh(2\cdot m\cdot\pi\cdot\tau\cdot\rho)}\right]$$

<u>Recommended</u>

$\alpha = \beta$	Lmax=Mmax
0.5	50
0.25	75
0.1	100
0.05	150
0.025	175
0.01	300
0.005	700
0.0025	1500
0.001	3000"

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$l := 1..Lmax$

$m := 1..Mmax$

$$LMterm_{l,m} := \frac{1}{l\cdot m}\cdot\sin(l\cdot\pi\cdot\alpha)\cdot\sin(m\cdot\pi\cdot\beta\cdot\rho)\cdot\left[\frac{\frac{2\cdot\pi\cdot\sqrt{l^2+m^2\cdot\rho^2}\cdot\tau}{Bi2\tau}+\tanh\left(2\cdot\pi\cdot\sqrt{l^2+m^2\cdot\rho^2}\cdot\tau\right)}{2\cdot\pi\cdot\sqrt{l^2+m^2\cdot\rho^2}\cdot\left[\left(1+\frac{Bi1\tau}{Bi2\tau}\right)+\left[\left(\frac{Bi1\tau}{2\cdot\pi\cdot\tau\cdot\sqrt{l^2+m^2\cdot\rho^2}}+\frac{2\cdot\pi\cdot\sqrt{l^2+m^2\cdot\rho^2}\cdot\tau}{Bi2\tau}\right)\cdot\tanh\left(2\cdot\pi\cdot\sqrt{l^2+m^2\cdot\rho^2}\cdot\tau\right)\right]\right]}\right]$$

$$Ltotal := \frac{\rho}{\pi^2}\cdot\sqrt{\frac{\beta}{\alpha}}\cdot\sum_{l=1}^{Lmax} Lterm_l \qquad Mtotal := \left(\frac{1}{\rho}\right)\cdot\left(\frac{1}{\pi^2}\right)\cdot\sqrt{\frac{\alpha}{\beta}}\cdot\sum_{m=1}^{Mmax} Mterm_m \qquad LMtotal := \frac{4}{\pi^2}\cdot\left(\frac{1}{\sqrt{\alpha\cdot\beta}}\right)\cdot\sum_{l=1}^{Lmax}\sum_{m=1}^{Mmax} LMterm_{l,m}$$

$\psi_{Sp} := Ltotal + Mtotal + LMtotal$ $\qquad\qquad \psi_{Sp} = 0.8052$

Calculation and Display of Convergence Plot of Thermal Spreading Resistance Ψ_{Sp}

$$\mathrm{LMsubtotal}_1 := \frac{4}{\pi^2}\cdot\left(\frac{1}{\sqrt{\alpha\cdot\beta}}\right)\cdot \mathrm{LMterm}_{1,1} \quad \mathrm{Lsubtotal}_1 := \frac{\rho}{\pi^2}\cdot\sqrt{\frac{\beta}{\alpha}}\cdot \mathrm{Lterm}_1 \quad \mathrm{Msubtotal}_1 := \left(\frac{1}{\rho}\right)\cdot\left(\frac{1}{\pi^2}\right)\cdot\left(\sqrt{\frac{\alpha}{\beta}}\right)\cdot \mathrm{Mterm}_1$$

$j := 2 .. \mathrm{Lmax}$

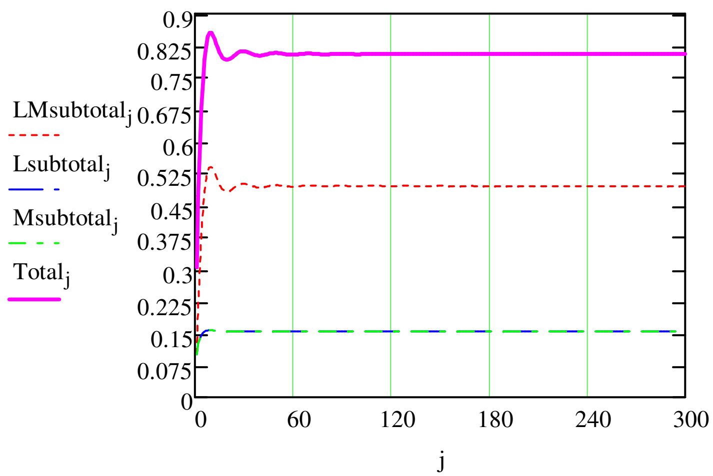
$$\mathrm{Total}_1 := \mathrm{LMsubtotal}_1 + \mathrm{Lsubtotal}_1 + \mathrm{Msubtotal}_1$$

$$\mathrm{LMsubtotal}_j := \mathrm{LMsubtotal}_{j-1} + \frac{4}{\pi^2}\cdot\left(\frac{1}{\sqrt{\alpha\cdot\beta}}\right)\cdot\sum_{l=1}^{j}\sum_{m=j}^{j}\mathrm{LMterm}_{l,m} + \frac{4}{\pi^2}\cdot\left(\frac{1}{\sqrt{\alpha\cdot\beta}}\right)\cdot\sum_{l=j}^{j}\sum_{m=1}^{j-1}\mathrm{LMterm}_{l,m}$$

$$\mathrm{Lsubtotal}_j := \frac{\rho}{\pi^2}\cdot\sqrt{\frac{\beta}{\alpha}}\cdot \mathrm{Lterm}_j + \mathrm{Lsubtotal}_{j-1} \qquad \mathrm{Msubtotal}_j := \frac{1}{\pi^2}\cdot\sqrt{\frac{\alpha}{\beta}}\cdot\frac{1}{\rho}\cdot \mathrm{Mterm}_j + \mathrm{Msubtotal}_{j-1}$$

$$\mathrm{Total}_j := \mathrm{LMsubtotal}_j + \mathrm{Lsubtotal}_j + \mathrm{Msubtotal}_j$$

$j := 1 .. \mathrm{Lmax}$



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Sum[(l=1 to 3, m=1 to 3), xlm] = {x11 + x12 + x21 + x22} + {x13 + x23 +x33} + {x31 + x32}

= {Sum[l=1 to 2, m=1 to 2), xlm]} + {Sum[(l=1 to 3, m=3 to 3), xlm]} + {Sum[l=3 to 3, m=1 to 3-1), xlm]}

Thus to get a single plot point, the first {} is the previous plot point and the second {} plus third {} are the additions required to get this point.

Then in general

Sum[(l=1 to j, m=1 to j), xlm] = Sum[(l=1 to j-1, m=1 to j-1), xlm] + Sum[(l=1 to j, m=j to j), xlm] + Sum[(l=j to j, m=1 to j-1), xlm]

Some Results for h=2.5x10^11:

α	β	ψSp	ψTotal	RTotal	TAMS(k=1)
0.05	0.05	0.427	0.429	8.58	8.6
0.05	0.10	0.361	0.364	5.15	5.2
0.05	1.00	0.115	0.1237	0.553	0.55

$a := 4$ $\quad b := 4$ $\quad T1 := 20$ $\quad h1 := 0.01$ $\quad Q := 10$

$k := 5$ $\quad \Delta x := 0.4$ $\quad \Delta y := 0.4$

$$\psi_U := \sqrt{\alpha \cdot \beta} \cdot \rho \cdot \left[\frac{\tau + \frac{\tau}{Bi2\tau}}{1 + Bi2\tau + \frac{Bi1\tau}{Bi2\tau}} + \left(1 + \frac{h1 \cdot a \cdot b \cdot T1}{Q}\right) \right] \qquad \psi_U = 2.514$$

$$R_U := \frac{\psi_U}{k \cdot \sqrt{\Delta x \cdot \Delta y}} \qquad R_U = 1.257 \qquad R_{Sp} := \frac{\psi_{Sp}}{k \cdot \sqrt{\Delta x \cdot \Delta y}} \qquad R_{Sp} = 0.403 \qquad R := R_U + R_{Sp} \qquad R = 1.659$$

Radiation Shape Factors for Parallel Plates

Graphical Results:

$x_0 := 0.09$ **x=L/S, y=W/S:**

$$y := 1000 \quad FParInf_0 := \left(\frac{2}{\pi \cdot x_0 \cdot y}\right) \cdot \left[\ln\left[\sqrt{\frac{\left[1+(x_0)^2\right]\cdot\left(1+y^2\right)}{1+(x_0)^2+y^2}}\right] + \left[y \cdot \sqrt{1+(x_0)^2} \cdot \mathrm{atan}\left[\frac{y}{\sqrt{1+(x_0)^2}}\right]\right] \dots \\ + \left(x_0 \cdot \sqrt{1+y^2} \cdot \mathrm{atan}\left(\frac{x_0}{\sqrt{1+y^2}}\right)\right) - y \cdot \mathrm{atan}(y) - x_0 \cdot \mathrm{atan}(x_0) \right]$$

$$y := 10 \quad FPar10_0 := \left(\frac{2}{\pi \cdot x_0 \cdot y}\right) \cdot \left[\ln\left[\sqrt{\frac{\left[1+(x_0)^2\right]\cdot\left(1+y^2\right)}{1+(x_0)^2+y^2}}\right] + \left[y \cdot \sqrt{1+(x_0)^2} \cdot \mathrm{atan}\left[\frac{y}{\sqrt{1+(x_0)^2}}\right]\right] \dots \\ + \left(x_0 \cdot \sqrt{1+y^2} \cdot \mathrm{atan}\left(\frac{x_0}{\sqrt{1+y^2}}\right)\right) - y \cdot \mathrm{atan}(y) - x_0 \cdot \mathrm{atan}(x_0) \right]$$

$$y := 4 \quad FPar4_0 := \left(\frac{2}{\pi \cdot x_0 \cdot y}\right) \cdot \left[\ln\left[\sqrt{\frac{\left[1+(x_0)^2\right]\cdot\left(1+y^2\right)}{1+(x_0)^2+y^2}}\right] + \left[y \cdot \sqrt{1+(x_0)^2} \cdot \mathrm{atan}\left[\frac{y}{\sqrt{1+(x_0)^2}}\right]\right] \dots \\ + \left(x_0 \cdot \sqrt{1+y^2} \cdot \mathrm{atan}\left(\frac{x_0}{\sqrt{1+y^2}}\right)\right) - y \cdot \mathrm{atan}(y) - x_0 \cdot \mathrm{atan}(x_0) \right]$$

$$y := 2 \quad FPar2_0 := \left(\frac{2}{\pi \cdot x_0 \cdot y}\right) \cdot \left[\ln\left[\sqrt{\frac{\left[1+(x_0)^2\right]\cdot\left(1+y^2\right)}{1+(x_0)^2+y^2}}\right] + \left[y \cdot \sqrt{1+(x_0)^2} \cdot \mathrm{atan}\left[\frac{y}{\sqrt{1+(x_0)^2}}\right]\right] \dots \\ + \left(x_0 \cdot \sqrt{1+y^2} \cdot \mathrm{atan}\left(\frac{x_0}{\sqrt{1+y^2}}\right)\right) - y \cdot \mathrm{atan}(y) - x_0 \cdot \mathrm{atan}(x_0) \right]$$

$$y := 1 \quad FPar1_0 := \left(\frac{2}{\pi \cdot x_0 \cdot y}\right) \cdot \left[\ln\left[\sqrt{\frac{\left[1+(x_0)^2\right]\cdot\left(1+y^2\right)}{1+(x_0)^2+y^2}}\right] + \left[y \cdot \sqrt{1+(x_0)^2} \cdot \mathrm{atan}\left[\frac{y}{\sqrt{1+(x_0)^2}}\right]\right] \dots \\ + \left(x_0 \cdot \sqrt{1+y^2} \cdot \mathrm{atan}\left(\frac{x_0}{\sqrt{1+y^2}}\right)\right) - y \cdot \mathrm{atan}(y) - x_0 \cdot \mathrm{atan}(x_0) \right]$$

$y := 0.5$ $\quad FParPt5_0 := \left(\frac{2}{\pi \cdot x_0 \cdot y}\right) \cdot \left[\ln\left[\sqrt{\frac{\left[1+(x_0)^2\right]\cdot\left(1+y^2\right)}{1+(x_0)^2+y^2}}\right] + \left[y \cdot \sqrt{1+(x_0)^2} \cdot \mathrm{atan}\left[\frac{y}{\sqrt{1+(x_0)^2}}\right]\right] \ldots \atop + \left(x_0 \cdot \sqrt{1+y^2} \cdot \mathrm{atan}\left(\frac{x_0}{\sqrt{1+y^2}}\right)\right) - y \cdot \mathrm{atan}(y) - x_0 \cdot \mathrm{atan}(x_0) \right]$

$y := 0.25$ $\quad FParPt25_0 := \left(\frac{2}{\pi \cdot x_0 \cdot y}\right) \cdot \left[\ln\left[\sqrt{\frac{\left[1+(x_0)^2\right]\cdot\left(1+y^2\right)}{1+(x_0)^2+y^2}}\right] + \left[y \cdot \sqrt{1+(x_0)^2} \cdot \mathrm{atan}\left[\frac{y}{\sqrt{1+(x_0)^2}}\right]\right] \ldots \atop + \left(x_0 \cdot \sqrt{1+y^2} \cdot \mathrm{atan}\left(\frac{x_0}{\sqrt{1+y^2}}\right)\right) - y \cdot \mathrm{atan}(y) - x_0 \cdot \mathrm{atan}(x_0) \right]$

$y := 0.1$ $\quad FParPt1_0 := \left(\frac{2}{\pi \cdot x_0 \cdot y}\right) \cdot \left[\ln\left[\sqrt{\frac{\left[1+(x_0)^2\right]\cdot\left(1+y^2\right)}{1+(x_0)^2+y^2}}\right] + \left[y \cdot \sqrt{1+(x_0)^2} \cdot \mathrm{atan}\left[\frac{y}{\sqrt{1+(x_0)^2}}\right]\right] \ldots \atop + \left(x_0 \cdot \sqrt{1+y^2} \cdot \mathrm{atan}\left(\frac{x_0}{\sqrt{1+y^2}}\right)\right) - y \cdot \mathrm{atan}(y) - x_0 \cdot \mathrm{atan}(x_0) \right]$

$\boxed{MAXxDIVS := 100}$ $\quad \boxed{STEP := 0.01}$ $\quad STEPS := \frac{MAXxDIVS}{STEP} \quad STEPS = 1 \times 10^4$

$i := 1, 2 .. STEPS$

$x_i := x_{i-1} + STEP$

$y := 10000$ $\quad FParInf_i := \left(\frac{2}{\pi \cdot x_i \cdot y}\right) \cdot \left[\ln\left[\sqrt{\frac{\left[1+(x_i)^2\right]\cdot\left(1+y^2\right)}{1+(x_i)^2+y^2}}\right] + \left[y \cdot \sqrt{1+(x_i)^2} \cdot \mathrm{atan}\left[\frac{y}{\sqrt{1+(x_i)^2}}\right]\right] \ldots \atop + \left(x_i \cdot \sqrt{1+y^2} \cdot \mathrm{atan}\left(\frac{x_i}{\sqrt{1+y^2}}\right)\right) - y \cdot \mathrm{atan}(y) - x_i \cdot \mathrm{atan}(x_i) \right]$

$y := 10$ $\quad FPar10_i := \left(\frac{2}{\pi \cdot x_i \cdot y}\right) \cdot \left[\ln\left[\sqrt{\frac{\left[1+(x_i)^2\right]\cdot\left(1+y^2\right)}{1+(x_i)^2+y^2}}\right] + \left[y \cdot \sqrt{1+(x_i)^2} \cdot \mathrm{atan}\left[\frac{y}{\sqrt{1+(x_i)^2}}\right]\right] \ldots \atop + \left(x_i \cdot \sqrt{1+y^2} \cdot \mathrm{atan}\left(\frac{x_i}{\sqrt{1+y^2}}\right)\right) - y \cdot \mathrm{atan}(y) - x_i \cdot \mathrm{atan}(x_i) \right]$

$y := 4$ $\qquad FPar4_i := \left(\frac{2}{\pi \cdot x_i \cdot y}\right) \cdot \left[\ln\left[\sqrt{\frac{\left[1+(x_i)^2\right]\cdot\left(1+y^2\right)}{1+(x_i)^2+y^2}}\right] + \left[y\cdot\sqrt{1+(x_i)^2}\cdot \mathrm{atan}\left[\frac{y}{\sqrt{1+(x_i)^2}}\right]\right] \dots \right.$
$\left. + \left(x_i\cdot\sqrt{1+y^2}\cdot \mathrm{atan}\left(\frac{x_i}{\sqrt{1+y^2}}\right)\right) - y\cdot \mathrm{atan}(y) - x_i\cdot \mathrm{atan}(x_i) \right]$

$y := 2$ $\qquad FPar2_i := \left(\frac{2}{\pi \cdot x_i \cdot y}\right) \cdot \left[\ln\left[\sqrt{\frac{\left[1+(x_i)^2\right]\cdot\left(1+y^2\right)}{1+(x_i)^2+y^2}}\right] + \left[y\cdot\sqrt{1+(x_i)^2}\cdot \mathrm{atan}\left[\frac{y}{\sqrt{1+(x_i)^2}}\right]\right] \dots \right.$
$\left. + \left(x_i\cdot\sqrt{1+y^2}\cdot \mathrm{atan}\left(\frac{x_i}{\sqrt{1+y^2}}\right)\right) - y\cdot \mathrm{atan}(y) - x_i\cdot \mathrm{atan}(x_i) \right]$

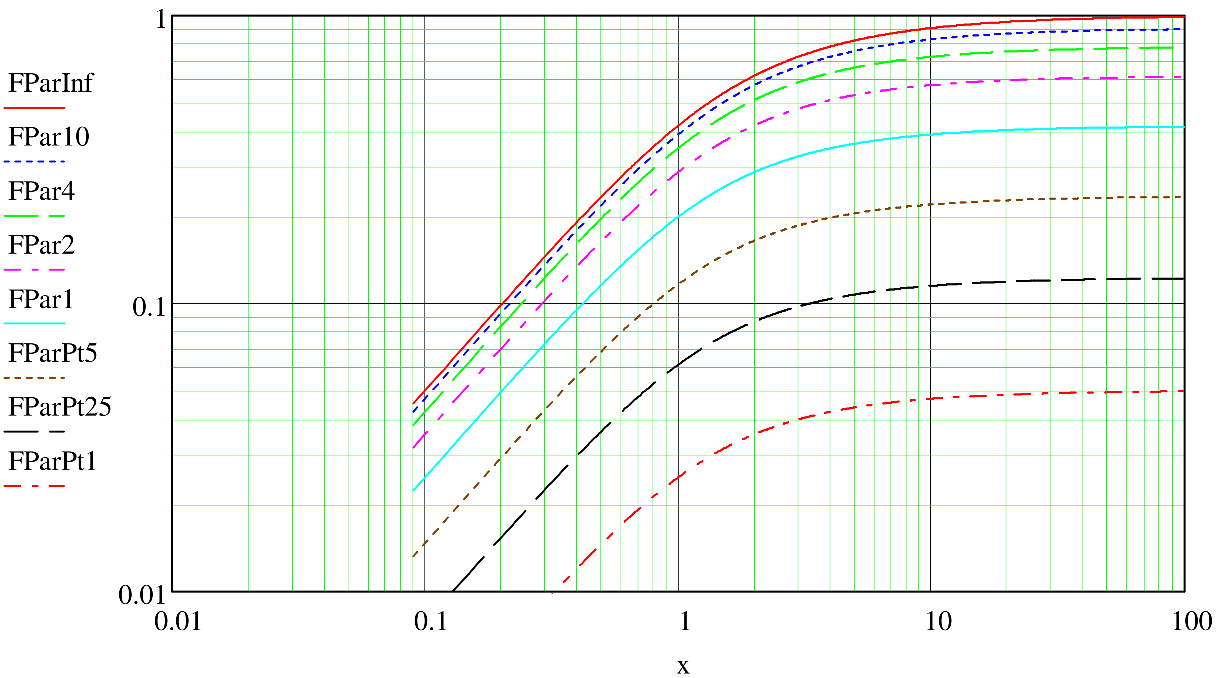
$y := 2$ $\qquad FPar2_i := \left(\frac{2}{\pi \cdot x_i \cdot y}\right) \cdot \left[\ln\left[\sqrt{\frac{\left[1+(x_i)^2\right]\cdot\left(1+y^2\right)}{1+(x_i)^2+y^2}}\right] + \left[y\cdot\sqrt{1+(x_i)^2}\cdot \mathrm{atan}\left[\frac{y}{\sqrt{1+(x_i)^2}}\right]\right] \dots \right.$
$\left. + \left(x_i\cdot\sqrt{1+y^2}\cdot \mathrm{atan}\left(\frac{x_i}{\sqrt{1+y^2}}\right)\right) - y\cdot \mathrm{atan}(y) - x_i\cdot \mathrm{atan}(x_i) \right]$

$y := 1$ $\qquad FPar1_i := \left(\frac{2}{\pi \cdot x_i \cdot y}\right) \cdot \left[\ln\left[\sqrt{\frac{\left[1+(x_i)^2\right]\cdot\left(1+y^2\right)}{1+(x_i)^2+y^2}}\right] + \left[y\cdot\sqrt{1+(x_i)^2}\cdot \mathrm{atan}\left[\frac{y}{\sqrt{1+(x_i)^2}}\right]\right] \dots \right.$
$\left. + \left(x_i\cdot\sqrt{1+y^2}\cdot \mathrm{atan}\left(\frac{x_i}{\sqrt{1+y^2}}\right)\right) - y\cdot \mathrm{atan}(y) - x_i\cdot \mathrm{atan}(x_i) \right]$

$y := 0.5$ $\qquad FParPt5_i := \left(\frac{2}{\pi \cdot x_i \cdot y}\right) \cdot \left[\ln\left[\sqrt{\frac{\left[1+(x_i)^2\right]\cdot\left(1+y^2\right)}{1+(x_i)^2+y^2}}\right] + \left[y\cdot\sqrt{1+(x_i)^2}\cdot \mathrm{atan}\left[\frac{y}{\sqrt{1+(x_i)^2}}\right]\right] \dots \right.$
$\left. + \left(x_i\cdot\sqrt{1+y^2}\cdot \mathrm{atan}\left(\frac{x_i}{\sqrt{1+y^2}}\right)\right) - y\cdot \mathrm{atan}(y) - x_i\cdot \mathrm{atan}(x_i) \right]$

$y := 0.25$ $\qquad FParPt25_i := \left(\frac{2}{\pi \cdot x_i \cdot y}\right) \cdot \left[\ln\left[\sqrt{\frac{\left[1+(x_i)^2\right]\cdot\left(1+y^2\right)}{1+(x_i)^2+y^2}}\right] + \left[y\cdot\sqrt{1+(x_i)^2}\cdot \mathrm{atan}\left[\frac{y}{\sqrt{1+(x_i)^2}}\right]\right] \dots \right.$
$\left. + \left(x_i\cdot\sqrt{1+y^2}\cdot \mathrm{atan}\left(\frac{x_i}{\sqrt{1+y^2}}\right)\right) - y\cdot \mathrm{atan}(y) - x_i\cdot \mathrm{atan}(x_i) \right]$

$y := 0.1$ $\qquad$ $$FParPt1_i := \left(\frac{2}{\pi \cdot x_i \cdot y}\right) \cdot \left[\ln\left[\sqrt{\frac{\left[1+(x_i)^2\right]\cdot\left(1+y^2\right)}{1+(x_i)^2+y^2}}\right] + \left[y \cdot \sqrt{1+(x_i)^2} \cdot \text{atan}\left[\frac{y}{\sqrt{1+(x_i)^2}}\right]\right] \ldots \right.$$
$$\left. + \left(x_i \cdot \sqrt{1+y^2} \cdot \text{atan}\left(\frac{x_i}{\sqrt{1+y^2}}\right)\right) - y \cdot \text{atan}(y) - x_i \cdot \text{atan}(x_i) \right]$$



Write to Files: ...\Par_Plate_X.dat $\qquad$ ...\Par_Plate_Inf.dat

x $\qquad$ FParInf

...\Par_Plate_10.dat $\qquad$ ...\Par_Plate_4.dat $\qquad$ ...\Par_Plate_2.dat

FPar10 $\qquad$ FPar4 $\qquad$ FPar2

...\Par_Plate_1.dat $\qquad$ ...\Par_Plate_Pt5.dat $\qquad$ C...\Par_Plate_Pt25.dat

FPar1 $\qquad$ FParPt5 $\qquad$ FParPt25

...\Par_Plate_Pt1.dat

FParPt1

Radiation Shape Factors for Perpendicular Plates

Graphical Results:

$x_0 := 0.09$ **$x=L_2/W$, $y=L_1/W$:**

$y := 0.1$ $z_0 := (x_0)^2 + y^2$

$$FPerPt1_0 := \left(\frac{1}{4\cdot\pi\cdot y}\right)\cdot\left[\ln\left[\left[\frac{\left[1+(x_0)^2\right]\cdot\left(1+y^2\right)}{1+z_0}\right]\cdot\left[\frac{y^2\cdot(1+z_0)}{\left(1+y^2\right)\cdot z_0}\right]^{y^2}\cdot\left[\frac{(x_0)^2\cdot(1+z_0)}{\left[1+(x_0)^2\right]\cdot z_0}\right]^{(x_0)^2}\right]\right]\ldots$$
$$+\left(\frac{1}{\pi\cdot y}\right)\cdot\left(y\cdot\operatorname{atan}\left(\frac{1}{y}\right)+x_0\cdot\operatorname{atan}\left(\frac{1}{x_0}\right)-\sqrt{z_0}\cdot\operatorname{atan}\left(\frac{1}{\sqrt{z_0}}\right)\right)$$

$y := 0.2$ $z_0 := (x_0)^2 + y^2$

$$FPerPt2_0 := \left(\frac{1}{4\cdot\pi\cdot y}\right)\cdot\left[\ln\left[\left[\frac{\left[1+(x_0)^2\right]\cdot\left(1+y^2\right)}{1+z_0}\right]\cdot\left[\frac{y^2\cdot(1+z_0)}{\left(1+y^2\right)\cdot z_0}\right]^{y^2}\cdot\left[\frac{(x_0)^2\cdot(1+z_0)}{\left[1+(x_0)^2\right]\cdot z_0}\right]^{(x_0)^2}\right]\right]\ldots$$
$$+\left(\frac{1}{\pi\cdot y}\right)\cdot\left(y\cdot\operatorname{atan}\left(\frac{1}{y}\right)+x_0\cdot\operatorname{atan}\left(\frac{1}{x_0}\right)-\sqrt{z_0}\cdot\operatorname{atan}\left(\frac{1}{\sqrt{z_0}}\right)\right)$$

$y := 0.4$ $z_0 := (x_0)^2 + y^2$

$$FPerPt4_0 := \left(\frac{1}{4\cdot\pi\cdot y}\right)\cdot\left[\ln\left[\left[\frac{\left[1+(x_0)^2\right]\cdot\left(1+y^2\right)}{1+z_0}\right]\cdot\left[\frac{y^2\cdot(1+z_0)}{\left(1+y^2\right)\cdot z_0}\right]^{y^2}\cdot\left[\frac{(x_0)^2\cdot(1+z_0)}{\left[1+(x_0)^2\right]\cdot z_0}\right]^{(x_0)^2}\right]\right]\ldots$$
$$+\left(\frac{1}{\pi\cdot y}\right)\cdot\left(y\cdot\operatorname{atan}\left(\frac{1}{y}\right)+x_0\cdot\operatorname{atan}\left(\frac{1}{x_0}\right)-\sqrt{z_0}\cdot\operatorname{atan}\left(\frac{1}{\sqrt{z_0}}\right)\right)$$

$y := 0.6 \qquad z_0 := (x_0)^2 + y^2$

$$FPerPt6_0 := \left(\frac{1}{4\cdot\pi\cdot y}\right)\cdot\left[\ln\left[\left[\frac{\left[1+(x_0)^2\right]\cdot\left(1+y^2\right)}{1+z_0}\right]\cdot\left[\frac{y^2\cdot(1+z_0)}{\left(1+y^2\right)\cdot z_0}\right]^{y^2}\cdot\left[\frac{(x_0)^2\cdot(1+z_0)}{\left[1+(x_0)^2\right]\cdot z_0}\right]^{(x_0)^2}\right]\ldots\right.$$
$$\left.+\left(\frac{1}{\pi\cdot y}\right)\cdot\left(y\cdot\text{atan}\left(\frac{1}{y}\right)+x_0\cdot\text{atan}\left(\frac{1}{x_0}\right)-\sqrt{z_0}\cdot\text{atan}\left(\frac{1}{\sqrt{z_0}}\right)\right)\right]$$

$y := 1 \qquad z_0 := (x_0)^2 + y^2$

$$FPer1_0 := \left(\frac{1}{4\cdot\pi\cdot y}\right)\cdot\left[\ln\left[\left[\frac{\left[1+(x_0)^2\right]\cdot\left(1+y^2\right)}{1+z_0}\right]\cdot\left[\frac{y^2\cdot(1+z_0)}{\left(1+y^2\right)\cdot z_0}\right]^{y^2}\cdot\left[\frac{(x_0)^2\cdot(1+z_0)}{\left[1+(x_0)^2\right]\cdot z_0}\right]^{(x_0)^2}\right]\ldots\right.$$
$$\left.+\left(\frac{1}{\pi\cdot y}\right)\cdot\left(y\cdot\text{atan}\left(\frac{1}{y}\right)+x_0\cdot\text{atan}\left(\frac{1}{x_0}\right)-\sqrt{z_0}\cdot\text{atan}\left(\frac{1}{\sqrt{z_0}}\right)\right)\right]$$

$y := 2 \qquad z_0 := (x_0)^2 + y^2$

$$FPer2_0 := \left(\frac{1}{4\cdot\pi\cdot y}\right)\cdot\left[\ln\left[\left[\frac{\left[1+(x_0)^2\right]\cdot\left(1+y^2\right)}{1+z_0}\right]\cdot\left[\frac{y^2\cdot(1+z_0)}{\left(1+y^2\right)\cdot z_0}\right]^{y^2}\cdot\left[\frac{(x_0)^2\cdot(1+z_0)}{\left[1+(x_0)^2\right]\cdot z_0}\right]^{(x_0)^2}\right]\ldots\right.$$
$$\left.+\left(\frac{1}{\pi\cdot y}\right)\cdot\left(y\cdot\text{atan}\left(\frac{1}{y}\right)+x_0\cdot\text{atan}\left(\frac{1}{x_0}\right)-\sqrt{z_0}\cdot\text{atan}\left(\frac{1}{\sqrt{z_0}}\right)\right)\right]$$

$y := 4 \qquad z_0 := (x_0)^2 + y^2$

$$FPer4_0 := \left(\frac{1}{4\cdot\pi\cdot y}\right)\cdot\left[\ln\left[\left[\frac{\left[1+(x_0)^2\right]\cdot\left(1+y^2\right)}{1+z_0}\right]\cdot\left[\frac{y^2\cdot(1+z_0)}{\left(1+y^2\right)\cdot z_0}\right]^{y^2}\cdot\left[\frac{(x_0)^2\cdot(1+z_0)}{\left[1+(x_0)^2\right]\cdot z_0}\right]^{(x_0)^2}\right]\ldots\right.$$
$$\left.+\left(\frac{1}{\pi\cdot y}\right)\cdot\left(y\cdot\text{atan}\left(\frac{1}{y}\right)+x_0\cdot\text{atan}\left(\frac{1}{x_0}\right)-\sqrt{z_0}\cdot\text{atan}\left(\frac{1}{\sqrt{z_0}}\right)\right)\right]$$

$y := 10 \qquad z_0 := (x_0)^2 + y^2$

$$FPer10_0 := \left(\frac{1}{4\cdot\pi\cdot y}\right)\cdot\left[\ln\left[\left[\frac{\left[1+(x_0)^2\right]\cdot\left(1+y^2\right)}{1+z_0}\right]\cdot\left[\frac{y^2\cdot(1+z_0)}{\left(1+y^2\right)\cdot z_0}\right]^{y^2}\cdot\left[\frac{(x_0)^2\cdot(1+z_0)}{\left[1+(x_0)^2\right]\cdot z_0}\right]^{(x_0)^2}\right]\right] \ldots$$
$$+\left(\frac{1}{\pi\cdot y}\right)\cdot\left(y\cdot\operatorname{atan}\left(\frac{1}{y}\right)+x_0\cdot\operatorname{atan}\left(\frac{1}{x_0}\right)-\sqrt{z_0}\cdot\operatorname{atan}\left(\frac{1}{\sqrt{z_0}}\right)\right)$$

$MAXxDIVS := 100 \qquad STEP := 0.1 \qquad STEPS := \frac{MAXxDIVS}{STEP} \qquad STEPS = 1\times10^3$

$i := 1, 2 .. STEPS$

$x_i := x_{i-1} + STEP$

$y := 0.1 \qquad z_i := (x_i)^2 + y^2$

$$FPerPt1_i := \left(\frac{1}{4\cdot\pi\cdot y}\right)\cdot\left[\ln\left[\left[\frac{\left[1+(x_i)^2\right]\cdot\left(1+y^2\right)}{1+z_i}\right]\cdot\left[\frac{y^2\cdot(1+z_i)}{\left(1+y^2\right)\cdot z_i}\right]^{y^2}\cdot\left[\frac{(x_i)^2\cdot(1+z_i)}{\left[1+(x_i)^2\right]\cdot z_i}\right]^{(x_i)^2}\right]\right] \ldots$$
$$+\left(\frac{1}{\pi\cdot y}\right)\cdot\left(y\cdot\operatorname{atan}\left(\frac{1}{y}\right)+x_i\cdot\operatorname{atan}\left(\frac{1}{x_i}\right)-\sqrt{z_i}\cdot\operatorname{atan}\left(\frac{1}{\sqrt{z_i}}\right)\right)$$

$y := 0.2 \qquad z_i := (x_i)^2 + y^2$

$$FPerPt2_i := \left(\frac{1}{4\cdot\pi\cdot y}\right)\cdot\left[\ln\left[\left[\frac{\left[1+(x_i)^2\right]\cdot\left(1+y^2\right)}{1+z_i}\right]\cdot\left[\frac{y^2\cdot(1+z_i)}{\left(1+y^2\right)\cdot z_i}\right]^{y^2}\cdot\left[\frac{(x_i)^2\cdot(1+z_i)}{\left[1+(x_i)^2\right]\cdot z_i}\right]^{(x_i)^2}\right]\right] \ldots$$
$$+\left(\frac{1}{\pi\cdot y}\right)\cdot\left(y\cdot\operatorname{atan}\left(\frac{1}{y}\right)+x_i\cdot\operatorname{atan}\left(\frac{1}{x_i}\right)-\sqrt{z_i}\cdot\operatorname{atan}\left(\frac{1}{\sqrt{z_i}}\right)\right)$$

$y := 0.4 \qquad z_i := (x_i)^2 + y^2$

$$FPerPt4_i := \left(\frac{1}{4\cdot\pi\cdot y}\right)\cdot\left[\ln\left[\left[\frac{\left[1+(x_i)^2\right]\cdot\left(1+y^2\right)}{1+z_i}\right]\cdot\left[\frac{y^2\cdot(1+z_i)}{\left(1+y^2\right)\cdot z_i}\right]^{y^2}\cdot\left[\frac{(x_i)^2\cdot(1+z_i)}{\left[1+(x_i)^2\right]\cdot z_i}\right]^{(x_i)^2}\right]\right] \dots$$
$$+\left(\frac{1}{\pi\cdot y}\right)\cdot\left(y\cdot \operatorname{atan}\left(\frac{1}{y}\right)+x_i\cdot\operatorname{atan}\left(\frac{1}{x_i}\right)-\sqrt{z_i}\cdot\operatorname{atan}\left(\frac{1}{\sqrt{z_i}}\right)\right)$$

$y := 0.6 \qquad z_i := (x_i)^2 + y^2$

$$FPerPt6_i := \left(\frac{1}{4\cdot\pi\cdot y}\right)\cdot\left[\ln\left[\left[\frac{\left[1+(x_i)^2\right]\cdot\left(1+y^2\right)}{1+z_i}\right]\cdot\left[\frac{y^2\cdot(1+z_i)}{\left(1+y^2\right)\cdot z_i}\right]^{y^2}\cdot\left[\frac{(x_i)^2\cdot(1+z_i)}{\left[1+(x_i)^2\right]\cdot z_i}\right]^{(x_i)^2}\right]\right] \dots$$
$$+\left(\frac{1}{\pi\cdot y}\right)\cdot\left(y\cdot \operatorname{atan}\left(\frac{1}{y}\right)+x_i\cdot\operatorname{atan}\left(\frac{1}{x_i}\right)-\sqrt{z_i}\cdot\operatorname{atan}\left(\frac{1}{\sqrt{z_i}}\right)\right)$$

$y := 1 \qquad z_i := (x_i)^2 + y^2$

$$FPer1_i := \left(\frac{1}{4\cdot\pi\cdot y}\right)\cdot\left[\ln\left[\left[\frac{\left[1+(x_i)^2\right]\cdot\left(1+y^2\right)}{1+z_i}\right]\cdot\left[\frac{y^2\cdot(1+z_i)}{\left(1+y^2\right)\cdot z_i}\right]^{y^2}\cdot\left[\frac{(x_i)^2\cdot(1+z_i)}{\left[1+(x_i)^2\right]\cdot z_i}\right]^{(x_i)^2}\right]\right] \dots$$
$$+\left(\frac{1}{\pi\cdot y}\right)\cdot\left(y\cdot \operatorname{atan}\left(\frac{1}{y}\right)+x_i\cdot\operatorname{atan}\left(\frac{1}{x_i}\right)-\sqrt{z_i}\cdot\operatorname{atan}\left(\frac{1}{\sqrt{z_i}}\right)\right)$$

$y := 2 \qquad z_i := (x_i)^2 + y^2$

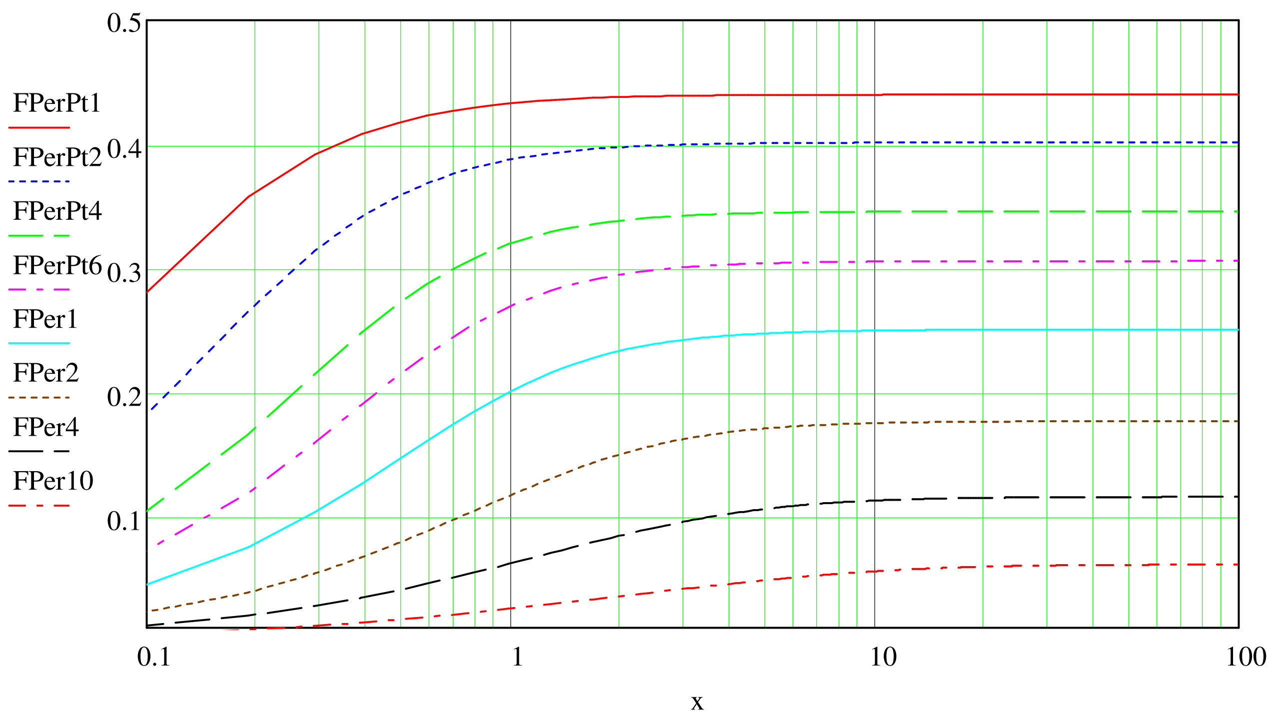
$$FPer2_i := \left(\frac{1}{4\cdot\pi\cdot y}\right)\cdot\left[\ln\left[\left[\frac{\left[1+(x_i)^2\right]\cdot\left(1+y^2\right)}{1+z_i}\right]\cdot\left[\frac{y^2\cdot(1+z_i)}{\left(1+y^2\right)\cdot z_i}\right]^{y^2}\cdot\left[\frac{(x_i)^2\cdot(1+z_i)}{\left[1+(x_i)^2\right]\cdot z_i}\right]^{(x_i)^2}\right]\right] \dots$$
$$+\left(\frac{1}{\pi\cdot y}\right)\cdot\left(y\cdot \operatorname{atan}\left(\frac{1}{y}\right)+x_i\cdot\operatorname{atan}\left(\frac{1}{x_i}\right)-\sqrt{z_i}\cdot\operatorname{atan}\left(\frac{1}{\sqrt{z_i}}\right)\right)$$

$y := 4 \qquad z_i := (x_i)^2 + y^2$

$$FPer4_i := \left(\frac{1}{4\cdot\pi\cdot y}\right)\cdot\left[\ln\left[\left[\frac{\left[1+(x_i)^2\right]\cdot\left(1+y^2\right)}{1+z_i}\right]\cdot\left[\frac{y^2\cdot(1+z_i)}{\left(1+y^2\right)\cdot z_i}\right]^{y^2}\cdot\left[\frac{(x_i)^2\cdot(1+z_i)}{\left[1+(x_i)^2\right]\cdot z_i}\right]^{(x_i)^2}\right]\right] \ldots$$
$$+\left(\frac{1}{\pi\cdot y}\right)\cdot\left(y\cdot\operatorname{atan}\left(\frac{1}{y}\right)+x_i\cdot\operatorname{atan}\left(\frac{1}{x_i}\right)-\sqrt{z_i}\cdot\operatorname{atan}\left(\frac{1}{\sqrt{z_i}}\right)\right)$$

$y := 10 \qquad z_i := (x_i)^2 + y^2$

$$FPer10_i := \left(\frac{1}{4\cdot\pi\cdot y}\right)\cdot\left[\ln\left[\left[\frac{\left[1+(x_i)^2\right]\cdot\left(1+y^2\right)}{1+z_i}\right]\cdot\left[\frac{y^2\cdot(1+z_i)}{\left(1+y^2\right)\cdot z_i}\right]^{y^2}\cdot\left[\frac{(x_i)^2\cdot(1+z_i)}{\left[1+(x_i)^2\right]\cdot z_i}\right]^{(x_i)^2}\right]\right] \ldots$$
$$+\left(\frac{1}{\pi\cdot y}\right)\cdot\left(y\cdot\operatorname{atan}\left(\frac{1}{y}\right)+x_i\cdot\operatorname{atan}\left(\frac{1}{x_i}\right)-\sqrt{z_i}\cdot\operatorname{atan}\left(\frac{1}{\sqrt{z_i}}\right)\right)$$



Write to Files:

...\Per_Plate_X.dat

x

...\Per_Plate_Pt1.dat

FPerPt1

...\Per_Plate_Pt2.dat

FPerPt2

...\Per_Plate_Pt4.dat

FPerPt4

...\Per_Plate_Pt6.dat

FPerPt6

C...\Per_Plate_1.dat

FPer1

C...\Per_Plate_2.dat

FPer2

C...\Per_Plate_4.dat

FPer4

...\Per_Plate_10.dat

FPer10

0

U-Channel Gray Body Radiation Shape Factors

Functions for Geometric Shape Factor for Perpendicular Plates:

W = common dimension of plates 1 and 2.
L1 and L2 are the other dimensions of plates 1 and 2.
$x=L_2/H$, $y=L_1/H$.

$$FPerp(x,y) := \left(\frac{1}{4\cdot\pi\cdot y}\right)\cdot\left[\ln\left[\left[\frac{(1+x^2)\cdot(1+y^2)}{1+x^2+y^2}\right]\cdot\left[\frac{y^2\cdot(1+x^2+y^2)}{(1+y^2)\cdot(x^2+y^2)}\right]^{y^2}\cdot\left[\frac{x^2\cdot(1+x^2+y^2)}{(1+x^2)\cdot(x^2+y^2)}\right]^{x^2}\right]\right] \ldots$$
$$+\left(\frac{1}{\pi\cdot y}\right)\cdot\left(y\cdot \operatorname{atan}\left(\frac{1}{y}\right)+x\cdot \operatorname{atan}\left(\frac{1}{x}\right)-\sqrt{x^2+y^2}\cdot \operatorname{atan}\left(\frac{1}{\sqrt{x^2+y^2}}\right)\right)$$

Functions for Geometric Shape Factor for Parallel Plates:

L and W are dimensions of identical plates, S = plate spacing.

$x=L/S$, $y=W/S$.

$$FPar(x,y) := \left(\frac{2}{\pi\cdot x\cdot y}\right)\cdot\left[\ln\left[\sqrt{\frac{(1+x^2)\cdot(1+y^2)}{1+x^2+y^2}}\right]+\left(y\cdot\sqrt{1+x^2}\cdot \operatorname{atan}\left(\frac{y}{\sqrt{1+x^2}}\right)\right) \ldots \right.$$
$$\left. +\left(x\cdot\sqrt{1+y^2}\cdot \operatorname{atan}\left(\frac{x}{\sqrt{1+y^2}}\right)\right)-y\cdot \operatorname{atan}(y)-x\cdot \operatorname{atan}(x)\right]$$

Function for C_{Net}:

a, b, c, d, e are dummy variable names for Resistances R_a, R_b, R_c, R_d, R_e, respect.

$$CNet(a,b,c,d,e) := \frac{(a+b+e)\cdot(c+d+e)-e^2}{(b+d)\cdot\left[(a+b+e)\cdot(c+d+e)-e^2\right] \ldots \\ +(-1)\cdot(b)\cdot[b\cdot(c+d+e)+e\cdot d]-d\cdot[d\cdot(a+b+e)+b\cdot e]}$$

Input Dimensions and Emissivity for Heat Sink Channel:

Height, Depth, Spacing: $H := 10$ $L := 1.0$ $S := .25$

Emissivity: $\varepsilon := 0.1$

Calculate Areas:

A1 = Back panel area, A3 = Side panel area.

$$A1 := H \cdot S \qquad A3 := H \cdot L$$

Calculate Geometric Shape Factors:

Surface 2 is front panel, surface 5 is (open) top panel.

$$F13 := FPerp\left(\frac{L}{H}, \frac{S}{H}\right) \qquad F35 := FPerp\left(\frac{S}{L}, \frac{H}{L}\right)$$

$$F15 := FPerp\left(\frac{L}{S}, \frac{H}{S}\right) \qquad F12 := FPar\left(\frac{S}{L}, \frac{H}{L}\right)$$

Calculate Various Resistances:

$$Ra := \frac{1-\varepsilon}{\varepsilon \cdot A3} \qquad Rb := \frac{2\cdot(1-\varepsilon)}{\varepsilon \cdot A1} \qquad Rc := \frac{1}{A1\cdot F13 + 2\cdot A3\cdot F35} \qquad Rd := \frac{2}{A1\cdot F12 + 2\cdot A1\cdot F15} \qquad Re := \frac{1}{A1\cdot F13}$$

Calculate CNet:

$$C := CNet(Ra, Rb, Rc, Rd, Re)$$

Calculate Radiation Gray Body Shape Factor:

$$F := 2\cdot\frac{C}{H\cdot(S + 2\cdot L)} \qquad F = 0.05952$$

Calculation of h_c/h_H from Van de Pol & Tierney

$T_A := 20$ $s := 0.3$ $L := 1.0$ $H := 5$ $\Delta T := 50$

$$T_M := \frac{2 \cdot T_A + \Delta T}{2} \quad T_S := T_A + \Delta T \quad z := \frac{H}{s} \quad x := \frac{L}{s}$$

$z = 16.667$ $x = 3.333$

$$C_1 := 5.454 \cdot 10^5 \cdot \exp\left(-9.254 \cdot 10^{-3} \cdot T_S\right) \quad \beta := \frac{1}{T_M + 273.16} \quad V := -11.8$$

$$GrPr_H := C_1 \cdot \beta \cdot \Delta T \cdot H^3 \quad r := \frac{2x \cdot s}{2 \cdot x + 1} \quad GrPr := C_1 \cdot \beta \cdot \Delta T \cdot r^3$$

$$PSI(a) := \frac{24 \cdot \left(1 - 0.483 \cdot \exp\left(\frac{-0.17}{a}\right)\right)}{\left[\left(1 + \frac{a}{2}\right) \cdot \left[1 + (1 - \exp(-0.83 \cdot a)) \cdot \left(9.14 \cdot \sqrt{a} \cdot \exp(V \cdot s) - 0.61\right)\right]\right]^3}$$

$$\psi := PSI\left(\frac{1}{x}\right) \quad RaChan := \left(\frac{r}{H}\right) \cdot GrPr$$

$\psi = 15.843$ $RaChan = 41.537$

$$Nur := \frac{RaChan}{\psi} \cdot \left[1 - \exp\left[-\psi \cdot \left(\frac{0.5}{RaChan}\right)^{\frac{3}{4}}\right]\right] \quad Nu_H := 0.595 \cdot GrPr_H^{\frac{1}{4}}$$

$$hRatio := \frac{H}{r} \frac{Nur}{Nu_H}$$

$Nur = 1.148$ $Nu_H = 28.952$ $hRatio = 0.76$

$$k_{Air} := 6.0583 \cdot 10^{-4} + 1.6906 \cdot 10^{-6} \cdot T_M \quad h_H := \frac{k_{Air}}{H} \cdot Nu_H \quad h_c := hRatio \cdot h_H$$

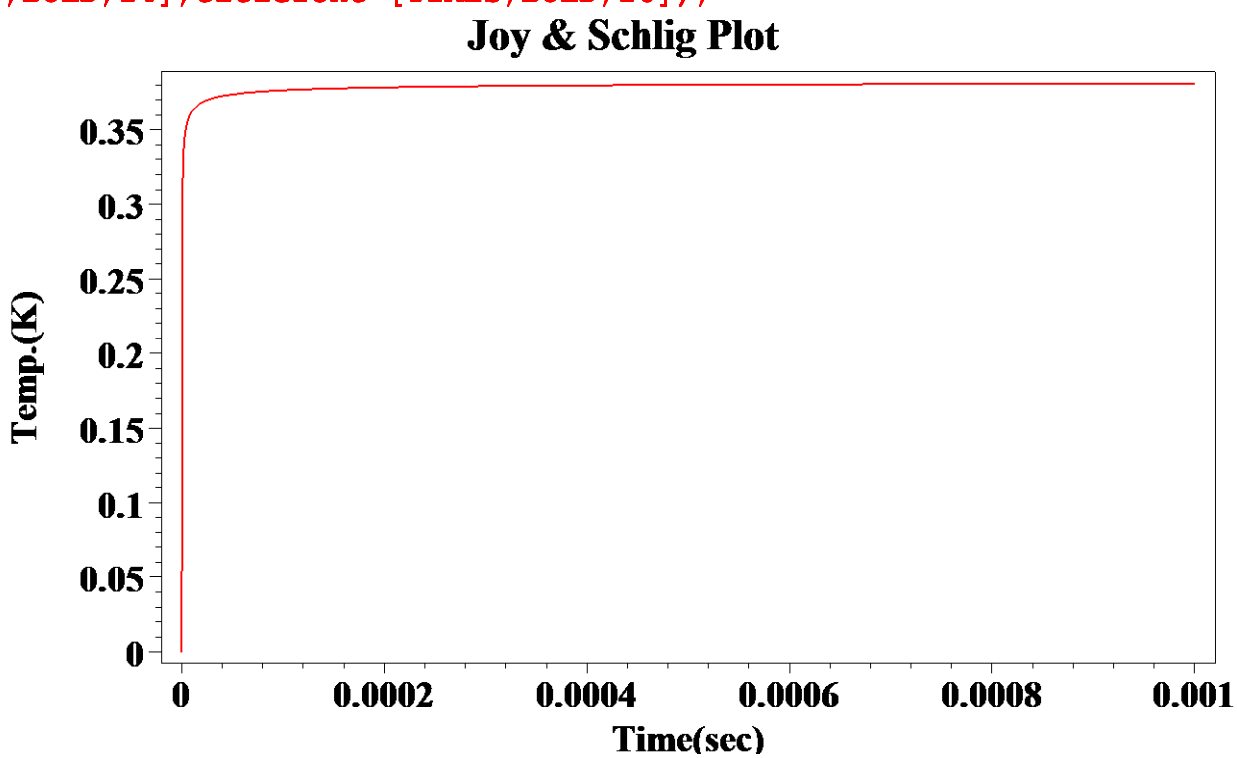
$k_{Air} = 6.819 \times 10^{-4}$ $h_H = 3.948 \times 10^{-3}$ $h_c = 3 \times 10^{-3}$

```
> start:
> rho:=2.33:
> C:=0.704:
> k:=1.465:
> W:=0.001:
> L:=1.0*W:
> H:=1.0*10^(-10):
> d:=0.0:
> xs:=0.0:
> ys:=0.0:
> zs:=0.0:
> x:=0:
> y:=0:
> z:=0.0:
> Q:=0.001:
> Steps:=10000:
> dt:=1.0*10^(-7):
> V:=L*H*W:
> alpha:=k/(rho*C):
> f:=t->(Q/(8*rho*C*V))*(erf((0.5*W+x-xs)/sqrt(4*alpha*t))+erf((0.5*
  W-x+xs)/sqrt(4*alpha*t)))\
>
  *(erf((0.5*L+y-ys)/sqrt(4*alpha*t))+erf((0.5*L-y+ys)/sqrt(4*alpha*
  t)))\
>
  *(erf((z+zs+d+H)/sqrt(4*alpha*t))+erf((-d-z-zs)/sqrt(4*alpha*t))\
> +erf((z-zs-d)/sqrt(4*alpha*t))+erf((d+H+zs-z)/sqrt(4*alpha*t)));
```

$$f := t \rightarrow \frac{1}{8} Q \left(\operatorname{erf}\left(\frac{0.5\,\mathrm{W} + x - xs}{\sqrt{4\,\alpha\,t}}\right) + \operatorname{erf}\left(\frac{0.5\,\mathrm{W} - x + xs}{\sqrt{4\,\alpha\,t}}\right)\right)$$
$$\left(\operatorname{erf}\left(\frac{0.5\,L + y - ys}{\sqrt{4\,\alpha\,t}}\right) + \operatorname{erf}\left(\frac{0.5\,L - y + ys}{\sqrt{4\,\alpha\,t}}\right)\right)$$
$$\left(\operatorname{erf}\left(\frac{z + zs + d + H}{\sqrt{4\,\alpha\,t}}\right) + \operatorname{erf}\left(\frac{-d - z - zs}{\sqrt{4\,\alpha\,t}}\right) + \operatorname{erf}\left(\frac{z - zs - d}{\sqrt{4\,\alpha\,t}}\right) + \operatorname{erf}\left(\frac{d + H + zs - z}{\sqrt{4\,\alpha\,t}}\right)\right) / (\rho\,C\,V)$$

```
> T:=array(1..Steps):
> t:=array(1..Steps):
> tMax:=0:
>
> for J from 1 to Steps do
>   T[J]:=evalf(Int(f(t),t=0..tMax)):
>   t[J]:=tMax:
>   tMax:=tMax+1*dt:
> end do:
```

```
>
> plot([t[M],T[M],M=1..Steps],thickness=2,labels=["Time(sec)","Temp.
  (K)"],title="Joy & Schlig
  Plot",axes=BOXED,labeldirections=[horizontal,vertical],font=[TIMES
  ,BOLD,14],titlefont=[TIMES,BOLD,16]);
```




```
> T[Steps];
```

$$0.3809523013$$

```
> C:=(T[Steps]/Q)*2*k*sqrt(L*W);
```

$$C := 1.116190243$$

```
> Rss:=C/(2*k*sqrt(L*W));
```

$$Rss := 380.9523014$$

```
> NDR:=k*sqrt(L*W)*Rss;
```

$$NDR := 0.5580951216$$

```
>
```

Application Example 13.2: Solution of Steady-State Network Using Gauss-Seidel Method - Relaxation - 10 Iterations

Input Data:

$t_1 := 0.1$ $k_1 := 1.0$ $w_1 := 1.0$ $l_1 := 1.0$

$t_2 := 0.05$ $k_2 := 0.02$ $w_2 := 1.0$ $l_2 := 1.0$

$t_3 := 0.5$ $k_3 := 10.0$ $w_3 := 1.0$ $l_3 := 0.5$

$t_4 := 0.5$ $k_4 := 4.0$ $w_4 := 1.0$ $l_4 := 0.5$

$h_8 := 2$ $h_9 := 1$ $T_7 := 20$ $\beta := 1.7$

T_7, Q_1 are matrix elements.

Calculate Conductances:

$i := 1, 2 .. 7$

$j := 1, 2 .. 7$

$C_{i,j} := 0.0$ $Q_i := 0$

$$C_{1,2} := \frac{k_1 \cdot w_1 \cdot l_1}{t_1} \quad C_{2,3} := \frac{k_2 \cdot w_2 \cdot \frac{l_2}{2}}{t_2} \quad C_{2,4} := \frac{k_2 \cdot w_2 \cdot \frac{l_2}{2}}{t_2} \quad C_{3,5} := \frac{k_3 \cdot w_3 \cdot l_3}{t_3}$$

$$C_{3,4} := \frac{1}{\dfrac{\frac{l_3}{2}}{k_3 \cdot \frac{t_3}{2} \cdot w_3} + \dfrac{\frac{l_4}{2}}{k_4 \cdot \frac{t_4}{2} \cdot w_4}} \quad C_{5,6} := \frac{1}{\dfrac{\frac{l_3}{2}}{k_3 \cdot \frac{t_3}{2} \cdot w_3} + \dfrac{\frac{l_4}{2}}{k_4 \cdot \frac{t_4}{2} \cdot w_4}}$$

$$C_{4,6} := \frac{k_4 \cdot w_4 \cdot l_4}{t_4} \quad C_{5,7} := h_8 \cdot w_3 \cdot l_3 \quad C_{6,7} := h_9 \cdot w_4 \cdot l_4$$

$C_{2,1} := C_{1,2}$ $C_{3,2} := C_{2,3}$ $C_{4,2} := C_{2,4}$ $C_{5,3} := C_{3,5}$ $C_{4,3} := C_{3,4}$

$C_{6,5} := C_{5,6}$ $C_{6,4} := C_{4,6}$ $C_{7,5} := C_{5,7}$ $C_{7,6} := C_{6,7}$

$$C = \begin{pmatrix} 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 10 & 0 & 0.2 & 0.2 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 2.857 & 10 & 0 & 0 \\ 0 & 0.2 & 2.857 & 0 & 0 & 4 & 0 \\ 0 & 0 & 10 & 0 & 0 & 2.857 & 1 \\ 0 & 0 & 0 & 4 & 2.857 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 1 & 0.5 & 0 \end{pmatrix}$$

$Q_1 := 10.0$

$$Q = \begin{pmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Solution Using Gauss-Seidel Showing Only Two Iterations:

Set Starting Temps:

$i := 1, 2 .. 6$

$T_i := 40$

First Iteration:

$TOLD := T_1$

$$T_1 := \frac{C_{1,2} \cdot T_2 + Q_1}{C_{1,2}} \qquad TR_1 := TOLD + \beta \cdot (T_1 - TOLD) \quad T_1 := TR_1$$

$TOLD := T_2$

$$T_2 := \frac{C_{2,1} \cdot TR_1 + C_{2,3} \cdot T_3 + C_{2,4} \cdot T_4}{C_{2,1} + C_{2,3} + C_{2,4}} \qquad TR_2 := TOLD + \beta \cdot (T_2 - TOLD) \quad T_2 := TR_2$$

$TOLD := T_3$

$$T_3 := \frac{C_{3,2} \cdot TR_2 + C_{3,4} \cdot T_4 + C_{3,5} \cdot T_5}{C_{3,2} + C_{3,4} + C_{3,5}} \qquad TR_3 := TOLD + \beta \cdot (T_3 - TOLD) \quad T_3 := TR_3$$

$TOLD := T_4$

$$T_4 := \frac{C_{4,2} \cdot TR_2 + C_{4,3} \cdot TR_3 + C_{4,6} \cdot T_6}{C_{4,2} + C_{4,3} + C_{4,6}} \qquad TR_4 := TOLD + \beta \cdot (T_4 - TOLD) \quad T_4 := TR_4$$

$TOLD := T_5$

$$T_5 := \frac{C_{5,3} \cdot TR_3 + C_{5,6} \cdot T_6 + C_{5,7} \cdot T_7}{C_{5,3} + C_{5,6} + C_{5,7}} \qquad TR_5 := TOLD + \beta \cdot (T_5 - TOLD) \quad T_5 := TR_5$$

$$TOLD := T_6$$

$$T_6 := \frac{C_{6,4} \cdot TR_4 + C_{6,5} \cdot TR_5 + C_{6,7} \cdot T_7}{C_{6,4} + C_{6,5} + C_{6,7}} \qquad TR_6 := TOLD + \beta \cdot (T_6 - TOLD) \quad T_6 := TR_6$$

$$T = \begin{pmatrix} 41.7 \\ 42.779 \\ 40.072 \\ 40.184 \\ 37.635 \\ 36.298 \\ 20 \end{pmatrix} \qquad TR = \begin{pmatrix} 41.7 \\ 42.779 \\ 40.072 \\ 40.184 \\ 37.635 \\ 36.298 \end{pmatrix}$$

Second Iteration:

$$TOLD := T_1$$

$$T_1 := \frac{C_{1,2} \cdot T_2 + Q_1}{C_{1,2}} \qquad TR_1 := TOLD + \beta \cdot (T_1 - TOLD) \quad T_1 := TR_1$$

$$TOLD := T_2$$

$$T_2 := \frac{C_{2,1} \cdot TR_1 + C_{2,3} \cdot T_3 + C_{2,4} \cdot T_4}{C_{2,1} + C_{2,3} + C_{2,4}} \qquad TR_2 := TOLD + \beta \cdot (T_2 - TOLD) \quad T_2 := TR_2$$

$$TOLD := T_3$$

$$T_3 := \frac{C_{3,2} \cdot TR_2 + C_{3,4} \cdot T_4 + C_{3,5} \cdot T_5}{C_{3,2} + C_{3,4} + C_{3,5}} \qquad TR_3 := TOLD + \beta \cdot (T_3 - TOLD) \quad T_3 := TR_3$$

$$TOLD := T_4$$

$$T_4 := \frac{C_{4,2} \cdot TR_2 + C_{4,3} \cdot TR_3 + C_{4,6} \cdot T_6}{C_{4,2} + C_{4,3} + C_{4,6}} \qquad TR_4 := TOLD + \beta \cdot (T_4 - TOLD) \quad T_4 := TR_4$$

$TOLD := T_5$

$$T_5 := \frac{C_{5,3} \cdot TR_3 + C_{5,6} \cdot T_6 + C_{5,7} \cdot T_7}{C_{5,3} + C_{5,6} + C_{5,7}} \qquad TR_5 := TOLD + \beta \cdot (T_5 - TOLD) \quad T_5 := TR_5$$

$TOLD := T_6$

$$T_6 := \frac{C_{6,4} \cdot TR_4 + C_{6,5} \cdot TR_5 + C_{6,7} \cdot T_7}{C_{6,4} + C_{6,5} + C_{6,7}} \qquad TR_6 := TOLD + \beta \cdot (T_6 - TOLD) \quad T_6 := TR_6$$

$$T = \begin{pmatrix} 45.234 \\ 46.619 \\ 37.111 \\ 34.635 \\ 34.36 \\ 31.598 \\ 20 \end{pmatrix} \qquad TR = \begin{pmatrix} 45.234 \\ 46.619 \\ 37.111 \\ 34.635 \\ 34.36 \\ 31.598 \end{pmatrix}$$

Third Iteration:

$TOLD := T_1$

$$T_1 := \frac{C_{1,2} \cdot T_2 + Q_1}{C_{1,2}} \qquad TR_1 := TOLD + \beta \cdot (T_1 - TOLD) \quad T_1 := TR_1$$

$TOLD := T_2$

$$T_2 := \frac{C_{2,1} \cdot TR_1 + C_{2,3} \cdot T_3 + C_{2,4} \cdot T_4}{C_{2,1} + C_{2,3} + C_{2,4}} \qquad TR_2 := TOLD + \beta \cdot (T_2 - TOLD) \quad T_2 := TR_2$$

$TOLD := T_3$

$$T_3 := \frac{C_{3,2} \cdot TR_2 + C_{3,4} \cdot T_4 + C_{3,5} \cdot T_5}{C_{3,2} + C_{3,4} + C_{3,5}} \qquad TR_3 := TOLD + \beta \cdot (T_3 - TOLD) \quad T_3 := TR_3$$

$TOLD := T_4$

$$T_4 := \frac{C_{4,2} \cdot TR_2 + C_{4,3} \cdot TR_3 + C_{4,6} \cdot T_6}{C_{4,2} + C_{4,3} + C_{4,6}} \qquad TR_4 := TOLD + \beta \cdot (T_4 - TOLD) \quad T_4 := TR_4$$

$TOLD := T_5$

$$T_5 := \frac{C_{5,3} \cdot TR_3 + C_{5,6} \cdot T_6 + C_{5,7} \cdot T_7}{C_{5,3} + C_{5,6} + C_{5,7}} \qquad TR_5 := TOLD + \beta \cdot \left(T_5 - TOLD\right) \quad T_5 := TR_5$$

$TOLD := T_6$

$$T_6 := \frac{C_{6,4} \cdot TR_4 + C_{6,5} \cdot TR_5 + C_{6,7} \cdot T_7}{C_{6,4} + C_{6,5} + C_{6,7}} \qquad TR_6 := TOLD + \beta \cdot \left(T_6 - TOLD\right) \quad T_6 := TR_6$$

$$T = \begin{pmatrix} 49.288 \\ 50.28 \\ 32.951 \\ 31.304 \\ 29.902 \\ 28.866 \\ 20 \end{pmatrix} \qquad TR = \begin{pmatrix} 49.288 \\ 50.28 \\ 32.951 \\ 31.304 \\ 29.902 \\ 28.866 \end{pmatrix}$$

Fourth Iteration:

$TOLD := T_1$

$$T_1 := \frac{C_{1,2} \cdot T_2 + Q_1}{C_{1,2}} \qquad TR_1 := TOLD + \beta \cdot \left(T_1 - TOLD\right) \quad T_1 := TR_1$$

$TOLD := T_2$

$$T_2 := \frac{C_{2,1} \cdot TR_1 + C_{2,3} \cdot T_3 + C_{2,4} \cdot T_4}{C_{2,1} + C_{2,3} + C_{2,4}} \qquad TR_2 := TOLD + \beta \cdot \left(T_2 - TOLD\right) \quad T_2 := TR_2$$

$TOLD := T_3$

$$T_3 := \frac{C_{3,2} \cdot TR_2 + C_{3,4} \cdot T_4 + C_{3,5} \cdot T_5}{C_{3,2} + C_{3,4} + C_{3,5}} \qquad TR_3 := TOLD + \beta \cdot \left(T_3 - TOLD\right) \quad T_3 := TR_3$$

$TOLD := T_4$

$$T_4 := \frac{C_{4,2}\cdot TR_2 + C_{4,3}\cdot TR_3 + C_{4,6}\cdot T_6}{C_{4,2} + C_{4,3} + C_{4,6}}$$ $$TR_4 := TOLD + \beta\cdot(T_4 - TOLD) \quad T_4 := TR_4$$

$$TOLD := T_5$$

$$T_5 := \frac{C_{5,3}\cdot TR_3 + C_{5,6}\cdot T_6 + C_{5,7}\cdot T_7}{C_{5,3} + C_{5,6} + C_{5,7}}$$ $$TR_5 := TOLD + \beta\cdot(T_5 - TOLD) \quad T_5 := TR_5$$

$$TOLD := T_6$$

$$T_6 := \frac{C_{6,4}\cdot TR_4 + C_{6,5}\cdot TR_5 + C_{6,7}\cdot T_7}{C_{6,4} + C_{6,5} + C_{6,7}}$$ $$TR_6 := TOLD + \beta\cdot(T_6 - TOLD) \quad T_6 := TR_6$$

$$T = \begin{pmatrix} 52.674 \\ 53.006 \\ 28.89 \\ 28.339 \\ 27.083 \\ 26.178 \\ 20 \end{pmatrix} \qquad TR = \begin{pmatrix} 52.674 \\ 53.006 \\ 28.89 \\ 28.339 \\ 27.083 \\ 26.178 \end{pmatrix}$$

Fifth Iteration:

$$TOLD := T_1$$

$$T_1 := \frac{C_{1,2}\cdot T_2 + Q_1}{C_{1,2}}$$ $$TR_1 := TOLD + \beta\cdot(T_1 - TOLD) \quad T_1 := TR_1$$

$$TOLD := T_2$$

$$T_2 := \frac{C_{2,1}\cdot TR_1 + C_{2,3}\cdot T_3 + C_{2,4}\cdot T_4}{C_{2,1} + C_{2,3} + C_{2,4}}$$ $$TR_2 := TOLD + \beta\cdot(T_2 - TOLD) \quad T_2 := TR_2$$

$$TOLD := T_3$$

$$T_3 := \frac{C_{3,2}\cdot TR_2 + C_{3,4}\cdot T_4 + C_{3,5}\cdot T_5}{C_{3,2} + C_{3,4} + C_{3,5}}$$ $$TR_3 := TOLD + \beta\cdot(T_3 - TOLD) \quad T_3 := TR_3$$

$$TOLD := T_4$$

$$T_4 := \frac{C_{4,2} \cdot TR_2 + C_{4,3} \cdot TR_3 + C_{4,6} \cdot T_6}{C_{4,2} + C_{4,3} + C_{4,6}} \qquad TR_4 := TOLD + \beta \cdot (T_4 - TOLD) \quad T_4 := TR_4$$

$TOLD := T_5$

$$T_5 := \frac{C_{5,3} \cdot TR_3 + C_{5,6} \cdot T_6 + C_{5,7} \cdot T_7}{C_{5,3} + C_{5,6} + C_{5,7}} \qquad TR_5 := TOLD + \beta \cdot (T_5 - TOLD) \quad T_5 := TR_5$$

$TOLD := T_6$

$$T_6 := \frac{C_{6,4} \cdot TR_4 + C_{6,5} \cdot TR_5 + C_{6,7} \cdot T_7}{C_{6,4} + C_{6,5} + C_{6,7}} \qquad TR_6 := TOLD + \beta \cdot (T_6 - TOLD) \quad T_6 := TR_6$$

$$T = \begin{pmatrix} 54.939 \\ 54.57 \\ 27.001 \\ 26.6 \\ 25.796 \\ 25.602 \\ 20 \end{pmatrix} \qquad TR = \begin{pmatrix} 54.939 \\ 54.57 \\ 27.001 \\ 26.6 \\ 25.796 \\ 25.602 \end{pmatrix}$$

Sixth Iteration:

$TOLD := T_1$

$$T_1 := \frac{C_{1,2} \cdot T_2 + Q_1}{C_{1,2}} \qquad TR_1 := TOLD + \beta \cdot (T_1 - TOLD) \quad T_1 := TR_1$$

$TOLD := T_2$

$$T_2 := \frac{C_{2,1} \cdot TR_1 + C_{2,3} \cdot T_3 + C_{2,4} \cdot T_4}{C_{2,1} + C_{2,3} + C_{2,4}} \qquad TR_2 := TOLD + \beta \cdot (T_2 - TOLD) \quad T_2 := TR_2$$

$TOLD := T_3$

$$T_3 := \frac{C_{3,2} \cdot TR_2 + C_{3,4} \cdot T_4 + C_{3,5} \cdot T_5}{C_{3,2} + C_{3,4} + C_{3,5}} \qquad TR_3 := TOLD + \beta \cdot (T_3 - TOLD) \quad T_3 := TR_3$$

$TOLD := T_4$

$$T_4 := \frac{C_{4,2} \cdot TR_2 + C_{4,3} \cdot TR_3 + C_{4,6} \cdot T_6}{C_{4,2} + C_{4,3} + C_{4,6}} \qquad TR_4 := TOLD + \beta \cdot (T_4 - TOLD) \quad T_4 := TR_4$$

$TOLD := T_5$

$$T_5 := \frac{C_{5,3} \cdot TR_3 + C_{5,6} \cdot T_6 + C_{5,7} \cdot T_7}{C_{5,3} + C_{5,6} + C_{5,7}} \qquad TR_5 := TOLD + \beta \cdot (T_5 - TOLD) \quad T_5 := TR_5$$

$TOLD := T_6$

$$T_6 := \frac{C_{6,4} \cdot TR_4 + C_{6,5} \cdot TR_5 + C_{6,7} \cdot T_7}{C_{6,4} + C_{6,5} + C_{6,7}} \qquad TR_6 := TOLD + \beta \cdot (T_6 - TOLD) \quad T_6 := TR_6$$

$$T = \begin{pmatrix} 56.012 \\ 55.112 \\ 26.015 \\ 26.61 \\ 25.285 \\ 25.677 \\ 20 \end{pmatrix} \qquad TR = \begin{pmatrix} 56.012 \\ 55.112 \\ 26.015 \\ 26.61 \\ 25.285 \\ 25.677 \end{pmatrix}$$

Seventh Iteration:

$TOLD := T_1$

$$T_1 := \frac{C_{1,2} \cdot T_2 + Q_1}{C_{1,2}} \qquad TR_1 := TOLD + \beta \cdot (T_1 - TOLD) \quad T_1 := TR_1$$

$TOLD := T_2$

$$T_2 := \frac{C_{2,1} \cdot TR_1 + C_{2,3} \cdot T_3 + C_{2,4} \cdot T_4}{C_{2,1} + C_{2,3} + C_{2,4}} \qquad TR_2 := TOLD + \beta \cdot (T_2 - TOLD) \quad T_2 := TR_2$$

$TOLD := T_3$

$$T_3 := \frac{C_{3,2} \cdot TR_2 + C_{3,4} \cdot T_4 + C_{3,5} \cdot T_5}{C_{3,2} + C_{3,4} + C_{3,5}} \qquad TR_3 := TOLD + \beta \cdot (T_3 - TOLD) \quad T_3 := TR_3$$

$$TOLD := T_4$$

$$T_4 := \frac{C_{4,2} \cdot TR_2 + C_{4,3} \cdot TR_3 + C_{4,6} \cdot T_6}{C_{4,2} + C_{4,3} + C_{4,6}} \qquad TR_4 := TOLD + \beta \cdot (T_4 - TOLD) \quad T_4 := TR_4$$

$$TOLD := T_5$$

$$T_5 := \frac{C_{5,3} \cdot TR_3 + C_{5,6} \cdot T_6 + C_{5,7} \cdot T_7}{C_{5,3} + C_{5,6} + C_{5,7}} \qquad TR_5 := TOLD + \beta \cdot (T_5 - TOLD) \quad T_5 := TR_5$$

$$TOLD := T_6$$

$$T_6 := \frac{C_{6,4} \cdot TR_4 + C_{6,5} \cdot TR_5 + C_{6,7} \cdot T_7}{C_{6,4} + C_{6,5} + C_{6,7}} \qquad TR_6 := TOLD + \beta \cdot (T_6 - TOLD) \quad T_6 := TR_6$$

$$T = \begin{pmatrix} 56.182 \\ 54.977 \\ 26.04 \\ 26.686 \\ 25.701 \\ 25.969 \\ 20 \end{pmatrix} \qquad TR = \begin{pmatrix} 56.182 \\ 54.977 \\ 26.04 \\ 26.686 \\ 25.701 \\ 25.969 \end{pmatrix}$$

Eigth Iteration:

$$TOLD := T_1$$

$$T_1 := \frac{C_{1,2} \cdot T_2 + Q_1}{C_{1,2}} \qquad TR_1 := TOLD + \beta \cdot (T_1 - TOLD) \quad T_1 := TR_1$$

$$TOLD := T_2$$

$$T_2 := \frac{C_{2,1} \cdot TR_1 + C_{2,3} \cdot T_3 + C_{2,4} \cdot T_4}{C_{2,1} + C_{2,3} + C_{2,4}} \qquad TR_2 := TOLD + \beta \cdot (T_2 - TOLD) \quad T_2 := TR_2$$

$$TOLD := T_3$$

$$T_3 := \frac{C_{3,2} \cdot TR_2 + C_{3,4} \cdot T_4 + C_{3,5} \cdot T_5}{C_{3,2} + C_{3,4} + C_{3,5}}$$

$TR_3 := TOLD + \beta \cdot (T_3 - TOLD)$ $T_3 := TR_3$

$TOLD := T_4$

$$T_4 := \frac{C_{4,2} \cdot TR_2 + C_{4,3} \cdot TR_3 + C_{4,6} \cdot T_6}{C_{4,2} + C_{4,3} + C_{4,6}}$$

$TR_4 := TOLD + \beta \cdot (T_4 - TOLD)$ $T_4 := TR_4$

$TOLD := T_5$

$$T_5 := \frac{C_{5,3} \cdot TR_3 + C_{5,6} \cdot T_6 + C_{5,7} \cdot T_7}{C_{5,3} + C_{5,6} + C_{5,7}}$$

$TR_5 := TOLD + \beta \cdot (T_5 - TOLD)$ $T_5 := TR_5$

$TOLD := T_6$

$$T_6 := \frac{C_{6,4} \cdot TR_4 + C_{6,5} \cdot TR_5 + C_{6,7} \cdot T_7}{C_{6,4} + C_{6,5} + C_{6,7}}$$

$TR_6 := TOLD + \beta \cdot (T_6 - TOLD)$ $T_6 := TR_6$

$$T = \begin{pmatrix} 55.834 \\ 54.507 \\ 26.579 \\ 27.262 \\ 26.174 \\ 26.61 \\ 20 \end{pmatrix} \qquad TR = \begin{pmatrix} 55.834 \\ 54.507 \\ 26.579 \\ 27.262 \\ 26.174 \\ 26.61 \end{pmatrix}$$

Ninth Iteration:

$TOLD := T_1$

$$T_1 := \frac{C_{1,2} \cdot T_2 + Q_1}{C_{1,2}}$$

$TR_1 := TOLD + \beta \cdot (T_1 - TOLD)$ $T_1 := TR_1$

$TOLD := T_2$

$$T_2 := \frac{C_{2,1} \cdot TR_1 + C_{2,3} \cdot T_3 + C_{2,4} \cdot T_4}{C_{2,1} + C_{2,3} + C_{2,4}}$$

$TR_2 := TOLD + \beta \cdot (T_2 - TOLD)$ $T_2 := TR_2$

$TOLD := T_3$

$$T_3 := \frac{C_{3,2} \cdot TR_2 + C_{3,4} \cdot T_4 + C_{3,5} \cdot T_5}{C_{3,2} + C_{3,4} + C_{3,5}} \qquad TR_3 := TOLD + \beta \cdot (T_3 - TOLD) \quad T_3 := TR_3$$

$$TOLD := T_4$$

$$T_4 := \frac{C_{4,2} \cdot TR_2 + C_{4,3} \cdot TR_3 + C_{4,6} \cdot T_6}{C_{4,2} + C_{4,3} + C_{4,6}} \qquad TR_4 := TOLD + \beta \cdot (T_4 - TOLD) \quad T_4 := TR_4$$

$$TOLD := T_5$$

$$T_5 := \frac{C_{5,3} \cdot TR_3 + C_{5,6} \cdot T_6 + C_{5,7} \cdot T_7}{C_{5,3} + C_{5,6} + C_{5,7}} \qquad TR_5 := TOLD + \beta \cdot (T_5 - TOLD) \quad T_5 := TR_5$$

$$TOLD := T_6$$

$$T_6 := \frac{C_{6,4} \cdot TR_4 + C_{6,5} \cdot TR_5 + C_{6,7} \cdot T_7}{C_{6,4} + C_{6,5} + C_{6,7}} \qquad TR_6 := TOLD + \beta \cdot (T_6 - TOLD) \quad T_6 := TR_6$$

$$T = \begin{pmatrix} 55.278 \\ 53.964 \\ 27.018 \\ 27.752 \\ 26.605 \\ 26.899 \\ 20 \end{pmatrix} \qquad TR = \begin{pmatrix} 55.278 \\ 53.964 \\ 27.018 \\ 27.752 \\ 26.605 \\ 26.899 \end{pmatrix}$$

Tenth Iteration:

$$TOLD := T_1$$

$$T_1 := \frac{C_{1,2} \cdot T_2 + Q_1}{C_{1,2}} \qquad TR_1 := TOLD + \beta \cdot (T_1 - TOLD) \quad T_1 := TR_1$$

$$TOLD := T_2$$

$$T_2 := \frac{C_{2,1} \cdot TR_1 + C_{2,3} \cdot T_3 + C_{2,4} \cdot T_4}{C_{2,1} + C_{2,3} + C_{2,4}} \qquad TR_2 := TOLD + \beta \cdot (T_2 - TOLD) \quad T_2 := TR_2$$

$$TOLD := T_3$$

$$T_3 := \frac{C_{3,2} \cdot TR_2 + C_{3,4} \cdot T_4 + C_{3,5} \cdot T_5}{C_{3,2} + C_{3,4} + C_{3,5}} \qquad TR_3 := TOLD + \beta \cdot (T_3 - TOLD) \quad T_3 := TR_3$$

$TOLD := T_4$

$$T_4 := \frac{C_{4,2} \cdot TR_2 + C_{4,3} \cdot TR_3 + C_{4,6} \cdot T_6}{C_{4,2} + C_{4,3} + C_{4,6}} \qquad TR_4 := TOLD + \beta \cdot (T_4 - TOLD) \quad T_4 := TR_4$$

$TOLD := T_5$

$$T_5 := \frac{C_{5,3} \cdot TR_3 + C_{5,6} \cdot T_6 + C_{5,7} \cdot T_7}{C_{5,3} + C_{5,6} + C_{5,7}} \qquad TR_5 := TOLD + \beta \cdot (T_5 - TOLD) \quad T_5 := TR_5$$

$TOLD := T_6$

$$T_6 := \frac{C_{6,4} \cdot TR_4 + C_{6,5} \cdot TR_5 + C_{6,7} \cdot T_7}{C_{6,4} + C_{6,5} + C_{6,7}} \qquad TR_6 := TOLD + \beta \cdot (T_6 - TOLD) \quad T_6 := TR_6$$

$$T = \begin{pmatrix} 54.744 \\ 53.501 \\ 27.443 \\ 27.958 \\ 26.926 \\ 27.098 \\ 20 \end{pmatrix} \qquad TR = \begin{pmatrix} 54.744 \\ 53.501 \\ 27.443 \\ 27.958 \\ 26.926 \\ 27.098 \end{pmatrix}$$

Application Example 13.2: Solution of Steady-State Network Using Simultaneous Equation Method

Input Data: $t_1 := 0.1$ $k_1 := 1.0$ $w_1 := 1.0$ $l_1 := 1.0$ $S1 := 10.0$

$t_2 := 0.05$ $k_2 := 0.02$ $w_2 := 1.0$ $l_2 := 1.0$ $t_3 := 0.5$ $k_3 := 10.0$ $w_3 := 1.0$ $l_3 := 0.5$

$t_4 := 0.5$ $k_4 := 4.0$ $w_4 := 1.0$ $l_4 := 0.5$ $h_8 := 2$ $h_9 := 1$ $TA := 20$

Calculate Conductances:

$i := 1,2..7$

$j := 1,2..7$

$C_{i,j} := 0.0$ $\quad Q_i := 0$

$$C_{1,2} := \frac{k_1 \cdot w_1 \cdot l_1}{t_1} \qquad C_{2,3} := \frac{k_2 \cdot w_2 \cdot \frac{l_2}{2}}{t_2} \qquad C_{2,4} := \frac{k_2 \cdot w_2 \cdot \frac{l_2}{2}}{t_2} \qquad C_{3,5} := \frac{k_3 \cdot w_3 \cdot l_3}{t_3}$$

$$C_{3,4} := \frac{1}{\dfrac{\frac{l_3}{2}}{k_3 \cdot \frac{t_3}{2} \cdot w_3} + \dfrac{\frac{l_4}{2}}{k_4 \cdot \frac{t_4}{2} \cdot w_4}} \qquad C_{5,6} := \frac{1}{\dfrac{\frac{l_3}{2}}{k_3 \cdot \frac{t_3}{2} \cdot w_3} + \dfrac{\frac{l_4}{2}}{k_4 \cdot \frac{t_4}{2} \cdot w_4}}$$

$$C_{4,6} := \frac{k_4 \cdot w_4 \cdot l_4}{t_4} \qquad C_{5,7} := h_8 \cdot w_3 \cdot l_3 \qquad C_{6,7} := h_9 \cdot w_4 \cdot l_4$$

$C_{2,1} := C_{1,2}$ $\quad C_{3,2} := C_{2,3}$ $\quad C_{4,2} := C_{2,4}$ $\quad C_{5,3} := C_{3,5}$ $\quad C_{4,3} := C_{3,4}$

$C_{6,5} := C_{5,6}$ $\quad C_{6,4} := C_{4,6}$ $\quad C_{7,5} := C_{5,7}$ $\quad C_{7,6} := C_{6,7}$

Set Up Equations:

Define Conductance, Source Matrices

$i := 1,2..6$

$j := 1,2..6$

$G_{i,j} := 0.0$ $\quad S_j := 0.0$ $\quad T_j := 30$ $\quad T_7 := TA$

$$S_1 := S1$$

Equation 1 Elements:

$G_{1,1} := C_{1,2}$ $\quad G_{1,2} := -C_{1,2}$ $\quad S_1 := 10.0$

Equation 2 Elements:

$G_{2,1} := -C_{2,1}$ $\quad G_{2,2} := C_{2,1} + C_{2,3} + C_{2,4}$ $\quad G_{2,3} := -C_{2,3}$ $\quad G_{2,4} := -C_{2,4}$

Equation 3 Elements:

$G_{3,2} := -C_{3,2}$ $\quad G_{3,3} := C_{3,2} + C_{3,4} + C_{3,5}$ $\quad G_{3,4} := -C_{3,4}$ $\quad G_{3,5} := -C_{3,5}$

Equation 4 Elements:

$G_{4,2} := -C_{4,2}$ $\quad G_{4,3} := -C_{4,3}$ $\quad G_{4,4} := C_{4,2} + C_{4,3} + C_{4,6}$ $\quad G_{4,6} := -C_{4,6}$

Equation 5 Elements:

$G_{5,3} := -C_{5,3}$ $\quad G_{5,6} := -C_{5,6}$ $\quad G_{5,5} := C_{5,3} + C_{5,6} + C_{5,7}$ $\quad S_5 := C_{5,7} \cdot T_7$

Equation 6 Elements:

$G_{6,4} := -C_{6,4}$ $\quad G_{6,5} := -C_{6,5}$ $\quad G_{6,6} := C_{6,4} + C_{6,5} + C_{6,7}$ $\quad S_6 := C_{6,7} \cdot T_7$

$$G = \begin{pmatrix} 10 & -10 & 0 & 0 & 0 & 0 \\ -10 & 10.4 & -0.2 & -0.2 & 0 & 0 \\ 0 & -0.2 & 13.057 & -2.857 & -10 & 0 \\ 0 & -0.2 & -2.857 & 7.057 & 0 & -4 \\ 0 & 0 & -10 & 0 & 13.857 & -2.857 \\ 0 & 0 & 0 & -4 & -2.857 & 7.357 \end{pmatrix}$$

Solve Problem by Matrix Inversion:

$$T := G^{-1} \cdot S$$

$$T = \begin{pmatrix} 53.467 \\ 52.467 \\ 27.252 \\ 27.682 \\ 26.625 \\ 26.749 \end{pmatrix}$$

$$r_1 := Q_1 - C_{1,2}\cdot(T_1 - T_2) \quad r_2 := -(T_2 - T_1)\cdot C_{2,1} - (T_2 - T_3)\cdot C_{2,3} - (T_2 - T_4)\cdot C_{2,4}$$

$$r_3 := -(T_3 - T_2)\cdot C_{3,2} - (T_3 - T_4)\cdot C_{3,4} - (T_3 - T_5)\cdot C_{3,5}$$

$$r_4 := -(T_4 - T_2)\cdot C_{4,2} - (T_4 - T_3)\cdot C_{4,3} - (T_4 - T_6)\cdot C_{4,6}$$

$$r_5 := -(T_5 - T_3)\cdot C_{5,3} - (T_5 - T_6)\cdot C_{5,6} - (T_5 - TA)\cdot C_{5,7}$$

$$r_6 := -(T_6 - T_4)\cdot C_{6,4} - (T_6 - T_5)\cdot C_{6,5} - (T_6 - TA)\cdot C_{6,7}$$

$$r := \begin{bmatrix} S_1 - C_{1,2}\cdot(T_1 - T_2) \\ -(T_2 - T_1)\cdot C_{2,1} - (T_2 - T_3)\cdot C_{2,3} - (T_2 - T_4)\cdot C_{2,4} \\ -(T_3 - T_2)\cdot C_{3,2} - (T_3 - T_4)\cdot C_{3,4} - (T_3 - T_5)\cdot C_{3,5} \\ -(T_4 - T_2)\cdot C_{4,2} - (T_4 - T_3)\cdot C_{4,3} - (T_4 - T_6)\cdot C_{4,6} \\ -(T_5 - T_3)\cdot C_{5,3} - (T_5 - T_6)\cdot C_{5,6} - (T_5 - TA)\cdot C_{5,7} \\ -(T_6 - T_4)\cdot C_{6,4} - (T_6 - T_5)\cdot C_{6,5} - (T_6 - TA)\cdot C_{6,7} \end{bmatrix}$$

$$EB1 := \frac{\sum_{i=1}^{6} r_i}{S_1}\cdot 100 \qquad EB2 := \frac{\sum_{i=1}^{6} \left|r_i\right|}{S_1}\cdot 100$$

$$EB1 = -9.237 \times 10^{-13} \qquad EB2 = 2.878 \times 10^{-12}$$

Application Example 13.4: Solution of Time-Dependent Network

Input Data: **Density units kg/m^3, CP J/(kg K), ΔV m^3 to give Capcitance units C* J/K or J/^{0}C.**

$t_1 := 0.1$ $k_1 := 1.0$ $w_1 := 1.0$ $l_1 := 1.0$ $Q1 := 10.0$ $\rho_1 := 4000$ $CP_1 := 800$

$t_2 := 0.05$ $k_2 := 0.02$ $w_2 := 1.0$ $l_2 := 1.0$ $\rho_2 := 2000$ $CP_2 := 700$

$t_3 := 0.5$ $k_3 := 10.0$ $w_3 := 1.0$ $l_3 := 0.5$ $\rho_3 := 10^4$ $CP_3 := 400$

$t_4 := 0.5$ $k_4 := 4.0$ $w_4 := 1.0$ $l_4 := 0.5$ $\rho_4 := 3000$ $CP_4 := 800$

$h_8 := 2$ $h_9 := 1$ $TA := 20$

Calculate Conductances:

$$C1\_2 := \frac{k_1 \cdot w_1 \cdot l_1}{t_1} \quad C2\_3 := \frac{k_2 \cdot w_2 \cdot \frac{l_2}{2}}{t_2} \quad C2\_4 := \frac{k_2 \cdot w_2 \cdot \frac{l_2}{2}}{t_2} \quad C3\_5 := \frac{k_3 \cdot w_3 \cdot l_3}{t_3}$$

$$C3\_4 := \frac{1}{\dfrac{\frac{l_3}{2}}{k_3 \cdot \frac{t_3}{2} \cdot w_3} + \dfrac{\frac{l_4}{2}}{k_4 \cdot \frac{t_4}{2} \cdot w_4}} \quad C5\_6 := \frac{1}{\dfrac{\frac{l_3}{2}}{k_3 \cdot \frac{t_3}{2} \cdot w_3} + \dfrac{\frac{l_4}{2}}{k_4 \cdot \frac{t_4}{2} \cdot w_4}}$$

$$C4\_6 := \frac{k_4 \cdot w_4 \cdot l_4}{t_4} \quad C5\_7 := h_8 \cdot w_3 \cdot l_3 \quad C6\_7 := h_9 \cdot w_4 \cdot l_4$$

$C2\_1 := C1\_2$ $C3\_2 := C2\_3$ $C4\_2 := C2\_4$ $C5\_3 := C3\_5$ $C4\_3 := C3\_4$

$C6\_5 := C5\_6$ $C6\_4 := C4\_6$ $C7\_5 := C5\_7$ $C7\_6 := C6\_7$

Calculate Total Capacitance of Each Node:

$$\Delta VN1 := \frac{t_1}{2} \cdot l_1 \cdot w_1 \cdot \left(\frac{2.54}{100}\right)^3 \quad CS1 := \rho_1 \cdot CP_1 \cdot \Delta VN1$$

$\Delta VN1 = 8.194 \times 10^{-7}$ $CS1 = 2.622$

$$\Delta VN2A := \frac{t_1}{2} \cdot l_1 \cdot w_1 \cdot \left(\frac{2.54}{100}\right)^3 \quad \Delta VN2B := \frac{t_2}{2} \cdot l_2 \cdot w_2 \cdot \left(\frac{2.54}{100}\right)^3 \quad CS2 := \rho_1 \cdot CP_1 \cdot \Delta VN2A + \rho_2 \cdot CP_2 \cdot \Delta VN2B$$

$\Delta VN2A = 8.194 \times 10^{-7}$ $\Delta VN2B = 4.097 \times 10^{-7}$ $CS2 = 3.195$

$$\Delta VN3A := \frac{t_2}{2} \cdot l_3 \cdot w_3 \cdot \left(\frac{2.54}{100}\right)^3 \quad \Delta VN3B := \frac{t_3}{2} \cdot l_3 \cdot w_3 \cdot \left(\frac{2.54}{100}\right)^3 \quad CS3 := \rho_2 \cdot CP_2 \cdot \Delta VN3A + \rho_3 \cdot CP_3 \cdot \Delta VN3B$$

$\Delta VN3A = 2.048 \times 10^{-7}$ $\Delta VN3B = 2.048 \times 10^{-6}$ $CS3 = 8.48$

$\Delta VN4A := \Delta VN3A$ $\quad$ $\Delta VN4B := \Delta VN3B$ $\quad$ $CS4 := \rho_2 \cdot CP_2 \cdot \Delta VN4A + \rho_4 \cdot CP_4 \cdot \Delta VN4B$

$\Delta VN4A = 2.048 \times 10^{-7}$ $\quad$ $\Delta VN4B = 2.048 \times 10^{-6}$ $\quad$ $CS4 = 5.203$

$$\Delta VN5 := \frac{t_3}{2} \cdot l_3 \cdot w_3 \cdot \left(\frac{2.54}{100}\right)^3 \qquad \Delta VN6 := \frac{t_4}{2} \cdot l_4 \cdot w_4 \cdot \left(\frac{2.54}{100}\right)^3 \qquad CS5 := \rho_3 \cdot CP_3 \cdot \Delta VN5 \qquad CS6 := \rho_4 \cdot CP_4 \cdot \Delta VN6$$

$\Delta VN5 = 2.048 \times 10^{-6}$ $\quad$ $\Delta VN6 = 2.048 \times 10^{-6}$ $\quad$ $CS5 = 8.194$ $\quad$ $CS6 = 4.916$

Calculate Maximum Time Step (for those methods that have stability criteria) for Each Node:

$$\Delta t1 := \frac{CS1}{C1\_2} \quad \Delta t2 := \frac{CS2}{C1\_2 + C2\_3 + C2\_4} \quad \Delta t3 := \frac{CS3}{C3\_2 + C3\_4 + C3\_5} \quad \Delta t4 := \frac{CS4}{C4\_2 + C4\_3 + C4\_6}$$

$$\Delta t5 := \frac{CS5}{C5\_3 + C5\_6 + C5\_7} \quad \Delta t6 := \frac{CS6}{C6\_4 + C6\_5 + C6\_7}$$

$\Delta t1 = 0.262$ $\quad$ $\Delta t2 = 0.307$ $\quad$ $\Delta t3 = 0.649$ $\quad$ $\Delta t4 = 0.737$ $\quad$ $\Delta t5 = 0.591$ $\quad$ $\Delta t6 = 0.668$

$\Delta t := 0.2$

$$S1 := \frac{\Delta t}{CS1} \cdot C1\_2 \qquad S2 := \frac{\Delta t}{CS2} \cdot (C1\_2 + C2\_3 + C2\_4) \qquad S3 := \frac{\Delta t}{CS3} \cdot (C3\_2 + C3\_4 + C3\_5)$$

$$S4 := \frac{\Delta t}{CS4} \cdot (C4\_2 + C4\_3 + C4\_6) \qquad S5 := \frac{\Delta t}{CS5} \cdot (C5\_3 + C5\_6 + C5\_7) \qquad S6 := \frac{\Delta t}{CS6} \cdot (C6\_4 + C6\_5 + C6\_7)$$

$S1 = 0.763$ $\quad$ $S2 = 0.651$ $\quad$ $S3 = 0.308$ $\quad$ $S4 = 0.271$ $\quad$ $S5 = 0.338$ $\quad$ $S6 = 0.299$

Set All Starting Temperatures:

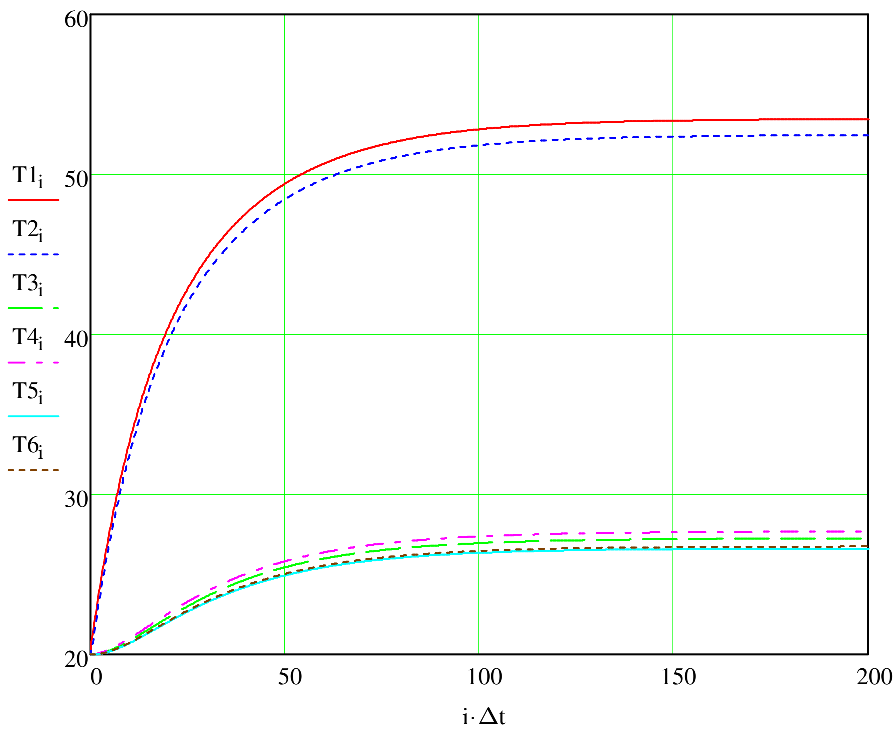
$$\begin{pmatrix} T1_0 \\ T2_0 \\ T3_0 \\ T4_0 \\ T5_0 \\ T6_0 \end{pmatrix} := \begin{pmatrix} TA \\ TA \\ TA \\ TA \\ TA \\ TA \end{pmatrix}$$

Solution Using Forward Finite Difference in Time:

$EndTime := 200$ $\quad$ $\Delta t := 0.2$ $\quad$ $MaxIt := \frac{EndTime}{\Delta t}$ $\quad$ $MaxIt = 1 \times 10^3$

$i := 0, 1 .. MaxIt$

$$
\begin{pmatrix} T1_{i+1} \\ T2_{i+1} \\ T3_{i+1} \\ T4_{i+1} \\ T5_{i+1} \\ T6_{i+1} \end{pmatrix} := \begin{bmatrix} T1_i\cdot(1-S1) + \dfrac{\Delta t}{CS1}\cdot\left(Q1 + C1\_2\cdot T2_i\right) \\ T2_i\cdot(1-S2) + \dfrac{\Delta t}{CS2}\cdot\left(C2\_1\cdot T1_i + C2\_3\cdot T3_i + C2\_4\cdot T4_i\right) \\ T3_i\cdot(1-S3) + \dfrac{\Delta t}{CS3}\cdot\left(C3\_2\cdot T2_i + C3\_4\cdot T4_i + C3\_5\cdot T5_i\right) \\ T4_i\cdot(1-S4) + \dfrac{\Delta t}{CS4}\cdot\left(C4\_2\cdot T2_i + C4\_3\cdot T3_i + C4\_6\cdot T6_i\right) \\ T5_i(1-S5) + \dfrac{\Delta t}{CS5}\cdot\left(C5\_3\cdot T3_i + C5\_6\cdot T6_i + C5\_7\cdot TA\right) \\ T6_i(1-S6) + \dfrac{\Delta t}{CS6}\cdot\left(C6\_4\cdot T4_i + C6\_5\cdot T5_i + C6\_7\cdot TA\right) \end{bmatrix}
$$



$T1_{MaxIt} = 53.45$

$T2_{MaxIt} = 52.45$

$T3_{MaxIt} = 27.245$

$T4_{MaxIt} = 27.674$

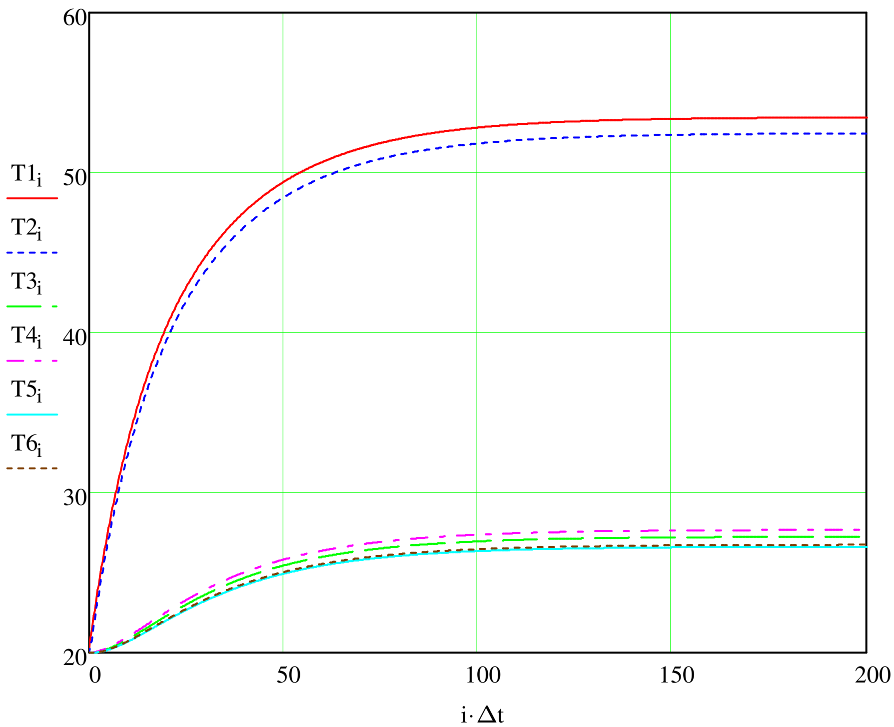
$T5_{MaxIt} = 26.618$

$T6_{MaxIt} = 26.742$

Forward Finite Difference in Time According to Holman:

$i := 0, 1 .. MaxIt$

$$
\begin{pmatrix} T1_{i+1} \\ T2_{i+1} \\ T3_{i+1} \\ T4_{i+1} \\ T5_{i+1} \\ T6_{i+1} \end{pmatrix} := \begin{bmatrix} \dfrac{\Delta t}{CS1}\cdot\left[Q1 + C1\_2\cdot\left(T2_i - T1_i\right)\right] + T1_i \\ \dfrac{\Delta t}{CS2}\cdot\left[C2\_1\cdot\left(T1_i - T2_i\right) + C2\_3\cdot\left(T3_i - T2_i\right) + C2\_4\cdot\left(T4_i - T2_i\right)\right] + T2_i \\ \dfrac{\Delta t}{CS3}\cdot\left[C3\_2\cdot\left(T2_i - T3_i\right) + C3\_4\cdot\left(T4_i - T3_i\right) + C3\_5\cdot\left(T5_i - T3_i\right)\right] + T3_i \\ \dfrac{\Delta t}{CS4}\cdot\left[C4\_2\cdot\left(T2_i - T4_i\right) + C4\_3\cdot\left(T3_i - T4_i\right) + C4\_6\cdot\left(T6_i - T4_i\right)\right] + T4_i \\ \dfrac{\Delta t}{CS5}\cdot\left[C5\_3\cdot\left(T3_i - T5_i\right) + C5\_6\cdot\left(T6_i - T5_i\right) + C5\_7\cdot\left(TA - T5_i\right)\right] + T5_i \\ \dfrac{\Delta t}{CS6}\cdot\left[C6\_4\cdot\left(T4_i - T6_i\right) + C6\_5\cdot\left(T5_i - T6_i\right) + C6\_7\cdot\left(TA - T6_i\right)\right] + T6_i \end{bmatrix}
$$



$T1_{MaxIt} = 53.45$

$T2_{MaxIt} = 52.45$

$T3_{MaxIt} = 27.245$

$T4_{MaxIt} = 27.674$

$T5_{MaxIt} = 26.618$

$T6_{MaxIt} = 26.742$

Backward Difference in Time Using First Method with Equations Resembling Gauss-Seidel:

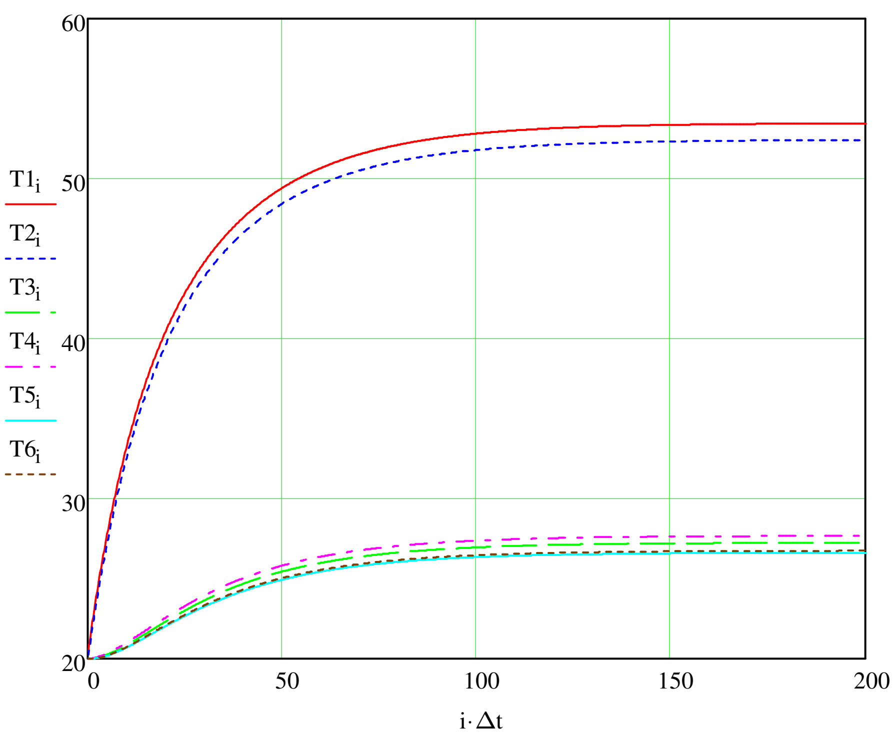
In this method we must also set for time t = 0 AND t = 0 + Δt.

$$\begin{pmatrix} T1_0 \\ T2_0 \\ T3_0 \\ T4_0 \\ T5_0 \\ T6_0 \end{pmatrix} := \begin{pmatrix} TA \\ TA \\ TA \\ TA \\ TA \\ TA \end{pmatrix}$$

$\Delta t := 0.2$ $\qquad MaxIt := \dfrac{EndTime}{\Delta t}$ $\qquad MaxIt = 1 \times 10^3$

$i := 1, 2 .. MaxIt$

$$
\begin{pmatrix} T1_{i} \\ T2_{i} \\ T3_{i} \\ T4_{i} \\ T5_{i} \\ T6_{i} \end{pmatrix} :=
\begin{bmatrix}
\dfrac{\left(C1\_2 \cdot T2_{i}\right) + \dfrac{CS1}{\Delta t} \cdot T1_{i-1} + Q1}{C1\_2 + \dfrac{CS1}{\Delta t}} \\[2ex]
\dfrac{\left(C2\_1 \cdot T1_{i} + C2\_3 \cdot T3_{i} + C2\_4 \cdot T4_{i}\right) + \dfrac{CS2}{\Delta t} \cdot T2_{i} - 1}{(C2\_1 + C2\_3 + C2\_4) + \dfrac{CS2}{\Delta t}} \\[2ex]
\dfrac{\left(C3\_2 \cdot T2_{i} + C3\_4 \cdot T4_{i} + C3\_5 \cdot T5_{i}\right) + \dfrac{CS3}{\Delta t} \cdot T3_{i-1}}{(C3\_2 + C3\_4 + C3\_5) + \dfrac{CS3}{\Delta t}} \\[2ex]
\dfrac{\left(C4\_2 \cdot T2_{i} + C4\_3 \cdot T3_{i} + C4\_6 \cdot T6_{i}\right) + \dfrac{CS4}{\Delta t} \cdot T4_{i-1}}{(C4\_2 + C4\_3 + C4\_6) + \dfrac{CS4}{\Delta t}} \\[2ex]
\dfrac{\left(C5\_3 \cdot T3_{i} + C5\_6 \cdot T6_{i} + C5\_7 \cdot TA\right) + \dfrac{CS5}{\Delta t} \cdot T5_{i-1}}{(C5\_3 + C5\_6 + C5\_7) + \dfrac{CS5}{\Delta t}} \\[2ex]
\dfrac{\left(C6\_4 \cdot T4_{i} + C6\_5 \cdot T5_{i} + C6\_7 \cdot TA\right) + \dfrac{CS6}{\Delta t} \cdot T6_{i-1}}{(C6\_4 + C6\_5 + C6\_7) + \dfrac{CS6}{\Delta t}}
\end{bmatrix}
$$



$T1_{MaxIt} = 53.45$

$T2_{MaxIt} = 52.413$

$T3_{MaxIt} = 27.245$

$T4_{MaxIt} = 27.674$

$T5_{MaxIt} = 26.618$

$T6_{MaxIt} = 26.742$

Backward Difference in Time Using Second Method with Equations Solvable as Simultaneous:

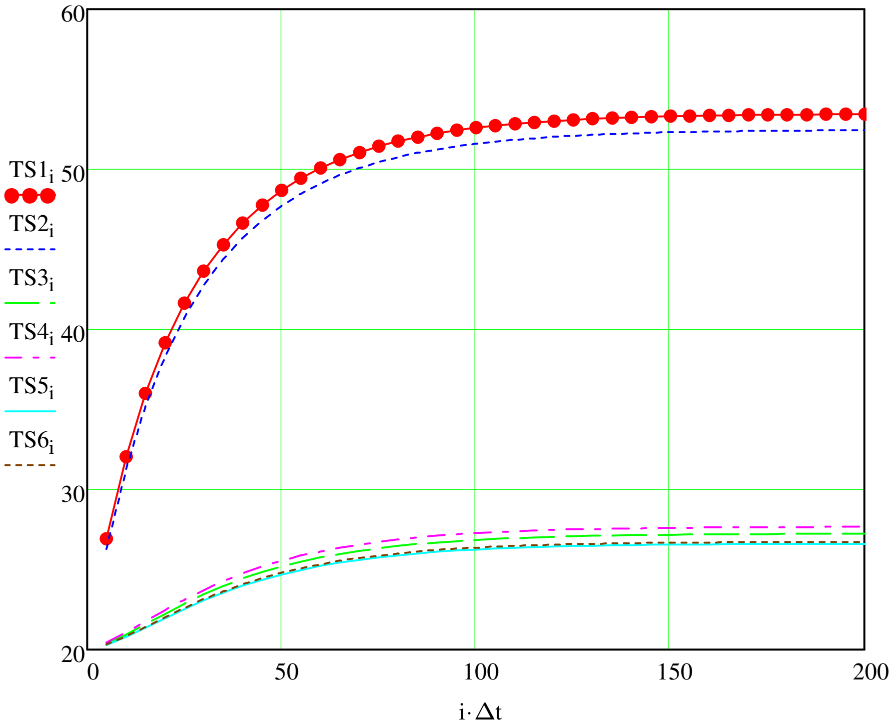
$\Delta t := 5$ $\quad$ $MaxIt := \dfrac{EndTime}{\Delta t}$ $\quad$ $MaxIt = 40$

$$\begin{pmatrix} TS1_0 \\ TS2_0 \\ TS3_0 \\ TS4_0 \\ TS5_0 \\ TS6_0 \end{pmatrix} := \begin{pmatrix} TA \\ TA \\ TA \\ TA \\ TA \\ TA \end{pmatrix} \qquad Time_0 := 0$$

$i := 1, 2 .. MaxIt$

$Time_i := i \cdot \Delta t$

$$\begin{pmatrix} TS1_i \\ TS2_i \\ TS3_i \\ TS4_i \\ TS5_i \\ TS6_i \end{pmatrix} := \begin{bmatrix} \left(\frac{CS1}{\Delta t} + C1\_2\right) & -C1\_2 & 0 & 0 \\ -C2\_1 & \left(\frac{CS2}{\Delta t} + C2\_1 + C2\_3 + C2\_4\right) & 0 - C2\_3 & -C2\_4 \\ 0 & -C3\_2 & \left(\frac{CS3}{\Delta t} + C3\_2 + C3\_4 + C3\_5\right) & -C3\_4 \\ 0 & -C4\_2 & -C4\_3 & \left(\frac{CS4}{\Delta t} + C4\_2 + C4\_3 + C4\_6\right) \\ 0 & 0 & -C5\_3 & 0 \\ 0 & 0 & 0 & -C6\_4 \end{bmatrix}$$



$TS1_{MaxIt} = 53.436$

$TS2_{MaxIt} = 52.436$

$TS3_{MaxIt} = 27.238$

$TS4_{MaxIt} = 27.667$

$TS5_{MaxIt} = 26.612$

$TS6_{MaxIt} = 26.736$

...\Example 13_4 Time.dat	...\Example 13_4 T1.dat	...\Example 13_4 T2.dat	...\Example 13_4 T3.dat	...\Example 13_4 T4.dat
Time	TS1	TS2	TS3	TS4

...\Example 13_4 T5.dat	...\Example 13_4 T6.dat
TS5	TS6

$$
\left.\begin{array}{cc}
0 & 0 \\
0 & 0 \\
-C3\_5 & 0 \\
0 & -C4\_6 \\
\left(\frac{CS5}{\Delta t} + C5\_3 + C5\_6 + C5\_7\right) & -C5\_6 \\
-C6\_5 & \left(\frac{CS6}{\Delta t} + C6\_4 + C6\_5 + C6\_7\right)
\end{array}\right]^{-1} \cdot \left(\begin{array}{c}
\frac{CS1}{\Delta t} \cdot TS1_{i-1} + Q1 \\
\frac{CS2}{\Delta t} \cdot TS2_{i-1} \\
\frac{CS3}{\Delta t} \cdot TS3_{i-1} \\
\frac{CS4}{\Delta t} \cdot TS4_{i-1} \\
\frac{CS5}{\Delta t} \cdot TS5_{i-1} + C5\_7 \cdot TA \\
\frac{CS6}{\Delta t} \cdot TS6_{i-1} + C6\_7 \cdot TA
\end{array}\right)
$$

Application Example 13.2: Solution of Steady-State Network Using Gauss-Seidel Method - No Relaxation

Input Data:

$t_1 := 0.1$ $k_1 := 1.0$ $w_1 := 1.0$ $l_1 := 1.0$ $Q1 := 10.0$

$t_2 := 0.05$ $k_2 := 0.02$ $w_2 := 1.0$ $l_2 := 1.0$

$t_3 := 0.5$ $k_3 := 10.0$ $w_3 := 1.0$ $l_3 := 0.5$

$t_4 := 0.5$ $k_4 := 4.0$ $w_4 := 1.0$ $l_4 := 0.5$

$h_8 := 2$ $h_9 := 1$ $TA := 20$

Calculate Conductances:

$$C1\_2 := \frac{k_1 \cdot w_1 \cdot l_1}{t_1} \quad C2\_3 := \frac{k_2 \cdot w_2 \cdot \frac{l_2}{2}}{t_2} \quad C2\_4 := \frac{k_2 \cdot w_2 \cdot \frac{l_2}{2}}{t_2} \quad C3\_5 := \frac{k_3 \cdot w_3 \cdot l_3}{t_3}$$

$$C3\_4 := \frac{1}{\dfrac{\frac{l_3}{2}}{k_3 \cdot \frac{t_3}{2} \cdot w_3} + \dfrac{\frac{l_4}{2}}{k_4 \cdot \frac{t_4}{2} \cdot w_4}} \quad C5\_6 := \frac{1}{\dfrac{\frac{l_3}{2}}{k_3 \cdot \frac{t_3}{2} \cdot w_3} + \dfrac{\frac{l_4}{2}}{k_4 \cdot \frac{t_4}{2} \cdot w_4}}$$

$$C4\_6 := \frac{k_4 \cdot w_4 \cdot l_4}{t_4} \quad C5\_7 := h_8 \cdot w_3 \cdot l_3 \quad C6\_7 := h_9 \cdot w_4 \cdot l_4$$

$C2\_1 := C1\_2$ $C3\_2 := C2\_3$ $C4\_2 := C2\_4$ $C5\_3 := C3\_5$ $C4\_3 := C3\_4$

$C6\_5 := C5\_6$ $C6\_4 := C4\_6$ $C7\_5 := C5\_7$ $C7\_6 := C6\_7$

Solution Using Gauss-Seidel: $\Delta T := 20$

$T1Start := \Delta T + TA$ $T2Start := \Delta T + TA$ $T3Start := \Delta T + TA$

$T4Start := \Delta T + TA$ $T5Start := \Delta T + TA$ $T6Start := \Delta T + TA$

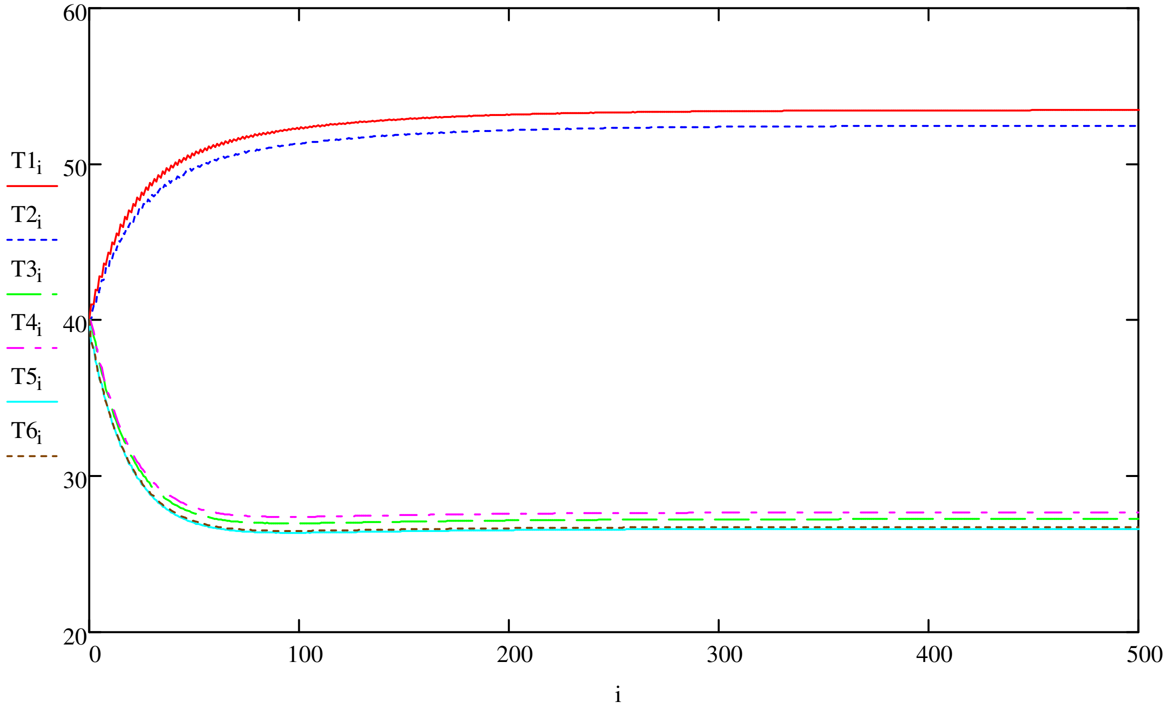
$$\begin{pmatrix} T1_0 \\ T2_0 \\ T3_0 \\ T4_0 \\ T5_0 \\ T6_0 \\ T7_0 \end{pmatrix} := \begin{pmatrix} T1Start \\ T2Start \\ T3Start \\ T4Start \\ T5Start \\ T6Start \\ TA \end{pmatrix}$$

The formulae must be put in a matrix to get variables to know other variable values, but have not found a way to make relaxation work in this method.

$MaxIt := 500$

$i := 0, 1 .. MaxIt$

$$\begin{pmatrix} T1_{i+1} \\ T2_{i+1} \\ T3_{i+1} \\ T4_{i+1} \\ T5_{i+1} \\ T6_{i+1} \\ T7_{i+1} \end{pmatrix} := \begin{pmatrix} \dfrac{C1\_2 \cdot T2_i + Q1}{C1\_2} \\ \dfrac{C2\_1 \cdot T1_i + C2\_3 \cdot T3_i + C2\_4 \cdot T4_i}{C2\_1 + C2\_3 + C2\_4} \\ \dfrac{C3\_2 \cdot T2_i + C3\_4 \cdot T4_i + C3\_5 \cdot T5_i}{C3\_2 + C3\_4 + C3\_5} \\ \dfrac{C4\_2 \cdot T2_i + C4\_3 \cdot T3_i + C4\_6 \cdot T6_i}{C4\_2 + C4\_3 + C4\_6} \\ \dfrac{C5\_3 \cdot T3_i + C5\_6 \cdot T6_i + C5\_7 \cdot T7_i}{C5\_3 + C5\_6 + C5\_7} \\ \dfrac{C6\_4 \cdot T4_i + C6\_5 \cdot T5_i + C6\_7 \cdot T7_i}{C6\_4 + C6\_5 + C6\_7} \\ T7_i \end{pmatrix}$$



$T1_{MaxIt} = 53.462$ $\quad T2_{MaxIt} = 52.462$ $\quad T3_{MaxIt} = 27.251$ $\quad T4_{MaxIt} = 27.68$

$T5_{MaxIt} = 26.624$ $\quad T6_{MaxIt} = 26.748$

$r_1 := Q1 - C1\_2 \cdot (T1_{MaxIt} - T2_{MaxIt}$ $\quad r_2 := -(T2_{MaxIt} - T1_{MaxIt}) \cdot C2\_1 - (T2_{MaxIt} - T3_{MaxIt}) \cdot C2\_3 - (T2_{MaxIt} - T4_{MaxIt}) \cdot C$

$$r_3 := -(T3_{MaxIt} - T2_{MaxIt}) \cdot C3\_2 - (T3_{MaxIt} - T4_{MaxIt}) \cdot C3\_4 - (T3_{MaxIt} - T5_{MaxIt}) \cdot C3\_5$$

$$r_4 := -(T4_{MaxIt} - T2_{MaxIt}) \cdot C4\_2 - (T4_{MaxIt} - T3_{MaxIt}) \cdot C4\_3 - (T4_{MaxIt} - T6_{MaxIt}) \cdot C4\_6$$

$$r_5 := -(T5_{MaxIt} - T3_{MaxIt}) \cdot C5\_3 - (T5_{MaxIt} - T6_{MaxIt}) \cdot C5\_6 - (T5_{MaxIt} - TA) \cdot C5\_7$$

$$r_6 := -(T6_{MaxIt} - T4_{MaxIt}) \cdot C6\_4 - (T6_{MaxIt} - T5_{MaxIt}) \cdot C6\_5 - (T6_{MaxIt} - TA) \cdot C6\_7$$

$$r := \begin{bmatrix} Q1 - C1\_2 \cdot (T1_{MaxIt} - T2_{MaxIt}) \\ -(T2_{MaxIt} - T1_{MaxIt}) \cdot C2\_1 - (T2_{MaxIt} - T3_{MaxIt}) \cdot C2\_3 - (T2_{MaxIt} - T4_{MaxIt}) \cdot C2\_4 \\ -(T3_{MaxIt} - T2_{MaxIt}) \cdot C3\_2 - (T3_{MaxIt} - T4_{MaxIt}) \cdot C3\_4 - (T3_{MaxIt} - T5_{MaxIt}) \cdot C3\_5 \\ -(T4_{MaxIt} - T2_{MaxIt}) \cdot C4\_2 - (T4_{MaxIt} - T3_{MaxIt}) \cdot C4\_3 - (T4_{MaxIt} - T6_{MaxIt}) \cdot C4\_6 \\ -(T5_{MaxIt} - T3_{MaxIt}) \cdot C5\_3 - (T5_{MaxIt} - T6_{MaxIt}) \cdot C5\_6 - (T5_{MaxIt} - TA) \cdot C5\_7 \\ -(T6_{MaxIt} - T4_{MaxIt}) \cdot C6\_4 - (T6_{MaxIt} - T5_{MaxIt}) \cdot C6\_5 - (T6_{MaxIt} - TA) \cdot C6\_7 \end{bmatrix}$$

$$r = \begin{pmatrix} 1.074 \times 10^{-3} \\ 3.74 \times 10^{-4} \\ 2.982 \times 10^{-4} \\ 1.706 \times 10^{-4} \\ 2.813 \times 10^{-4} \\ 1.505 \times 10^{-4} \end{pmatrix}$$

$$EB1 := \frac{\sum_{i=0}^{5} r_i}{Q1} \cdot 100 \qquad EB2 := \frac{\sum_{i=0}^{5} \left| r_i \right|}{Q1} \cdot 100$$

$$EB1 = 2.348 \times 10^{-2} \qquad EB2 = 0.023$$

Application Example 13.2: Solution of Steady-State Network Using Gauss-Seidel Method - Relaxation - 10 Iterations

Input Data:

$t_1 := 0.1$ $\quad k_1 := 1.0$ $\quad w_1 := 1.0$ $\quad l_1 := 1.0$

$t_2 := 0.05$ $\quad k_2 := 0.02$ $\quad w_2 := 1.0$ $\quad l_2 := 1.0$

$t_3 := 0.5$ $\quad k_3 := 10.0$ $\quad w_3 := 1.0$ $\quad l_3 := 0.5$

$t_4 := 0.5$ $\quad k_4 := 4.0$ $\quad w_4 := 1.0$ $\quad l_4 := 0.5$

$h_8 := 2$ $\quad h_9 := 1$ $\quad T_7 := 20$ $\quad \beta := 1.7$

T_7, Q_1 are matrix elements.

Calculate Conductances:

$i := 1, 2 .. 7$

$j := 1, 2 .. 7$

$C_{i,j} := 0.0 \qquad Q_i := 0$

$$C_{1,2} := \frac{k_1 \cdot w_1 \cdot l_1}{t_1} \qquad C_{2,3} := \frac{k_2 \cdot w_2 \cdot \frac{l_2}{2}}{t_2} \qquad C_{2,4} := \frac{k_2 \cdot w_2 \cdot \frac{l_2}{2}}{t_2} \qquad C_{3,5} := \frac{k_3 \cdot w_3 \cdot l_3}{t_3}$$

$$C_{3,4} := \frac{1}{\dfrac{\frac{l_3}{2}}{k_3 \cdot \frac{t_3}{2} \cdot w_3} + \dfrac{\frac{l_4}{2}}{k_4 \cdot \frac{t_4}{2} \cdot w_4}} \qquad C_{5,6} := \frac{1}{\dfrac{\frac{l_3}{2}}{k_3 \cdot \frac{t_3}{2} \cdot w_3} + \dfrac{\frac{l_4}{2}}{k_4 \cdot \frac{t_4}{2} \cdot w_4}}$$

$$C_{4,6} := \frac{k_4 \cdot w_4 \cdot l_4}{t_4} \qquad C_{5,7} := h_8 \cdot w_3 \cdot l_3 \qquad C_{6,7} := h_9 \cdot w_4 \cdot l_4$$

$$C_{2,1} := C_{1,2} \quad C_{3,2} := C_{2,3} \quad C_{4,2} := C_{2,4} \quad C_{5,3} := C_{3,5} \quad C_{4,3} := C_{3,4}$$

$$C_{6,5} := C_{5,6} \quad C_{6,4} := C_{4,6} \quad C_{7,5} := C_{5,7} \quad C_{7,6} := C_{6,7}$$

$$C = \begin{pmatrix} 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 10 & 0 & 0.2 & 0.2 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 2.857 & 10 & 0 & 0 \\ 0 & 0.2 & 2.857 & 0 & 0 & 4 & 0 \\ 0 & 0 & 10 & 0 & 0 & 2.857 & 1 \\ 0 & 0 & 0 & 4 & 2.857 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 1 & 0.5 & 0 \end{pmatrix}$$

$Q_1 := 10.0$

$$Q = \begin{pmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Solution Using Gauss-Seidel Showing Only Two Iterations:

Set Starting Temps:

$i := 1, 2 .. 6$

$T_i := 40$

First Iteration:

$TOLD := T_1$

$$T_1 := \frac{C_{1,2} \cdot T_2 + Q_1}{C_{1,2}} \qquad TR_1 := TOLD + \beta \cdot (T_1 - TOLD) \quad T_1 := TR_1$$

$TOLD := T_2$

$$T_2 := \frac{C_{2,1} \cdot TR_1 + C_{2,3} \cdot T_3 + C_{2,4} \cdot T_4}{C_{2,1} + C_{2,3} + C_{2,4}} \qquad TR_2 := TOLD + \beta \cdot (T_2 - TOLD) \quad T_2 := TR_2$$

$TOLD := T_3$

$$T_3 := \frac{C_{3,2} \cdot TR_2 + C_{3,4} \cdot T_4 + C_{3,5} \cdot T_5}{C_{3,2} + C_{3,4} + C_{3,5}} \qquad TR_3 := TOLD + \beta \cdot (T_3 - TOLD) \quad T_3 := TR_3$$

$TOLD := T_4$

$$T_4 := \frac{C_{4,2} \cdot TR_2 + C_{4,3} \cdot TR_3 + C_{4,6} \cdot T_6}{C_{4,2} + C_{4,3} + C_{4,6}} \qquad TR_4 := TOLD + \beta \cdot (T_4 - TOLD) \quad T_4 := TR_4$$

$TOLD := T_5$

$$T_5 := \frac{C_{5,3} \cdot TR_3 + C_{5,6} \cdot T_6 + C_{5,7} \cdot T_7}{C_{5,3} + C_{5,6} + C_{5,7}} \qquad TR_5 := TOLD + \beta \cdot (T_5 - TOLD) \quad T_5 := TR_5$$

$$TOLD := T_6$$

$$T_6 := \frac{C_{6,4} \cdot TR_4 + C_{6,5} \cdot TR_5 + C_{6,7} \cdot T_7}{C_{6,4} + C_{6,5} + C_{6,7}} \qquad TR_6 := TOLD + \beta \cdot (T_6 - TOLD) \quad T_6 := TR_6$$

$$T = \begin{pmatrix} 41.7 \\ 42.779 \\ 40.072 \\ 40.184 \\ 37.635 \\ 36.298 \\ 20 \end{pmatrix} \qquad TR = \begin{pmatrix} 41.7 \\ 42.779 \\ 40.072 \\ 40.184 \\ 37.635 \\ 36.298 \end{pmatrix}$$

Second Iteration:

$$TOLD := T_1$$

$$T_1 := \frac{C_{1,2} \cdot T_2 + Q_1}{C_{1,2}} \qquad TR_1 := TOLD + \beta \cdot (T_1 - TOLD) \quad T_1 := TR_1$$

$$TOLD := T_2$$

$$T_2 := \frac{C_{2,1} \cdot TR_1 + C_{2,3} \cdot T_3 + C_{2,4} \cdot T_4}{C_{2,1} + C_{2,3} + C_{2,4}} \qquad TR_2 := TOLD + \beta \cdot (T_2 - TOLD) \quad T_2 := TR_2$$

$$TOLD := T_3$$

$$T_3 := \frac{C_{3,2} \cdot TR_2 + C_{3,4} \cdot T_4 + C_{3,5} \cdot T_5}{C_{3,2} + C_{3,4} + C_{3,5}} \qquad TR_3 := TOLD + \beta \cdot (T_3 - TOLD) \quad T_3 := TR_3$$

$$TOLD := T_4$$

$$T_4 := \frac{C_{4,2} \cdot TR_2 + C_{4,3} \cdot TR_3 + C_{4,6} \cdot T_6}{C_{4,2} + C_{4,3} + C_{4,6}} \qquad TR_4 := TOLD + \beta \cdot (T_4 - TOLD) \quad T_4 := TR_4$$

$TOLD := T_5$

$$T_5 := \frac{C_{5,3} \cdot TR_3 + C_{5,6} \cdot T_6 + C_{5,7} \cdot T_7}{C_{5,3} + C_{5,6} + C_{5,7}} \qquad TR_5 := TOLD + \beta \cdot \left(T_5 - TOLD\right) \quad T_5 := TR_5$$

$TOLD := T_6$

$$T_6 := \frac{C_{6,4} \cdot TR_4 + C_{6,5} \cdot TR_5 + C_{6,7} \cdot T_7}{C_{6,4} + C_{6,5} + C_{6,7}} \qquad TR_6 := TOLD + \beta \cdot \left(T_6 - TOLD\right) \quad T_6 := TR_6$$

$$T = \begin{pmatrix} 45.234 \\ 46.619 \\ 37.111 \\ 34.635 \\ 34.36 \\ 31.598 \\ 20 \end{pmatrix} \qquad TR = \begin{pmatrix} 45.234 \\ 46.619 \\ 37.111 \\ 34.635 \\ 34.36 \\ 31.598 \end{pmatrix}$$

Third Iteration:

$TOLD := T_1$

$$T_1 := \frac{C_{1,2} \cdot T_2 + Q_1}{C_{1,2}} \qquad TR_1 := TOLD + \beta \cdot \left(T_1 - TOLD\right) \quad T_1 := TR_1$$

$TOLD := T_2$

$$T_2 := \frac{C_{2,1} \cdot TR_1 + C_{2,3} \cdot T_3 + C_{2,4} \cdot T_4}{C_{2,1} + C_{2,3} + C_{2,4}} \qquad TR_2 := TOLD + \beta \cdot \left(T_2 - TOLD\right) \quad T_2 := TR_2$$

$TOLD := T_3$

$$T_3 := \frac{C_{3,2} \cdot TR_2 + C_{3,4} \cdot T_4 + C_{3,5} \cdot T_5}{C_{3,2} + C_{3,4} + C_{3,5}} \qquad TR_3 := TOLD + \beta \cdot \left(T_3 - TOLD\right) \quad T_3 := TR_3$$

$TOLD := T_4$

$$T_4 := \frac{C_{4,2} \cdot TR_2 + C_{4,3} \cdot TR_3 + C_{4,6} \cdot T_6}{C_{4,2} + C_{4,3} + C_{4,6}} \qquad TR_4 := TOLD + \beta \cdot \left(T_4 - TOLD\right) \quad T_4 := TR_4$$

$TOLD := T_5$

$$T_5 := \frac{C_{5,3} \cdot TR_3 + C_{5,6} \cdot T_6 + C_{5,7} \cdot T_7}{C_{5,3} + C_{5,6} + C_{5,7}} \qquad TR_5 := TOLD + \beta \cdot (T_5 - TOLD) \quad T_5 := TR_5$$

$TOLD := T_6$

$$T_6 := \frac{C_{6,4} \cdot TR_4 + C_{6,5} \cdot TR_5 + C_{6,7} \cdot T_7}{C_{6,4} + C_{6,5} + C_{6,7}} \qquad TR_6 := TOLD + \beta \cdot (T_6 - TOLD) \quad T_6 := TR_6$$

$$T = \begin{pmatrix} 49.288 \\ 50.28 \\ 32.951 \\ 31.304 \\ 29.902 \\ 28.866 \\ 20 \end{pmatrix} \qquad TR = \begin{pmatrix} 49.288 \\ 50.28 \\ 32.951 \\ 31.304 \\ 29.902 \\ 28.866 \end{pmatrix}$$

Fourth Iteration:

$TOLD := T_1$

$$T_1 := \frac{C_{1,2} \cdot T_2 + Q_1}{C_{1,2}} \qquad TR_1 := TOLD + \beta \cdot (T_1 - TOLD) \quad T_1 := TR_1$$

$TOLD := T_2$

$$T_2 := \frac{C_{2,1} \cdot TR_1 + C_{2,3} \cdot T_3 + C_{2,4} \cdot T_4}{C_{2,1} + C_{2,3} + C_{2,4}} \qquad TR_2 := TOLD + \beta \cdot (T_2 - TOLD) \quad T_2 := TR_2$$

$TOLD := T_3$

$$T_3 := \frac{C_{3,2} \cdot TR_2 + C_{3,4} \cdot T_4 + C_{3,5} \cdot T_5}{C_{3,2} + C_{3,4} + C_{3,5}} \qquad TR_3 := TOLD + \beta \cdot (T_3 - TOLD) \quad T_3 := TR_3$$

$TOLD := T_4$

$$T_4 := \frac{C_{4,2} \cdot TR_2 + C_{4,3} \cdot TR_3 + C_{4,6} \cdot T_6}{C_{4,2} + C_{4,3} + C_{4,6}} \qquad TR_4 := TOLD + \beta \cdot (T_4 - TOLD) \quad T_4 := TR_4$$

$TOLD := T_5$

$$T_5 := \frac{C_{5,3} \cdot TR_3 + C_{5,6} \cdot T_6 + C_{5,7} \cdot T_7}{C_{5,3} + C_{5,6} + C_{5,7}} \qquad TR_5 := TOLD + \beta \cdot (T_5 - TOLD) \quad T_5 := TR_5$$

$TOLD := T_6$

$$T_6 := \frac{C_{6,4} \cdot TR_4 + C_{6,5} \cdot TR_5 + C_{6,7} \cdot T_7}{C_{6,4} + C_{6,5} + C_{6,7}} \qquad TR_6 := TOLD + \beta \cdot (T_6 - TOLD) \quad T_6 := TR_6$$

$$T = \begin{pmatrix} 52.674 \\ 53.006 \\ 28.89 \\ 28.339 \\ 27.083 \\ 26.178 \\ 20 \end{pmatrix} \qquad TR = \begin{pmatrix} 52.674 \\ 53.006 \\ 28.89 \\ 28.339 \\ 27.083 \\ 26.178 \end{pmatrix}$$

Fifth Iteration:

$TOLD := T_1$

$$T_1 := \frac{C_{1,2} \cdot T_2 + Q_1}{C_{1,2}} \qquad TR_1 := TOLD + \beta \cdot (T_1 - TOLD) \quad T_1 := TR_1$$

$TOLD := T_2$

$$T_2 := \frac{C_{2,1} \cdot TR_1 + C_{2,3} \cdot T_3 + C_{2,4} \cdot T_4}{C_{2,1} + C_{2,3} + C_{2,4}} \qquad TR_2 := TOLD + \beta \cdot (T_2 - TOLD) \quad T_2 := TR_2$$

$TOLD := T_3$

$$T_3 := \frac{C_{3,2} \cdot TR_2 + C_{3,4} \cdot T_4 + C_{3,5} \cdot T_5}{C_{3,2} + C_{3,4} + C_{3,5}} \qquad TR_3 := TOLD + \beta \cdot (T_3 - TOLD) \quad T_3 := TR_3$$

$TOLD := T_4$

$$T_4 := \frac{C_{4,2} \cdot TR_2 + C_{4,3} \cdot TR_3 + C_{4,6} \cdot T_6}{C_{4,2} + C_{4,3} + C_{4,6}} \qquad TR_4 := TOLD + \beta \cdot (T_4 - TOLD) \quad T_4 := TR_4$$

$TOLD := T_5$

$$T_5 := \frac{C_{5,3} \cdot TR_3 + C_{5,6} \cdot T_6 + C_{5,7} \cdot T_7}{C_{5,3} + C_{5,6} + C_{5,7}} \qquad TR_5 := TOLD + \beta \cdot (T_5 - TOLD) \quad T_5 := TR_5$$

$TOLD := T_6$

$$T_6 := \frac{C_{6,4} \cdot TR_4 + C_{6,5} \cdot TR_5 + C_{6,7} \cdot T_7}{C_{6,4} + C_{6,5} + C_{6,7}} \qquad TR_6 := TOLD + \beta \cdot (T_6 - TOLD) \quad T_6 := TR_6$$

$$T = \begin{pmatrix} 54.939 \\ 54.57 \\ 27.001 \\ 26.6 \\ 25.796 \\ 25.602 \\ 20 \end{pmatrix} \qquad TR = \begin{pmatrix} 54.939 \\ 54.57 \\ 27.001 \\ 26.6 \\ 25.796 \\ 25.602 \end{pmatrix}$$

Sixth Iteration:

$TOLD := T_1$

$$T_1 := \frac{C_{1,2} \cdot T_2 + Q_1}{C_{1,2}} \qquad TR_1 := TOLD + \beta \cdot (T_1 - TOLD) \quad T_1 := TR_1$$

$TOLD := T_2$

$$T_2 := \frac{C_{2,1} \cdot TR_1 + C_{2,3} \cdot T_3 + C_{2,4} \cdot T_4}{C_{2,1} + C_{2,3} + C_{2,4}} \qquad TR_2 := TOLD + \beta \cdot (T_2 - TOLD) \quad T_2 := TR_2$$

$TOLD := T_3$

$$T_3 := \frac{C_{3,2} \cdot TR_2 + C_{3,4} \cdot T_4 + C_{3,5} \cdot T_5}{C_{3,2} + C_{3,4} + C_{3,5}} \qquad TR_3 := TOLD + \beta \cdot (T_3 - TOLD) \quad T_3 := TR_3$$

$TOLD := T_4$

$$T_4 := \frac{C_{4,2} \cdot TR_2 + C_{4,3} \cdot TR_3 + C_{4,6} \cdot T_6}{C_{4,2} + C_{4,3} + C_{4,6}} \qquad TR_4 := TOLD + \beta \cdot (T_4 - TOLD) \quad T_4 := TR_4$$

$TOLD := T_5$

$$T_5 := \frac{C_{5,3} \cdot TR_3 + C_{5,6} \cdot T_6 + C_{5,7} \cdot T_7}{C_{5,3} + C_{5,6} + C_{5,7}} \qquad TR_5 := TOLD + \beta \cdot (T_5 - TOLD) \quad T_5 := TR_5$$

$TOLD := T_6$

$$T_6 := \frac{C_{6,4} \cdot TR_4 + C_{6,5} \cdot TR_5 + C_{6,7} \cdot T_7}{C_{6,4} + C_{6,5} + C_{6,7}} \qquad TR_6 := TOLD + \beta \cdot (T_6 - TOLD) \quad T_6 := TR_6$$

$$T = \begin{pmatrix} 56.012 \\ 55.112 \\ 26.015 \\ 26.61 \\ 25.285 \\ 25.677 \\ 20 \end{pmatrix} \qquad TR = \begin{pmatrix} 56.012 \\ 55.112 \\ 26.015 \\ 26.61 \\ 25.285 \\ 25.677 \end{pmatrix}$$

Seventh Iteration:

$TOLD := T_1$

$$T_1 := \frac{C_{1,2} \cdot T_2 + Q_1}{C_{1,2}} \qquad TR_1 := TOLD + \beta \cdot (T_1 - TOLD) \quad T_1 := TR_1$$

$TOLD := T_2$

$$T_2 := \frac{C_{2,1} \cdot TR_1 + C_{2,3} \cdot T_3 + C_{2,4} \cdot T_4}{C_{2,1} + C_{2,3} + C_{2,4}} \qquad TR_2 := TOLD + \beta \cdot (T_2 - TOLD) \quad T_2 := TR_2$$

$TOLD := T_3$

$$T_3 := \frac{C_{3,2} \cdot TR_2 + C_{3,4} \cdot T_4 + C_{3,5} \cdot T_5}{C_{3,2} + C_{3,4} + C_{3,5}} \qquad TR_3 := TOLD + \beta \cdot (T_3 - TOLD) \quad T_3 := TR_3$$

$TOLD := T_4$

$$T_4 := \frac{C_{4,2} \cdot TR_2 + C_{4,3} \cdot TR_3 + C_{4,6} \cdot T_6}{C_{4,2} + C_{4,3} + C_{4,6}} \qquad TR_4 := TOLD + \beta \cdot (T_4 - TOLD) \quad T_4 := TR_4$$

$TOLD := T_5$

$$T_5 := \frac{C_{5,3} \cdot TR_3 + C_{5,6} \cdot T_6 + C_{5,7} \cdot T_7}{C_{5,3} + C_{5,6} + C_{5,7}} \qquad TR_5 := TOLD + \beta \cdot (T_5 - TOLD) \quad T_5 := TR_5$$

$TOLD := T_6$

$$T_6 := \frac{C_{6,4} \cdot TR_4 + C_{6,5} \cdot TR_5 + C_{6,7} \cdot T_7}{C_{6,4} + C_{6,5} + C_{6,7}} \qquad TR_6 := TOLD + \beta \cdot (T_6 - TOLD) \quad T_6 := TR_6$$

$$T = \begin{pmatrix} 56.182 \\ 54.977 \\ 26.04 \\ 26.686 \\ 25.701 \\ 25.969 \\ 20 \end{pmatrix} \qquad TR = \begin{pmatrix} 56.182 \\ 54.977 \\ 26.04 \\ 26.686 \\ 25.701 \\ 25.969 \end{pmatrix}$$

Eigth Iteration:

$TOLD := T_1$

$$T_1 := \frac{C_{1,2} \cdot T_2 + Q_1}{C_{1,2}} \qquad TR_1 := TOLD + \beta \cdot (T_1 - TOLD) \quad T_1 := TR_1$$

$TOLD := T_2$

$$T_2 := \frac{C_{2,1} \cdot TR_1 + C_{2,3} \cdot T_3 + C_{2,4} \cdot T_4}{C_{2,1} + C_{2,3} + C_{2,4}} \qquad TR_2 := TOLD + \beta \cdot (T_2 - TOLD) \quad T_2 := TR_2$$

$TOLD := T_3$

$$T_3 := \frac{C_{3,2} \cdot TR_2 + C_{3,4} \cdot T_4 + C_{3,5} \cdot T_5}{C_{3,2} + C_{3,4} + C_{3,5}} \qquad TR_3 := TOLD + \beta \cdot (T_3 - TOLD) \quad T_3 := TR_3$$

$TOLD := T_4$

$$T_4 := \frac{C_{4,2} \cdot TR_2 + C_{4,3} \cdot TR_3 + C_{4,6} \cdot T_6}{C_{4,2} + C_{4,3} + C_{4,6}} \qquad TR_4 := TOLD + \beta \cdot (T_4 - TOLD) \quad T_4 := TR_4$$

$TOLD := T_5$

$$T_5 := \frac{C_{5,3} \cdot TR_3 + C_{5,6} \cdot T_6 + C_{5,7} \cdot T_7}{C_{5,3} + C_{5,6} + C_{5,7}} \qquad TR_5 := TOLD + \beta \cdot (T_5 - TOLD) \quad T_5 := TR_5$$

$TOLD := T_6$

$$T_6 := \frac{C_{6,4} \cdot TR_4 + C_{6,5} \cdot TR_5 + C_{6,7} \cdot T_7}{C_{6,4} + C_{6,5} + C_{6,7}} \qquad TR_6 := TOLD + \beta \cdot (T_6 - TOLD) \quad T_6 := TR_6$$

$$T = \begin{pmatrix} 55.834 \\ 54.507 \\ 26.579 \\ 27.262 \\ 26.174 \\ 26.61 \\ 20 \end{pmatrix} \qquad TR = \begin{pmatrix} 55.834 \\ 54.507 \\ 26.579 \\ 27.262 \\ 26.174 \\ 26.61 \end{pmatrix}$$

Ninth Iteration:

$TOLD := T_1$

$$T_1 := \frac{C_{1,2} \cdot T_2 + Q_1}{C_{1,2}} \qquad TR_1 := TOLD + \beta \cdot (T_1 - TOLD) \quad T_1 := TR_1$$

$TOLD := T_2$

$$T_2 := \frac{C_{2,1} \cdot TR_1 + C_{2,3} \cdot T_3 + C_{2,4} \cdot T_4}{C_{2,1} + C_{2,3} + C_{2,4}} \qquad TR_2 := TOLD + \beta \cdot (T_2 - TOLD) \quad T_2 := TR_2$$

$TOLD := T_3$

$$T_3 := \frac{C_{3,2} \cdot TR_2 + C_{3,4} \cdot T_4 + C_{3,5} \cdot T_5}{C_{3,2} + C_{3,4} + C_{3,5}} \qquad TR_3 := TOLD + \beta \cdot (T_3 - TOLD) \quad T_3 := TR_3$$

$$TOLD := T_4$$

$$T_4 := \frac{C_{4,2} \cdot TR_2 + C_{4,3} \cdot TR_3 + C_{4,6} \cdot T_6}{C_{4,2} + C_{4,3} + C_{4,6}} \qquad TR_4 := TOLD + \beta \cdot (T_4 - TOLD) \quad T_4 := TR_4$$

$$TOLD := T_5$$

$$T_5 := \frac{C_{5,3} \cdot TR_3 + C_{5,6} \cdot T_6 + C_{5,7} \cdot T_7}{C_{5,3} + C_{5,6} + C_{5,7}} \qquad TR_5 := TOLD + \beta \cdot (T_5 - TOLD) \quad T_5 := TR_5$$

$$TOLD := T_6$$

$$T_6 := \frac{C_{6,4} \cdot TR_4 + C_{6,5} \cdot TR_5 + C_{6,7} \cdot T_7}{C_{6,4} + C_{6,5} + C_{6,7}} \qquad TR_6 := TOLD + \beta \cdot (T_6 - TOLD) \quad T_6 := TR_6$$

$$T = \begin{pmatrix} 55.278 \\ 53.964 \\ 27.018 \\ 27.752 \\ 26.605 \\ 26.899 \\ 20 \end{pmatrix} \qquad TR = \begin{pmatrix} 55.278 \\ 53.964 \\ 27.018 \\ 27.752 \\ 26.605 \\ 26.899 \end{pmatrix}$$

Tenth Iteration:

$$TOLD := T_1$$

$$T_1 := \frac{C_{1,2} \cdot T_2 + Q_1}{C_{1,2}} \qquad TR_1 := TOLD + \beta \cdot (T_1 - TOLD) \quad T_1 := TR_1$$

$$TOLD := T_2$$

$$T_2 := \frac{C_{2,1} \cdot TR_1 + C_{2,3} \cdot T_3 + C_{2,4} \cdot T_4}{C_{2,1} + C_{2,3} + C_{2,4}} \qquad TR_2 := TOLD + \beta \cdot (T_2 - TOLD) \quad T_2 := TR_2$$

$$TOLD := T_3$$

$$T_3 := \frac{C_{3,2} \cdot TR_2 + C_{3,4} \cdot T_4 + C_{3,5} \cdot T_5}{C_{3,2} + C_{3,4} + C_{3,5}} \qquad TR_3 := TOLD + \beta \cdot (T_3 - TOLD) \quad T_3 := TR_3$$

$TOLD := T_4$

$$T_4 := \frac{C_{4,2} \cdot TR_2 + C_{4,3} \cdot TR_3 + C_{4,6} \cdot T_6}{C_{4,2} + C_{4,3} + C_{4,6}} \qquad TR_4 := TOLD + \beta \cdot (T_4 - TOLD) \quad T_4 := TR_4$$

$TOLD := T_5$

$$T_5 := \frac{C_{5,3} \cdot TR_3 + C_{5,6} \cdot T_6 + C_{5,7} \cdot T_7}{C_{5,3} + C_{5,6} + C_{5,7}} \qquad TR_5 := TOLD + \beta \cdot (T_5 - TOLD) \quad T_5 := TR_5$$

$TOLD := T_6$

$$T_6 := \frac{C_{6,4} \cdot TR_4 + C_{6,5} \cdot TR_5 + C_{6,7} \cdot T_7}{C_{6,4} + C_{6,5} + C_{6,7}} \qquad TR_6 := TOLD + \beta \cdot (T_6 - TOLD) \quad T_6 := TR_6$$

$$T = \begin{pmatrix} 54.744 \\ 53.501 \\ 27.443 \\ 27.958 \\ 26.926 \\ 27.098 \\ 20 \end{pmatrix} \qquad TR = \begin{pmatrix} 54.744 \\ 53.501 \\ 27.443 \\ 27.958 \\ 26.926 \\ 27.098 \end{pmatrix}$$

Application Example 13.2: Solution of Steady-State Network Using Simultaneous Equation Method

Input Data: $t_1 := 0.1$ $k_1 := 1.0$ $w_1 := 1.0$ $l_1 := 1.0$ $S1 := 10.0$

$t_2 := 0.05$ $k_2 := 0.02$ $w_2 := 1.0$ $l_2 := 1.0$ $t_3 := 0.5$ $k_3 := 10.0$ $w_3 := 1.0$ $l_3 := 0.5$

$t_4 := 0.5$ $k_4 := 4.0$ $w_4 := 1.0$ $l_4 := 0.5$ $h_8 := 2$ $h_9 := 1$ $TA := 20$

Calculate Conductances:

$i := 1, 2 .. 7$

$j := 1, 2 .. 7$

$C_{i,j} := 0.0$ $\quad Q_i := 0$

$$C_{1,2} := \frac{k_1 \cdot w_1 \cdot l_1}{t_1} \quad C_{2,3} := \frac{k_2 \cdot w_2 \cdot \frac{l_2}{2}}{t_2} \quad C_{2,4} := \frac{k_2 \cdot w_2 \cdot \frac{l_2}{2}}{t_2} \quad C_{3,5} := \frac{k_3 \cdot w_3 \cdot l_3}{t_3}$$

$$C_{3,4} := \frac{1}{\dfrac{\frac{l_3}{2}}{k_3 \cdot \frac{t_3}{2} \cdot w_3} + \dfrac{\frac{l_4}{2}}{k_4 \cdot \frac{t_4}{2} \cdot w_4}} \quad C_{5,6} := \frac{1}{\dfrac{\frac{l_3}{2}}{k_3 \cdot \frac{t_3}{2} \cdot w_3} + \dfrac{\frac{l_4}{2}}{k_4 \cdot \frac{t_4}{2} \cdot w_4}}$$

$$C_{4,6} := \frac{k_4 \cdot w_4 \cdot l_4}{t_4} \quad C_{5,7} := h_8 \cdot w_3 \cdot l_3 \quad C_{6,7} := h_9 \cdot w_4 \cdot l_4$$

$C_{2,1} := C_{1,2}$ $\quad C_{3,2} := C_{2,3}$ $\quad C_{4,2} := C_{2,4}$ $\quad C_{5,3} := C_{3,5}$ $\quad C_{4,3} := C_{3,4}$

$C_{6,5} := C_{5,6}$ $\quad C_{6,4} := C_{4,6}$ $\quad C_{7,5} := C_{5,7}$ $\quad C_{7,6} := C_{6,7}$

Set Up Equations:

Define Conductance, Source Matrices

$i := 1, 2 .. 6$

$j := 1, 2 .. 6$

$G_{i,j} := 0.0$ $\quad S_j := 0.0$ $\quad T_j := 30$ $\quad T_7 := TA$

$S_1 := S1$

Equation 1 Elements:

$G_{1,1} := C_{1,2} \quad G_{1,2} := -C_{1,2} \quad S_1 := 10.0$

Equation 2 Elements:

$G_{2,1} := -C_{2,1} \quad G_{2,2} := C_{2,1} + C_{2,3} + C_{2,4} \quad G_{2,3} := -C_{2,3} \quad G_{2,4} := -C_{2,4}$

Equation 3 Elements:

$G_{3,2} := -C_{3,2} \quad G_{3,3} := C_{3,2} + C_{3,4} + C_{3,5} \quad G_{3,4} := -C_{3,4} \quad G_{3,5} := -C_{3,5}$

Equation 4 Elements:

$G_{4,2} := -C_{4,2} \quad G_{4,3} := -C_{4,3} \quad G_{4,4} := C_{4,2} + C_{4,3} + C_{4,6} \quad G_{4,6} := -C_{4,6}$

Equation 5 Elements:

$G_{5,3} := -C_{5,3} \quad G_{5,6} := -C_{5,6} \quad G_{5,5} := C_{5,3} + C_{5,6} + C_{5,7} \quad S_5 := C_{5,7} \cdot T_7$

Equation 6 Elements:

$G_{6,4} := -C_{6,4} \quad G_{6,5} := -C_{6,5} \quad G_{6,6} := C_{6,4} + C_{6,5} + C_{6,7} \quad S_6 := C_{6,7} \cdot T_7$

$$G = \begin{pmatrix} 10 & -10 & 0 & 0 & 0 & 0 \\ -10 & 10.4 & -0.2 & -0.2 & 0 & 0 \\ 0 & -0.2 & 13.057 & -2.857 & -10 & 0 \\ 0 & -0.2 & -2.857 & 7.057 & 0 & -4 \\ 0 & 0 & -10 & 0 & 13.857 & -2.857 \\ 0 & 0 & 0 & -4 & -2.857 & 7.357 \end{pmatrix}$$

Solve Problem by Matrix Inversion:

$T := G^{-1} \cdot S$

$$T = \begin{pmatrix} 53.467 \\ 52.467 \\ 27.252 \\ 27.682 \\ 26.625 \\ 26.749 \end{pmatrix}$$

$r_1 := Q_1 - C_{1,2} \cdot (T_1 - T_2) \quad r_2 := -(T_2 - T_1) \cdot C_{2,1} - (T_2 - T_3) \cdot C_{2,3} - (T_2 - T_4) \cdot C_{2,4}$

$r_3 := -(T_3 - T_2) \cdot C_{3,2} - (T_3 - T_4) \cdot C_{3,4} - (T_3 - T_5) \cdot C_{3,5}$

$r_4 := -(T_4 - T_2) \cdot C_{4,2} - (T_4 - T_3) \cdot C_{4,3} - (T_4 - T_6) \cdot C_{4,6}$

$r_5 := -(T_5 - T_3) \cdot C_{5,3} - (T_5 - T_6) \cdot C_{5,6} - (T_5 - TA) \cdot C_{5,7}$

$r_6 := -(T_6 - T_4) \cdot C_{6,4} - (T_6 - T_5) \cdot C_{6,5} - (T_6 - TA) \cdot C_{6,7}$

$$r := \begin{bmatrix} S_1 - C_{1,2} \cdot (T_1 - T_2) \\ -(T_2 - T_1) \cdot C_{2,1} - (T_2 - T_3) \cdot C_{2,3} - (T_2 - T_4) \cdot C_{2,4} \\ -(T_3 - T_2) \cdot C_{3,2} - (T_3 - T_4) \cdot C_{3,4} - (T_3 - T_5) \cdot C_{3,5} \\ -(T_4 - T_2) \cdot C_{4,2} - (T_4 - T_3) \cdot C_{4,3} - (T_4 - T_6) \cdot C_{4,6} \\ -(T_5 - T_3) \cdot C_{5,3} - (T_5 - T_6) \cdot C_{5,6} - (T_5 - TA) \cdot C_{5,7} \\ -(T_6 - T_4) \cdot C_{6,4} - (T_6 - T_5) \cdot C_{6,5} - (T_6 - TA) \cdot C_{6,7} \end{bmatrix}$$

$$EB1 := \frac{\sum_{i=1}^{6} r_i}{S_1} \cdot 100 \qquad EB2 := \frac{\sum_{i=1}^{6} |r_i|}{S_1} \cdot 100$$

$$EB1 = -9.237 \times 10^{-13} \qquad EB2 = 2.878 \times 10^{-12}$$

Application Example 13.4: Solution of Time-Dependent Network

Input Data: **Density units kg/m^3, CP J/(kg K), ΔV m^3 to give Capcitance units C* J/K or J/^{0}C.**

$t_1 := 0.1$ $k_1 := 1.0$ $w_1 := 1.0$ $l_1 := 1.0$ $Q1 := 10.0$ $\rho_1 := 4000$ $CP_1 := 800$

$t_2 := 0.05$ $k_2 := 0.02$ $w_2 := 1.0$ $l_2 := 1.0$ $\rho_2 := 2000$ $CP_2 := 700$

$t_3 := 0.5$ $k_3 := 10.0$ $w_3 := 1.0$ $l_3 := 0.5$ $\rho_3 := 10^4$ $CP_3 := 400$

$t_4 := 0.5$ $k_4 := 4.0$ $w_4 := 1.0$ $l_4 := 0.5$ $\rho_4 := 3000$ $CP_4 := 800$

$h_8 := 2$ $h_9 := 1$ $TA := 20$

Calculate Conductances:

$$C1\_2 := \frac{k_1 \cdot w_1 \cdot l_1}{t_1} \qquad C2\_3 := \frac{k_2 \cdot w_2 \cdot \frac{l_2}{2}}{t_2} \qquad C2\_4 := \frac{k_2 \cdot w_2 \cdot \frac{l_2}{2}}{t_2} \qquad C3\_5 := \frac{k_3 \cdot w_3 \cdot l_3}{t_3}$$

$$C3\_4 := \frac{1}{\dfrac{\frac{l_3}{2}}{k_3 \cdot \frac{t_3}{2} \cdot w_3} + \dfrac{\frac{l_4}{2}}{k_4 \cdot \frac{t_4}{2} \cdot w_4}} \qquad C5\_6 := \frac{1}{\dfrac{\frac{l_3}{2}}{k_3 \cdot \frac{t_3}{2} \cdot w_3} + \dfrac{\frac{l_4}{2}}{k_4 \cdot \frac{t_4}{2} \cdot w_4}}$$

$$C4\_6 := \frac{k_4 \cdot w_4 \cdot l_4}{t_4} \qquad C5\_7 := h_8 \cdot w_3 \cdot l_3 \qquad C6\_7 := h_9 \cdot w_4 \cdot l_4$$

$$C2\_1 := C1\_2 \quad C3\_2 := C2\_3 \quad C4\_2 := C2\_4 \quad C5\_3 := C3\_5 \quad C4\_3 := C3\_4$$

$$C6\_5 := C5\_6 \quad C6\_4 := C4\_6 \quad C7\_5 := C5\_7 \quad C7\_6 := C6\_7$$

Calculate Total Capacitance of Each Node:

$$\Delta VN1 := \frac{t_1}{2} \cdot l_1 \cdot w_1 \cdot \left(\frac{2.54}{100}\right)^3 \qquad CS1 := \rho_1 \cdot CP_1 \cdot \Delta VN1$$

$\Delta VN1 = 8.194 \times 10^{-7}$ $CS1 = 2.622$

$$\Delta VN2A := \frac{t_1}{2} \cdot l_1 \cdot w_1 \cdot \left(\frac{2.54}{100}\right)^3 \qquad \Delta VN2B := \frac{t_2}{2} \cdot l_2 \cdot w_2 \cdot \left(\frac{2.54}{100}\right)^3 \qquad CS2 := \rho_1 \cdot CP_1 \cdot \Delta VN2A + \rho_2 \cdot CP_2 \cdot \Delta VN2B$$

$\Delta VN2A = 8.194 \times 10^{-7}$ $\Delta VN2B = 4.097 \times 10^{-7}$ $CS2 = 3.195$

$$\Delta VN3A := \frac{t_2}{2} \cdot l_3 \cdot w_3 \cdot \left(\frac{2.54}{100}\right)^3 \qquad \Delta VN3B := \frac{t_3}{2} \cdot l_3 \cdot w_3 \cdot \left(\frac{2.54}{100}\right)^3 \qquad CS3 := \rho_2 \cdot CP_2 \cdot \Delta VN3A + \rho_3 \cdot CP_3 \cdot \Delta VN3B$$

$\Delta VN3A = 2.048 \times 10^{-7}$ $\Delta VN3B = 2.048 \times 10^{-6}$ $CS3 = 8.48$

$\Delta VN4A := \Delta VN3A$ $\Delta VN4B := \Delta VN3B$ $CS4 := \rho_2 \cdot CP_2 \cdot \Delta VN4A + \rho_4 \cdot CP_4 \cdot \Delta VN4B$

$\Delta VN4A = 2.048 \times 10^{-7}$ $\Delta VN4B = 2.048 \times 10^{-6}$ $CS4 = 5.203$

$$\Delta VN5 := \frac{t_3}{2} \cdot l_3 \cdot w_3 \cdot \left(\frac{2.54}{100}\right)^3 \qquad \Delta VN6 := \frac{t_4}{2} \cdot l_4 \cdot w_4 \cdot \left(\frac{2.54}{100}\right)^3 \qquad CS5 := \rho_3 \cdot CP_3 \cdot \Delta VN5 \qquad CS6 := \rho_4 \cdot CP_4 \cdot \Delta VN6$$

$\Delta VN5 = 2.048 \times 10^{-6}$ $\Delta VN6 = 2.048 \times 10^{-6}$ $CS5 = 8.194$ $CS6 = 4.916$

Calculate Maximum Time Step (for those methods that have stability criteria) for Each Node:

$$\Delta t1 := \frac{CS1}{C1\_2} \quad \Delta t2 := \frac{CS2}{C1\_2 + C2\_3 + C2\_4} \quad \Delta t3 := \frac{CS3}{C3\_2 + C3\_4 + C3\_5} \quad \Delta t4 := \frac{CS4}{C4\_2 + C4\_3 + C4\_6}$$

$$\Delta t5 := \frac{CS5}{C5\_3 + C5\_6 + C5\_7} \quad \Delta t6 := \frac{CS6}{C6\_4 + C6\_5 + C6\_7}$$

$\Delta t1 = 0.262$ $\Delta t2 = 0.307$ $\Delta t3 = 0.649$ $\Delta t4 = 0.737$ $\Delta t5 = 0.591$ $\Delta t6 = 0.668$

$\Delta t := 0.2$

$$S1 := \frac{\Delta t}{CS1} \cdot C1\_2 \qquad S2 := \frac{\Delta t}{CS2} \cdot (C1\_2 + C2\_3 + C2\_4) \qquad S3 := \frac{\Delta t}{CS3} \cdot (C3\_2 + C3\_4 + C3\_5)$$

$$S4 := \frac{\Delta t}{CS4} \cdot (C4\_2 + C4\_3 + C4\_6) \qquad S5 := \frac{\Delta t}{CS5} \cdot (C5\_3 + C5\_6 + C5\_7) \qquad S6 := \frac{\Delta t}{CS6} \cdot (C6\_4 + C6\_5 + C6\_7)$$

$S1 = 0.763$ $S2 = 0.651$ $S3 = 0.308$ $S4 = 0.271$ $S5 = 0.338$ $S6 = 0.299$

Set All Starting Temperatures:

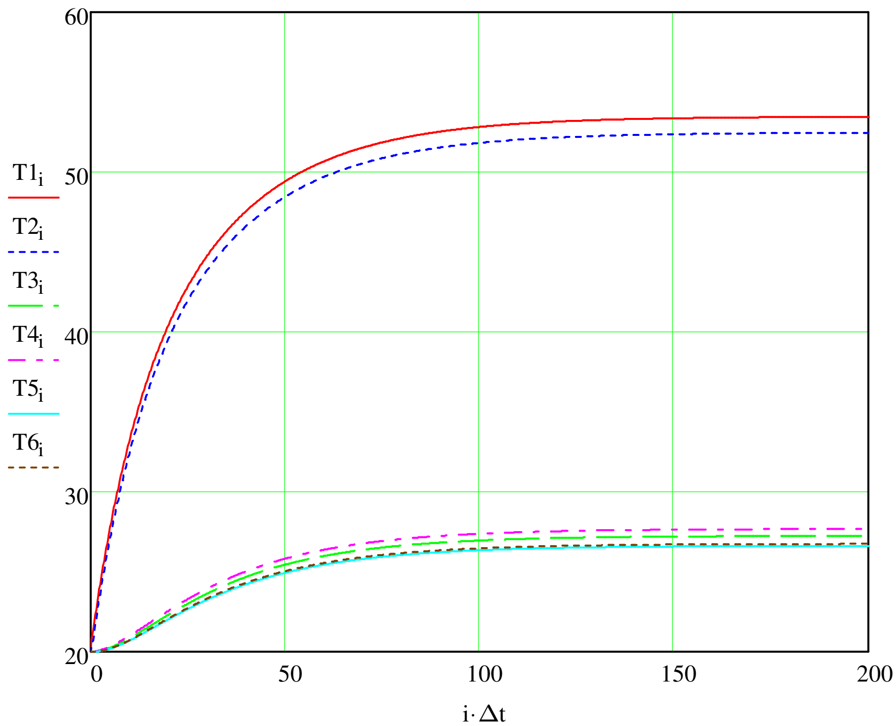
$$\begin{pmatrix} T1_0 \\ T2_0 \\ T3_0 \\ T4_0 \\ T5_0 \\ T6_0 \end{pmatrix} := \begin{pmatrix} TA \\ TA \\ TA \\ TA \\ TA \\ TA \end{pmatrix}$$

Solution Using Forward Finite Difference in Time:

$EndTime := 200$ $\Delta t := 0.2$ $MaxIt := \dfrac{EndTime}{\Delta t}$ $MaxIt = 1 \times 10^3$

$i := 0, 1 .. MaxIt$

$$\begin{pmatrix} T1_{i+1} \\ T2_{i+1} \\ T3_{i+1} \\ T4_{i+1} \\ T5_{i+1} \\ T6_{i+1} \end{pmatrix} := \begin{bmatrix} T1_i \cdot (1 - S1) + \frac{\Delta t}{CS1} \cdot \left(Q1 + C1\_2 \cdot T2_i\right) \\ T2_i \cdot (1 - S2) + \frac{\Delta t}{CS2} \cdot \left(C2\_1 \cdot T1_i + C2\_3 \cdot T3_i + C2\_4 \cdot T4_i\right) \\ T3_i \cdot (1 - S3) + \frac{\Delta t}{CS3} \cdot \left(C3\_2 \cdot T2_i + C3\_4 \cdot T4_i + C3\_5 \cdot T5_i\right) \\ T4_i \cdot (1 - S4) + \frac{\Delta t}{CS4} \cdot \left(C4\_2 \cdot T2_i + C4\_3 \cdot T3_i + C4\_6 \cdot T6_i\right) \\ T5_i (1 - S5) + \frac{\Delta t}{CS5} \cdot \left(C5\_3 \cdot T3_i + C5\_6 \cdot T6_i + C5\_7 \cdot TA\right) \\ T6_i (1 - S6) + \frac{\Delta t}{CS6} \cdot \left(C6\_4 \cdot T4_i + C6\_5 \cdot T5_i + C6\_7 \cdot TA\right) \end{bmatrix}$$



$T1_{MaxIt} = 53.45$

$T2_{MaxIt} = 52.45$

$T3_{MaxIt} = 27.245$

$T4_{MaxIt} = 27.674$

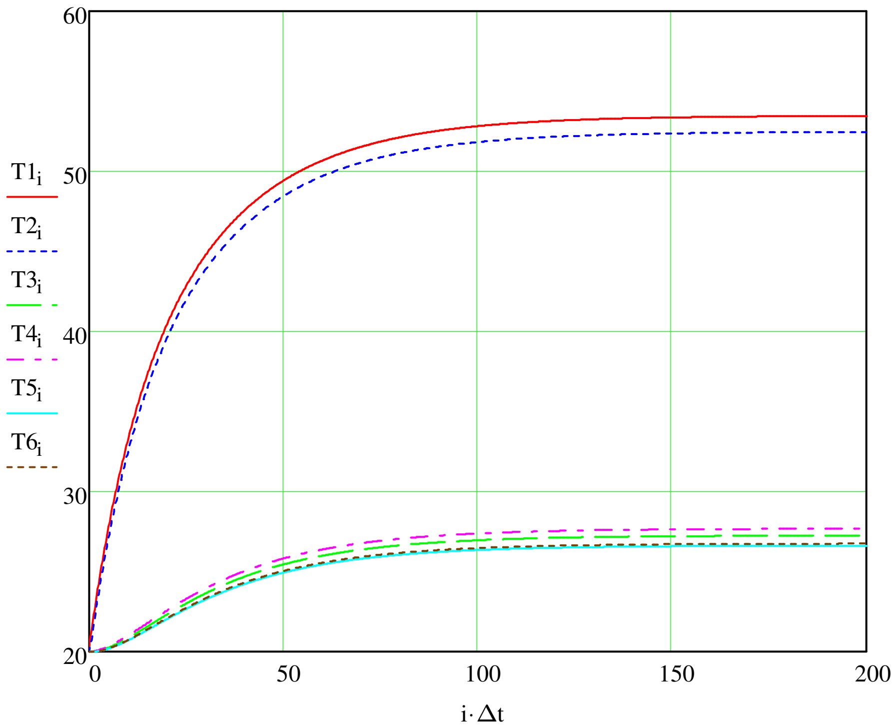
$T5_{MaxIt} = 26.618$

$T6_{MaxIt} = 26.742$

Forward Finite Difference in Time According to Holman:

$i := 0, 1 .. MaxIt$

$$\begin{pmatrix} T1_{i+1} \\ T2_{i+1} \\ T3_{i+1} \\ T4_{i+1} \\ T5_{i+1} \\ T6_{i+1} \end{pmatrix} := \begin{bmatrix} \frac{\Delta t}{CS1} \cdot \left[Q1 + C1\_2 \cdot \left(T2_i - T1_i\right)\right] + T1_i \\ \frac{\Delta t}{CS2} \cdot \left[C2\_1 \cdot \left(T1_i - T2_i\right) + C2\_3 \cdot \left(T3_i - T2_i\right) + C2\_4 \cdot \left(T4_i - T2_i\right)\right] + T2_i \\ \frac{\Delta t}{CS3} \cdot \left[C3\_2 \cdot \left(T2_i - T3_i\right) + C3\_4 \cdot \left(T4_i - T3_i\right) + C3\_5 \cdot \left(T5_i - T3_i\right)\right] + T3_i \\ \frac{\Delta t}{CS4} \cdot \left[C4\_2 \cdot \left(T2_i - T4_i\right) + C4\_3 \cdot \left(T3_i - T4_i\right) + C4\_6 \cdot \left(T6_i - T4_i\right)\right] + T4_i \\ \frac{\Delta t}{CS5} \cdot \left[C5\_3 \cdot \left(T3_i - T5_i\right) + C5\_6 \cdot \left(T6_i - T5_i\right) + C5\_7 \cdot \left(TA - T5_i\right)\right] + T5_i \\ \frac{\Delta t}{CS6} \cdot \left[C6\_4 \cdot \left(T4_i - T6_i\right) + C6\_5 \cdot \left(T5_i - T6_i\right) + C6\_7 \cdot \left(TA - T6_i\right)\right] + T6_i \end{bmatrix}$$



$T1_{MaxIt} = 53.45$

$T2_{MaxIt} = 52.45$

$T3_{MaxIt} = 27.245$

$T4_{MaxIt} = 27.674$

$T5_{MaxIt} = 26.618$

$T6_{MaxIt} = 26.742$

Backward Difference in Time Using First Method with Equations Resembling Gauss-Seidel:

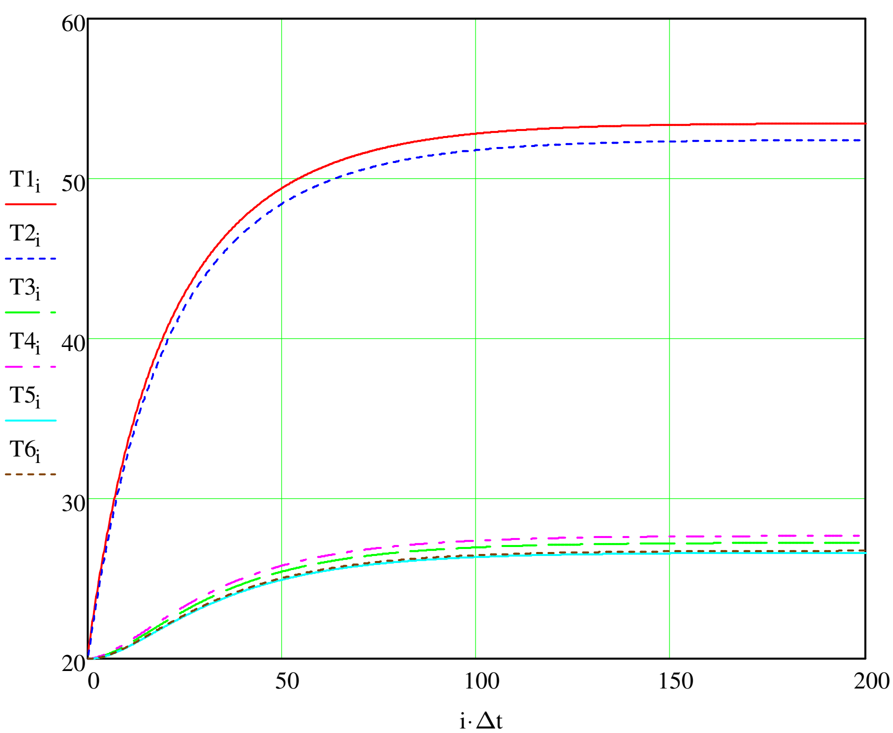
In this method we must also set for time t = 0 AND t = 0 + Δt.

$$\begin{pmatrix} T1_0 \\ T2_0 \\ T3_0 \\ T4_0 \\ T5_0 \\ T6_0 \end{pmatrix} := \begin{pmatrix} TA \\ TA \\ TA \\ TA \\ TA \\ TA \end{pmatrix}$$

$\Delta t := 0.2$ $\qquad$ $MaxIt := \dfrac{EndTime}{\Delta t}$ $\qquad$ $MaxIt = 1 \times 10^3$

$i := 1, 2 .. MaxIt$

$$\begin{pmatrix} T1_i \\ T2_i \\ T3_i \\ T4_i \\ T5_i \\ T6_i \end{pmatrix} := \begin{bmatrix} \dfrac{\left(C1\_2 \cdot T2_i\right) + \dfrac{CS1}{\Delta t} \cdot T1_{i-1} + Q1}{C1\_2 + \dfrac{CS1}{\Delta t}} \\ \dfrac{\left(C2\_1 \cdot T1_i + C2\_3 \cdot T3_i + C2\_4 \cdot T4_i\right) + \dfrac{CS2}{\Delta t} \cdot T2_i - 1}{(C2\_1 + C2\_3 + C2\_4) + \dfrac{CS2}{\Delta t}} \\ \dfrac{\left(C3\_2 \cdot T2_i + C3\_4 \cdot T4_i + C3\_5 \cdot T5_i\right) + \dfrac{CS3}{\Delta t} \cdot T3_{i-1}}{(C3\_2 + C3\_4 + C3\_5) + \dfrac{CS3}{\Delta t}} \\ \dfrac{\left(C4\_2 \cdot T2_i + C4\_3 \cdot T3_i + C4\_6 \cdot T6_i\right) + \dfrac{CS4}{\Delta t} \cdot T4_{i-1}}{(C4\_2 + C4\_3 + C4\_6) + \dfrac{CS4}{\Delta t}} \\ \dfrac{\left(C5\_3 \cdot T3_i + C5\_6 \cdot T6_i + C5\_7 \cdot TA\right) + \dfrac{CS5}{\Delta t} \cdot T5_{i-1}}{(C5\_3 + C5\_6 + C5\_7) + \dfrac{CS5}{\Delta t}} \\ \dfrac{\left(C6\_4 \cdot T4_i + C6\_5 \cdot T5_i + C6\_7 \cdot TA\right) + \dfrac{CS6}{\Delta t} \cdot T6_{i-1}}{(C6\_4 + C6\_5 + C6\_7) + \dfrac{CS6}{\Delta t}} \end{bmatrix}$$



$T1_{MaxIt} = 53.45$

$T2_{MaxIt} = 52.413$

$T3_{MaxIt} = 27.245$

$T4_{MaxIt} = 27.674$

$T5_{MaxIt} = 26.618$

$T6_{MaxIt} = 26.742$

Backward Difference in Time Using Second Method with Equations Solvable as Simultaneous:

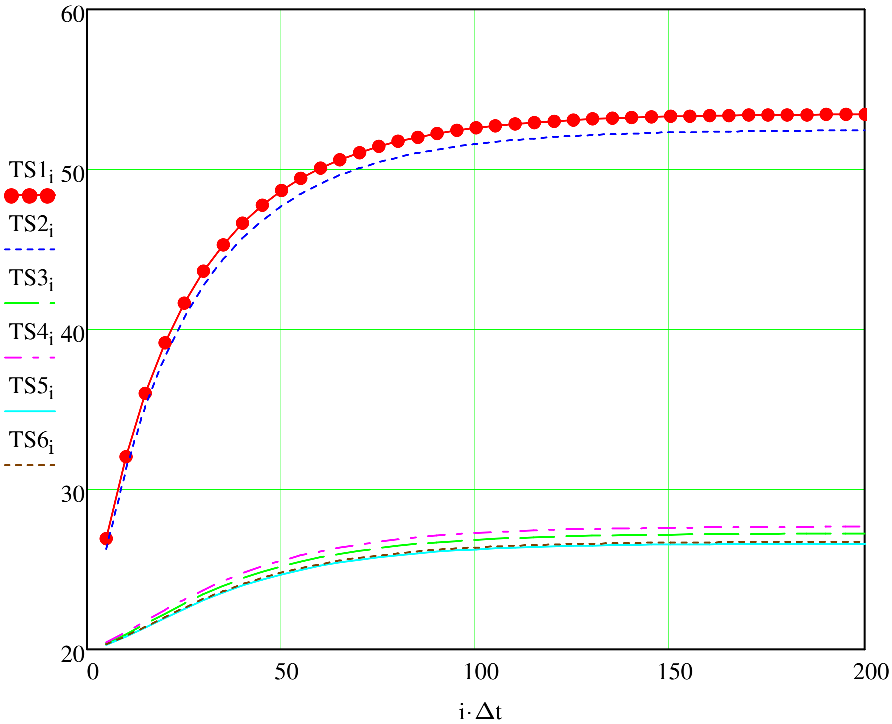
$$\Delta t := 5 \qquad MaxIt := \frac{EndTime}{\Delta t} \qquad MaxIt = 40$$

$$\begin{pmatrix} TS1_0 \\ TS2_0 \\ TS3_0 \\ TS4_0 \\ TS5_0 \\ TS6_0 \end{pmatrix} := \begin{pmatrix} TA \\ TA \\ TA \\ TA \\ TA \\ TA \end{pmatrix} \qquad Time_0 := 0$$

$$i := 1, 2 .. MaxIt$$

$$Time_i := i \cdot \Delta t$$

$$\begin{pmatrix} TS1_i \\ TS2_i \\ TS3_i \\ TS4_i \\ TS5_i \\ TS6_i \end{pmatrix} := \left[\begin{matrix} \left(\frac{CS1}{\Delta t} + C1\_2\right) & -C1\_2 & 0 & 0 \\ -C2\_1 & \left(\frac{CS2}{\Delta t} + C2\_1 + C2\_3 + C2\_4\right) & 0 - C2\_3 & -C2\_4 \\ 0 & -C3\_2 & \left(\frac{CS3}{\Delta t} + C3\_2 + C3\_4 + C3\_5\right) & -C3\_4 \\ 0 & -C4\_2 & -C4\_3 & \left(\frac{CS4}{\Delta t} + C4\_2 + C4\_3 + C4\_6\right) \\ 0 & 0 & -C5\_3 & 0 \\ 0 & 0 & 0 & -C6\_4 \end{matrix} \right.$$



$TS1_{MaxIt} = 53.436$

$TS2_{MaxIt} = 52.436$

$TS3_{MaxIt} = 27.238$

$TS4_{MaxIt} = 27.667$

$TS5_{MaxIt} = 26.612$

$TS6_{MaxIt} = 26.736$

...\Example 13_4 Time.dat	...\Example 13_4 T1.dat	...\Example 13_4 T2.dat	...\Example 13_4 T3.dat	...\Example 13_4 T4.dat
Time	TS1	TS2	TS3	TS4

...\Example 13_4 T5.dat	...\Example 13_4 T6.dat
TS5	TS6

$$\left.\begin{matrix} 0 & 0 \\ 0 & 0 \\ -C3\_5 & 0 \\ 0 & -C4\_6 \\ \left(\frac{CS5}{\Delta t} + C5\_3 + C5\_6 + C5\_7\right) & -C5\_6 \\ -C6\_5 & \left(\frac{CS6}{\Delta t} + C6\_4 + C6\_5 + C6\_7\right) \end{matrix}\right]^{-1} \cdot \begin{pmatrix} \frac{CS1}{\Delta t} \cdot TS1_{i-1} + Q1 \\ \frac{CS2}{\Delta t} \cdot TS2_{i-1} \\ \frac{CS3}{\Delta t} \cdot TS3_{i-1} \\ \frac{CS4}{\Delta t} \cdot TS4_{i-1} \\ \frac{CS5}{\Delta t} \cdot TS5_{i-1} + C5\_7 \cdot TA \\ \frac{CS6}{\Delta t} \cdot TS6_{i-1} + C6\_7 \cdot TA \end{pmatrix}$$

Application Example 13.2: Solution of Steady-State Network Using Gauss-Seidel Method - No Relaxation

Input Data:

$t_1 := 0.1$ $k_1 := 1.0$ $w_1 := 1.0$ $l_1 := 1.0$ $Q1 := 10.0$

$t_2 := 0.05$ $k_2 := 0.02$ $w_2 := 1.0$ $l_2 := 1.0$

$t_3 := 0.5$ $k_3 := 10.0$ $w_3 := 1.0$ $l_3 := 0.5$

$t_4 := 0.5$ $k_4 := 4.0$ $w_4 := 1.0$ $l_4 := 0.5$

$h_8 := 2$ $h_9 := 1$ $TA := 20$

Calculate Conductances:

$$C1\_2 := \frac{k_1 \cdot w_1 \cdot l_1}{t_1} \quad C2\_3 := \frac{k_2 \cdot w_2 \cdot \frac{l_2}{2}}{t_2} \quad C2\_4 := \frac{k_2 \cdot w_2 \cdot \frac{l_2}{2}}{t_2} \quad C3\_5 := \frac{k_3 \cdot w_3 \cdot l_3}{t_3}$$

$$C3\_4 := \frac{1}{\dfrac{\frac{l_3}{2}}{k_3 \cdot \frac{t_3}{2} \cdot w_3} + \dfrac{\frac{l_4}{2}}{k_4 \cdot \frac{t_4}{2} \cdot w_4}} \quad C5\_6 := \frac{1}{\dfrac{\frac{l_3}{2}}{k_3 \cdot \frac{t_3}{2} \cdot w_3} + \dfrac{\frac{l_4}{2}}{k_4 \cdot \frac{t_4}{2} \cdot w_4}}$$

$$C4\_6 := \frac{k_4 \cdot w_4 \cdot l_4}{t_4} \quad C5\_7 := h_8 \cdot w_3 \cdot l_3 \quad C6\_7 := h_9 \cdot w_4 \cdot l_4$$

$C2\_1 := C1\_2$ $C3\_2 := C2\_3$ $C4\_2 := C2\_4$ $C5\_3 := C3\_5$ $C4\_3 := C3\_4$

$C6\_5 := C5\_6$ $C6\_4 := C4\_6$ $C7\_5 := C5\_7$ $C7\_6 := C6\_7$

Solution Using Gauss-Seidel: $\Delta T := 20$

$T1Start := \Delta T + TA$ $T2Start := \Delta T + TA$ $T3Start := \Delta T + TA$

$T4Start := \Delta T + TA$ $T5Start := \Delta T + TA$ $T6Start := \Delta T + TA$

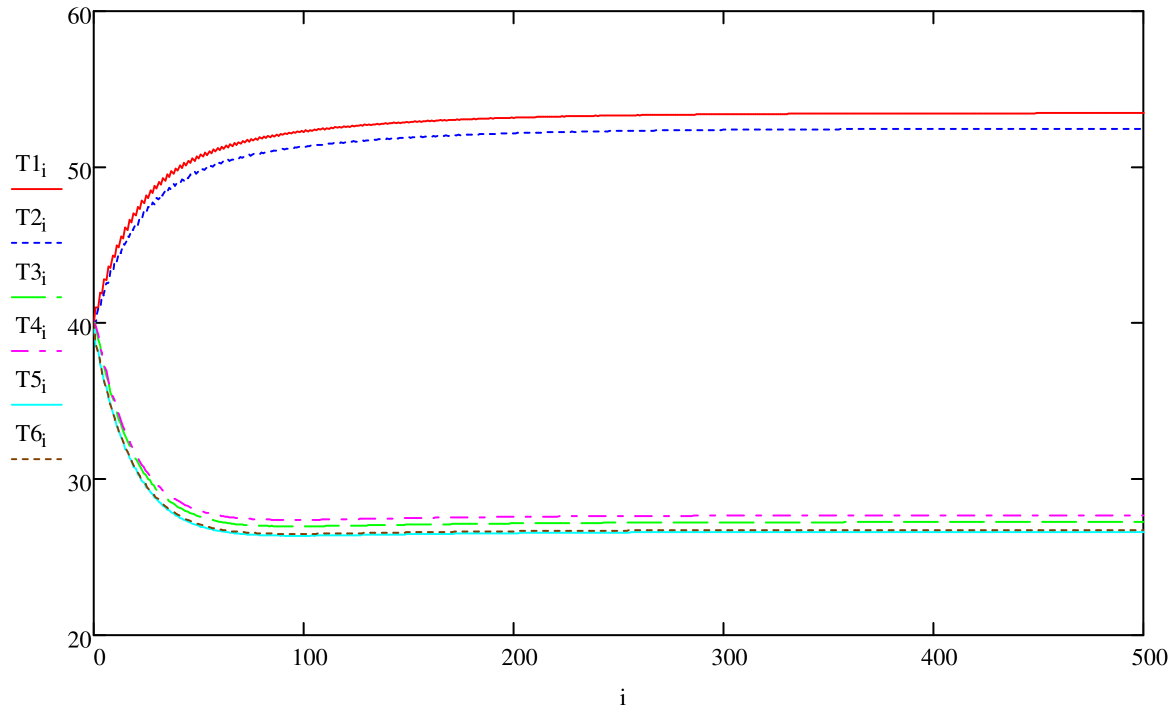
$$\begin{pmatrix} T1_0 \\ T2_0 \\ T3_0 \\ T4_0 \\ T5_0 \\ T6_0 \\ T7_0 \end{pmatrix} := \begin{pmatrix} T1Start \\ T2Start \\ T3Start \\ T4Start \\ T5Start \\ T6Start \\ TA \end{pmatrix}$$

The formulae must be put in a matrix to get variables to know other variable values, but
have not found a way to make relaxation work in this method.

MaxIt := 500

i := 0, 1 .. MaxIt

$$\begin{pmatrix} T1_{i+1} \\ T2_{i+1} \\ T3_{i+1} \\ T4_{i+1} \\ T5_{i+1} \\ T6_{i+1} \\ T7_{i+1} \end{pmatrix} := \begin{pmatrix} \dfrac{C1\_2 \cdot T2_i + Q1}{C1\_2} \\ \dfrac{C2\_1 \cdot T1_i + C2\_3 \cdot T3_i + C2\_4 \cdot T4_i}{C2\_1 + C2\_3 + C2\_4} \\ \dfrac{C3\_2 \cdot T2_i + C3\_4 \cdot T4_i + C3\_5 \cdot T5_i}{C3\_2 + C3\_4 + C3\_5} \\ \dfrac{C4\_2 \cdot T2_i + C4\_3 \cdot T3_i + C4\_6 \cdot T6_i}{C4\_2 + C4\_3 + C4\_6} \\ \dfrac{C5\_3 \cdot T3_i + C5\_6 \cdot T6_i + C5\_7 \cdot T7_i}{C5\_3 + C5\_6 + C5\_7} \\ \dfrac{C6\_4 \cdot T4_i + C6\_5 \cdot T5_i + C6\_7 \cdot T7_i}{C6\_4 + C6\_5 + C6\_7} \\ T7_i \end{pmatrix}$$



$T1_{MaxIt} = 53.462$ $\quad T2_{MaxIt} = 52.462$ $\quad T3_{MaxIt} = 27.251$ $\quad T4_{MaxIt} = 27.68$

$T5_{MaxIt} = 26.624$ $\quad T6_{MaxIt} = 26.748$

$r_1 := Q1 - C1\_2 \cdot \left(T1_{MaxIt} - T2_{MaxIt}\right.$ $r_2 := -\left(T2_{MaxIt} - T1_{MaxIt}\right) \cdot C2\_1 - \left(T2_{MaxIt} - T3_{MaxIt}\right) \cdot C2\_3 - \left(T2_{MaxIt} - T4_{MaxIt}\right) \cdot C$

$$r_3 := -\left(T3_{MaxIt} - T2_{MaxIt}\right) \cdot C3\_2 - \left(T3_{MaxIt} - T4_{MaxIt}\right) \cdot C3\_4 - \left(T3_{MaxIt} - T5_{MaxIt}\right) \cdot C3\_5$$

$$r_4 := -\left(T4_{MaxIt} - T2_{MaxIt}\right) \cdot C4\_2 - \left(T4_{MaxIt} - T3_{MaxIt}\right) \cdot C4\_3 - \left(T4_{MaxIt} - T6_{MaxIt}\right) \cdot C4\_6$$

$$r_5 := -\left(T5_{MaxIt} - T3_{MaxIt}\right) \cdot C5\_3 - \left(T5_{MaxIt} - T6_{MaxIt}\right) \cdot C5\_6 - \left(T5_{MaxIt} - TA\right) \cdot C5\_7$$

$$r_6 := -\left(T6_{MaxIt} - T4_{MaxIt}\right) \cdot C6\_4 - \left(T6_{MaxIt} - T5_{MaxIt}\right) \cdot C6\_5 - \left(T6_{MaxIt} - TA\right) \cdot C6\_7$$

$$r := \begin{bmatrix} Q1 - C1\_2 \cdot \left(T1_{MaxIt} - T2_{MaxIt}\right) \\ -\left(T2_{MaxIt} - T1_{MaxIt}\right) \cdot C2\_1 - \left(T2_{MaxIt} - T3_{MaxIt}\right) \cdot C2\_3 - \left(T2_{MaxIt} - T4_{MaxIt}\right) \cdot C2\_4 \\ -\left(T3_{MaxIt} - T2_{MaxIt}\right) \cdot C3\_2 - \left(T3_{MaxIt} - T4_{MaxIt}\right) \cdot C3\_4 - \left(T3_{MaxIt} - T5_{MaxIt}\right) \cdot C3\_5 \\ -\left(T4_{MaxIt} - T2_{MaxIt}\right) \cdot C4\_2 - \left(T4_{MaxIt} - T3_{MaxIt}\right) \cdot C4\_3 - \left(T4_{MaxIt} - T6_{MaxIt}\right) \cdot C4\_6 \\ -\left(T5_{MaxIt} - T3_{MaxIt}\right) \cdot C5\_3 - \left(T5_{MaxIt} - T6_{MaxIt}\right) \cdot C5\_6 - \left(T5_{MaxIt} - TA\right) \cdot C5\_7 \\ -\left(T6_{MaxIt} - T4_{MaxIt}\right) \cdot C6\_4 - \left(T6_{MaxIt} - T5_{MaxIt}\right) \cdot C6\_5 - \left(T6_{MaxIt} - TA\right) \cdot C6\_7 \end{bmatrix}$$

$$r = \begin{pmatrix} 1.074 \times 10^{-3} \\ 3.74 \times 10^{-4} \\ 2.982 \times 10^{-4} \\ 1.706 \times 10^{-4} \\ 2.813 \times 10^{-4} \\ 1.505 \times 10^{-4} \end{pmatrix}$$

$$EB1 := \frac{\sum_{i=0}^{5} r_i}{Q1} \cdot 100 \qquad EB2 := \frac{\sum_{i=0}^{5} \left| r_i \right|}{Q1} \cdot 100$$

$$EB1 = 2.348 \times 10^{-2} \qquad EB2 = 0.023$$

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