

! For an efficient use of these tables, first read [HowTo.pdf](#).

T2.18B. Integrands of the form $\frac{1}{x^4 \sqrt{(a^2 \pm x^2)(b^2 \pm x^2)}}$ on the intervals (y, b) and (b, y) .

Notation used: $\delta = \arccos \frac{y}{b}$, $\varepsilon = \arccos \frac{b}{y}$,

$$\zeta = \arcsin \frac{a}{b} \sqrt{\frac{b^2 - y^2}{a^2 - y^2}}, \quad \kappa = \arcsin \frac{a}{y} \sqrt{\frac{y^2 - b^2}{a^2 - b^2}},$$

$$q = \frac{\sqrt{a^2 - b^2}}{a}, \quad r = \frac{b}{\sqrt{a^2 + b^2}}, \quad s = \frac{a}{\sqrt{a^2 + b^2}}, \quad t = \frac{b}{a}.$$

$$\begin{aligned} 1. \int_y^b \frac{dx}{x^4 \sqrt{(x^2 + a^2)(b^2 - x^2)}} &= \frac{1}{3a^4 b^4 \sqrt{a^2 + b^2}} \{a^2(2a^2 - b^2)F(\delta, r) - 2(a^4 - b^4)E(\delta, r)\} \\ &+ \frac{a^2 b^2 + 2y^2(a^2 - b^2)}{3a^4 b^4 y^3} \sqrt{(b^2 - y^2)(a^2 + y^2)}, \quad b > y > 0. \end{aligned}$$

$$\begin{aligned} 2. \int_b^y \frac{dx}{x^4 \sqrt{(x^2 + a^2)(x^2 - b^2)}} &= \frac{2b^2 - a^2}{3a^4 b^2 \sqrt{a^2 + b^2}} F(\varepsilon, s) + \frac{2(a^2 - b^2)\sqrt{a^2 + b^2}}{3a^4 b^4} E(\varepsilon, s) \\ &+ \frac{1}{3a^2 b^2 y^3} \sqrt{(y^2 + a^2)(y^2 - b^2)}, \quad y > b > 0. \end{aligned}$$

$$\begin{aligned} 3. \int_y^b \frac{dx}{x^4 \sqrt{(a^2 - x^2)(b^2 - x^2)}} &= \frac{1}{3a^3 b^4} \left\{ (2a^2 + b^2)F(\zeta, t) - 2(a^2 + b^2)E(\zeta, t) \right. \\ &\left. + \frac{[(2a^2 + b^2)y^2 + a^2 b^2]a}{y^3} \sqrt{\frac{b^2 - y^2}{a^2 - y^2}} \right\}, \quad a > b > y > 0. \end{aligned}$$

$$4. \int_b^y \frac{dx}{x^4 \sqrt{(a^2 - x^2)(x^2 - b^2)}} = \frac{1}{3a^3b^4} \{2(a^2 + b^2)E(\kappa, q) - b^2F(\kappa, q)\} \\ + \frac{1}{3a^2b^2y^3} \sqrt{(a^2 - y^2)(y^2 - b^2)}, \quad a \geq y > b > 0.$$
