! For an efficient use of these tables, first read `HowTo.pdf`.

T2.05A. Integrands of the form $\dfrac{1}{(r-x)\sqrt{(a-x)(b-x)(c-x)}}$ on the intervals (y,a), (a,y), (y,b), (b,y), (y,c) and (c,y).

Notation used: $\beta = \arcsin\sqrt{\dfrac{c-y}{b-y}}$, $\gamma = \arcsin\sqrt{\dfrac{y-c}{b-c}}$,

$$\delta = \arcsin\sqrt{\frac{(a-c)(b-y)}{(b-c)(a-y)}}, \quad \kappa = \arcsin\sqrt{\frac{(a-c)(y-b)}{(a-b)(y-c)}},$$

$$\lambda = \arcsin\sqrt{\frac{a-y}{a-b}}, \quad \mu = \arcsin\sqrt{\frac{y-a}{y-b}},$$

$$p = \sqrt{\frac{a-b}{a-c}}, \quad q = \sqrt{\frac{b-c}{a-c}}.$$

1. $$\int_y^a \frac{dx}{(x-r)\sqrt{(a-x)(x-b)(x-c)}} = \frac{2}{(a-r)\sqrt{a-c}}\Pi\left(\lambda,\, \frac{a-b}{a-r},\, p\right), \quad a > y \geq b > c,\ r \neq a.$$

2. $$\int_a^y \frac{dx}{(x-r)\sqrt{(x-a)(x-b)(x-c)}} = \frac{2}{(b-r)(a-r)\sqrt{a-c}} \times \left[(b-a)\Pi\left(\mu,\, \frac{b-r}{a-b},\, q\right) + (a-p)\,F(\mu,\, q)\right], \quad y > a > b > c,\ r \neq a.$$

3. $$\int_y^b \frac{dx}{(r-x)\sqrt{(a-x)(b-x)(x-c)}} = \frac{2}{(r-a)(r-b)\sqrt{a-c}} \times \left[(b-a)\Pi\left(\delta,\, q^2\frac{r-a}{r-b},\, q\right) + (r-b)\,F(\delta,\, q)\right], \quad a > b > y \geq c,\ r \neq b.$$

4. $$\int_b^y \frac{dx}{(x-r)\sqrt{(a-x)(x-b)(x-c)}} = \frac{2}{(c-r)(b-r)\sqrt{a-c}} \times \left[(c-b)\Pi\left(\kappa,\, p^2\frac{c-r}{b-r},\, p\right) + (b-r)\,F(\kappa,\, p)\right], \quad a \geq y > b > c,\ r \neq b.$$

5. $$\int_y^c \frac{dx}{(r-x)\sqrt{(a-x)(b-x)(c-x)}} = \frac{2(c-b)}{(r-b)(r-c)\sqrt{a-c}}\,\Pi\left(\beta,\, \frac{r-b}{r-c},\, p\right) + \frac{2}{(r-b)\sqrt{a-c}}F(\beta,\, p), \quad a > b > c > y,\ r \neq 0.$$

6. $$\int_c^y \frac{dx}{(r-x)\sqrt{(a-x)(b-x)(x-c)}} = \frac{2}{(r-c)\sqrt{a-c}}\Pi\left(\gamma,\, \frac{b-c}{r-c},\, q\right), \quad a > b \geq y > c,\, r \neq c.$$
