SOLUTIONS MANUAL FOR

Driveline Systems of Ground Vehicles: Theory and Design

by

Alexandr F. Andreev
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CRC Press
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Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business

CRC Press
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

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Printed in the United States of America on acid-free paper
10 9 8 7 6 5 4 3 2 1

International Standard Book Number: 978-1-4398-2153-4 (Paperback)

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Chapter 1

1.1. The first wheeled vehicles employed a differential-less, that is, positively engaged driveline system.

1.2. The wheels of one axle of the vehicle should be able to roll at different angular velocities (different rpm) even on a straight road section to accommodate different road-surface irregularities over which each of the wheels is to travel.

1.3. The fact that the driveline system of the drive wheels of the first motor vehicles was locked caused one of the wheels of axle to slip and the other wheel of the same axle to slide due to road irregularities that are not the same on the paths traveled by the left and right wheels.

1.4. Rolling of the drive wheels of the drive axis at different angular velocities is made possible by installing an open differential.

1.5. The differential was invented by the French engineer Onesiphore Pecquer in 1827 and he was also the first to construct a differential in 1828; he installed it in his private steam-driven vehicle.

1.6. The kinematic and dynamic properties of an open differential are:

- allows the wheels to rotate at different angular velocities (different rpm), which is particularly important when the vehicle makes a turn;
- transmits virtually equal torques to the drive-axle wheels, if the slight internal friction in the mechanism is neglected.

1.7. The open differential has a detrimental effect on the vehicle's mobility, since both wheels develop equal torques. Their numerical values are the same and are equal to the torque of the wheel subject to poor gripping conditions. The wheel that rolls under better gripping conditions is unable to develop a greater torque and to thus increase the vehicle's traction force and improve its mobility.

1.8. Gripping of the support surface by a vehicle's drive wheels is improved by:

- reducing the air pressure in tires;
- using double drive wheels;

- selecting an optimal design and outline of the tire's tread;

- using chains with special soil-gripping tread projections, semi-track drive and similar devices.

1.9. They are locking differentials, free-running differentials, limited slip differentials, viscous couplings, etc.

1.10. The term locking differentials applies to all those that are locked by means of a locking device actuated manually or automatically.

1.11. A limited slip differential is one in which the internal friction between the components is higher than in an open differential (for example, by including friction clutches into the differential's design). This provides partial locking of the differential.

1.12. Differentials are classified on the basis of their location within the driveline system into interwheel and interaxle differentials.

1.13. The driveline system is part of the powertrain located between the transmission and the vehicle's drive wheels, that is starting with powertrain units located directly following the transmission (in many vehicle designs the engine's power flow starts being divided between the axles precisely starting with these units), and ending with the half axles of the vehicle's drive wheels.

The driveline system of a vehicle with two drive axles and a transfer case includes the transfer case itself, driveshafts of the front and rear axles and both drive axles with all the elements that transmit power to the drive wheels.

The driveline system of a vehicle with three drive axles, where two of the rear axles form a tandem with a pass-through forward rear axle is made up of the transfer case, driveshafts that connect the transfer case to the front and forward rear axles, the driveshaft that connects the rear and forward rear axles and the three drive axles themselves.

The driveline system of a vehicle with a single rear drive axle consists of the driveshaft that connects the transmission to the rear drive axle and the rear drive axle itself to all its elements that transmit power to the left and right drive wheels.

The drive gear of a vehicle with a front drive axle includes the gears that connect the output shaft of the transmission to the front-axle differential, the differential itself and the half axles of the left and right wheels.

1.14. Its principal purpose consists in distributing the flow of power supplied by the engine between the drive wheels.

1.15. The simple driveline system is a system consisting of power dividing units of the same type (for example, consisting solely of open differentials). The combined driveline system consists of power dividing units of different types (for example, open differentials with limited slip differentials); an integrated system is one (simple or combined) that is operationally integrated with other vehicle systems (suspension, steering, brakes, etc.).

1.16. The wheel formula is an indicator expressing the relationship between the total number of wheels and the number of drive wheels. As defined by this indicator, vehicles may be: non-all-wheel drive, in which only two of the wheels are drive wheels (for example, 4x2 or 6x2), all-wheel drive – in which all the wheels are permanently or temporarily connected to the drive gear (for example, 4x4 full time, 4x4 part time, 6x6, 8x8, 12x12); multi-wheel drive vehicles are those in which the number of drive wheels is four and more, but the total number of wheels is greater than that of the number of drive wheels (for example, 6x4, 8x4, 8x6, 14x12).

1.17. The drive formula gives the location of the driving axles within the overall design of the vehicle. For example, the designation 0034 corresponds to an 8x4 vehicle with the third and forth axle powered. A 16x16 vehicle has a drive formula written as 12345678, whereas a 6x4 vehicle with a rear-wheel drive is designated as 023.

1.18. The axle formula symbolizes the number of axles situated in a row. For example, a vehicle with four axles (such as 8x8, 8x6, 8x4) that form two tandems have an axle formula of 2-2, whereas vehicles with four axles in which the second and the third axles are located close to one another have the axle formula 1-2-1. If a vehicle has wheels of different dimensions on the different axles, then the height of numbers in the axle formula should differ. For example, an agricultural tractor with two axles and smaller tires on the front wheels has the axle formula 1-1.

1.19. The steering formula gives the ordinal number of the steered axle. For example, the steering formula for a vehicle with three axles in which the second and third axles form a tandem, whereas the front axle is steered, is 1-00. A vehicle with two axles, of which the front and rear axles are steered, has a steering formula of 1-1.

1.20. The steering formula $0\overset{s}{-}0$ corresponds to two articulated non-steered axles coupled by a "fifth" wheel.

1.21. The power-dividing unit is a component of the driveline system of a vehicle that has one input and two outputs. The PDU divides the input power between its output shafts. The examples are: the open differential, the free-running differential, the limited slip differential, and a locked drive of two axles.

1.22. The number of power dividing units is ($2n - 1$). The number of power dividing units in vehicles with wheel formulae 4x4, 8x8, 12x12 and 16x16 is 3, 7, 11 and 15, respectively.

1.23. Driven, neutral, free, driving and braking rolling modes.

1.24. The static radius is the distance from the center of a wheel at rest subjected to a normal load to the support surface. The dynamic radius is the same, but with the wheel rolling.

1.25. The rolling radius (or the effective rolling radius) r_w is a radius that links the actual velocity V_x of the center of the wheel to the wheel's angular velocity ω_w

$$r_w = \frac{V_x}{\omega_w}$$

This is a conditional radius of the wheel that defines the location of the instantaneous center of zero velocity. For this reason this radius cannot be determined by direct measurement on the wheel. It is obtained by calculations on the basis of the length of path and the number of revolutions of a wheel when traveling this path.

The number of revolutions of a wheel over a given path depends on the torque and on the forces applied to the wheel. The application of torque and of forces to the wheel causes (i) longitudinal (tangential) deflections of the tire; (ii) longitudinal (tangential) deflections of the soil on deformable surfaces; and (iii) relative displacement between the tire and soil (skid). Because of these factors the wheel may perform a different number of revolutions over the same path. Application and increase of torque increases the deformations listed in items (i) and (ii), whereas at high values of the torque or on slippery roads there may appear displacements listed in item (iii).

1.26. The rolling radius is equal to zero. The wheel does not move forward but only rotates about its axis. The instantaneous zero velocity center is located on the wheel's axis of rotation.

1.27. The rolling radius is equal to infinity. The wheel is locked, that is, it does not rotate about its axis (translates only). The instantaneous center of zero velocity is located on a straight line that passes through the axis of the wheel and it located at infinity, measured in the direction of the surface of motion, from this axis.

1.28. The slip ratio is a measure of change in the linear velocity of the wheel center caused by changes in the rolling radius and in the angular velocity of the wheel due to any combination of the longitudinal deflections of the tire, tire and soil on deformable surfaces, and due to the skid between the tire and the soil:

$$s_\delta = \frac{V_t - V_x}{V_t} = \frac{V_\delta}{V_t}$$

Here, V_t is the theoretical linear velocity of the wheel ($s_\delta = 0$); V_x is the actual linear velocity of the wheel ($s_\delta > 0$); $V_\delta = V_t - V_x$ is the slip velocity. The slip ratio changes from 0 to 1.

1.29. The slippage is 1 (or 100%).

1.30. The answer to all four questions is positive – YES, wheel slippage depends on all the listed factors.

1.31. The rolling radius of the wheel in the driven mode. See formula (1.21).

1.32. The following two definitions of the peak friction factor are encountered in engineering literature:

$$\mu_{px} = \frac{F_x^{\max}}{R_z}$$

where $F_x^{\max}$ is the maximum circumferential wheel force that can be attained in the contact between the wheel and the surface of motion. Such an approach to determining the peak friction factor is usually typical of engineering publications pertaining to tractors and other off-road vehicles.

The peak friction factor is defined in automotive literature as the ratio

$$\mu_p = \frac{F_w^{\max}}{R_z}$$

where $F_w^{\max}$ is the maximum wheel traction force.

1.33. $P_\delta = F_x \omega_w (r_w^0 - r_w)$ is the slip power, i.e., the power lost in slippage (in deflecting the tire and the soil in the longitudinal direction) that results in loss of the linear velocity of the wheel given by the expression $V_\delta = V_t - V_x$ in Eq. (1.9).

1.34. The tractive efficiency, transportation efficiency and the total efficiency.

1.35. Solution.

$$\eta_{fw} = \frac{F_w}{F_x} \text{ where } F_x = W_w \mu_{px}\left(1 - e^{-ks_\delta}\right) \text{ thus: } \eta_{fw} = \frac{F_w}{W_w \mu_{px}\left(1 - e^{-ks_\delta}\right)}$$

$$\eta_{\delta w} = 1 - s_\delta$$

$$\eta_w^t = \eta_{fw}\eta_{\delta w} = \frac{F_w}{W_w \mu_{px}\left(1 - e^{-ks_\delta}\right)}\left(1 - s_\delta\right)$$

$F_x = R_x + F_w = W_w \mu_{px}\left(1 - e^{-ks_\delta}\right)$. Solving for s_δ yields:

$$\frac{R_x + F_w}{W_w \mu_{px}} = 1 - e^{-ks_\delta}$$

$$e^{-ks_\delta} = 1 - \frac{R_x + F_w}{W_w \mu_{px}}$$

$$s_\delta = -\frac{\ln\left(1 - \frac{R_x + F_w}{W_w \mu_{px}}\right)}{k}.$$

The rolling resistance force is $R_x = f_i W_w = 0.08\left[1 - 0.85\log_{10}(i)\right]W_w$

The solution algorithm is given below.

1. For a given value of the net tractive force F_w, determine the rolling resistance factor and the rolling resistance force for four passages over the same track.

2. Determine the circumferential wheel force as a function of the tractive force and rolling resistance force for four passages over the same track.

3. Calculate wheel slip ratio as a function of the circumferential force for four passages in the same track.

4. Determine the wheel efficiency factors.

5. Plot and analyze the results.

Continue on next page.

Rolling Resistance Efficiency:

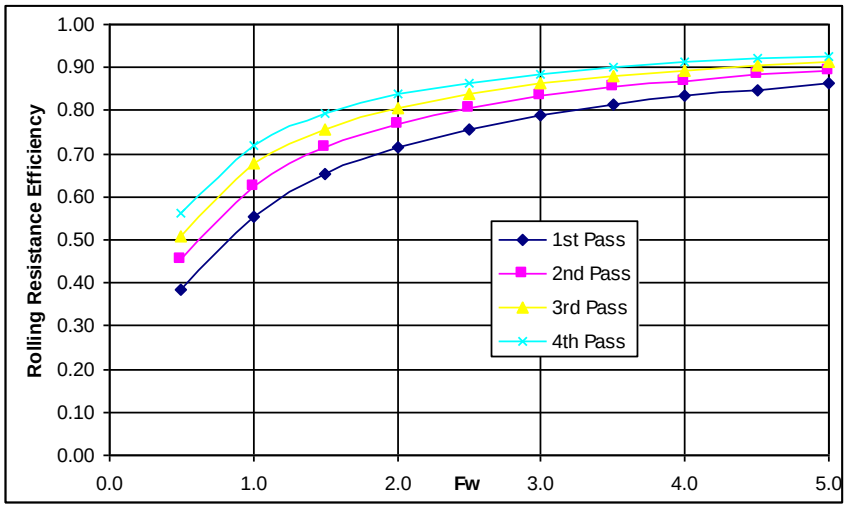

Fig. 1.35a Q and P. Rolling Resistance Efficiency

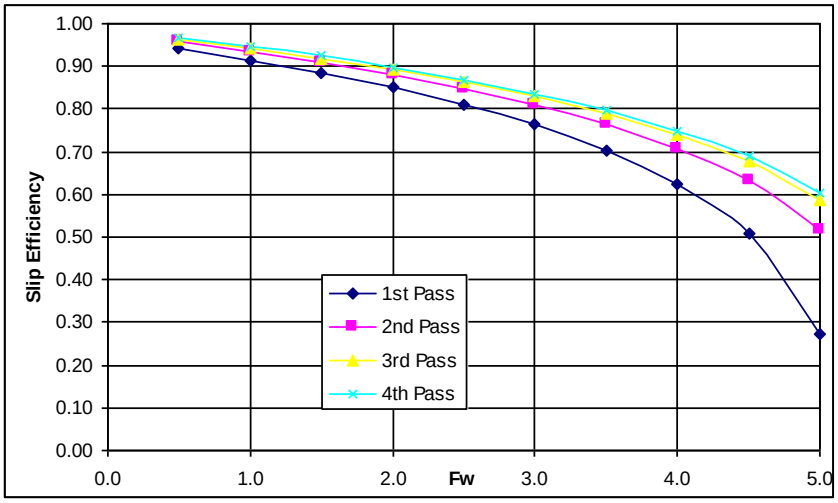

Fig. 1.35b Q and P. Slip Efficiency

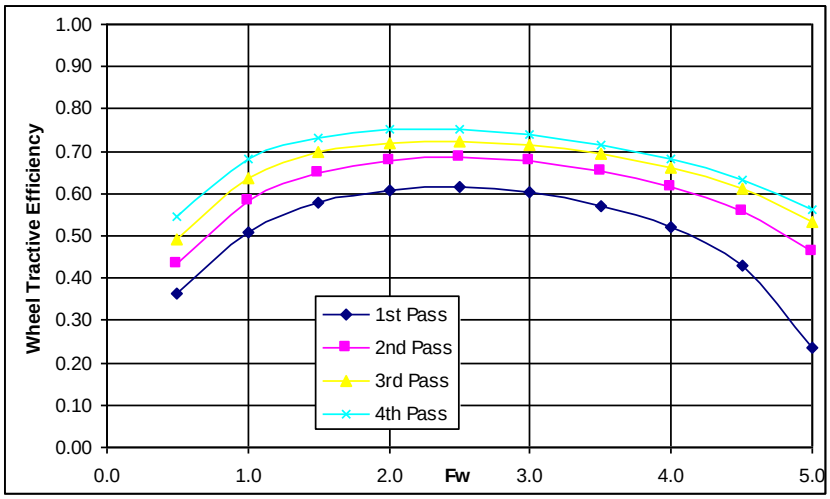

Fig. 1.35c Q and P. Wheel Tractive Efficiency

The rolling resistance increases with the tractive force while slip efficiency decreases. The wheel tractive efficiency is the product of the rolling resistance and slip efficiencies therefore it first increases then decreases. There are optimal values of the tractive force F_w at which the wheel tractive efficiency is

at maximum. All efficiencies increase with the number of passes because the soil becomes more compacted as the number of passes increases. This decreases the rolling resistance and tire slip.

1.36. It may be assumed that the rolling radius of the wheel in the driven mode is independent of the wheel slip angle and when the latter angle changes it remains equal to its value determined in rolling without sideslip.

1.37. The appearance and increase in the side slip angle causes an increase in the tangential elasticity of the tire and this reduces the rolling radius of the wheel in the driving mode and consequently increases its slip.

1.38. The application of torque to the wheel reduces the tire's resistance to lateral slip. As a result, the slip angle of a wheel subjected to a constant applied force increases, since the lateral reaction exerted on the wheel by the road that balances the applied lateral force is equal to the product of the slip resistance factor and the slip angle.

1.39. Tractive vehicles, subjected to tractive loads and, accordingly, the traction mode, consist of farm tractors that have coupled to them implements that penetrate the soil and thus produce a tractive force.

Transportation vehicles that transport useful cargo and, accordingly, the transportation mode, consist of passenger cars, trucks, and vehicular trains.

Transportation-tractive vehicles, and accordingly, the tractive-transportation mode, consist of vehicles that transport useful cargo and are loaded by a tractive load. For example, an armored personnel carrier, when it transports military personnel and at the same time extricates itself by means of a winch. A farm tractor equipped by a cultivator and a harrow that generate a tractive force when the tractor moves and with a trailer with containers from where fertilizer is spread.

1.40. There exist two main forms of mechanical power losses in driveline systems:

1. Losses that do not depend on the distribution of power between the drive wheels, i.e., losses that are independent of the driveline system characteristics. These include losses in seals, hydraulic losses in mixing, spraying and squeezing out of lubricating oil and also a part of power losses in bearings.

2. Losses that depend on the distribution of power between the drive wheels, i.e., losses that depend on the driveline system characteristics. The physical substance of losses making up this second group consists in the fact that the change in the load torques and angular velocities of power dividing unit components changes the friction torques in gearing and modifies that part of losses in bearings that depend on the torque transmitted by the shaft.

1.41. The driveline system affects the slippage power losses in a vehicle's wheel system. These slippage power losses depend on (i) the total power supplied by the engine to all drive wheels and (ii) on the manner in which this total power is divided between the drive wheels, which is precisely determined by the type of the driveline system and by the properties of its power-dividing units.

1.42. The mobility of vehicles is their ability to move under road-less conditions, while still performing their functions. Recall that vehicles were classified by their functional attributes into three groups (see Table 1.19 in Sec. 1.4.1). A distinction is made between the topographical and support-surface mobility. The topographical (geometric) mobility represents the ability of a vehicle to surmount various obstacles including ditches and trenches, hillsides, man-made obstacles (this applies to combat and tactical vehicles), etc. The support-surface mobility is determined by the ability of the vehicle to move over deformable surfaces and over snow.

1.43. Turnability is the ability of a vehicle to change the curvature of its path of motion upon changes in its design parameters (in our case the driveline system) and operating factors (velocity, drawbar pull, conditions of gripping between the wheels and the surface of motion, etc.). For example, locking of the interwheel differential causes the circumferential wheel forces to become unequal and brings about the appearance of a moment of resistance to turning in the plane of the road, which generates lateral forces

and angular wheel slip. This increases the turning radius R_a (the path curvature $1/R_a$ decreases); the location of the instantaneous turn center and the path of the vehicle's motion change.

1.44. If the relationship between slip angles α_1 and α_2 is determined at values of operating factors that are fixed in time and at unchanged design parameters of the vehicle, one refers to the *static turnability* of the vehicle. An example of investigating the static turnability is determining the curvature $1/R_a$ at a fixed value of the steering angle δ of the front axle and constant vehicle speed (note that angle δ is related to the steering angles of the right and left wheels δ'' and δ').

1.45. The *dynamic turnability of a vehicle*, as the ability to change the path of motion is determined upon change in the operating factors and design parameters of the vehicle in time. For example, the path curvature is investigated at a specified mode of change in the turn angles of steered wheels, with the wheel traction control system operating, under unstable road conditions or upon changes in the velocity of the vehicle.

1.46. Stability of motion is an operational property of a vehicle to maintain specified differential parameters of motion or a specified course of variation in them after perturbing forces are removed. The term perturbing forces is here used to denote forces that have not been included in the vehicle's equation of motion due to lack of knowledge of their directions and their magnitudes.

1.47. Handling of a vehicle is an operational property applied to the ability of a vehicle to respond to the steering input with the required precision and speed.

1.48. Detailed answers are contained in Sec. 1.6.

Chapter 2.

2.1. The gear ratio of an open differential is the ratio of the number of teeth of the mechanism's side gears. In symmetrical differentials, unlike their non-symmetrical counterparts, the internal gear ratio is unity.

Answer – NO. The gear ratio of a differential is a parameter of the interaxial differential drive that determines the distribution of torques between the axles of a 4x4 vehicle with an interaxle differential.

Answer – NO. The second wheel rotates twice as fast as the differential's case (see formula (2.10)).

2.2. The correct answer is *a*.

2.3. Answer: No.

2.4. Correct answers:

a) Yes, the torques are equal.

b) No, the circumferential wheel forces are not equal to one another.

2.5. It is sensible to use pinions with odd number of teeth and the number of side-gear teeth divided by the number of pinions (two or four) – fractional.

2.6. The strength and stiffness of a component can be controlled by modifying the shape of its cross section, changing its configuration, changing its manufacturing technology and changing its location within the assembly.

2.7. Changes in the drive axle design may cause a half unloaded axle to become fully free of the vertical loads of wheels that are taken up in unloaded designs by the drive axle housing.

2.8. Components of bolt and rivet connections of the differential's case and the final-drive bevel wheel can be relieved of tension. For this the final-drive wheel should be supported on the case in a manner that this support, rather than the bolt (rivet) connection, would take up the axial force exerted on the final-drive wheel by the final-drive pinion. This axial force is then taken up by the bearing of the differential's case.

2.9. Spring washers should not be used for fixing screwed connections because they at times break when tightened and may fall out from underneath the bolt or nut head in the course of operation and fall between the teeth of the geared transmission. This may cause seizing and breakage of teeth.

2.10. In order to allow disassembly of bearings, the collars on the shafts and in the bearing seats should be made such that the ends of the bearing races that rest on them should be half open. If this cannot be done, then 2-3 slits should be made in the collars. This is done to allow grabbing the end of the bearing race by means of a special device.

2.11. Slots of the same size for the same Woodruff keys on shaft segments having different diameters for improving the shaft manufacturability (for reducing the cost of its manufacture) in the case when the same torque is applied to each of the segments.

2.12. The energy-loading factor K_N, kW/cm^2 of a differential, is obtained by dividing the power P_0 supplied to the differential, by the conditional area A_d. Since power is work per unit time, it may be assumed that K_N reflects to some extent the theoretical level of wear of elements of differentials of the vehicles being compared over the same time of operation. It is assumed here that the magnitude of wear is proportional to the friction work in the differential and with it – to the power.

2.13. The velocity modes of the differential's components will increase in proportion to the gear ratio of the wheel-hub gear set. The sliding velocities of the rubbing surfaces will also increase.

2.14. The lubrication systems of differentials of drive axles can be subdivided into three groups by their operating principle:

- passive systems;
- active systems;
- forced circulation systems.

The operating principle of passive systems consists in placing various oil-trapping pockets on segments with vortical flows of the oil being sprayed, from which the oil flows by gravity over special grooves to the friction areas. Active type systems provide for placing on the rotating components of additional elements, for example, in the form of disks, from which the oil is pushed off in the course of rotation by oil traps and is directed over appropriate grooves to the rubbing surfaces. Forced circulation systems are those in which oil is supplied to difficult-to-reach rubbing couples by all kinds of pumps and other devices for which pumping oil through themselves is not a direct function. There also exist composite systems that combine within themselves elements of the above three types of lubrication systems.

2.15. Basically, use is made of planetary-type non-symmetrical differentials with spur cylindrical gears. Non-symmetrical spur-gear differentials occupy less space in axial direction than bevel-gear differentials. This improves the stiffness of the transfer case. As a rule, spur gears cost less to manufacture.

2.16. The principal design feature of the supports of intermediate shafts of pass-through axles consists in the fact that they should be self-positioning.

2.17. The principal requirements put to lubricating oils are two: the first – they should be pure and the second – their temperature should not rise excessively (usually should not be higher than 80ºC). Satisfaction of the first requirement involves providing for various kinds of filters and settling containers within the lubrication system. Satisfaction of the second requirement involves including oil-cooling devices within the lubricating system.

2.18. Complete air tightness of lubrication systems is prevented by using a breather. The breather connects the lubrication system with the atmosphere and equalizes the pressure in the gear shafts to the atmosphere, thus preventing the squeezing out of lubricating oil through seals.

2.19. The universal joint drive consists of unequal angular velocity universal joints, drive shafts and intermediate supports.

2.20. Universal joint drives are used between units of the vehicle's drive gear for compensating linear and angular displacements of these units to which they are subjected from twisting relative to the elastic frame when the vehicle moves over irregularly surfaced roads and particularly over rugged terrain.

2.21. The following types of universal joints are used in driveline systems: non equal angular velocity rigid joints, equal angular velocity rigid joints (also known as double universal joints) and flexible joints with elastic elements.

2.22. Flexible universal joints are used for connecting two shafts when the angle between them is close to zero and precisely when the angle between these shafts remains unchanged in the course of the vehicle's operation.

2.23. At the same values of torques on the moving and non-moving splined ends of the universal joint the specific load on the splines of the moving end (former) should be 2-3 fold lower than on the splines of the non-moving end (latter) in order to ensure that the lubricant should not be squeezed out from the space between the splines something that would cause accelerated wear of the splines.

2.24. The size of the telescopic joint of the drive shaft is made with a larger diameter according to the number of splines and their height. The purpose of this is to reduce 2-3 fold the pressure on the its splines and of its shank as compared with the splines of the shank of the final drive pinion in order to ensure a relative light movement of the telescopic splined joint. It is known that increasing the spline diameter increases the angular stiffness, i.e., it reduces the angular micro-deformation and, as a result prevents seizing of the telescopic splined joint.

2.25. This is because the universal joint is called upon to perform three functions: a) provide for transmission of power from one unit to another while allowing for their relative displacement in the course of the vehicle's motion; b) compensate for the inevitable longitudinal and angular errors installation between components; c) provide for uniform angular motion of the shafts of components in

the case of deformation of the vehicle's frame that occur every time the vehicle travels over road micro-irregularities.

2.26. When the wheel turns, two elements of the case of the double universal joint and the joint itself form a triangle, with at least one side changing in size, and this is possible only with a moving telescopic joint.

2.27. The moving splined shaft of the universal joint should have a torsional moment resistance that is 1.5-2 fold higher than that of the non-moving shaft in order to prevent its angular distortion and seizing in the mating splined bushing of the non-moving shaft.

2.28. In all cases the universal joint forks should be positioned in the same plane.

2.29. To attain complete equality of the speeds of rotation of two shafts connected by a universal joint, an effort should be made that they would be located strictly parallel relative to one another and also that the angles between them and the universal joint be equal, if the design does not provide for parallel positioning of the two output shafts.

2.30. In general, for drive shafts rotating at 25 to 50 s^{-1} the allowable installation angles range from 3 to 15º. In tandems of multi-axle vehicles these angles may be at times as large as 30º.

2.31. Zero angles are not recommended for drive shaft installation because they cause brinelling.

2.32. When universal joints are used in tandems of multiaxle vehicles their forks should be made longer.

2.33. The intermediate supports in drive shafts are used for suspending intermediate drive shafts.

2.34. The drive wheel gears in vehicles employ equal angular velocity joints because they provide for uniform rotation of the steered drive wheels when making a turn.

2.35. The following requirement must be satisfied in designing the turning device of the steered drive wheel with equal angular velocity joints: the center of the constant velocities joint must be very precisely aligned with the center of the axis of the kingpin.

2.36. The tandems of drive axles of multi-wheel drive vehicles are comprised of paired drive axles with a single suspension.

2.37. The balancing tandems of drive axles allow the drive wheels to properly adapt to terrain roughness and provide high mobility of the vehicle.

2.38. Solution.

It is known that

$$F_{x1} + F_{x2} = F_{x\Sigma}.$$

The interaxle differential distributes the input torque T_0 to its output shafts as follows:

$$T_1 = \frac{T_0}{1+u_d}$$

$$T_2 = \frac{T_0 u_d}{1+u_d}$$

Hence, the circumferential forces are

$$F_{x1} = T_1 u_f / r_{w1}^0 = \frac{T_0}{1+u_d}\frac{u_f}{r_{w1}^0}$$

$$F_{x2} = T_2 u_f / r_{w2}^0 = \frac{T_0 u_d}{1+u_d}\frac{u_f}{r_{w2}^0}$$

Substituting these forces in the above formula for $F_{x\Sigma}$, one finds

$$F_{x1} + F_{x2} = \frac{T_0}{1+u_d}\frac{u_f}{r_{w1}^0} + \frac{T_0 u_d}{1+u_d}\frac{u_f}{r_{w2}^0} = F_{x\Sigma}$$

The input torque T_0 is

$$T_0 = \frac{F_{x\Sigma} r_{w1}^0 r_{w2}^0 (1+u_d)}{u_f (r_{w2}^0 + u_d r_{w1}^0)}$$

Substituting T_0 in the formulae for the circumferential wheel forces, one finds

$$F_{x1} = T_1 u_f / r_{w1}^0 = \frac{T_0}{1+u_d}\frac{u_f}{r_{w1}^0} = \frac{F_{x\Sigma} r_{w1}^0 r_{w2}^0 (1+u_d)}{u_f (r_{w2}^0 + u_d r_{w1}^0)} \frac{u_f}{(1+u_d) r_{w1}^0} = F_{x\Sigma} \frac{r_{w2}^0}{(r_{w2}^0 + u_d r_{w1}^0)}$$

$$F_{x2} = T_2 u_f / r_{w2}^0 = \frac{T_0 u_d}{1+u_d}\frac{u_f}{r_{w2}^0} = \frac{F_{x\Sigma} r_{w1}^0 r_{w2}^0 (1+u_d)}{u_f (r_{w2}^0 + u_d r_{w1}^0)} \frac{u_d u_f}{(1+u_d) r_{w2}^0} = F_{x\Sigma} \frac{u_d r_{w1}^0}{(r_{w2}^0 + u_d r_{w1}^0)}$$

2.39. Three supports cannot be perfectly concentric (coaxial). For this reason when a shaft is installed it usually bends in the course of installation. This produces significant installation loads both on the shaft itself and on its support. This causes premature failure of shafts and of their supports.

2.40. The lateral force on the rear wheel does not depend on the type of the interaxle driveline system (see formula (2.392)). Hence, the differential gear ratio does not affect the lateral force acting on the rear wheels.

2.41. Solution.

1. Consider the empty SUV.

Find static loads on the axle:

$W_{c1} + W_{c2} = 11270$ N

$W_{c1} = 1.68\ W_{c2}$

$2.68\ W_{c2} = 11270$ N

$W_{c2} = 4205$ N

$W_{c1} = 7065$ N

To find the center of gravity, take the moments about the front wheel for static conditions

$$\Sigma M_1 = W_c l_a - W_{c2} l_2 = 11270 l_a - (4205)(2.2) = 0$$

l_a = 0.82 m

Find the height of the center of gravity based on the formula provided in the *Hints*

h = (0.4095 * 2.2 – 0.82) / tan 7°

h = 0.66 m

Account for the inertia forces:

F_a = (2.6 m/s^2)(1150 kg)

F_a = 2990 N

Again sum the moments about the front wheels, this time accounting for acceleration, and find normal reactions of the wheels

$\Sigma M_1 = 11270 * 0.82 + 2990 * 0.66 - R_{z2} * 2.2 = 0$

R_{z2} = 5098 N

R_{z1} = 11270 – 5098 = 6172 N

Further determine

$f_2 = f_1(1 - 0.85 \log_{10} 2)$

f_2 = 0.074

R_{x2} = 0.074 * 5098 = 379 N

R_{x1} = 0.1 * 6172 = 617 N

Now we can find the total circumferential force required for the given conditions:

ΣF_x = 2990 + 379 + 617 = 3986 N

Since $r_{a1}^0 = r_{a2}^0$, we have

$F_{x1} = \Sigma F_x/(1 + u_d)$

$F_{x2} = \Sigma F_x u_d/(1 + u_d)$

For $u_d = 1$,

$F_{x1} = 1993$ N

$F_{x2} = 1993$ N

Assuming that the relationship between the slip coefficient and circumferential force is exponential, solve for slip coefficient as follows

$F_x = R_z \mu_{px} (1 - \exp(-k s_\delta))$

$s_\delta = - (1/ k) \ln (1 - (F_x/R_z \mu_{px}))$

Using our values for the circumferential force and the given empirical wheel values the slip coefficients for the front and rear wheels are:

$s_{\delta 1} = 0.11$

$s_{\delta 2} = 0.13$

Using formula (2.365), calculate the slip efficiency

$$\eta_\delta = \frac{(1+u_d)(1-s_{\delta a1})(1-s_{\delta a2})}{1-s_{\delta a2}+u_d(1-s_{\delta a1})} = \frac{(1+1)(1-0.11)(1-0.13)}{1-0.13+1-0.11} = 0.875$$

2. Consider the fully loaded SUV.

$W_1 + W_2 = 15190$ N

$W_1 = 0.93\ W_2$

$1.93\ W_2 = 15190$ N

$W_2 = 7870$ N

$W_1 = 7320$ N

To find the longitudinal coordinate of the center of gravity

$$\Sigma M_1 = W_a l_a - W_2 l_2 = 15190 l_a - (7870)(2.2) = 0$$

$l_a = 1.14$ m

Find the height of the center of gravity based on the formula provided in the *Hints*

$h = (0.5504 * 2.2 - 1.14) / \tan 7°$

$h = 0.58$ m

Account for the inertia forces:

$F_a = (2.6\ \text{m/s}^2)(1550\ \text{kg})$

$F_a = 4030$ N

Sum the moments about the front wheels, this time accounting for acceleration, and find

$W_2 = 8934$ N

$W_1 = 15190 - 8934 = 6256$ N

The rolling resistance factors are the same as for the empty vehicle

$f_2 = f_1(1 - 0.85 \log_{10} 2)$

$f_2 = 0.074$

The rolling resistance forces are

$R_{x2} = 0.074 * 8934 = 661$ N

$R_{x1} = 0.1 * 6256 = 626$ N

Now we can find the total circumferential force required for the given conditions:

$\Sigma F_x = 4030 + 661 + 626 = 5317$ N

Distribute the total circumferential force between the axles

$F_{x1} = \Sigma F_x / (1 + u_d)$

$F_{x2} = \Sigma F_x u_d / (1 + u_d)$

For $u_d = 1$,

$F_{x1} = 2659$ N

$F_{x2} = 2659$ N

Given that the relationship between the slip coefficient and circumferential force is exponential, we can solve for slip coefficient as follows

$F_x = R_z \mu_{px} (1 - \exp(-k s_\delta))$

$s_\delta = - (1/k) \ln (1 - (F_x/R_z \mu_{px}))$

Using our values for the circumferential force and the given wheel empirical values the slip coefficient for the front and rear wheels are:

$s_{\delta 1} = 0.18$

$s_{\delta 2} = 0.09$

Using formula (2.365), calculate the slip efficiency

$$\eta_\delta = \frac{(1+u_d)(1-s_{\delta a1})(1-s_{\delta a2})}{1-s_{\delta a2}+u_d(1-s_{\delta a1})} = \frac{(1+1)(1-0.18)(1-0.09)}{1-0.18+1-0.09} = 0.863$$

The optimal gear ratio of the interaxle differential is obtained by setting the slip coefficients equal to one another (also see a graph below),

$s_{\delta 1} = s_{\delta 2}$

$(1/k_1) \ln (1 - (F_{x1}/R_{z1}\mu_{px\,1})) = (1/k_2) \ln (1 - (F_{x2}/R_{z2}\ \mu_{px2}))$

Loaded Vehicle

Ratio	F_{x1}	F_{x2}	$S_{\delta 1}$	$S_{\delta 2}$
0.6	3323.1	1993.9	0.309	0.059
0.7	3127.6	2189.4	0.256	0.066
0.8	2953.9	2363.1	0.221	0.073
0.9	2798.4	2518.6	0.196	0.080
1.0	2658.5	2658.5	0.176	0.086
1.1	2531.9	2785.1	0.160	0.092
1.2	2416.8	2900.2	0.147	0.097
1.3	2311.7	3005.3	0.137	0.102
1.4	2215.4	3101.6	0.127	0.108
1.5	2126.8	3190.2	0.119	0.112
1.6	2045.0	3272.0	0.112	0.117
1.7	1969.3	3347.7	0.106	0.121
1.8	1898.9	3418.1	0.101	0.126
1.9	1833.4	3483.6	0.096	0.130
2.0	1772.3	3544.7	0.091	0.134

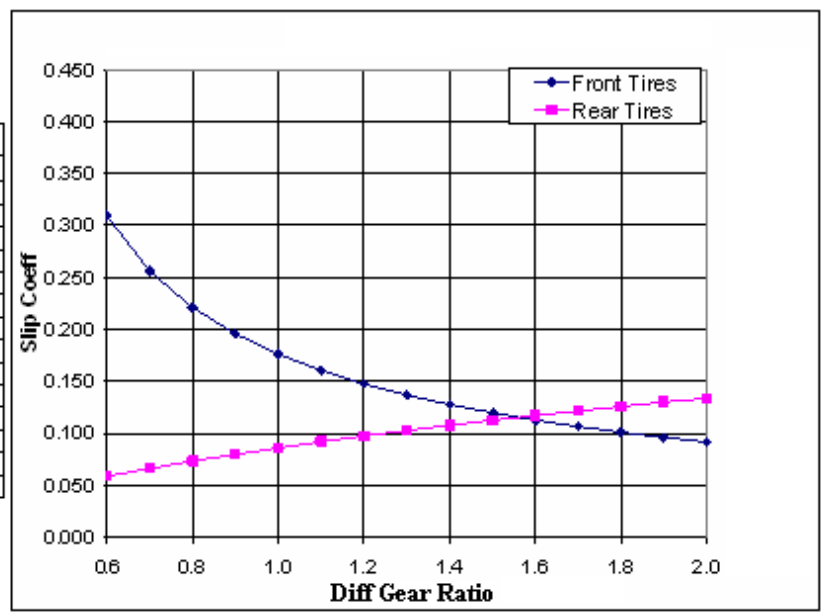

Fig. 2.41 Q and P. Optimal gear ratio

Chapter 3

3.1. The ratio of angular velocities of the output shafts of a locked power-dividing unit may be constant or may be controllable by the driver or by a control system.

3.2. A double-side action overrunning clutch is a double single-side action overrunning clutch with toothed locking clutch.

3.3. The theoretical velocities of the wheels are

$$V_{ti} = \omega_{wi} r_{wi}^{0} \text{ , } i = 1, n,$$

The velocities can be different because of differences in (i) the angular velocities of the wheels (rpm of the wheels can be different due to different gear ratios from the transfer case to the front and rear wheels) and (ii) the rolling radii in the driven mode due to different tire size, inflation pressure and normal loads on the front and rear wheels.

3.4. No. The kinematic discrepancy determines the distribution of power between the front and rear wheels of a 4x4 vehicle with a positively locked driveline system.

3.5. The extent of disagreement of the theoretical velocities V_{t1} and V_{t2} is assessed by introducing the kinematic discrepancy factor (see formula (3.6.)):

$$m_{ru} = \frac{V_{t2} - V_{t1}}{V_{t2}} = \frac{r_{a2}^{0} u_1 - r_{a1}^{0} u_2}{r_{a2}^{0} u_1}$$

that incorporates the inequality of the rolling radii r_{a1}^{0} *and* r_{a2}^{0} of the gear ratios u_1 and u_2.

Design kinematic discrepancy is that that stems from the very design of the vehicle (different gear ratios u_1 and u_2, different size of the front and rear wheels, $r_{a1}^{0} \neq r_{a2}^{0}$).

Operational-manufacturing kinematic discrepancy is that that arises from the inevitable difference in the dimensions of the tires due to manufacturing tolerances, unequal static and dynamic weight loads on the wheels and differences in tire air pressure and in tire wear.

3.6. The kinematic discrepancy is set in a positively engaged driveline system in which automatic engagement/disengagement of one axle or a group of axles is brought about by a device that is actuated by differences in the angular velocities of its elements (for example, a free-running clutch).

3.7. The appearance of operational-manufacturing kinematic discrepancy in a positively engaged driveline system may be caused by non-equal rolling radii in the driven mode (the reasons for this may be different normal wheel loads, different tire inflation pressures, different tire wear) and unequal paths of wheels when moving on a road with irregularities and when making a turn.

3.8. The difference in the lengths of paths traveled by the front and rear wheels. The front wheels travel over a curve with a larger radius than the rear wheels.

3.9. According to the method suggested в Sec. 3.2.2, the difference between the theoretical velocity V_{ti} of each axle and the theoretical velocity V_a of the vehicle is the kinematic discrepancy of the *i*th axle of the vehicle. And the ratio of this difference to velocity V_{ti} determines the kinematic discrepancy factor m_{Hi} of the given axle (see formula (3.20)):

$$m_{Hi} = \frac{V_{ti} - V_a}{V_{ti}}$$

It is quite obvious that this method of assessing the kinematic discrepancy of the velocities of the driving axles appears to be more natural. The method immediately compares the two principal parameters of the axle: its actual velocity V_a as a part of the vehicle (e.g. theoretical velocity of the vehicle) and the theoretical velocity V_{ti}, which it would have had, had it moved separately from the vehicle.

3.10. If a negative torque will appear at least one of the drive wheel/axles of a vehicle, then the vehicle together with the surface of the road will form a loop through power known as parasitic will start circulating.

3.11. The rolling radii of two wheels will equalize in the plane of the road by virtue of tangential deformation of each tire; these deformations will be of equal magnitude but of opposite direction.

3.12. Experience shows that a vehicle with a locked interaxle driveline has higher fuel consumption than that with a differential driveline (usually by up to 10-12%). This is attributed to additional tangential deformations of tires in contact with the road by virtue of kinematic discrepancy that arises in the drive gear of the vehicle's axles.

3.13. As a result of forces arising in the teeth meshing the intermediate gear is acted upon by two transverse forces: the resultant of two circumferential forces (the larger force) and the resultant of two radial forces (smaller force). In arrangement *a* (see Fig. 3.13 Q and P) they act in opposite directions, whereas in arrangement *b* – in the same direction. For this reason arrangement *a* is preferable from the point of view of load on the bearings. However, it was noted that in the case of very small clearance and very high loads in the meshing of teeth, in the case of arrangement *a* the intermediate gear undergoes an internal micro-displacement that produces a clearance-less meshing leading to failure of the reduction gear. In order to avoid this failure, the design clearance in the meshing of teeth in arrangement *a* should be increased.

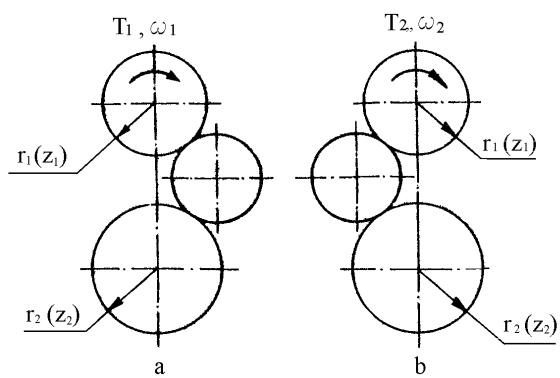

Fig. 3.13 Q and P. Diagrams of gear sets

3.14. The slip factors of the front and rear tires are equal if the kinematic discrepancy is zero. This is seen from formula (3.61).

3.15. The same. Since the length of the front and rear frames are equal to each other, the radii in the driven mode of the tires are equal (the normal loads and the inflation pressure are the same), and also the gear ratios are the same, so the kinematic discrepancy is zero and slippage of the front tires will be equal to the slippage of the rear tires when the vehicle is making a turn.

3.16. Solution.

The kinematic discrepancy factor, $m_{ru} = \dfrac{r_{w2}^0 u_1 - r_{w1}^0 u_2}{r_{w2}^0 u_1}$

Assuming $u_1 = u_2$, m_{ru} reduces to: $m_r = \dfrac{r_{w2}^0 - r_{w1}^0}{r_{w2}^0} = 1 - \dfrac{r_{w1}^0}{r_{w2}^0}$.

For a 15.00-20 tire, the wheel diameter is 20" and the tire height is 15", therefore the non-loaded tire radius r is equal to 25" or 0.635 m. Thus:

$$\boxed{r_w^0 = 0.635 \frac{0.635 p_w + 0.0004310 W_w}{0.635 p_w + 0.0004965 W_w}}$$

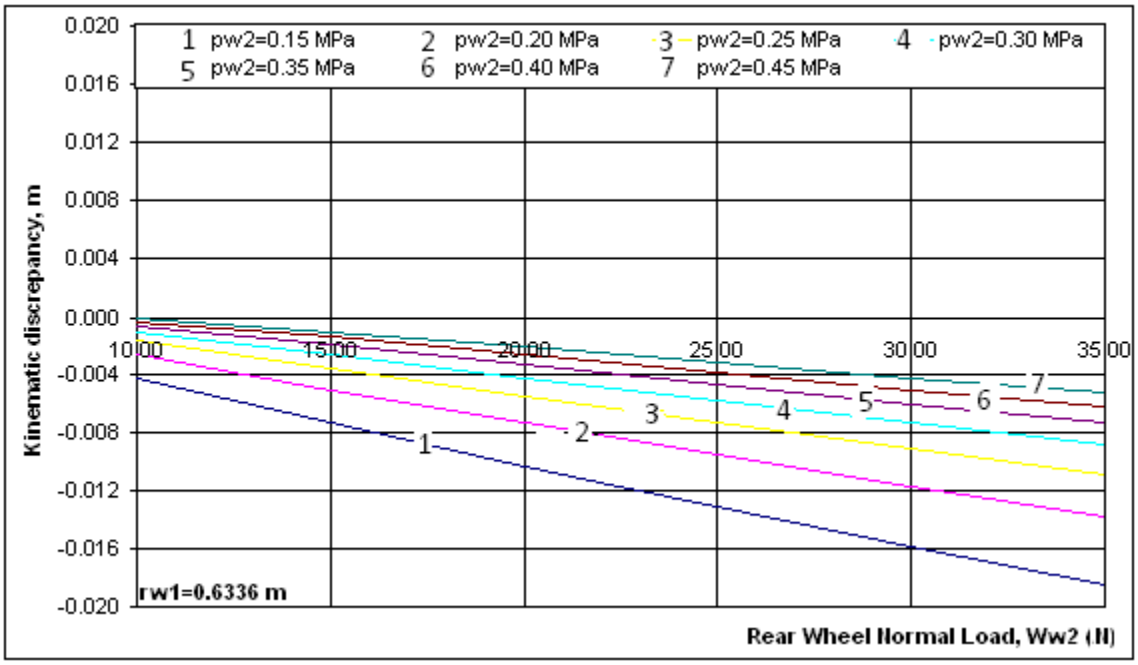

Fig. 3.16a Q and P. Kinematic discrepancy for the case of r_{w1}^{0} = 0.6336 m (p_{w1} = 0.45 MPa, W_{w1} = 1 kN) (maximum effective rolling radius)

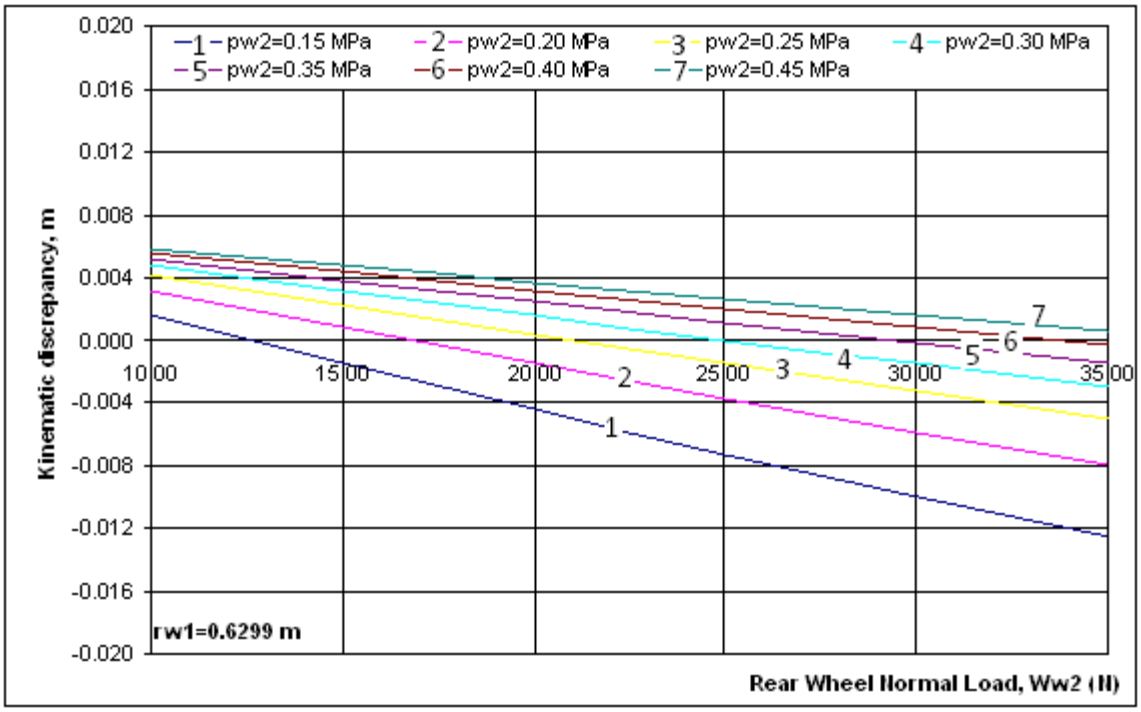

Fig. 3.16b Q and P. Kinematic discrepancy for the case of r_{w1}^{0} = 0.6299 m (p_{w1} = 0.30 MPa, W_{w1} = 2.5 kN) (mid range effective rolling radius)

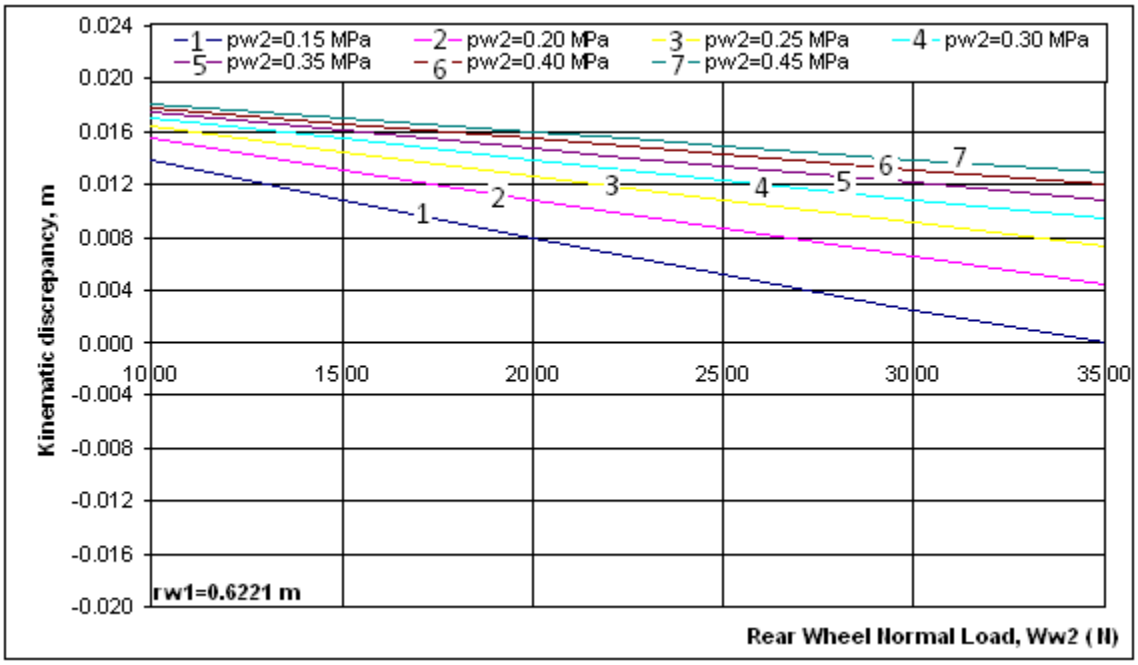

Fig. 3.16c Q and P. Kinematic discrepancy for the case of r_{w1}^{0} = 0.6221 m (p_{w1} = 0.15 MPa, W_{w1} = 3.5 kN) (minimum effective rolling radius)

3.17. Algorithm for solving the problem:

Determine the kinematic discrepancy factor m_R from formula (3.16).

Calculate the slippage of the front and rear wheels from formula (3.69), in which m_{ru} should be replaced by m_R.

Calculate the circumferential forces of the front and rear wheels from formulae (3.70).

3.18. If the design kinematic discrepancy and the operational-manufacturing kinematic discrepancy are of the same magnitude and opposite sign, then the overall kinematic discrepancy is equal to zero. This has a negative effect on the performance of the interaxle power-dividing unit, the normal operation of which requires kinematic discrepancy (the additional axle will in this case constantly engage and disengage).

If the design kinematic discrepancy and the operational-manufacturing kinematic discrepancy in will be of the same magnitude and the same sign, this will increase the overall kinematic discrepancy. As a result, the additional axle will be engaged upon significant slippage of the principal axes (see formula (3.61)). This will cause elevated power loss in slippage. For this reason the design kinematic discrepancy is usually held to 5-8%.

3.19. Equality of the angular velocities of the two drive wheels with the locked differential causes equality or almost equality of their slippage when moving in a straight line, which brings about more economical operation of the tractor's engine when performing field work (lowest slippage power losses). In the case of a differential drive, on the other hand, the wheel slippages may be unequal, which causes elevated slippage power losses. However, when turning with the differential locked, there arises a torque that resists the turn. For this reason differentials are unlocked when making a turn.

3.20. The turning of an axle with a locked interwheel drive, that is, with the wheels rotating at the same angular velocities, will cause elevated slippage and involve the appearance of an elevated circumferential force at the wheel that is inner relative to the vehicle's center of turn, if the gripping conditions of the wheels are the same or if the inner wheel moves over soil that is harder than that on which the outer wheel moves. The higher the overall traction force of the vehicle (e.g. the total circumferential force of the vehicle), the larger the difference between the circumferential forces of wheels for the same turn radius.

If the inner wheel moves over soil that is softer than that under the outer wheel, then at high turning radii the circumferential force of the inner wheel is lower than that of the outer wheel. As the turning radius decreases, the magnitudes of the circumferential forces become close to one another and then the circumferential force of the inner wheel becomes higher than that of the outer wheel. The higher the overall traction force of the vehicle, the smaller the turning radius at which the circumferential forces become equal.

3.21. It is preferable to have zero kinematic discrepancy, at which the fuel consumption is at minimum. The gear ratio u_1 at which zero kinematic discrepancy is attained it determined from formula (3.6)

$$m_{ru} = \frac{V_{t2} - V_{t1}}{V_{t2}} = \frac{r_{a2}^0 u_1 - r_{a1}^0 u_2}{r_{a2}^0 u_1} = \frac{0.75u_1 - (0.55)(27.9)}{0.75u_1} = 0$$

$$u_1 = (0.55)(27.9)/(0.75) = 20.46$$

3.22. Since $u_1 = u_2 = u_f$, the kinematic discrepancy is

$$m_r = \frac{r^0{}_{a2} - r^0{}_{a1}}{r^0{}_{a2}} = \frac{278 - 286}{278} = -0.0288$$

It is seen from formula (3.6) that zero kinematic discrepancy can be provided by changing either the gear ratio of the final drive in the front axle, or in rear axle, or in both

$$m_{ru} = \frac{V_{t2} - V_{t1}}{V_{t2}} = \frac{r_{a2}^0 u_1 - r_{a1}^0 u_2}{r_{a2}^0 u_1}$$

The most reasonable way is to change u_1 or u_2. Determine a new value for u_1 that makes $m_{ru} = 0$:

$$m = \frac{V_{t2} - V_{t1}}{V_{t2}} \Rightarrow \frac{r^0{}_{a2} u_1 - r^0{}_{a1} u_2}{r^0{}_{a2} u_1}$$

$$u_1 = \frac{r^0{}_{a1} u_2}{\left(1 - m_{ru}\right) r^0{}_{a2}}$$

from this

$$u_1 = \frac{r^0{}_{a1} u_2}{r^0{}_{a2}} = \frac{(286)(4.3)}{278} = 4.42 \quad \text{when } m_{ru} = 0$$

When u_1 = const,

$$u_2 = \frac{r^0{}_{a2} u_1}{r^0{}_{a1}} = \frac{(278)(4.3)}{286} = 4.18 \quad \text{when } m_{ru} = 0$$

3.23. The kinematic discrepancy is defined as:

$$m_{ruR} = (r_{a2}^{0}\, u_1\, R_{t1} - r_{a1}^{0}\, u_2\, R_{t2})/r_{a2}^{0} u_1\, R_{t1}$$

However, in this case we have it that the effective front and rear wheel radii, as well as the front and rear drive ratios are equal, and hence:

Since

$$r_{w2}^{0} = r_{w1}^{0}$$

$$u_1 = u_{2,}$$

the kinematic discrepancy is

$$m_R = (R_{t1} - R_{t2})/R_{t1}$$

$$= 1 - R_{t2}/R_{t1}$$

$$= 1 - \cos\delta_1$$

This would indicate that the kinematic discrepancy is a function of the turn angle only. No other vehicle parameters are involved.

Yes, we get the same values of the kinematic discrepancy for any vehicle with positive locking axle engagement and front steering system.

δ_1	m
0	0.000
5	0.004
10	0.015
15	0.034
20	0.060
25	0.094
30	0.134
35	0.181
40	0.234
45	0.293
50	0.357
55	0.427
60	0.500
65	0.578
70	0.658
75	0.741
80	0.827
85	0.913
90	1.000

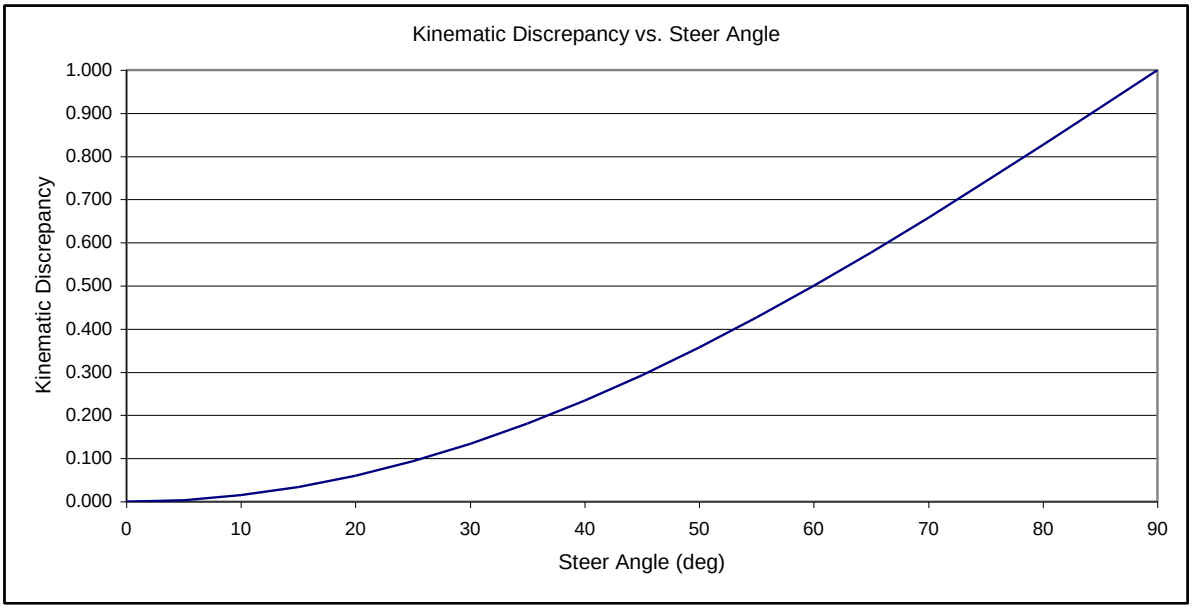

Fig. 3.23 Q and P. Kinematic discrepancy vs. steering angle.

3.24. The kinematic discrepancy is (use Fig. 2.13)

$$m_R = (R_{t1} - R_{t2})/R_{t1}$$

$$= (1/\cos\delta_1 - 1/\cos\delta_2)/(1/\cos\delta_1)$$

$$m_R = 1 - \cos\delta_1/\cos\delta_2$$

The front steering angle can be specified, but the manner in which the rear steering angle reacts to the front steering angle is unknown. To assist us, let's define a geometric parameter (ratio) *RR* as:

$$RR = O_1C/O_1O_2$$

$$= \tan\delta_1/(\tan\delta_1 + \tan\delta_2)$$

This in turn allows us to define the rear steering angle as a function of this geometric parameter:

$$\delta_2 = \tan^{-1}((1/RR - 1)\tan\delta_1)$$

Now we can define the kinematic discrepancy in terms of the front steering angle and *RR*:

$$m_R = 1 - \cos\delta_1/\cos(\tan^{-1}((1/RR - 1)\tan\delta_1))$$

When RR = 0.0, the vehicle is effectively a front steered only vehicle, and this implies that the center of turn is in line with the rear axle. In this case, the relationship of kinematic discrepancy and front steer angle matches the results of problem 3.23.

For RR = 0.5, the kinematic discrepancy is zero. This is because the turn radius is identical for the front and rear wheels.

Chapter 4

4.1. No, the reduced internal slip (e.g. the increased internal friction) in the limited slip differential does not markedly affect the power losses in this mechanism. This is seen from analysis of formula (4.3) and Fig. 4.2.

4.2. The ratio of torques (of the higher to the lower) at the output shafts of limited slip differentials in their relative rotation is known as the locking factor or the torque bias:

$$K_d = \frac{T''}{T'} = \frac{T_0 + T_r}{T_0 - T_r}$$

The locking factor represents the degree of redistribution of torques between the output shafts of an limited slip differential (see formulae (4.10)).

4.3. The ratio of torques T'' and T' at the instant of actuation of the differential, i.e., at the instant of appearance of relative rotation of the output shafts, is represented by the so-called static locking factor K_{dp}, determined by the static friction torque T_{rs} and static friction factor μ_s:

$$K_{dp} = \frac{T''_{(2)}}{T'_{(2)}} = \frac{T_0 + T_{rs}}{T_0 - T_{rs}} = \frac{1 + \mu_s P}{1 - \mu_s P}$$

Since the sliding (dynamic) friction factor μ is somewhat smaller than μ_s, then after the differential is actuated, torques T'' and T' change. The ratio of torques in these points is equal to the value of torque

bias K_d from formulae (4.6) and (4.14) that is determined at relative rotations of the differential's output shafts.

4.4. This problem should be solved using the approach demonstrated by Fig. 4.15. This figure should be reconstructed in a manner that points *B* and *D* in the square *ABCD* would have as their respective coordinates (0.1, 0.8) and (0.8, 0.1). Then formula (4.16) should be used for calculating the percentage of road conditions under which the vehicle can develop maximum traction.

4.5. The formula to calculate the torque bias is (4.31)

$$K_d = \frac{(1+\mu A)(1+\mu B)}{(1-\mu A)(1-\mu B)}$$

As seen from the formula, the geometric parameter A should be decreased to keep the same value of K_d when the friction coefficient increases from 0.1 to 0.14.

Since

$$A = \frac{r_M i_M}{r_g} \tan\alpha_w \cos\delta_c,$$

we see that the following differential geometric parameters can be changed to keep the same value of K_d:

$$r_M,\ i_M\ ,r_g.$$

Also, we can change the pressure angle α_w and the pinion pitch angle δ_c. However, the pressure angle is standard. Changing angle δ_c requires changing the number of the pinion's teeth, something that involves many of problems related to the differential's durability. The easiest way to decrease the parameter A is to reduce the number of friction pairs i_M.

4.6. Formula (4.32) for calculating the torque bias is:

$$K_d = \frac{1+\mu P_1}{1-\mu P_1}$$

The differential is fully locked when the torque bias goes to infinity: $K_d = \infty$.

Form this we have:

$$1-\mu P_1 = 0.$$

Hence,

$$\mu P_1 = 1$$

and the value of the friction factor at which the differential is fully locked is

$$\mu = 1/P_1.$$

When designing a limited slip differential, one has to choose the materials for the plate disks, i.e. to choose the friction factor μ, and to package the differential, i.e. to choose the differential parameters r_M, i_M ,r_g, α_w, δ_c that provide a value of the parameter P_1 in such a way that provide $\mu P_1 < 1$.

4.7. The substance of differentials with constant friction torque consists in the fact that the disks of the clutches of these differentials are subjected to constant compression force, primarily by means of springs. For this reason the friction torque is constant, but the torque bias, as seen from formulae (4.10) and (4.126) turns out to be variable upon variation in the torque, decreasing with increase in the latter.

4.8. In such differentials the internal friction torque depends on the torque applied to the case or on the relative rotational velocity of the principal elements in such a manner that the locking factor from formula (4.10) will turn out not to be constant, but variable as a function either of the applied torque or of the above relative rotational velocity.

4.9. The design feature specific to the worm-gear differential consists in the fact that it uses worm gear pairs rather than bevel gearing. The locking factor is constant.

4.10. The cam-plunger differential is a cam-plunger mechanism that obeys the principal equation of differential kinematics. The elevated friction is not produced in friction clutches, but in the plunger-sprocket rubbing pairs.

4.11. The feature specific to a differential with hydraulic resistance is the fact that its design is based on a hydraulic pump with a system of plungers that interact with the sprockets. The resistance to the liquid flow produces a resistance to the relative rotation of the mechanism's output shafts.

4.12. See formula (4.161) for the resisting yaw moment. As seen from the formula, the yaw moment is constant if the torque bias is constant. However, the yaw moment is increasing when the slip coefficient of the rear wheel rises. Since the vehicle has two positively engaged axles, the circumferential force at the rear wheels is increasing and the yaw moment also becomes larger.

4.13. Solution:

$$s''_{\delta 2} = s'_{\delta 2} = \frac{F_{x\Sigma}}{K'_{x2} + K''_{x2}} = \frac{30}{100 + 200} = 0.1$$

$$F''_{x2} = K''_{x2} s''_{\delta 2} = (200)(0.1) = 20kN$$

$$F'_{x2} = K'_{x2} s'_{\delta 2} = (100)(0.1) = 10kN$$

$$M_{ZR} = (0.5)(1.8)(20 - 10) = 9kNm$$

$$F^{w}_{y1} = F^{w}_{y2} = M_{ZR} / L = 9 / 4.2 = 2.14kN$$

4.14. Solution.

The torque bias is

$$K_d = K_0 \frac{(1 - \mu A)T_0 + \mu H}{(1 + \mu T_0) - \mu H}$$

From this we have

$$3 = (1.18)\frac{(1-(0.1)(4.5)(3200)+0.1H}{(1+(0.1)(4.5)(3200)-0.1H}$$

and the geometric parameter H is equal to 28,321.

The radius of turn when the differential becomes actuated is as follows

$$R_d = 0.5t_B \frac{(K_d+1)}{(K_d-1)}\frac{(1-s_\delta)}{s_\delta}$$

The slip factor s_δ of the wheel before the axle started making the turn, is determined from the formula

$$F_x = K_x s_\delta .$$

The wheel circumferential force is $F_x = T_0 u_k /(2r_w^0) = (3200)(3)/((2)(0.685)) = 7007.3N$.

The slip factor is $s_\delta = \frac{F_x}{K_x} = \frac{7007.3}{140150} = 0.0499$.

The radius of turn is

$$R_d = 0.5t_B \frac{(K_d+1)}{(K_d-1)}\frac{(1-s_\delta)}{s_\delta} = (0.5)(2.2)\frac{(3+1)}{(3-1)}\frac{(1-0.0499)}{0.0499} = 41.8m$$

The 20% increase in torque causes the input torque to increase as

$$T_0 = (1.2)(3200) = 3840Nm .$$

Calculate a new value of the torque bias

$$K_d = K_0 \frac{(1-\mu A)T_0 + \mu H}{(1+\mu A)T_0 - \mu H} = 1.18\frac{(1-(0.1)(4.5))3840+(0.1)(28321)}{(1+(0.1)(4.5))3840-(0.1)(28321)} = 2.13$$

The wheel circumferential force is $F_x = T_0 u_k /(2r_w^0) = (3840)(3)/((2)(0.685)) = 8408.8N$.

The slip factor is $s_\delta = \frac{F_x}{K_x} = \frac{8408.8}{140150} = 0.0599$.

The radius of turn is

$$R_d = 0.5 t_B \frac{(K_d + 1)}{(K_d - 1)} \frac{(1 - s_\delta)}{s_\delta} = (0.5)(2.2) \frac{(2.13 + 1)}{(2.13 - 1)} \frac{(1 - 0.0599)}{0.0599} = 47.8m$$

4.15. Determine the value of torque bias if the friction factor is $\mu = 0.12$;

$$K_d = \frac{1 + \mu P_1}{1 - \mu P_1} = \frac{1 + (0.12)(4.5)}{1 - (0.12)(4.5)} = 3.35$$

The total circumferential force of the drive axle is

$$F_{x\Sigma} = T_0 / r_w^0 = 3,200 / 0.44 = 7272.72N$$

The axle slippage is

$$s_{\delta a} = \frac{F_{x\Sigma}}{2K_x} = \frac{7272.72}{(2)(150000)} = 0.024$$

Calculate the turn radius R_d of the axle at which the limited slip differential loaded with $T_0 = 3,200Nm$ becomes actuated

$$R_{td} = 0.5 t_B \frac{K_d + 1}{K_d - 1} \frac{1 - s_{\delta a}}{s_{\delta a}} = (0.5)(2) \frac{3.35 + 1}{3.35 - 1} \frac{1 - 0.024}{0.024} = 75.26m$$

4.16. These two differentials affect the turnability of a vehicle in a different manner. The point is that the differential with constant internal friction torque creates a yaw resistance moment that is constant and independent of the drive torque supplied to the differential's case. The differential with internal friction torque and accordingly, the locking factor, that decrease with increasing T_0 produces in the plane of the road a yaw resisting moment that is not constant, but decreases with increasing T_0 (see formulae (4.163) and (4.164). This is the substance of the difference in the effect of the above differentials on a vehicle's turnability.

Chapter 5

5.1. A free-running differential may also be called a free-running toothed clutch.

5.2. The free-running differential rigidly locks the half axles of the drive axle (axles) when the vehicle travels in a straight path and disengages the advancing wheel of the drive axle (advancing axle) when it moves in the course of a turn.

5.3. The principal drive element of the free-running differential is a drive clutch with two rings of straight teeth.

5.4. Each of the two gear rings of the drive clutch of the free-running differential has usually 12 to 18 teeth located one opposite the other.

5.5. The advancing wheel of the drive axle when a vehicle turns is disengaged by an elongated key-like tooth of the drive clutch, trapezoidal (arc-shaped) teeth of the central ring and of the inner gear ring of the clutch halves, a circular slot in the central gear ring.

5.6. Noiseless operation of modern free-running differentials is achieved by a split spring lock ring with trapezoidal teeth.

5.7. The design of currently produced free-running differentials is based on that of the NoSPIN differential.

5.8. In the course of change of direction of the vehicle (forward-reverse and vice versa) when the vehicle moving in a straight line in the traction mode the differential becomes separated into the two kinematic links *A* and *B* (see Section 5.1).

5.9. Link *A* is made up of the drive clutch and the case of the free-running differential, whereas link *B* is comprised of both driven half clutches with split spring lock rings, the central ring and both side gears (see Section 5.1).

5.10. When a vehicle moving forward or backward in a straight line is braked, the free-running differential behaves in the same manner as upon a switch from forward to reverse motion in the traction mode, i.e., the mechanism becomes separated into the two kinematic links *A* and *B*.

5.11. When the vehicle makes a turn, the mechanism of the free-running differential becomes separated into kinematic links *A′* and *B′* (see Section 5.1).

5.12. Link *A′* is comprised on the drive clutch with the differential's case, the central ring, the half clutch of the inward (lagging in making a turn) wheel of the vehicle with split spring lock ring and the outer side gear (see Section 5.1).

5.13. The wheel of a drive axle with a free-running differential that is disengaged when turning in the traction mode is the one that is external relative to the vehicle's center of turn.

5.14. The wheel of a drive axle with a free-running differential that is disengaged when a turning vehicle is braked by the engine is the lagging inward wheel relative to the vehicle's center of turn.

5.15. When a vehicle turns while coasting either the left or the right wheel of a drive axle with a free-running differential may become disengaged.

5.16. The reason why, when a vehicle is turning while coasting any wheel of a drive axle with a free-running differential may disengage is the absence at that time of a circumferential force that would act on the straight power teeth or on the drive clutch and of the driven clutch halves. In order to ensure that at least one of the wheels with a free-running differential would definitely disengage when a coasting vehicle is turning it would have been necessary to round off the shapes of the trapezoidal teeth of the center ring of the drive clutch. This must have been done, since otherwise, when a coasting vehicle would made a turn, the axle's free-running differential would have continued maintaining a rigid linkage between the axle's wheels.

5.17. When a vehicle traveling in the traction mode makes a turn neither the left nor the right wheel of a drive axle with a free-running differential may disengage, as happens while coasting because the meshing

of the straight teeth of the half clutch of the inner (lagging) wheel with those of the drive clutch is acted upon by a large circumferential force that forms a large axial friction force in the meshing that prevents disengagement of the inner wheel.

5.18. Springs for free-running differential should be selected both in the compressed and in the no-load state. In the compressed state they are selected to have the same compression force and in the non-load state they are selected to have the same length.

5.19. In order to provide for precise disengagement at the proper moment of one of the driven half clutches of the free-running differential, its straight power teeth and the teeth of the drive clutch should be made 0.2-0.3 mm shorter than the trapezoidal teeth of the center ring.

5.20. To ensure precise disengagement of the driven half clutches of a free-running differential their trapezoidal teeth should 0.15-0.20 mm higher than those of the teeth of the driven half clutches of the free-running differential.

5.21. To prevent the trapezoidal teeth of the driven half clutch of a free-running differential from sliding off the ends of the teeth of the center ring earlier than the teeth of the split spring lock ring, the teeth of the split spring lock ring should be made 1.2-1.3 mm narrower than the trapezoidal teeth of the driven clutches.

5.22. In order not to weaken the coupling between the split spring ring with the driven half clutch of a free-running differential due to the circumferential force exerted on it, the split spring lock ring is installed in the driven half clutch with a pre-tension. The split spring ring is here seated in such a manner that it would bear by its outer cylindrical surface on the collar of the driven half clutch.

5.23. The width of the slot in the lock (slit) of the split ring lock ring placed in the driven half clutch of a free-running differential should be equal to the distance between two adjoining tips of its adjoining teeth.

5.24. It is not permissible to modify the design of free-running differentials and their elements in a manner that would entirely change their layout.

5.25. As a rule free-running differentials are used in running gear of heavy-duty off-road vehicles, road-construction machines and heavy-duty wheeled tractors.

5.26. Free-running differentials are used as interwheel and interaxle power dividing units.

5.27. Yes, free-running differentials are used in front steered axles of multi-wheel drive wheeled vehicles.

5.28. Yes, a roller-type overrunning (free-running) clutch may be used instead of a free-running differential in the interaxle drive gear of a 4x4 vehicle, but this requires introducing design kinematic discrepancy between the drive axles of the vehicle.

5.29. Yes, free-running differentials can operate in the interaxle drive gear without kinematic discrepancy between the drive axles.

Chapter 6

6.1. The fully locked drives have the greatest effect on the range of distribution of circumferential forces between the wheels of a multi-wheel drive vehicle; fully differential drives have the least effect. Driveline systems containing open and free-running differentials take an intermediate position.

6.2. Yes, the motion will be stable, since the value 0.6 does not exceed excessively the allowed value of 0.5 and three axles are capable of holding the vehicle from skidding, since the gripping-force utilization factor $K_{\mu i}$ of the wheels of these axles is smaller than 0.5.

6.3. When a vehicle with a positively engaged interaxle driveline system turns, the circumferential force of the front wheels becomes negative (parasitic power circulation). This causes the lateral reaction of the front wheels to change direction and to markedly increase in the magnitude. This increases the slip angle of the wheels of a vehicle with a positively engaged interaxle driveline system and increases its turning radius. This does not occur when using an open interaxle differential because then the circumferential force of the front wheels is always positive and reduces the lateral reaction of these wheels.

6.4. This is a multi-criterion problem. Chapter 1 and Sec. 6.2 discussed the operating properties of vehicles that depend on the type of the driveline system, i.e., on the distribution of power between the wheels. For this reason the optimization of distribution of power between wheels is a multi-criterion problem.

6.5. Optimization of the power distribution includes specifying a program of the vehicle's motion, that is, a set of kinematic parameters that must be provided for in the motion. For example, a functional relationship is specified between acceleration and the velocity of the accelerating vehicle – formula (6.20).

6.6. This should be done using equations (6.45)

$$\begin{cases} 1/(1-s_{\delta i}^{('')}) + (\exp(k_i^{('')} s_{\delta i}^{('')}) - 1)/(k_i^{('')}(1-s_{\delta i}^{('')})^2) = -\lambda \\ \sum_{i=1}^{n} b_i^{('')}(1-\exp(-k_i^{('')} s_{\delta i}^{('')})) = A \end{cases}$$

In these equations number n of drive axles should be 3, the number i ranges from 1 to 3, since we are dealing with a 6x6 vehicle. Parameter A in this system is the overall circumferential force $F_{x\Sigma}$ of the drive wheels, equal to the applied resistance to motion. The value of $F_{x\Sigma}$ is determined from equation (6.31), in which $n = m = 3$. This leaves the slippage of wheels and the parameter A as the unknowns in the seven equations.

6.7. Irrespective of whether the gripping factors between the vehicle's wheels and the surface of motion are the same or different, the optimum slippages corresponding to maximum slippage efficiency and maximum running gear efficiency are slightly not equal to one another when the relationship between the circumferential force of the wheel and its slippage is not linear.

6.8. It is not advisable; the difference between the optimum slippages is negligible. Making allowance for it will highly complicate the control system and will not have a positive effect on the fuel efficiency of the vehicle.

6.9. The scooping up of material into the bucket is the most energy intensive process and requires higher traction properties as compared with other motion modes of the loader.

6.10. No, constraints (6.19) are the same.

6.11. The properties of the interwheel drive are treated as additional constrains in optimizing the power distribution between the wheels.

6.12. Strictly speaking, the slippages of the left and right wheels of the front axle will not be equal and those of the left and right wheels of the rear axle may also not be equal when the power distribution between the wheels is optimal (see Sec. 6.3). However, the difference in the optimal slippages is negligible, which was also shown in Sec. 6.3.

6.13. No. The generalized slippages of the axles are not equal to one another, since the interaxle drive is positively engaged. When material is scooped up into the bucket, the normal loads on the axles and, consequently, the rolling radii in the driven mode of the front and rear wheels will be different. This will cause kinematic discrepancy between the axles, which means that the slippages of the axles will be different (see Chapter 3).

6.14. When material is scooped up into the bucket the weight of the loader increases and with it the rolling resistance of the wheels.

6.15. This happens because the wheels of the axles move along the same track,, whereas the rolling resistance factors of all the wheels are different – they are highest at the front and lowest at the rear wheels. On the other hand, the distribution of normal loads shown in Fig. 6.22 controls the manner of variation in the rolling resistance forces of the three axles and in the total wheel rolling resistance.

6.16. As seen from Fig. 6.31, the best arrangement is placing the middle wheels close to the rear wheels.

6.17. As seen from Fig. 6.33*a* and *b*, in these arrangements the optimal distribution of torques between the front and middle axle remains virtually constant, which may highly simplify the design of the power-

dividing unit that will be placed between the front and middle axle and also the algorithm of control of this power-dividing unit. In this case only unit 2 needs be controlled.

These advantages are lost when using the arrangement in Fig. 6.33*c*, which shows marked changes in all power-distribution ratios, something that makes it necessary to control both units.

6.18. Off-road multi-wheel drive vehicles operate under off-road conditions when the mechanical properties of the soil and the geometric properties of the surface of motion vary within wide ranges. If these variations are not accounted form, the results of optimization may be in error.

Variations in the mechanical properties of the soil cause the rolling resistance of the wheels to vary within wide limits, for which reason the wheel rolling resistance factor should be treated as random quantity and its numerical values for wheels of the different axles should be determined also with allowance for travel of wheels over the same track. The wheel gripping factor and any other parameters pertaining to the properties of the soil should be simulated in the same manner. It should be emphasized that the resistance and gripping factors vary randomly also when moving over paved roads. The micro- and macro-topography of the support surface is also random (both when moving off-road and on paved roads).

6.19. This requires compiling two mathematical models of the same vehicle: one model for optimizing the power distribution between the wheels and in the second model to insert the existing driveline system that can be modeled on the basis of the theoretical postulates of Chapters 1-5. The motion of both vehicle models should be modeled under the same highway (off-road) conditions and when moving with the same kinematic parameters (velocity and acceleration).

6.20. Yes, it is significant. The difference for an 8x8 vehicle ranges from 3 to 15% depending on the highway (off-road) motion conditions.

6.21. This should be done using the modified limiting optimality criterion (LOM). Its use will ensure full utilization of optimum torques under 77% of road conditions (for the 8x8 vehicle investigated in Sec. 6.6.2). This is sufficient for ensuring mobility of a multi-wheel drive vehicle.

6.22. The reason for this is the difference in wheel rolling resistance factors – these factors usually decrease from the front to the rear. This makes it possible to develop a higher torque at the wheels of the rear tandem and a lower torque at the wheels of the front tandem with simultaneous reduction in the wheel slippage that corresponds to optimum distribution of power between the wheels.

Chapter 7

7.1. A simple mechatronic driveline system consists of power-dividing units of the same type. For example, the mechatronic system of a vehicle with two drives axles, in which the interaxle and interwheel differentials have electronic control systems, engagement/disengagement modes. The combined system includes various power-dividing units with electronic control. For example, the front axle of a 4x4 vehicle is engaged/disengaged by an electronically controlled clutch, whereas the interwheel differentials are equipped by electronic lockers.

7.2. Integrated mechatronic driveline system consists of power-dividing units that have a common control algorithm with other mechatronic systems of the vehicle. For example, traction control is a system that controls the locking of the differential, brakes and the engine fuel feed system.

7.3. Cars equipped with mechatronic systems are capable of prolonged curvilinear motion at steering-wheel turning angles of 65-100º with lateral acceleration exceeding by 14-32% the lateral accelerations of a similar car with mechanical drives of the four wheels. The use of mechatronic driveline systems allows significant control of the vehicle's yaw moment. As a result, cars with mechatronic driveline systems require 20-60% smaller angles of turn of the steering wheel when switching lane than similar cars with systems based on braking of the wheels. At the same time, the time of performing this maneuver decreases by approximately 30-35%. The yaw rate (the angular velocity of turn of the vehicle about the vertical axis) is 5-15% smaller. The actuation time of mechatronic systems (also known as reaction time)

is usually 100 msec or somewhat more. Some companies even claim a reaction time of the order of 80 msec. The significant "flexibility" of mechatronic systems in controlling the wheel torques makes it possible to effectively control the dynamic behavior of vehicles. For example, when performing a 90º turn, a vehicle with control of the torque distribution between the front and rear axles may perform in the same manner as a rear-wheel drive vehicle when starting the turn, whereas when completing the turn a large fraction of the torque is transmitted to the front wheels. The re-distribution of torques between the front and rear, left and right wheels may markedly improve the traction performance of the vehicle and increase the rate of acceleration.

7.4. The substance of algorithms of control of the force parameters of a wheel constructed on the basis of the inverse wheel dynamics approach consists in the fact that control is carried out by controlling the angular acceleration and the angular velocity of the wheel and not of the power parameters alone. Such algorithms are stable and adaptable to changing motion conditions.

7.5. It becomes possible to control the wheel slippage power losses that occur not only because of changes in the rotational speed, but also as a result of changes in the wheel's rolling radius. These changes in the rolling radius stem from the tangential elasticity of the tire and the soil.

7.6. The two principal engineering problems are still being solved today by different methods: (i) determination of the peak friction (gripping) factor and (ii) determination of the slope of the tractive/brake force – slip ratio functions that are frequently termed the "slippage curves."

7.7. Various approaches and methods were developed for solving this problem. One set of methods consists in investigating the state of the surface of motion by means of optical tools. For example, a TV camera is used for assessing the road's pavement: dry or wet, is there a film of water, snow or ice on the road. Different road states are determined based on differences in the rate of reflection of light from the different surfaces. This property is used for determining the friction factor. Another set of methods is

based on analytic determination of the functional relationships between the deformations of the tire and the peak friction factor followed by utilizing this in control logarithms.

7.8. This follows from Fig. 7.11. The curves shown in this figure have different slopes under different gripping conditions. In addition, the rate of variation in ratio $\Delta r_w / \Delta T_w$ allows assessing how close is the wheel slippage close to unity, i.e., to total slippage.

7.9. Figure 7.13 establishes a relationship between the vehicle's velocity, its base and the time interval equal to the fraction obtained by dividing the base on the velocity. When the vehicle's velocity and base are known, this graph makes it always possible to determine the time interval, needed so that the rear axle wheels will find themselves under the same gripping conditions to which the front wheels are subjected at the given point in time. This time interval can be used for making a decision and controlling the interwheel power-dividing unit of the rear axle.

7.10. This parameter is equal to the ratio of the teeth of the output gear of the planetary gear set to the number of teeth of the input gear with the carrier stopped. This design parameter is always positive ($K \geq 1$) and is used for determining the gear ratio of planetary transmissions. In an open bevel gear differential K is equal to unity.

7.11. When an open bevel-gear differential is used as the turning mechanism in a tracked vehicle the engine is additionally loaded by a higher torque for making the turn that would be needed for overcoming for the resistance to motion. This stems from the properties of an open differential: when the inner track is braked in order to perform the turn, a high torque is transmitted to the outer track. This increase in torque at the outer track is equal to the braking torque applied to the brake of the inward track. This forces the engine to develop a high torque, the magnitude of which, when reduced to the differential's case, is twice the value of the braking torque.

7.12. Yes, it will manifest itself. This means that the engine will be subjected to an additional torque when braking the slipping wheel. The wheeled vehicle will suffer from the same problems as described for the case of a tracked vehicle in Question 7.11.

7.13. No, it does not depend on it (see formula 7.39)).

7.14. It is advantageous to use the ratio of the angular velocities of the output shafts as the locking input signal because this ratio is not affected by changes in the vehicle's velocity, i.e., the rpm of the differential's case, when making a turn (see formula (7.39)). On the other hand, the difference in angular velocities of the output shafts does depend on the rotational velocity of the differential's case when the vehicle makes a turn.

The ratio of angular velocities to the velocity of the case is uniquely determined by the geometric parameters of the vehicle (wheelbase and wheel tread) and the turning angle of the steered wheels, which can be always measured by the control system's sensor when a vehicle turns. For this reason the interwheel differential locking control algorithm will lock this mechanism when the ratio of the angular velocities of the output shafts will exceed the value obtained from formula (7.39).

7.15. Yes, it can. This is seen from formulae (7.40) and (7.41). The turning radius with the braked (stopped) inner wheel is equal to half of the wheel tread, that is, the turn occurs about the stopped inner wheel.

7.16. No. When one of the gears of the outer differential in a double differential is braked, the load on the engine does not occur as is the case upon stopping one of the gears of the open differential. This is seen from formulae (7.67).

7.17. No, it cannot. This is seen from formula (7.57). The turning radius will be greater than one half of the axle's wheel tread, since the gear ratio is $u_{dd} > 1$.

7.18. Yes, it is possible. This follows from formulae (7.75)-(7.77). For this it is necessary to select the number of teeth of the differential's gears in such a manner that parameter K should become equal to

$$K = \frac{z_{f'}}{z_{b'}} \frac{z_{a'}}{z_{g'}} = \frac{2R_{t\min}}{t_B} = \rho_r$$

7.19. For this purpose it is necessary to synthesize two models of the vehicle. One for determining the optimal distribution of power between the axle's wheels, which corresponds to the maximum possible energy efficiency of the vehicle under the specified conditions of motion. This model will make it possible to determine the optimal ratio of torques between the wheels that corresponds to the optimal distribution of power. This ratio is found as a function of time (see $K_{mki} = f(t)$ at $i = 1$ in formula (7.142)). The second mathematical model simulates the motion of the vehicle under the same conditions, but now in the when the interwheel differential is modeled as an open differential. This second model allows determining the ratio of the angular velocities of the output shafts, i.e., to find $\omega' / \omega'' = f(t)$. Then $K_{mki} = f(\omega' / \omega'')$ is found by eliminating the time. Then criteria of energy effectiveness based on both models are calculated concurrently. Further, analysis of computational results based on the second mathematical model is used for determining the ratio of the angular velocities of the output shafts at which the reduction in the performance of the vehicle with the open differential as compared with the ratio corresponding to the optimum distribution of power between the wheels becomes perceptible. This ratio of the angular velocities of output shafts is taken as the threshold value, upon attaining which the differential should become locked in order to prevent reduction in the vehicle's energy effectiveness.

7.20. During the first seconds after starting from rest the vehicle's acceleration is at maximum, which increases the resistance to its motion and hence requires higher wheel circumferential forces. When the gripping conditions of the drive wheels are different, this causes intense changes in their slippage. This calls for a control system capable of analyzing wheel slippage.

Chapter 8

8.1. Batch produced power-dividing units are tested in order to monitor the stability of their manufacturing technology, checking their operating capabilities and in order to partially run them in.

8.2. Batch produced power-dividing units are usually tested on stands under manufacturing conditions.

8.3. General-purpose stands should be designed and used for testing power dividing units in production for some time now, newly designed and improved power-dividing units.

8.4. Yes, in a number of cases the design features of a power-dividing unit may require conditions corresponding to their operation as a part of a vehicle when the power-dividing unit interacts functionally with other systems of the vehicle. For example, the algorithm for controlling the locking modes of the differential is functionally related to the suspension operation modes. Or, for example, it may be necessary to investigate the operation of the interaxle differential of the drive tandem with allowance for the stiffness properties of the crankcase and other components that comprise this interaxle power-dividing unit. An example of such a stand is shown in Fig. 8.14.

8.5. The first parameters of newly designed limited slip differentials to be tested on stands are their locking performance and torque bias – at the instant of appearance of relative rotation of the output shafts and upon their relative rotation.

8.6. Tests on the stand shown in Figs. 8.1 and 8.2 are carried out by applying braking torques to the differential's output shafts.

8.7. Braking torques applied to output shafts of differentials during stand tests simulate the rolling resistance of wheels coupled to the left and right output shafts.

8.8. The locking properties of differentials on the stand from Figs. 8.1 and 8.2 are investigated for determining the effect of the geometric parameters of the differentials (such as the number of rubbing pairs, dimensions of gears, friction clutches and other design elements). The investigation includes types

of materials used in the differentials, types of lubricating oil and its temperature, the magnitude of the applied torque and the mechanism's rotational velocities.

8.9. An additional direction of power-dividing unit testing consists of stand tests of their reliability and operating life.

8.10. The main purposes of stand testing of differential lubricating systems consist in determining:

- whether the oil indeed flows to the differential;
- whether the oil flows through the differential, lubricates its principal elements and is discharged from the differential;
- whether the products of wear are removed from the differential and, if this is incorporated into the mechanism's design, whether the products of wear are accumulated in the vessel provided for this purpose;
- whether the temperature of the oil is maintained within the specified range.

8.11. In such differentials, the centrifugal forces deposit products of wear on the internal surfaces preventing oil from reaching friction sites.

8.12. In order to prevent products of wear from plugging up access to friction surfaces of the differential, the lubricating system should be of the flowage type.

8.13. The effectiveness (or non-effectiveness) of a differential's lubricating system can be ascertained visually by adding to the lubricating oil some quantity of a material consisting of fine particles (for example, abrasive powder that remains after polishing components on an emery wheel). Measuring the quantity of abrasive and the pattern of its deposition inside the differential make it possible to visually assess the flowage performance of the lubricating system.

8.14. The principal factors affecting the flowage of oil through the differential consist of the specifics of the lubrication system design, rpm, quantity of oil in the axle housing, its temperature and viscosity.

8.15. The flowage lubrication system operates poorly of a differential operates poorly at very low and very high rpm of the latter and totally fails to operate at very low rpm.

8.16. Gear pumps are simple to manufacture, compact, operate reliably and have a very long operating life, do not fail at relatively high rotor rpm and at short-time overloads.

8.17. Gear pumps are incapable of self-suction, meaning that they cannot operate when air is present in their cavity or in the intake piping.

8.18. In order for a gear pump to operate stably it should be placed within the oil bath with the level of the axis of its rotors located not above the oil bath level.

8.19. A gear pump can operate with air-permeated oil, but this steeply reduces its output.

8.20. To avoid a steep reduction in the output of a gear pump because of centrifugal forces at the rotor teeth, it is recommended that the flow velocity in its intake cavity should be maintained at between 1 and 2 m/sec.

8.21. The standard route is a segment of test grounds, passing through one or another kind of soil, with one or another pavement or without a pavement at all, with typical obstacles on it or without them.

8.22. The presence of kinematic discrepancy in the tandem of a three-axle vehicle causes redistribution of the traction load (torque) between the axles. Kinematic discrepancy causes one of the axles to be under-loaded and the other – overloaded.

8.23. Locking differentials do not provide a perceptible advantage as compared with open differentials when the all wheels grip the surface of motion in the same manner. When the gripping conditions of all the wheels are the same, all of them slip in the same manner and develop the same torques. Under these conditions there is no need and possibility to redistribute the torque and, accordingly, to improve the vehicle's mobility.

8.24. Positive locking of a differential or the use of limited slip differentials with high locking factors may be detrimental to the mobility of a wheeled vehicle when the latter moves over soggy meadowland with a

soft interstice as a result of tearing off the moss cover and sinking of the drive wheel with a high torque into the soft soil.

8.25. This indicates that even three axles with open differentials in them provide poorer mobility to an all-wheel drive vehicle than a single axle with a locking differential in a vehicle with only one drive axle, that is that the effect of the interwheel differential in this case turned out in this case more significant than that of the interaxle differential.

8.26. The free-running differential provides a better performance with respect to a vehicle's mobility than other differentials.

8.27. When a vehicle takes a turn the free-running differential may disengage the advancing wheel and then the entire torque will be transmitted to the wheel internal relative to the center of turn. This, in its turn may increase the vehicle's turn resistance torque.

8.28. The free-running differential affects least the turnability of multi-wheel drive vehicles.

8.29. When the locking factor is high, the turn resistance torque of an of-road vehicle increases and its turnability deteriorates. At locking factors of 3-4 the effect of the interwheel differential on the turnability is within the acceptable limits.

8.30. The highest loads on components of a locked drive and of a drive with locking differential are attained when driving over roads with high-grade pavement.

8.31. When operating vehicles on roads with high-grade pavement it is advisable to disengage the locking of differentials and in positively engaged driveline systems – disconnect a part of the axles from the running gear.

K11141

CRC Press
Taylor & Francis Group
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