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1. General comment

In teaching from *Engineering BGM* since publication in November 2007, some *typos* and several mathematical mistakes have become evident. Fortunately only one of those mistakes is *uncorrectable* and it is not crucial. Also my email addresses

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were somehow deleted from the preface during the final stages of publication.

This note details mistakes discovered so far and will be updated as and when further defects appear. The author offers his sincere apologies for any inconvenience caused to readers.

2. Typing mistakes

- **page 3 line 11** 'forwards' should be 'forward'
- **page 6 line -2** replace $v\left(t, F_T\left(0, T_1\right)\right)$ by $v\left(t, F_T\left(t, T_1\right)\right)$
- **page 13 line 3** equation should read $\lambda\left(t\right) = \left(\lambda^{(1)}\left(t\right),\ \lambda^{(2)}\left(t\right)\right)^*$
- **page 19 line 2** equation should read

$$\omega\left(t\right) = L^{(N)}\left(t, T_0\right) = \ .....$$

- **page 25 line -1** delete $\sqrt{V}$ from the equation

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- **page 41 line 5** insert level (t) so equation reads

$$\text{level}\,(t) = \sum_{i=0}^{M-1} \overline{\delta}_i B\left(t, \overline{T}_{i+1}\right).$$

- **page 51 line -2** equation should read

$$..... = \sigma\,(j-1) \left\{ \begin{array}{c} \sum_{\ell=0}^{j} B\,(0, x_{\ell+1})\, K\,(0, x_\ell) \\ -B\,(0, x_{j+1})\, K\,(0, x_j) \end{array} \right\},$$

- **page 57 line 7** (3.3.5) should be (3.5) **line 14** (3.4.16) should be (4.16)
- **page 87 line -1** delete full stop between 'each' and X_{T_L}
- **page 177 line 9** should read 'Section-A.2.1'
- **page 200 line -8** delete $P\,(y, \lambda)$

3. Correctable math mistake - page 33

In three equations on this page the cash forward $K\,(t, T_j)$, which can be negative, appears in the denominator. The **middle paragraph** on **page 33** should run >>>

Start by considering how changes in $v_j\,(t)$ arise. We have

$$v_j\,(t) = u_j\,(t)\, H\,(t, T_j) = u_j\,(t)\,[K\,(t, T_j) + a\,(T_j)]\,,$$

where $a\,(T_j)$ may be (much) larger than $K\,(t, T_j)$ and is time independent, while the $u_j\,(t)$ are virtually constant. So the $u_j\,(t)\, a\,(T_j)$ term varies little in contrast to $u_j\,(t)\, K\,(t, T_j)$, but is much larger, suggesting it should be immediately isolated. That leads to the approximation

$$\begin{aligned} & v_j\,(t) \cong u_j\,(t)\, K\,(t, T_j) + u_j\,(0)\, a\,(T_j) \\ \Rightarrow \quad & \frac{dv_j\,(t)}{v_j\,(t)} = \frac{d\,[u_j\,(t)\, K\,(t, T_j)]}{u_j\,(t)\, H\,(t, T_j)} \qquad j = 0, .., N-1. \end{aligned}$$

Because both $v_j\,(t)$ and $u_j\,(t)$ are $\widetilde{\mathbb{P}}_T$-martingales, and

$$d\,[u_j\,(t)\, K\,(t, T_j)] = dv_j\,(t) - a\,(T_j)\;\; du_j\,(t)\,,$$

clearly $u_j\,(t)\, K\,(t, T_j)$ must also be a $\widetilde{\mathbb{P}}_T$-martingale. Also, because the $\mathbb{P}_{T_j}$-forward and $\widetilde{\mathbb{P}}$-swaprate measures are equivalent, we can write

$$dK\,(t, T_j) = dH\,(t, T_j) = (\textit{drift})\, dt + H\,(t, T_j)\, \xi^*\,(t, T_j)\, d\widetilde{W}_T\,(t)\,.$$

Hence, using the SDE (4.7) for $u_j(t)$, the SDE for $u_j(t)K(t,T_j)$ must be

$$d\left[u_j(t)K(t,T_j)\right] = \left\{ \begin{array}{c} u_j(t)H(t,T_j)\xi(t,T_j) + \\ u_j(t)\left[\begin{array}{c} -b(t,T,T_{j+1}) \\ +\sum_{i=0}^{M-1}\overline{u}_i(t)b\left(t,T,\overline{T}_{i+1}\right)\end{array}\right]K(t,T_j) \end{array} \right\}^* d\widetilde{W}_T(t),$$

and, dividing this by $u_j(t)H(t,T_j)$, it follows that an approximate SDE for $v_j(t)$ is

$$\frac{dv_j(t)}{v_j(t)} \cong \left[\begin{array}{c} \xi(t,T_j) \\ +\frac{K(t,T_j)}{H(t,T_j)}\left(\begin{array}{c} -b(t,T,T_{j+1}) \\ +\sum_{i=0}^{M-1}\overline{u}_i(t)b\left(t,T,\overline{T}_{i+1}\right)\end{array}\right)\end{array}\right]^* d\widetilde{W}_T(t). \qquad (4.14)$$

Note that the two SDEs (4.12) and (4.14) for the *kosher* and approximate $v_j(t)$ both coincide in the flat case.

4. Correctable math mistake - page 36&37

Throughout Section-4.5 the A's that appear as denominators need to be positive. So on **page 36 line -8** insert >>>

... in which, we emphasize, the $A_j(t)$ are stochastic. Note that because $w_\ell(t)$ and $\overrightarrow{u}_\ell(t)$ are weights summing to unity

$$A_0(t) = w_0(t) - h_0(t)\sum_{\ell=0}^{N-1}\left(w_\ell(t) - \overrightarrow{u}_\ell(t)\right) = w_0(t) > 0,$$

allowing the forward volatility $\xi(t,T_0)$ to be calculated in terms of the swaprate volatility $\sigma(t)$ and later forward volatilities $\xi(t,T_j)$ $(j>0)$ in a *backward fashion* via

$$\xi(t,T_0) = \frac{\sigma(t) - \sum_{j=1}^{N-1}A_j(t)\xi(t,T_j)}{A_0(t)}.$$

5. Uncorrectable math mistake - page 38

As noted, the A's in the denominators must be positive and that can only be so if the union of the tenors is increasing to the left. **Example 4.2** on **page 38** should be replaced by >>>

Example 4.2

We now show how to make a set of two forwards and the swaprates of two swaps jointly lognormal. Start with the swaps/forwards

$$\begin{aligned}
\text{pSwap}_4(t) &= \delta B(t,T_5)\left[K(t,T_4)-\kappa\right], \\
\text{pSwap}_{3,4}(t) &= \delta\left\{B(t,T_4)+B(t,T_5)\right\}\left[\omega_{3,4}(t)-\kappa\right], \\
\text{pSwap}_2(t) &= \delta B(t,T_3)\left[K(t,T_2)-\kappa\right], \\
\text{pSwap}_{1,3}(t) &= \delta\left\{B(t,T_2)+B(t,T_3)+B(t,T_4)\right\}\left[\omega_{1,3}(t)-\kappa\right],
\end{aligned}$$

and notice that the union of their tenors strictly increases to the left

$$\begin{aligned}
(T_4, T_5] &= (T_4, T_5], \\
(T_3, T_5] \cup (T_4, T_5] &= (T_3, T_5], \\
(T_2, T_3] \cup (T_3, T_5] \cup (T_4, T_5] &= (T_2, T_5], \\
(T_1, T_4] \cup (T_2, T_3] \cup (T_3, T_5] \cup (T_4, T_5] &= (T_1, T_5].
\end{aligned}$$

Proceed as follows:
[1] Make the forward $K(t, T_4)$ lognormal under the forward measures $\mathbb{P}_{T_5}$ by letting $\xi(t, T_4)$ be deterministic.
[2]The swaprate $\omega_{3,4}(t)$ for $\text{pSwap}_{3,4}(t)$ has volatility

$$\sigma_{3,4}(t) = A_3^{(3,4)}(t)\,\xi(t, T_3) + A_4^{(3,4)}(t)\,\xi(t, T_4)$$

in which $\xi(t, T_4)$ is already determined and $A_3^{(3,4)} > 0$. Make $\omega_{3,4}(t)$ lognormal under its unique swaprate measure $\widetilde{\mathbb{P}}_{3,4}$ by letting $\sigma_{3,4}(t)$ be deterministic and setting

$$\xi(t, T_3) = \frac{\sigma_{3,4}(t) - A_4^{(3,4)}(t)\,\xi(t, T_4)}{A_3^{(3,4)}}.$$

[3] Make the forward $K(t, T_2)$ lognormal under the forward measures $\mathbb{P}_{T_3}$ by letting $\xi(t, T_2)$ be deterministic.
[4] The swaprate $\omega_{1,3}(t)$ for $\text{pSwap}_{1,3}(t)$ has volatility

$$\sigma_{1,3}(t) = A_1^{(1,3)}(t)\,\xi(t, T_1) + A_2^{(1,3)}(t)\,\xi(t, T_2) + A_3^{(1,3)}(t)\,\xi(t, T_3)$$

in which $\xi(t, T_2)$ and $\xi(t, T_3)$ are already determined and $A_1^{(1,3)}(t) > 0$. Make $\omega_{1,3}(t)$ lognormal under its unique swaprate measure $\widetilde{\mathbb{P}}_{1,3}$ by letting $\sigma_{1,3}(t)$ be deterministic and setting

$$\xi(t, T_3) = \frac{\sigma_{1,3}(t) - A_2^{(1,3)}(t)\,\xi(t, T_2) - A_3^{(1,3)}(t)\,\xi(t, T_3)}{A_3^{(1,3)}}. \qquad \square$$

Clearly the construction method used in these two examples will work as long as there is always a new $\xi(t, T_j)$ available on the left to make the volatility of the next swaprate deterministic, that is, so long as the union of the tenors of the underlying swaps is strictly increasing to the left. A further example is that the swaprates of a set of swaps with identical tenors can be made jointly lognormal.