Errata: Linear Algebra and Matrix Analysis

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1. Page 10, Line 13: Thus if $\boldsymbol{AB}$ and $\boldsymbol{BA}$ are both well-defined, $\boldsymbol{AB}$ and $\boldsymbol{BA}$ are necessarily both square matrices.

2. Page 12, Theorem 1.2 part *(iv)*: *Left-hand distributive law:* Let $\boldsymbol{A}$ be an $m \times p$ matrix, $\boldsymbol{B}$ and $\boldsymbol{C}$ both be $p \times n$ matrices. Then, $\boldsymbol{A}(\boldsymbol{B}+\boldsymbol{C}) = \boldsymbol{AB}+\boldsymbol{AC}$.

3. Page 37, Line 10: **Step 1:** Solve for $x_n = b_n/u_{nn}$;

4. Page 46, Definition 2.5, part *(iii)*, Equation (2.10):

$$\boldsymbol{E}_{ij}(\beta) = \boldsymbol{I} + \beta \boldsymbol{e}_i \boldsymbol{e}_j' \ .$$

5. Page 47, Lines 1 and 2: and Type-III operations is obtained by premultiplying with $\boldsymbol{E}_{ik}(\beta)$

$$\boldsymbol{E}_{ik}(\beta)\boldsymbol{A} = (\boldsymbol{I} + \beta \boldsymbol{e}_i \boldsymbol{e}_k')\boldsymbol{A} = \boldsymbol{A} + \beta \boldsymbol{e}_i \boldsymbol{a}_{k*}' = \begin{bmatrix} \boldsymbol{a}_{1*}' \\ \vdots \\ \boldsymbol{a}_{i*}' + \beta \boldsymbol{a}_{k*}' \\ \vdots \\ \boldsymbol{a}_{k*}' \\ \vdots \\ \boldsymbol{a}_{m*}' \end{bmatrix}$$

6. Page 47, statement of Theorem 2.2: ...and $E_{jk}(\beta)'$ on the matrix $\boldsymbol{A}$...

7. Page 53, Proof of (iii):...

$$\boldsymbol{A}'\boldsymbol{X} = \boldsymbol{A}'(\boldsymbol{A}^{-1})' = \cdots$$

8. Page 56, statement of Lemma 2.2: ...

$$(\boldsymbol{I} + \boldsymbol{u}\boldsymbol{v}')^{-1} = \boldsymbol{I} - \frac{\boldsymbol{u}\boldsymbol{v}'}{1 + \boldsymbol{v}'\boldsymbol{u}} \ .$$

9. Page 70, Line -2 (second last line):

$$\ldots \Longrightarrow \boldsymbol{L}_1 = \boldsymbol{L}_2 \ \text{ and } \ \boldsymbol{U}_1 = \boldsymbol{U}_2 \ .$$

10. Page 94, Line 11 (line 3 of Section 4.5): ...and $\alpha_1, \alpha_2 \in \Re^1$.

11. Page 96, in the line above Equation 4.5: Then, it is easy to verify that $\boldsymbol{a}_3 = 2\boldsymbol{a}_1 + 3\boldsymbol{a}_2$.

12. Page 96, Equation 4.5:
$$2\boldsymbol{a}_1 + 3\boldsymbol{a}_2 - \boldsymbol{a}_3 = \mathbf{0} \; .$$

13. Page 144, Definition 5.3. Let $\boldsymbol{A}$ be an $m \times n$ matrix such that $\rho(\boldsymbol{A}) = r \geq 1$.

14. Page 152, Line 5:
$$\cdots \Longrightarrow \begin{bmatrix} \boldsymbol{I}_r & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{O} \end{bmatrix} \begin{bmatrix} \boldsymbol{Q}_1 \\ \boldsymbol{Q}_2 \end{bmatrix} \boldsymbol{x} = \mathbf{0} \Longrightarrow \boldsymbol{Q}_1 \boldsymbol{x} = \mathbf{0} \Longrightarrow \boldsymbol{x} \in \mathcal{N}(\boldsymbol{Q}_1) \; ,$$

15. Page 157, statement of Theorem 6.2: Let $\mathcal{V}$ be a vector space in $\Re^m$ and let $\mathcal{S}_1$ and $\mathcal{S}_2$ be two subspaces of $\mathcal{V}$. Suppose $\mathcal{V} = \mathcal{S}_1 + \mathcal{S}_2$. The following statements are equivalent:...

16. Page 178, Problem 28: Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be two $n \times n$ projectors. Prove that...

17. Page 197, Lemma 7.6 part *(i)*: If $\boldsymbol{u} \in \mathcal{X}^{\perp}$, then $\boldsymbol{u} \in \{Sp(\mathcal{X})\}^{\perp}$.

18. Page 211, statement of Theorem 8.2: Let $\boldsymbol{X}$ be an $n \times p$ matrix with $p \leq n$...

19. Page 217, Line -3 (third from last): ...can be used efficiently to solve the normal equations in (8.7) when...

20. Page 259, Lines 13 and 14 (last sentence under **Second proof (using orthogonality)**): ...(recall Lemma 7.7).

21. Page 260, Line 15: is equal to the number of variables.

22. Page 260, stament of Theorem 9.4: ...solution if and only if the columns...

23. Page 261, Line 10: where β is any real number between 0 and 1.

24. Page 263, Line 13 in Equation (9.1):
$$\boldsymbol{A}'\boldsymbol{A}\boldsymbol{x}_0 = \boldsymbol{A}'\boldsymbol{b} \Longrightarrow \cdots$$

25. Page 264, proof of Theorem 9.8: ...Let $\boldsymbol{B}$ be a left inverse of $\boldsymbol{C}$ and $\boldsymbol{D}$ be a right inverse of $\boldsymbol{R}$. If $\boldsymbol{b} \in \mathcal{C}(\boldsymbol{A})$, then $\boldsymbol{b} = \boldsymbol{CRy}$ for some vector $\boldsymbol{y}$. Let $\boldsymbol{G} = \boldsymbol{DB}$. Then,
$$\begin{aligned} \boldsymbol{AGb} = \boldsymbol{CRDBb} = \boldsymbol{C}(\boldsymbol{RD})\boldsymbol{Bb} = \boldsymbol{CI}_r\boldsymbol{Bb} = \boldsymbol{CBb} = \\ = \boldsymbol{CB}(\boldsymbol{CRy}) = \boldsymbol{C}(\boldsymbol{BC})\boldsymbol{Ry} = \boldsymbol{CRy} = \boldsymbol{b} \; . \end{aligned}$$
Therefore, $\boldsymbol{G}$ is a generalized inverse of $\boldsymbol{A}$.

26. Page 264, proof of Lemma 9.1: Clearly, if $\boldsymbol{C} = \boldsymbol{AGC}$, then $\mathcal{C}(\boldsymbol{C}) \subseteq \mathcal{C}(\boldsymbol{A})$...

27. Page 268, Line 1: Based upon Corollary 9.2 and Lemma 9.2, , we can define the orthogonal projector onto $\mathcal{C}(\boldsymbol{A})$ more generally as...

28. Page 272, Line 7: The verification for...

29. Page 272, Line 9: We now verify that $\boldsymbol{A}^{+}$ is the Moore-Penrose inverse...

30. Page 297, Line -3 (last unnumbered equation toward the bottom of the page):

$$\begin{aligned} |\boldsymbol{A}| &= \cdots \\ &= a_{12}A_{12} + a_{22}A_{22} + a_{32}A_{32} \ . \end{aligned}$$

31. Page 306, Example 10.4:

$$A_{IJ} = (-1)^{(1+3)+(1+2)} |\boldsymbol{A}_{\bar{I}\bar{J}}| = \cdots$$

32. Page 306, statement of Theorem 10.16:

$$|\boldsymbol{A}| = \sum_J |\boldsymbol{A}_{IJ}| A_{IJ} = \sum_J (-1)^{i_1+\cdots+i_k+j_1+\cdots+j_k} |\boldsymbol{A}_{\bar{I}\bar{J}}|,$$

33. Page 309, Problem 14: If $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$ are all $n \times n$ and $\boldsymbol{AC} = \boldsymbol{CA}$ and $\boldsymbol{A}$ is nonsingular, then prove that

$$\begin{vmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{D} \end{vmatrix} = |\boldsymbol{AD} - \boldsymbol{CB}| \ .$$

34. Page 335, proof of Theorem 11.14 (in the second paragraph of the proof just above the unnumbered equation): Observe that

$$\begin{aligned} \boldsymbol{AP} &= \boldsymbol{A}\left[\boldsymbol{x} : \boldsymbol{P}_2\right] = \left[\boldsymbol{Ax} : \boldsymbol{AP}_2\right] = \left[\lambda \boldsymbol{x} : \boldsymbol{AP}_2\right] \\ &= \left[\boldsymbol{x} : \boldsymbol{P}_2\right] \begin{bmatrix} \lambda & \boldsymbol{v}' \\ \boldsymbol{0} & \boldsymbol{B} \end{bmatrix} = \boldsymbol{P} \begin{bmatrix} \lambda & \boldsymbol{v}' \\ \boldsymbol{0} & \boldsymbol{B} \end{bmatrix} , \end{aligned}$$

where $\boldsymbol{v}'$ is the $1 \times (n-1)$ vector and $\boldsymbol{B}$ is the $(n-1) \times (n-1)$ matrix such that $\boldsymbol{AP}_2 = \left[\boldsymbol{x} : \boldsymbol{P}_2\right] \begin{bmatrix} \boldsymbol{v}' \\ \boldsymbol{B} \end{bmatrix} = \boldsymbol{P} \begin{bmatrix} \boldsymbol{v}' \\ \boldsymbol{B} \end{bmatrix}$ and, hence, $\begin{bmatrix} \boldsymbol{v}' \\ \boldsymbol{B} \end{bmatrix} = \boldsymbol{P}^{-1}\boldsymbol{AP}_2$. Therefore,...

35. Page 338, Line 4 (spelling of Caley should be Cayley, here and elsewhere): Another application of these results is the Cayley-Hamilton theorem

36. Page 338, title of Section 11.6: Cayley-Hamilton

37. Page 342, Line 2: Cayley-Hamilton

38. Page 348, statement of Theorem 11.26: ...Then $\boldsymbol{x}_1 \perp \boldsymbol{x}_2$.

39. Page 426, the last unnumbered equation in the proof of (ii) for Theorem 13.20:

$$(\boldsymbol{x}'\boldsymbol{y})^2 = (\boldsymbol{u}'\boldsymbol{v})^2 \leq (\boldsymbol{u}'\boldsymbol{u})^2(\boldsymbol{v}'\boldsymbol{v})^2 = (\boldsymbol{x}'\boldsymbol{Ax})(\boldsymbol{y}'\boldsymbol{A}^{-1}\boldsymbol{y}) \ .$$

40. Page 426, proof of Corollary 13.10: ...we obtain

$$a_{ij}^2 = (\boldsymbol{e}_i'\boldsymbol{Ae}_j)^2 \leq (\boldsymbol{e}_i'\boldsymbol{Ae}_i)(\boldsymbol{e}_j'\boldsymbol{Ae}_j) = a_{ii}a_{jj} \leq \max a_{ii}^2, a_{jj}^2$$

41. Page 500, Line 21, (proof of Theorem 15.11): Writing $\boldsymbol{A}_1 := \boldsymbol{U}_1'\boldsymbol{AV}_1$, we obtain

$$\frac{1}{\sigma_1^2 + \|\boldsymbol{w}\|^2} \left\| \boldsymbol{A}_1 \left[\begin{array}{c} \sigma_1 \\ \boldsymbol{w} \end{array} \right] \right\|_2^2 = \frac{1}{\sigma_1^2 + \|\boldsymbol{w}\|^2} \left\| \left[\begin{array}{c} \sigma_1^2 + \|\boldsymbol{w}\|^2 \\ \boldsymbol{Bw} \end{array} \right] \right\|_2^2 \geq \sigma_1^2 + \|\boldsymbol{w}\|^2 \ .$$