! For an efficient use of these tables, first read `HowTo.pdf`.

T2.63A. Integrands involving logarithm functions of complicated arguments, like $\left(1+a^2/x^2\right)$, $(1\pm e^{-x})$, $\left(1+2e^{-x}\cos t+e^{-2x}\right)$ and others, on the interval $(0,1)$.

1. $\displaystyle\int_0^1 \ln x \ln(1-x)\,dx = 2-\frac{\pi^2}{6}.$

2. $\displaystyle\int_0^1 \ln x \ln(1+x)\,dx = 2-\frac{\pi^2}{12}-2\ln 2.$

3. $\displaystyle\int_0^1 \ln\frac{1-ax}{1-a}\frac{dx}{\ln x} = -\sum_{k=1}^{\infty} a^k\,\frac{\ln(1+k)}{k}, \quad a<1.$

4. $\displaystyle\int_0^1 \ln\left(\ln\frac{1}{x}\right)dx = -\gamma_e.$

5. $\displaystyle\text{p.v.}\int_0^1 \frac{dx}{\ln\left(\ln\frac{1}{x}\right)}\,dx = \text{p.v.}\int_0^\infty \frac{e^{-u}}{\ln u}\,du \approx -0.154479.$

6. $\displaystyle\int_0^1 \ln\left(\ln\frac{1}{x}\right)\frac{dx}{\sqrt{\ln\frac{1}{x}}}\,dx = -\left(\gamma_e+2\ln 2\right)\sqrt{\pi}.$

7. $\displaystyle\int_0^1 \ln\left(\ln\frac{1}{x}\right)\left(\ln\frac{1}{x}\right)^{\mu-1}dx = \psi(\mu)\,\Gamma(\mu), \quad \Re\{\mu\}>0.$

8. $\displaystyle\int_0^1 \ln(a + \ln x)\, dx = \ln a - e^{-a}\,\mathrm{Ei}\,(a), \quad a > 0.$

9. $\displaystyle\int_0^1 \ln(a - \ln x)\, dx = \ln a - e^{a}\,\mathrm{Ei}\,(-a), \quad a > 0.$