Using the "Tool for Interactive Fourier Transforms" for Understanding the VSRT Labs

A deeper understanding of the results of the VSRT labs and how interferometric data relate to images of radio sources can be obtained with a bit of exploration with Fourier Transforms. This tool creates the Fourier Transforms of input functions and the exercises discussed below are designed to help explain the relation of the visibility functions in the VSRT labs to the source structure, and by extrapolation how data from an array of antennas is converted into an image.

First, a brief discussion about Fourier Transforms, in general.

A. Introduction to Fourier Transforms

Fourier Transforms are extremely useful and far more common than most people realize. Since there are extensive presentations of Fourier Transforms in numerous textbooks, we won't bother to give an explanation here. We will, instead, just give a quick summary of the basic idea and the more important aspects and concepts.

The most familiar example of a Fourier Transform is the conversion of a function of time to a function of frequency. For example, consider the detection of a signal that oscillates with time at a frequency of 10 Hz, so that it cycles up and down every tenth of a second. If this signal vs. time is Fourier Transformed one gets a function that contains spikes at +10 Hz and -10 Hz, and zero at all other frequencies, showing that all the power in this signal occurs at a frequency of 10 Hz. If the signal was more complicated than a single frequency cosine function, the Fourier Transform will produce a "power spectrum" which reveals all the frequencies involved with that signal, and how much power occurs at each frequency.

Mathematically, in principle, any real function, f(x), can be expressed as a series sum of cosine or sine functions, e.g.

$$f(x) = \sum_m A_m cos(mx)$$

When the sum is turned into an integral, this becomes something like

$$f(x) = \int g(m)cos(mx)dm,$$

where the series of A_m's becomes the continuous function g(m). Here, f(x) is considered a cosine transform of g(m); a cosine transform is one specific type of Fourier Transform. Now, the interesting thing is that this transform is easily reversed, i.e. g(m) is also the cosine transform of f(x), i.e.

$$g(m) = \int f(x) cos(mx) dx.$$

The cosine transform, however, only considers math along the real axis and is not as powerful as a transform that occurs in the complex plane (with real and imaginary numbers). You won't need to have any familiarity with complex numbers to find this tutorial informative, but a brief explanation here will be helpful for understanding one limitation of the Tool for Interactive Fourier Transform, (discussed below). In general, a full Fourier Transform involves a real part, which includes a cosine transform, and an imaginary part, using sines. The full Fourier Transform is given by

$$f(x) = \int g(m)(cos(mx) - i\ sin(mx))dm = \int g(m) e^{-imx} dm$$

and its inverse is given by

$$g(m) = \int f(x) e^{imx} dx.$$

(NOTE: for sake of brevity, we have intentionally neglected the limits of integration and the normalization factors in front. We assume that anyone interested in knowing the details of Fourier Transforms will find a complete presentation elsewhere.)

Important Things To Know:

1. Since the arguments of cosine and sine, and of exponentials, must be unitless, the independent variables of functions which are Fourier transforms of each other must be the inverse of each other. For example, frequency = 1/time. And, in our general expressions above, m = 1/x.

2. In the Fourier Transform, since all cosine functions are "even" about the origin and sines are "odd," (i.e. cos(-x) = cos(x) while sin(-x) = -sin(x)), the imaginary part of the transform is needed to pick up asymmetries in the initial function. That is, the limitation of a cosine-only transform is that it cannot produce an f(x) in which f(-x) does NOT equal f(x).

B. Getting Familiar with the TIFT Window Buttons and Controls

1. Start by clicking on the "TIFT_Chooser" shortcut. You'll then see a small pop-up window with two buttons; click on "Full Complex TIFT."

2. A new display window will pop up with with four graphs. The top two graphs show the magnitude and phase for any complex function of time, f(t), and the lower graphs show the magnitude and phase for a corresponding function of frequency, F(ν). One can choose to the change the display to represent the real and imaginary parts by clicking "Show Rectangular," and return to magnitude and phase by clicking "Show Polar." The lower graphs represent the "Fourier Transform" of the function represented in the top two graphs. Similarly, the function in the top graphs are the "inverse Fourier Transform" of the function shown in the bottom graphs.

3. To the right are two set of boxes for inputting parameters. The inputs of the top two boxes affect the Fourier Transform calculation (one sets the step size along the x-axis of the input function and the other sets the number of points). The bottom two boxes only affect how much of the display windows are shown and can be used for zooming in on either or both graphs. Be sure to hit "Enter" after changing a value in any input box.

4. Scroll the mouse over the topmost graph and click when the cursor becomes a "+" sign. Notice that this moves the data point at that x-value to the position of the cursor. Alternatively, you can grab a data point and pull it up or down.

5. Try tracing a curve (rather than changing every individual point) by holding the left-button down while you move the mouse (slowly) along the desired curve.

6. You'll find that as you change the function in the upper graph(s), changes occur instantly in the lower graphs. This provides an interactive way for you to see the relation between the Fourier transform and the initial function.

7. The x and y values of any data point can be read by moving the cursor onto the data point,

8. The "Reset" button undoes any changes you made to any data points, resetting the data values to that given by the equation, but does not change the values of the input parameters. The "Full Reset" erases the input function and sets all data values to zero.

9. If you get a plot that you'd like to save, you can right-click on the graph and save the image as a JPEG file.

10. At any point, you can click "Save File." You can, then, at a later date use "Open File" to read these data back in.

C. Exercises

1. A Single Source of Varying Size

1. In "TIFT_Chooser" click on "Full Complex TIFT"

2. In the top-most graph, move the point at t=0 upward (to a magnitude $\approx$ 1). Note what happened to the Magnitude of F(ν) (the third graph).

3. Move the second point in the top graph up to the same magnitude as the first point, and note, again, how the magnitude of F(ν) changes.

4. Keep moving subsequent points up to the same magnitude, watching the magnitude of F(ν) as you do so.

5. Consider the top graph as representing the CFL in your VSRTI Observations of a Resolved Source lab. Recall the shape of the Visibility function with a single source and how the Visibility function depended on the angular size of the source. Does the magnitude of F(ν) look like the Visibility function you measured in the VSRTI Lab?

2. A Double Source of Varying Separation

1. First hit "Reset" to move all the points back to zero.

2. In the top-most graph, move the point at t=0 upward. Then move the point at t = 0.3 to the same height. What does the magnitude of F(ν) look like now? Count the number of cycles from $\nu = 0$ to $\nu = 10$. What is the period of oscillation in F(ν)?

3. Move the t = 0.3 point back to zero and move the t = 0.2 point up to the same height as the t = 0 point. How does this change the magnitude of F(ν)? Count the number of cycles between $\nu = 0$ to $\nu = 10$. What is the new period of F(ν)?

4. Considering the top graph, now, as representing the pair of CFLs in your "VSRTI Observations of a Binary Source" lab, how does the magnitude of F(ν) compare with the shape of the Visibility function you observed in the lab?

5. What is the relation between the period of oscillation in F(ν) and the positions of the spikes in f(t)?

D. Fourier Transforms and VSRT data

1. Discuss the relation of the visibilities in the VSRT labs to the Fourier transforms you encountered in the exercises above. Is there a relation between the visibility function, $V(b/lambda)$, and the the intensity of the radio-emission of the CFLs as a function of angle, $I(\theta)$ analogous to that between f(t) and F(ν)?

2. Consider the independent variable in the visibility function. Is it unitless? Consider inverting it to find its Fourier co-variable ... do you get the independent variable needed to describe the distribution of radio emission in the source?

3. Considering that any function's Fourier Transform can be inverted to yield the original function, discuss the recovering of the radio source brightness distribution from the visibility function, i.e. what must radio astronomers do to obtain an image, of even complex sources, from the interferometric data?

4. Why do the VSRT labs not include this final step? In the "mystery source" lab, for example, why not just do an inverse Fourier Transform to obtain the result? (Hint, consider the fact that in the TIFT exercises you saw 4 windows, not 2.)