! For an efficient use of these tables, first read `HowTo.pdf`.

T2.01A. Definitions and General Formulas.

1. $\displaystyle\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx, \quad f \in F[a,b].$

2. $\displaystyle\int_a^b k\,dx = k\,(b-a), \quad k = \text{const}.$

3. $\displaystyle\int_a^b [f(x)+g(x)]\;dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx.$

4. $\displaystyle m(b-a) \le \int_a^b f(x)\,dx \le M(b-a), \quad \text{where } m \le f(x) \le M \text{ for all } x \in (a,b).$

5. $\displaystyle\int_a^b f(t)\,dt = F(b) - F(a), \quad \text{where } F'(x) = f(x) \text{ on } [a,b].$

6. $\displaystyle\int_a^b g\,(f(x))\; f'(x)\,dx = \int_{f(a)}^{f(b)} g(u)\,du.$

7.
$$\begin{aligned}
\int_a^b f(x)\,e^{i\,px}\,dx &= e^{i\,pb}\left[-\frac{i\,f(b)}{p} + \frac{f'(b)}{p^2} + \frac{i\,f''(b)}{p^3} - \frac{f'''(b)}{p^4} - \cdots\right] \\
&\quad - e^{i\,pa}\left[-\frac{i\,f(a)}{p} + \frac{f'(a)}{p^2} + \frac{i\,f''(a)}{p^3} - \frac{f'''(a)}{p^4} - \cdots\right] \\
&= e^{i\,p(a+b)/2}\left\{e^{i\,p(b-a)/2}\left[-\frac{i\,f(b)}{p} + \frac{f'(b)}{p^2} + \frac{i\,f''(b)}{p^3} - \frac{f'''(b)}{p^4} - \cdots\right]\right. \\
&\quad \left. - e^{-i\,p(b-a)/2}\left[-\frac{i\,f(a)}{p} + \frac{f'(a)}{p^2} + \frac{i\,f''(a)}{p^3} - \frac{f'''(a)}{p^4} - \cdots\right]\right\}.
\end{aligned}$$

8. $$\int_a^b f(x)\,\cos px\,dx = \cos\frac{p(a+b)}{2}\left\{\sin\frac{p(b-a)}{2}\left[\frac{f(b)+f(a)}{p} - \frac{f''(b)+f''(a)}{p^3} + \cdots\right]\right.$$
$$\left. + \cos\frac{p(b-a)}{2}\left[\frac{f'(b)-f'(a)}{p^2} - \frac{f'''(b)-f'''(a)}{p^4} + \cdots\right]\right\}$$
$$+ \sin\frac{p(a+b)}{2}\left\{\cos\frac{p(b-a)}{2}\left[\frac{f(b)-f(a)}{p} - \frac{f''(b)-f''(a)}{p^3} + \cdots\right]\right.$$
$$\left. + \sin\frac{p(b-a)}{2}\left[\frac{f'(b)+f'(a)}{p^2} - \frac{f'''(b)+f'''(a)}{p^4} + \cdots\right]\right\}.$$

9. $$\int_a^b f(x)\,\sin px\,dx = \sin\frac{p(a+b)}{2}\left\{\sin\frac{p(b-a)}{2}\left[\frac{f(b)+f(a)}{p} - \frac{f''(b)+f''(a)}{p^3} + \cdots\right]\right.$$
$$\left. + \cos\frac{p(b-a)}{2}\left[\frac{f'(b)-f'(a)}{p^2} - \frac{f'''(b)-f'''(a)}{p^4} + \cdots\right]\right\}$$
$$- \cos\frac{p(a+b)}{2}\left\{\cos\frac{p(b-a)}{2}\left[\frac{f(b)-f(a)}{p} - \frac{f''(b)-f''(a)}{p^3} + \cdots\right]\right.$$
$$\left. - \sin\frac{p(b-a)}{2}\left[\frac{f'(b)+f'(a)}{p^2} - \frac{f'''(b)+f'''(a)}{p^4} + \cdots\right]\right\}.$$

10. If a function f is continuous on on the interval $(-a, a)$, then
$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\displaystyle\int_0^a f(x)\,dx, & \text{if } f(-x) = f(x), \\ 0, & \text{if } f(-x) = -f(x). \end{cases}$$

11. $$\int_0^{\pi/2} f(\sin x)\,dx = \int_0^{\pi/2} f(\cos x)\,dx.$$

12. $$\int_0^{2\pi} f(a\,\cos x + b\,\sin x)\,dx = 2\int_0^{\pi} f(\sqrt{a^2+b^2}\,\cos x)\,dx, \quad \text{if } f \in (-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}).$$

13. $$\int_0^{\pi/2} f(\sin 2x)\,\cos x\,dx = \int_0^{\pi/2} f(\cos^2 x)\,\cos x\,dx, \quad \text{if } f \in F(0,1).$$

14. If $f(x+\pi) = f(x)$ and $f(-x) = f(x)$, then

15. $$\int_0^{\infty} f(x)\frac{\sin x}{x}\,dx = \begin{cases} \displaystyle\int_0^{\pi/2} f(x)\,dx, & \text{if } f(x+\pi) = f(x) \text{ and } f(-x) = f(x), \\ \displaystyle\int_0^{\pi/2} f(x)\,\cos x\,dx, & \text{if } f(x+\pi) = -f(x) \text{ and } f(-x) = f(x), \end{cases}$$

provided the integral on the left exists.

16. $$\int_0^\pi \frac{f(\alpha+e^{xi})+f(\alpha+e^{-xi})}{1+2p\,\cos x+p^2}\,dx=\frac{2\pi}{1-p^2}f(\alpha+p),\quad |p|<1.$$

17. $$\int_0^\pi \frac{1-p\,\cos x}{1-2p\,\cos x+p^2}\{f(\alpha+e^{xi})+f(\alpha+e^{-xi})\}\,dx=\pi\{f(\alpha+p)+f(\alpha)\},\quad |p|<1.$$

18. $$\int_0^\pi \frac{f(\alpha+e^{-xi})-f(\alpha+e^{xi})}{1-2p\,\cos\,x+p^2}\,\sin\,x\,dx=\frac{\pi}{ip}\{f(\alpha+p)-f(\alpha)\},\quad |p|<1.$$

In formulas 16-18, it is assumed that the function f is analytic in the closed unit disk with center at the point α.

19. $$\int_0^\pi f\left(\frac{\sin^2\,x}{1+2p\,\cos\,x+p^2}\right)\,dx=\begin{cases}\displaystyle\int_0^\pi f(\sin^2\,x)\,dx, & p^2\geq 1,\\ \displaystyle\int_0^\pi f\left(\frac{\sin^2 x}{p^2}\right)\,dx, & p^2<1.\end{cases}$$

20. $$\int_0^\pi F^{(n)}(\cos\,x)\,\sin^{2n}\,x\,dx=(2n-1)!!\int_0^\pi F(\cos\,x)\,\cos\,nx\,dx.$$
