! For an efficient use of these tables, first read `HowTo.pdf`.

T2.72D. Integrands involving logarithms and exponentials on the interval $(0, \pi)$.

1. $$\int_0^\pi \ln\sin\, x \cos[2m(x-n)]\,dx = -\frac{\pi\cos 2mn}{2m}.$$

2. $$\int_0^\pi \frac{\ln(1+p\cos x)}{\cos x}\,dx = \pi\ \arcsin p, \quad p^2<1.$$

3. $$\int_0^\pi \ln\sin\, x\frac{dx}{1-2a\cos x+a^2} = \begin{cases} \dfrac{\pi}{1-a^2}\ln\dfrac{1-a^2}{2}, & a^2<1, \\ \dfrac{\pi}{a^2-1}\ln\dfrac{a^2-1}{2a^2}, & a^2>1. \end{cases}$$

4. $$\int_0^\pi \ln\sin\, bx\frac{dx}{1-2a\cos x+a^2} = \frac{\pi}{1-a^2}\ln\frac{1-a^{2b}}{2}, \quad a^2<1.$$

5. $$\int_0^\pi \ln\cos\, bx\frac{dx}{1-2a\cos x+a^2} = \frac{\pi}{1-a^2}\ln\frac{1+a^{2b}}{2}, \quad a^2<1.$$

6. $$\int_0^\pi \ln\sin\, bx\frac{dx}{1-2a\cos 2x+a^2} = \frac{\pi}{1-a^2}\ln\frac{1-a^{b}}{2}, \quad a^2<1.$$

7. $$\int_0^\pi \ln\cos\, bx\frac{dx}{1-2a\cos 2x+a^2} = \frac{\pi}{1-a^2}\ln\frac{1+a^{b}}{2}, \quad a^2<1.$$

8. $$\int_0^\pi \ln\sin\, x\frac{\cos x\,dx}{1-2a\cos x+a^2} = \begin{cases} \dfrac{\pi}{2a}\dfrac{1+a^2}{1-a^2}\ln(1-a^2) - \dfrac{a\pi\ln 2}{1-a^2}, & a^2<1, \\ \dfrac{\pi}{2a}\dfrac{a^2+1}{a^2-1}\ln\dfrac{a^2-1}{a^2} - \dfrac{\pi\ln 2}{a(a^2-1)}, & a^2>1. \end{cases}$$

9. $$\int_0^\pi \ln\sin\, bx\frac{\cos x\,dx}{1-2a\cos 2x+a^2} = \int_0^\pi \ln\cos\, bx\frac{\cos x\,dx}{1-2a\cos 2x+a^2} = 0, \quad 0<a<1.$$

10. $$\int_0^\pi \ln\sin\,x\frac{\cos^2 x\,dx}{1-2a\cos 2x+a^2} = \begin{cases} \dfrac{\pi}{4a}\dfrac{1+a}{1-a}\ln(1-a)-\dfrac{\pi\ln 2}{2(1-a)}, & 0<a<1, \\ \dfrac{\pi}{4a}\dfrac{a+1}{a-1}\ln\dfrac{a-1}{a}-\dfrac{\pi\ln 2}{2a(a-1)}, & a>1. \end{cases}$$

11. $$\int_0^\pi \ln\sin\,x\frac{dx}{a+b\cos x} = \frac{\pi}{\sqrt{a^2-b^2}}\ln\frac{\sqrt{a^2-b^2}}{a+\sqrt{a^2-b^2}}, \quad a>0,\ a>b.$$

12. $$\int_0^\pi \ln\sin\,x\sin^{2n}2x\cos 2x\,dx = -\frac{(2n-1)!!}{(2n)!!}\frac{\pi}{4n+2}.$$

13. $$\int_0^\pi \frac{\ln\tan\,bx\,dx}{1-2a\,\cos 2x+a^2} = \frac{\pi}{1-a^2}\ln\frac{1-a^b}{1+a^b}, \quad 0<a<1,\ b>0.$$

14. $$\int_0^\pi \frac{\ln\tan\,bx\cos x\,dx}{1-2a\cos 2x+a^2} = 0, \quad 0<a<1.$$

15. $$\int_0^\pi \ln(1+p\cos x)\frac{dx}{\cos x} = \pi\ \arcsin p, \quad p^2<1.$$

16. $$\int_0^\pi \ln(1-2a\cos x+a^2)\cos nx\,dx = \begin{cases} -\dfrac{\pi\,a^n}{n}, & a^2<1, \\ -\dfrac{\pi}{na^n}, & a^2>1. \end{cases}$$

17. $$\int_0^\pi \ln(1-2a\cos x+a^2)\sin nx\sin x\,dx = \frac{\pi}{2}\left(\frac{a^{n+1}}{n+1}-\frac{a^{n-1}}{n-1}\right), \quad a^2>1.$$

18. $$\int_0^\pi \ln(1-2a\cos x+a^2)\sin nx\sin x\,dx = -\frac{\pi}{2}\left(\frac{a^{n+1}}{n+1}+\frac{a^{n-1}}{n-1}\right), \quad a^2<1.$$

19. $$\int_0^\pi \ln(1-2a\cos 2x+a^2)\ \cos(2n-1)x\,dx = 0, \quad a^2<1.$$

20. $$\int_0^\pi \ln(1-2a\cos 2x+a^2)\ \sin 2nx\sin x\,dx = 0, \quad a^2<1.$$

21. $$\int_0^\pi \ln(1-2a\cos 2x+a^2)\ \sin(2n-1)\,x\sin x\,dx = \frac{\pi}{2}\left(\frac{a^n}{n}-\frac{a^{n-1}}{n-1}\right), \quad a^2<1.$$

22. $\displaystyle\int_0^\pi \ln(1-2a\cos 2x+a^2)\,\cos 2nx\cos x\,dx = 0, \quad a^2<1.$

23. $\displaystyle\int_0^\pi \ln(1-2a\cos 2x+a^2)\,\cos(2n-1)\,x\cos x\,dx = -\frac{\pi}{2}\left(\frac{a^n}{n}+\frac{a^{n-1}}{n-1}\right), \quad a^2<1.$

24. $\displaystyle\int_0^\pi \frac{\ln(1-2a\cos x+a^2)}{1-2b\cos x+b^2}\,dx = \frac{2\pi\ln(1-ab)}{1-b^2}, \quad a^2\le 1,\ b^2<1.$

25. $\displaystyle\int_0^\pi \ln\frac{1+2a\cos x+a^2}{1-2a\cos x+a^2}\sin(2n+1)x\,dx = (-1)^n\frac{2\pi a^{2n+1}}{2n+1}, \quad a^2<1.$

26. $\displaystyle\int_0^\pi \ln\frac{1+2a\cos 2x+a^2}{1+2a\cos 2nx+a^2}\cot x\,dx = 0.$

27. $\displaystyle\int_0^\pi \ln\frac{1+\sin x}{1+\cos b\,\sin x}\,\frac{dx}{\sin x} = b^2, \quad b^2<\pi^2.$

28. $\displaystyle\int_0^\pi \frac{\ln\,\tan\,rx\,dx}{1-2p\,\cos x+p^2} = \frac{\pi}{1-p^2}\ln\frac{1-p^{2r}}{1+p^{2r}}, \quad p^2<1.$
