Quantum Mechanics
5th Edition

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Contents

1 Coloured Versions of Some Figures 3

2 Updates and Corrections 6
   Chapter 2 ................................................................. 6
   Chapter 3 ................................................................. 6
   Chapter 4 ................................................................. 7
   Chapter 5 ................................................................. 8
   Chapter 6 ................................................................. 8
   Chapter 7 ................................................................. 9
   Chapter 8 ............................................................... 10
   Chapter 9 ................................................................. 11
   Chapter 11 ............................................................. 11
   Chapter 12 ............................................................. 11
   Chapter 13 ............................................................. 13

3 Bibliography 14
   Background ........................................................... 14
   Complementary ....................................................... 15
   Advanced ............................................................. 15
   The conceptual problems of quantum mechanics .......... 16
   Additions .............................................................. 17
FIGURE 2.6
An image of the (111) surface of silicon obtained by scanning tunnelling microscopy. The bright peaks correspond to silicon atoms. The hexagonal symmetry is a characteristic feature of this surface. (Supplied by P. A. Sloan and R. E. Palmer of the Nanoscale Physics Research Laboratory in the University of Birmingham.)
FIGURE 3.3
Representations of the shapes of the spherical harmonics with quantum numbers $l$, $m$, where $l \leq 2$ and the $z$ axis is vertical. In the case of $m = 0$, the dark and light regions are of opposite sign; when $m \neq 0$, the function is complex and its phase changes by $2m\pi$ during a complete circuit of the $z$ axis.
FIGURE 10.1
The distribution of atomic velocities in a collection of rubidium atoms held in a trap for three temperatures that span the Bose condensation temperature of 170 nK. Zero velocity corresponds to the centre of the peak in each case. The condensate is absent at 200 nK, well established at 100 nK and very nearly complete at 20 nK. (As illustrated for the JILA Bose-Einstein condensate group in 1995 by M. R. Matthews, and reproduced by permission).
Updates and Corrections

This section contains an updated list of corrections to known misprints in the text. The majority have been corrected in the 2010 reprint of the book, but those marked with ⊗ were identified too late to include in the reprint.

If you detect any misprints or mistakes that are not listed here, please let us know by e-mailing john.navas@informa.com.

Chapter 2

• Worked Example 2.1 In the solution on page 24, the magnitude of the proton mass is wrong. The correct expression for $E_n$ is

$$E_n = \pi^2 \times (1.05 \times 10^{-34})^2 n^2 / (8 \times 1.7 \times 10^{-27} \times 4 \times 10^{-30}) = 2.0 \times 10^{-12} n^2 \text{ J}$$

• In the solution to worked Example 2.4 on page 34, $a$ should be $a^2$

Chapter 3

• On the second last line of page 51, $p' = p + 2$ should be $p' = p - 2$

• In the last paragraph of the solution to Worked Example 3.2 on page 56, right-hand should be left-hand

• The expression for $r'$ on the first line of page 57 should be $r' = (m\omega/h)^{1/2}r$

• The first term in square brackets in the final equation of Worked Example 3.3 should not have a minus sign
Updates and Corrections

Chapter 4

• The final equation in Worked Example 4.2 on page 78 should read

\[ \frac{3}{5} \exp[ik_0x] + \frac{4}{5} \exp[2ik_0x] \]

• In the solution to Worked Example 4.3 on page 81, the final expression for the integral in part (iii) should be

\[ a^2 \left( \frac{1}{3} - \frac{2}{\pi^2} \right) = 0.13a^2 \]

• Worked example 4.4: None of the minus signs should be shown and neither should the \( \hbar^2 \)s in the first three lines, and some factors of \( \hbar^2 \) and one of \( \hat{P}_x \) should not appear. The correct example is shown below.

Find an expression for the commutator bracket of the \( x \) coordinate of a particle and its kinetic energy.

Solution

\[
[\hat{X}, \hat{T}] = \frac{1}{2m}[\hat{X}, \hat{P}^2]
\]

\[ = \frac{1}{2m} \left[ \hat{X} \hat{P}_x^2 - \hat{P}_x^2 \hat{X} + \hat{X} \hat{P}_y^2 - \hat{P}_y^2 \hat{X} + \hat{X} \hat{P}_z^2 - \hat{P}_z^2 \hat{X} \right] \]

\[ = \frac{1}{2m} \left[ (\hat{P}_x \hat{X} + i\hbar)\hat{P}_x - \hat{P}_x (\hat{X} \hat{P}_x - i\hbar) \hat{P}_x \right] \]

\[ = i\hbar \hat{P}_x / m \]

where we have used (4.42).

• Worked Example 4.5 on page 89 should refer back to Worked Example 4.3. When the corrections to the latter are included in the former, the correct expression is

\[ \Delta x \Delta p_x = a \left( \frac{1}{3} - \frac{2}{\pi^2} \right)^{1/2} \times \frac{\pi \hbar}{2a} \]

\[ = 0.36 \frac{\pi \hbar}{2} = 0.57 \hbar > \frac{1}{2} \hbar \]

• The sentence beginning on the line following (4.89) on page 99 should read

As expected, therefore, the probability distribution for the momentum of an electron in the ground state of the hydrogen atom . . .
Chapter 5

- In the solution to Worked Example 5.2 on page 109, \( \hbar \) in the final expression should be \( \frac{\hbar}{2} \).
- In the solution to Worked Example 5.4 on page 117, the final expression should read:

\[
Y_{1, \pm 1} = \hat{L}_x Y_{1, 0} = \pm \hbar \exp ( \pm i \phi ) \frac{\partial}{\partial \theta} \cos \theta = \mp \hbar \sin \theta \exp ( \pm i \phi )
\]

- Also on page 117, the bracketed term in the third expression in the last equation should be

\[
(\hat{L}_x^* + i \hat{L}_y^*).
\]

Chapter 6

- Figure 6.1 The lower rightmost beam from the final SGZ should be \( \beta_z \) not \( \beta_x \). The correct figure is shown below.

- Equation (6.22) on page 128 should read:

\[
c_+ = \alpha_x^+ \alpha_c = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \\

\]

\[
c_- = \beta_x^+ \alpha_c = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \\

\]

- In the solution to Worked Example 6.2 on page 130, \( \sin (\phi/2) \) should be \( \sin (\theta/2) \).
- Also on page 130, Dirac was born and educated in Bristol.
- On page 133, equation (6.44) should read:

\[
\hat{H}' = f(r)(\hat{J}^2 - \hat{L}^2 - \hat{S}^2) + (\mu_B / \hbar) B_0 (\hat{L}_z + 2 \hat{S}_z)
\]
In the caption to figure 6.2 on page 135, the word ‘squared’ should be inserted before ‘magnitudes’.

In figure 6.3 on page 136, the right hand label on the lowest energy state should be $j = 1/2$ (not $j = 0$).

In table 6.2 on page 143, $2/\sqrt{3}$ should be $\sqrt{2/3}$ in lines 2 and 3 of the table.

On page 145, some factors of $\mu_B$ have been omitted from equation (6.73)

\[
E = \varepsilon \pm 2 \mu_B B \\
E = \frac{1}{2} \left[ -\varepsilon \pm \mu_B B + (9 \varepsilon^2 \pm 2 \varepsilon \mu_B B + \mu_B^2 B^2)^{1/2} \right] \\
E = \frac{1}{2} \left[ -\varepsilon \pm \mu_B B - (9 \varepsilon^2 \pm 2 \varepsilon \mu_B B + \mu_B^2 B^2)^{1/2} \right]
\]

The final two paragraphs in the solution to Worked example 6.5 should read

If $B$ is not zero, but $\mu_B B << \varepsilon$, we can ignore the terms in $B^2$ in (6.73) and expand the round brackets to get

\[
E = \varepsilon \pm 2 \mu_B B \\
E = \frac{1}{2} \left[ -\varepsilon \pm \mu_B B + 3 \varepsilon \left( 1 \pm \frac{\mu_B B}{9 \varepsilon} \right) \right] = \varepsilon \pm \frac{2}{3} \mu_B B \\
E = \frac{1}{2} \left[ -\varepsilon \pm \mu_B B - 3 \varepsilon \left( 1 \pm \frac{\mu_B B}{9 \varepsilon} \right) \right] = -2 \varepsilon \pm \frac{1}{3} \mu_B B
\]

which agrees with the weak-field values obtained earlier (6.53).

In the strong-field limit, we can ignore the term in $\varepsilon^2$ and expand the square bracket assuming $\mu_B B >> \varepsilon$ to get

\[
E = \pm 2 \mu_B B + \varepsilon \\
E = \frac{1}{2} \left[ -\varepsilon \pm \mu_B B + \mu_B B \left( 1 \pm \frac{\varepsilon}{\mu_B B} \right) \right] = \mu_B B \text{ or } -\varepsilon \\
E = \frac{1}{2} \left[ -\varepsilon \pm \mu_B B - \mu_B B \left( 1 \pm \frac{\varepsilon}{\mu_B B} \right) \right] = -\mu_B B \text{ or } -\varepsilon
\]

These results are identical with our previous calculation.

Chapter 7

$\beta$ is put equal to one after it has fulfilled its role in setting up equations (7.7) to (7.9).
• Equation (7.18) on page 152 should read

\[ |n⟩ = |0n⟩ + \sum_{k \neq n} \frac{⟨0k|H'|0n⟩}{E_{0n} - E_{0k}} |0k⟩ + \text{higher order terms} \quad (7.18) \]

• In Worked Example 7.3 on pages 158 and 159, the expressions for the matrix elements should read

\[
H'_{12,12} = a^{-4}V_0 \int_{-a}^{a} \int_{-a}^{a} \cos^2 \left( \frac{\pi x}{2a} \right) \sin^2 \left( \frac{\pi y}{2a} \right) xy \, dx \, dy = 0
\]
\[
H'_{21,12} = H'_{12,21} = a^{-4}V_0 \int_{-a}^{a} \int_{-a}^{a} \cos \left( \frac{\pi x}{2a} \right) \sin \left( \frac{\pi y}{2a} \right) \sin \left( \frac{\pi x}{2a} \right) \cos \left( \frac{\pi y}{2a} \right) xy \, dx \, dy = \frac{1024}{81\pi^3} V_0
\]
\[
H'_{21,21} = a^{-4}V_0 \int_{-a}^{a} \int_{-a}^{a} \sin^2 \left( \frac{\pi x}{2a} \right) \cos^2 \left( \frac{\pi y}{2a} \right) xy \, dx \, dy = 0
\]

• Equation (7.57) on page 164 should read

\[ E = \frac{\hbar^2}{2m_e} \left[ k^2 - \frac{2\pi k}{a} + \frac{2\pi^2}{a^2} \right] \pm \frac{1}{2} \left[ \frac{\hbar^2}{2m_e} \left( \frac{4\pi k}{a} - \frac{4\pi^2}{a^2} \right)^2 + V_0^2 \right]^{1/2} \quad (7.57) \]

Chapter 8

• On page 177, equation (8.7) and the following line should be deleted because they reproduce (8.6) and the last line on page 176.

• Equation (8.11) on page 177 should read

\[
\begin{aligned}
\frac{\partial a_1}{\partial t} &= -\frac{1}{2} \omega \rho a_2 \\
\frac{\partial a_2}{\partial t} &= \frac{1}{2} \omega \rho a_1
\end{aligned}
\quad (8.11)
\]

• In Worked Example 8.2 on pages 179 and 180, the expression in the last line of text on page 179 should read \( k' = k - i\alpha'x \) (not \( k' = k + i\alpha'x \)).

• In the first line of the equation at the top of page 180 \( (2\pi)^{-1/2} \) should be \( (2\pi)^{-1} \)

• The last few words on page 184 should read

If, however, we had used the second instead of the first term on the . . .
Chapter 9

- In Worked Example 9.3 on page 210, the equation on the second last line of the page should read

\[ \beta = \frac{2Ze^2}{4\pi \varepsilon_0} \]

- Equation (9.52) on page 214 should read

\[ u_{klm} = \frac{A}{r} \left[ \exp(-ikr) - \exp(i(kr - t\pi + 2\delta_{lm})) \right] Y_{lm}(\theta, \phi) \quad (9.52) \]

Chapter 11

- The second equation in (11.28) on page 251 should be

\[ \sigma \cdot (\hat{P} + eA) \dot{c}u_+ - mc^2 u_+ + Vu_+ = Eu_+ \]

Chapter 12

- Table 12.1 on page 267 should read

<table>
<thead>
<tr>
<th>Alice</th>
<th>Bob (Eve inactive)</th>
<th>Eve</th>
<th>Bob (Eve active)</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>S</td>
<td>M</td>
<td>O</td>
</tr>
<tr>
<td>z</td>
<td>α 0</td>
<td></td>
<td>z</td>
</tr>
<tr>
<td>z</td>
<td>β 1</td>
<td></td>
<td>z</td>
</tr>
<tr>
<td>x</td>
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<tr>
<td>z</td>
<td>β 1</td>
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</tr>
<tr>
<td>z</td>
<td>β 1</td>
<td></td>
<td>z</td>
</tr>
</tbody>
</table>

- Equation (12.7) on page 270 should read

\[ \phi(1)\psi(2) \rightarrow \psi(1)\psi(2) \]
\[ \phi(1)\chi(2) \rightarrow \chi(1)\chi(2) \quad (12.7) \]
Updates and Corrections

• In the second equation (12.15) on page 272, $A(0)$ should be $B(0)$.

• Equation (12.18) on page 273 should read

$$2^{-1/2} [d(2)c(3) - c(2)d(3)],$$

if $i = 1$

and

$$2^{-1/2} [c(2)c(3) - d(2)d(3)],$$

otherwise. (12.18)

• Equation (12.20) There is a small misprint in equation (12.20) on page 276, which should read

$$\chi = \alpha_1(1) \alpha_2(2) \alpha_3(3)$$

$$= 2^{-3/2} (\alpha_1 + \beta_1)(\alpha_2 + \beta_2)(\alpha_3 + \beta_3)$$

$$= 2^{-3/2} [\alpha_1 \alpha_2 \alpha_3 + \alpha_1 \alpha_2 \beta_3 + \alpha_1 \beta_2 \alpha_3 + \alpha_1 \beta_2 \beta_3 + \beta_1 \alpha_2 \alpha_3 + \beta_1 \alpha_2 \beta_3 + \beta_1 \beta_2 \alpha_3 + \beta_1 \beta_2 \beta_3].$$ (12.20)

• Equation (12.24) should read

$$\tilde{\gamma}_k = N^{-1} \sum_{l=0}^{m-1} \sum_{j=0}^{r-1} \exp \left( \frac{2\pi i}{N} k(j + lr) \right) y_{j+lr}$$

$$= N^{-1} \sum_{l=0}^{m-1} \exp \left( \frac{2\pi i}{N} klr \right) \sum_{j=0}^{r-1} \exp \left( \frac{2\pi i}{N} jk \right) y_j$$

$$= N^{-1} \left[ 1 - \exp \left( \frac{2\pi ik}{m} \right) \right] ^{r-1} \sum_{j=0}^{r-1} \exp \left( \frac{2\pi i}{N} jk \right) y_j$$ (12.24)

where the expression in square brackets equals the sum over $l$ in the line above because the latter is a geometric series. Unless $k = pm$, where $p$ is an integer $\leq r$, this term equals zero, because the numerator is zero and the denominator is finite. However, if $k = pm$ both are zero and their ratio equals $m$.

• The first summation in the second line of equation (12.26) on page 279 should be over the index $j$ (not $k$).

• In the first line of equation (12.31) on page 281, $2^{n/2}$ inside the brackets should be $2^n$.

• The last few lines at the foot of page 279, continuing to page 280 should read

The probability of obtaining a particular value of $k$ following such a measurement is $|\langle k|\psi \rangle|^2 = \langle \tilde{\gamma}_k | \tilde{\gamma}_k \rangle$, using (12.26). Referring to (12.28) we see that the term in square brackets is zero unless $k = pn$, where $p$ is an
integer, in which case it equals \( m \), so that

\[
\langle \tilde{y}_k | \tilde{y}_k \rangle = \frac{m^2 r^{r-1} r^{r-1}}{N^2} \sum_{j=0}^{r-1} \sum_{j'=0}^{r-1} \exp \left( \frac{2\pi i}{N} (j - j') k \right) \langle y_j | y_{j'} \rangle
\]

\[
= \frac{m^2 r}{N^2} \Rightarrow \frac{1}{r} = \quad (12.29)
\]

remembering that \( N = mr \) and that the \( |y_j\rangle \) are orthonormal.

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**Chapter 13**

- On page 289 \( V(X) \) should be \( V(x) \) in equations (13.4) and (13.6) and \( V(r) \) in equation (13.9).
- On page 297, \( P_- \) in (13.7) and the preceding text should read \( P_{--} \). In the first term of the RHS of the last line of equation (13.18), \( 3_- \) should be \( 3_+ \).
- Equation (13.17) In the last line \( P_- \) should be \( P_{--} \)

\[
\begin{align*}
P_{++}(\theta) &= \sin^2(\theta/2) \\
P_{+-}(\theta) &= \cos^2(\theta/2) \\
P_{-+}(\theta) &= \cos^2(\theta/2) \\
P_{--}(\theta) &= \sin^2(\theta/2)
\end{align*}
\]

(13.17)

- Equation (13.20) There is a small misprint in equation (13.20) on page 298, which should read

\[
n(1_+, 2_+ - n(1_+, 3_+) + n(2_-, 3_+) \geq 0.
\]

(13.20)

- Equation (13.21) The \( P_{++} \) should be replaced with \( P_{--} \), i.e.,

\[
P_{+-}(\theta_{12}) - P_{--}(\theta_{13}) + P_{--}(\theta_{23}) \geq 0.
\]

(13.21)

- On page 298, \( n(2_+, 3_-) \) should read \( n(2_-, 3_+) \) in both equations (13.19) and (13.20). In equation (13.21) \( P_{++}(\theta_{23}) \) should read \( P_{--}(\theta_{23}) \)
- On pages 310 onwards, references to equation (13.36) should be to equation (13.35).
- On page 317, the reference to figure 13.6 should be to figure 13.7.
3

Bibliography

The early editions of *Quantum Mechanics* contained a Bibliography. Over the years, not only have many of the references in it become out of date, but the amount of relevant published material has exploded to the point where there is a great danger that important works will be left out. Moreover, the availability of web searches and other data retrieving tools means that readers can much more easily build up their own databases of relevant material. For these reasons, a bibliography has not been included in recent editions.

However, the original bibliography is published here along with some more recent titles that are labelled ‘addition. If you think a useful reference has been left out, please let us know by e-mailing john.navas@informa.com.

Background

A more extensive treatment of the experimental evidence for the need for quantum mechanics than is given in Chapter 1 is contained in many textbooks on atomic physics, of which the following are good examples.


A general familiarity with mathematical techniques, particularly calculus and elementary matrix algebra, up to a level typical of that of a first-year undergraduate physics course, is assumed; suitable textbooks covering this material are


Complementary

Some of the textbooks on quantum mechanics which treat the subject at approximately the same level as the present volume are listed below with short comments.

- **Bohm, D., Quantum Theory**, Prentice-Hall, New York, 1951. This thorough, discursive text is one of the few at this level to attempt a detailed discussion of the conceptual problems of the subject. It is of course rather out of date by now.

- **Dicke, R. H., and Wittke, J. R., Introduction to Quantum Mechanics**, Addison-Wesley, Reading, Massachusetts, 1960. In many ways the approach of this book is rather similar to that adopted in the present work, but it relies on a more detailed understanding of formal classical mechanics and uses rather complex mathematical arguments at times.

- **Feynman, R. P., Leighton, R. B., and Sands, M., The Feynman Lectures in Physics (vol. III, Quantum Mechanics)**, Addison-Wesley, Reading, Massachusetts, 1965. This book contains many physical insights and is written in an attractive informal style. However, from the start it uses an abstract, matrix formulation which some students find difficult.

- **French, A. P., and Taylor, E. F., An Introduction to Quantum Physics**, Nelson, Middlesex, 1978. This book describes the physical principles behind many quantum processes in considerable detail, and this discussion is illustrated by a large number of examples. However, detailed mathematical arguments are avoided, which somewhat limits its scope.


- **Sillito, R. M., Non-Relativistic Quantum Mechanics** (2nd ed.), University Press, Edinburgh, 1967. A good reliable text, if a little over detailed and mathematical in places, by the person who taught the subject to the present author.

Advanced

The following is a small selection of the many books on quantum mechanics which treat the subject at a more advanced level.

- **Dirac, P. A. M., Quantum Mechanics** (4th ed.), Oxford University Press, London, 1967. This classic text describes the formal, rigorous version of quantum mechanics that was first developed by its author.
Bibliography


Addition


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The conceptual problems of quantum mechanics

The following constitute a selection of some publications on the topics discussed in Chapter 11. They all contain references to further published work.


• Clauser, J. F., and Shimony, A., ‘Bell’s theorem: experimental tests and implications’, *Reports on Progress in Physics*, vol. 41, pp. 1881–1927, 1978. This review article contains a thorough discussion of Bell’s theorem and carefully analyses the various experiments that have been performed to test it.

• D’Espagnat, B., *Conceptual Foundations of Quantum Mechanics*, Benjamin, Massachusetts, 1976. This is probably the nearest there is to an authoritative reference work on this subject. It reviews and analyses the whole field, including non-separability, measurement theories, and the associated philosophical problems.


Bibliography

• Rae, A. I. M., *Quantum Physics, Illusion or Reality*, University Press, Cambridge, 1986. This book discusses the conceptual problems of quantum mechanics at greater length, although at a somewhat less advanced level than is done in Chapter 11.

Additions

• Auletta, G., *Foundations and Interpretations of Quantum Mechanics*, World Scientific, Singapore, 2001. A wideranging text that discusses many topics in considerable depth, but (despite its 981 pages) only touches on others.


• Griffiths, R. B. *Consistent Quantum Theory*, Cambridge University Press, Cambridge, 2002. A text by one of the inventors of the consistent histories interpretation, which aims to teach the whole subject from this point of view. Contains illuminating insights which are sometimes masked by mathematical detail.

Applications

The applications of quantum mechanics discussed in this book are mainly, if not entirely, drawn from the fields of atomic, nuclear, particle, and solid-state physics. More detailed discussion of these subjects can be found in a number of textbooks of which the following are typical examples.


