! For an efficient use of these tables, first read `HowTo.pdf`.

T2.72C. Integrands involving logarithms and exponentials on the interval $(0, \pi/2)$.

1. $\displaystyle\int_0^{\pi/2} \ln \sin x \sin x \, dx = \ln 2 - 1.$

2. $\displaystyle\int_0^{\pi/2} \ln \sin x \cos x \, dx = -1.$

3. $\displaystyle\int_0^{\pi/2} \ln \sin x \, \cos 2nx \, dx = \begin{cases} -\pi/(4n), & n > 0, \\ -\pi \ln 2/2, & n = 0. \end{cases}$

4. $\displaystyle\int_0^{\pi/2} \ln \sin x \, \sin^2 x \, dx = \frac{\pi}{8}(1 - \ln 4).$

5. $\displaystyle\int_0^{\pi/2} \ln \sin x \, \cos^2 x \, dx = -\frac{\pi}{8}(1 + \ln 4).$

6. $\displaystyle\int_0^{\pi/2} \ln \sin x \, \sin x \cos^2 x \, dx = \frac{1}{9}(\ln 8 - 4).$

7. $\displaystyle\int_0^{\pi/2} \ln \sin x \, \tan x \, dx = -\frac{\pi^2}{24}.$

8. $\displaystyle\int_0^{\pi/2} \ln \sin 2x \, \sin x \, dx = \int_0^{\pi/2} \ln \sin 2x \, \cos x \, dx = 2(\ln 2 - 1).$

9. $$\int_0^{\pi/2} \ln\sin x \frac{dx}{1-2a\cos 2x + a^2} = \frac{1}{2}\int_0^{\pi} \ln\sin x \frac{dx}{1-2a\cos 2x+a^2}$$
$$= \begin{cases} \dfrac{\pi}{2(1-a^2)} \ln \dfrac{1-a}{2}, & a^2 < 1, \\ \dfrac{\pi}{2(a^2-1)} \ln \dfrac{a-1}{2a}, & a^2 > 1. \end{cases}$$

10. $$\int_0^{\pi/2} \ln\sin x \frac{\cos 2x\,dx}{1-2a\cos 2x + a^2} = \frac{1}{2}\int_0^{\pi} \ln\sin x \frac{\cos 2x\,dx}{1-2a\cos 2x+a^2}$$
$$= \begin{cases} \dfrac{\pi}{2a(1-a^2)} \left\{ \dfrac{1+a^2}{2} \ln(1-a) - a^2 \ln 2 \right\}, & a^2 < 1, \\ \dfrac{\pi}{2a(a^2-1)} \left\{ \dfrac{1+a^2}{2} \ln \dfrac{a-1}{a} - \ln 2 \right\}, & a^2 > 1. \end{cases}$$

11. $$\int_0^{\pi/2} \ln\cos x \frac{\cos 2x\,dx}{1-2a\cos 2x + a^2} = \begin{cases} \dfrac{\pi}{2a(1-a^2)} \left\{ \dfrac{1+a^2}{2} \ln(1+a) - a^2 \ln 2 \right\}, & a^2 < 1, \\ \dfrac{\pi}{2a(a^2-1)} \left\{ \dfrac{1+a^2}{2} \ln \dfrac{1+a}{a} - \ln 2 \right\}, & a^2 > 1. \end{cases}$$

12. $$\int_0^{\pi/2} \ln\sin x \frac{dx}{(a\sin x \pm b\cos x)^2} = \int_0^{\pi/2} \ln\cos x \frac{dx}{(a\cos x \pm b\sin x)^2}$$
$$= \frac{1}{b(a^2+b^2)}\left(\mp a \ln\frac{a}{b} - \frac{b\pi}{2} \right), \quad a>0,\ b>0.$$

13. $$\int_0^{\pi/2} \frac{\ln\sin x\,dx}{a^2\sin^2 x + b^2\cos^2 x} = \int_0^{\pi/2} \frac{\ln\cos x\,dx}{b^2\sin^2 x + a^2\cos^2 x} = \frac{\pi}{2ab}\ln\frac{b}{a+b}, \quad a>0,\ b>0.$$

14. $$\int_0^{\pi/2} \ln\sin x \frac{\sin 2x\,dx}{(a\sin^2 x + b\cos^2 x)^2} = \int_0^{\pi/2} \ln\cos x \frac{\sin 2x\,dx}{(b\sin^2 x + a\cos^2 x)^2}$$
$$= \frac{1}{2b(b-a)}\ln\frac{a}{b}, \quad a>0,\ b>0.$$

15. $$\int_0^{\pi/2} \ln\sin x \frac{a^2\sin^2 x - b^2\cos^2 x}{(a^2\sin^2 x + b^2\cos^2 x)^2}\,dx = \int_0^{\pi/2} \ln\cos x \frac{a^2\cos^2 x - b^2\sin^2 x}{(a^2\cos^2 x + b^2\sin^2 x)^2}\,dx$$
$$= \frac{\pi}{2b(a+b)}, \quad a>0,\ b>0.$$

16. $$\int_0^{\pi/2} \ln\sin x \frac{\sin x}{\sqrt{1+\sin^2 x}}\,dx = \int_0^{\pi/2} \frac{\cos x\,\ln\cos x}{\sqrt{1+\cos^2 x}}\,dx = -\frac{\pi}{8}\ln 2.$$

17. $$\int_0^{\pi/2} \frac{\sin^3 x \ln \sin x}{\sqrt{1+\sin^2 x}}\, dx = \int_0^{\pi/2} \frac{\cos^3 x \ln \cos x}{\sqrt{1+\cos^2 x}}\, dx = \frac{\ln 2 - 1}{4}.$$

18. $$\int_0^{\pi/2} \ln \sin x \frac{dx}{\sqrt{1-k^2 \sin^2 x}} = -\frac{1}{2}\mathbf{K}(k)\ln k - \frac{\pi}{4}\mathbf{K}(k').$$

19. $$\int_0^{\pi/2} \frac{\ln \cos x\, dx}{\sqrt{1-k^2 \sin^2 x}} = \frac{1}{2}\mathbf{K}(k)\ln \frac{k'}{k} - \frac{\pi}{4}\mathbf{K}(k').$$

20. $$\int_0^{\pi/2} \ln \sin x \sin^\mu x \cos^\nu x\, dx = \int_0^{\pi/2} \ln \cos x \cos^\mu x \sin^\nu x\, dx$$
$$= \frac{1}{4}B\left(\frac{\mu+1}{2}, \frac{\nu+1}{2}\right)\left[\psi\left(\frac{\mu+1}{2}\right) - \psi\left(\frac{\mu+\nu+2}{2}\right)\right], \quad \Re\{\mu\} > -1,\ \Re\{\nu\} > -1.$$

21. $$\int_0^{\pi/2} \ln \sin x \sin^{\mu-1} x\, dx = \frac{\sqrt{\pi}\,\Gamma\left(\frac{\mu}{2}\right)}{4\Gamma\left(\frac{\mu+1}{2}\right)}\left[\psi\left(\frac{\mu}{2}\right) - \psi\left(\frac{\mu+1}{2}\right)\right], \quad \Re\{\mu\} > 0.$$

22. $$\int_0^{\pi/2} \ln \sin x \cos^{\nu-1} x\, dx = \frac{\sqrt{\pi}\,\Gamma\left(\frac{\nu}{2}\right)}{4\Gamma\left(\frac{\nu+1}{2}\right)}\left[\psi\left(\frac{\nu}{2}\right) - \psi\left(\frac{\nu+1}{2}\right)\right], \quad \Re\{\nu\} > 0.$$

23. $$\int_0^{\pi/2} \ln \sin x \sin^{2n} x\, dx = \frac{(2n-1)!!}{(2n)!!}\frac{\pi}{2}\left\{\sum_{k=1}^{2n}\frac{(-1)^{k+1}}{k} - \ln 2\right\}.$$

24. $$\int_0^{\pi/2} \ln \sin x \sin^{2n+1} x\, dx = \frac{(2n)!!}{(2n+1)!!}\left\{\sum_{k=1}^{2n+1}\frac{(-1)^k}{k} + \ln 2\right\}.$$

25. $$\int_0^{\pi/2} \ln \sin x \cos^{2n} x\, dx = -\frac{(2n-1)!!}{(2n)!!}\frac{\pi}{4}\left[\sum_{k=1}^{n}\frac{1}{k} + \ln 4\right]$$
$$= -\frac{(2n-1)!!}{(2n)!!}\frac{\pi}{4}\left[\psi(n+1) + \gamma_e + \ln 4\right].$$

26. $$\int_0^{\pi/2} \ln \sin x \cos^{2n+1} x\, dx = -\frac{(2n)!!}{(2n+1)!!}\sum_{k=0}^{n}\frac{1}{2k+1}$$
$$= -\frac{(2n)!!}{2(2n+1)!!}\left[\psi\left(n+\frac{3}{2}\right) - \psi\left(\frac{1}{2}\right)\right].$$

27. $\displaystyle\int_0^{\pi/2} \ln\cos\,x\,\sin^{2n}x\,dx = -\frac{(2n-1)!!}{2^{n+1}\,n!}\,\frac{\pi}{2}\,\left[\gamma_e + 2\ln 2 + \psi(n+1)\right].$

28. $\displaystyle\int_0^{\pi/2} \ln\cos\,x\,\cos^{2n}x\,dx = -\frac{(2n-1)!!}{2^n n!}\frac{\pi}{2}\left\{\ln 2 + \sum_{k=1}^{2n}\frac{(-1)^k}{k}\right\}.$

29. $\displaystyle\int_0^{\pi/2} \ln\cos\,x\,\cos^{2n-1}x\,dx = \frac{2^{n-1}(n-1)!}{(2n-1)!!}\left[\ln 2 + \sum_{k=1}^{2n-1}\frac{(-1)^k}{k}\right].$

30. $\displaystyle\int_0^{\pi/2} \ln\sin\,x\frac{\sin^{p-1}x}{\cos^{p+1}x}\,dx = -\frac{\pi}{2p}\csc\frac{p\pi}{2}, \quad 0 < p < 2.$

31. $\displaystyle\int_0^{\pi/2} \ln\sin\,x\frac{dx}{\tan^{p-1}x\sin 2x} = \frac{1}{4}\frac{\pi}{p-1}\,\sec\frac{p\pi}{2}, \quad p^2 < 1.$

32. $\displaystyle\int_0^{\pi/2} \ln\sin\,x\,\sin^{\mu-1}x\cos x\,dx = \int_0^{\frac{\pi}{2}} \ln\cos\,x\cos^{\mu-1}x\sin x\,dx = -\frac{1}{\mu^2}, \quad \Re\{\mu\} > 0.$

33. $\displaystyle\int_{-\pi/2}^{\pi/2} \ln\cos\,x\cos^p x\cos px\,dx = \frac{\pi}{2^{p+1}}\left[\gamma_e + \psi(p+1) - 2\ln 2\right], \quad p > -1.$

34. $\displaystyle\int_0^{\pi/2} \ln\cos\,x\,\cos^{p-1}x\sin px\sin x\,dx = \frac{\pi}{2^{p+2}}\left[\gamma_e + \psi(p) - \frac{1}{p} - 2\ln 2\right], \quad p > 0.$

35. $\displaystyle\int_0^{\pi/2} \ln\tan\,x\sin x\,dx = \ln 2.$

36. $\displaystyle\int_0^{\pi/2} \ln\tan\,x\cos x\,dx = -\ln 2.$

37. $\displaystyle\int_0^{\pi/2} \ln\tan\,x\sin^2 x\,dx = -\int_0^{\pi/2} \ln\tan\,x\cos^2 x\,dx = \frac{\pi}{4}.$

38. $\displaystyle\int_0^{\pi/2} \sin x\ln\cot\,\frac{x}{2}\,dx = \ln 2.$

39. $$\int_0^{\pi/2} \frac{\ln\tan\, x\, dx}{1 - 2a\cos 2x + a^2} = \begin{cases} \dfrac{\pi}{2(1-a^2)} \ln \dfrac{1-a}{1+a}, & a^2 < 1, \\ \dfrac{\pi}{2(a^2-1)} \ln \dfrac{a-1}{a+1}, & a^2 > 1. \end{cases}$$

40. $$\int_0^{\pi/2} \frac{\ln\tan\, x \cos 2x\, dx}{1 - 2a\cos 2x + a^2} = \begin{cases} \dfrac{\pi}{4a} \dfrac{1+a^2}{1-a^2} \ln \dfrac{1-a}{1+a}, & a^2 < 1, \\ \dfrac{\pi}{4a} \dfrac{a^2+1}{a^2-1} \ln \dfrac{a-1}{a+1}, & a^2 > 1. \end{cases}$$

41. $$\int_0^{\pi/2} \frac{\ln\tan\, x\, dx}{\sqrt{1 - k^2 \sin^2 x}} = -\ln k' \mathbf{K}(k).$$

42. $$\int_0^{\pi/2} \ln(a\, \tan x)\, \sin^{\mu-1} 2x\, dx = 2^{\mu-2} \ln a \frac{\left\{\Gamma\left(\frac{a}{2}\right)\right\}^2}{\Gamma(a)}, \quad a > 0,\ \Re\{\mu\} > 0.$$

43. $$\int_0^{\pi/2} \ln\tan\, x \cos^{2(\mu-1)}\, x\, dx = -\frac{\sqrt{\pi}}{4} \frac{\Gamma\left(u - \frac{1}{2}\right)}{\Gamma(\mu)} \left[\gamma_e + \psi\left(\frac{2\mu - 1}{2}\right) + \ln 4\right], \quad \Re\{\mu\} > \tfrac{1}{2}.$$

44. $$\int_0^{\pi/2} \ln\tan\, x\, \cos^{q-1} x \cot x \sin[(q+1)x]\, dx = -\frac{\pi}{2}[\gamma_e + \psi(q+1)], \quad q > -1.$$

45. $$\int_0^{\pi/2} \ln\tan\, x\, \cos^{q-1} x \cos[(q+1)x]\, dx = -\frac{\pi}{2q}, \quad q > 0.$$

46. $$\int_0^{\pi/2} (\ln\tan\, x)^{2n-1} \frac{dx}{\cos 2x} = \frac{1 - 2^{2n}}{2n}\, \pi^{2n}\, |B_{2n}|, \quad n = 1, 2, \ldots.$$

47. $$\int_0^{\pi/2} \ln(1 + p\sin x) \frac{dx}{\sin x} = \frac{\pi^2}{8} - \frac{1}{2}(\arccos p^2), \quad p^2 < 1.$$

48. $$\int_0^{\pi/2} \ln(1 + p\cos x) \frac{dx}{\cos x} = \frac{\pi^2}{8} - \frac{1}{2}(\arccos p)^2, \quad p^2 < 1.$$

49. $$\int_0^{\pi/2} \frac{\cos x \ln(1 + \cos\alpha \cos x)}{1 - \cos^2\alpha \cos^2 x}\, dx = \frac{L\left(\frac{\pi}{2} - \alpha\right) - \alpha \ln\sin\, \alpha}{\sin\alpha \cos\alpha}, \quad 0 < \alpha < \frac{\pi}{2}.$$

50. $$\int_0^{\pi/2} \frac{\cos x \ln(1 - \cos\alpha \cos x)}{1 - \cos^2\alpha \cos^2 x}\, dx = \frac{L\left(\frac{\pi}{2} - \alpha\right) + (\pi - \alpha) \ln\sin\, \alpha}{\sin\alpha \cos\alpha}, \quad 0 < \alpha < \frac{\pi}{2}.$$

51. $$\int_0^{\pi/2} \ln(1+2a\cos 2x + a^2)\sin^2 x\,dx = \begin{cases} -\dfrac{a\pi}{4}, & a^2 < 1, \\ \dfrac{\pi \ln a^2}{4} - \dfrac{\pi}{4a}, & a^2 > 1. \end{cases}$$

52. $$\int_0^{\pi/2} \ln(1+2a\cos 2x + a^2)\cos^2 x\,dx = \begin{cases} \dfrac{a\pi}{4}, & a^2 < 1, \\ \dfrac{\pi \ln a^2}{4} + \dfrac{\pi}{4a}, & a^2 > 1. \end{cases}$$

53. $$\int_0^{\pi/2} \ln\left(1+a\sin^2 x\right)\sin^2 x\,dx = \frac{\pi}{2}\left(\ln\frac{1+\sqrt{1+a}}{2} - \frac{1-\sqrt{1+a}}{2\left(1+\sqrt{1+a}\right)}\right), \quad a > -1.$$

54. $$\int_0^{\pi/2} \ln\left(1+a\sin^2 x\right)\cos^2 x\,dx = \frac{\pi}{2}\left(\ln\frac{1+\sqrt{1+a}}{2} + \frac{1-\sqrt{1+a}}{2\left(1+\sqrt{1+a}\right)}\right), \quad a > -1.$$

55. $$\int_0^{\pi/2} \frac{\ln(1-\cos^2\beta\,\cos^2 x)}{1-\cos^2\alpha\cos^2 x}\,dx = -\frac{\pi}{\sin\alpha}\ln\frac{1+\sin\alpha}{\sin\alpha+\sin\beta}, \quad 0<\beta<\frac{\pi}{2},\ 0<\alpha<\frac{\pi}{2}.$$

56. $$\int_0^{\pi/2} \ln\frac{p+q\sin ax}{p-q\sin ax}\,\frac{dx}{\sin ax} = \int_0^{\pi/2} \ln\frac{p+q\,\cos ax}{p-q\cos ax}\,\frac{dx}{\cos ax} = \int_0^{\pi/2} \ln\frac{p+q\,\tan ax}{p-q\,\tan ax}\,\frac{dx}{\tan ax}$$
$$= \pi\,\arcsin\frac{q}{p}, \quad p>q>0.$$

57. $$\int_0^{\pi/2} \frac{\cos x}{1-\cos^2\alpha\,\cos^2 x}\ln\frac{1+\cos\beta\cos x}{1-\cos\beta\cos x}\,dx = \frac{2\pi}{\sin 2\alpha}\ln\frac{\cos\frac{\alpha-\beta}{2}}{\sin\frac{\alpha+\beta}{2}}, \quad 0<\alpha\le\beta<\frac{\pi}{2}.$$

58. $$\int_0^{\pi/2} \ln\left(p^2+q^2\,\tan^2 x\right)\frac{dx}{a^2\sin^2 x + b^2\cos^2 x} = \frac{\pi}{ab}\ln\frac{ap+bq}{a},$$
$$a>0,\ b>0,\ p>0,\ q>0.$$

59. $$\int_0^{\pi/2} \ln\left(1+q^2\,\tan^2 x\right)\frac{1}{p^2\sin^2 x + r^2\cos^2 x}\,\frac{dx}{s^2\sin^2 x + t^2\cos^2 x}$$
$$= \frac{\pi}{p^2t^2-s^2r^2}\left\{\frac{p^2-r^2}{pr}\,\ln\left(1+\frac{qr}{p}\right) + \frac{t^2-s^2}{st}\ln\left(1+\frac{qt}{s}\right)\right\},$$
$$q>0,\ p>0,\ r>0,\ s>0,\ t>0.$$

60. $$\int_0^{\pi/2} \ln\left(1+q^2\,\tan^2 x\right) \frac{\sin^2 x}{p^2\sin^2 x + r^2\,\cos^2 x}\,\frac{dx}{s^2\sin^2 x + t^2\cos^2 x}$$
$$= \frac{\pi}{p^2t^2 - s^2r^2}\left\{\frac{t}{s}\ln\left(1+\frac{qt}{s}\right) - \frac{r}{p}\ln\left(1+\frac{qr}{p}\right)\right\},$$
$$q>0,\ p>0,\ r>0,\ s>0,\ t>0.$$

61. $$\int_0^{\pi/2} \ln\left(1+q^2\,\tan^2 x\right) \frac{\cos^2 x}{p^2\sin^2 x + r^2\,\cos^2 x}\,\frac{dx}{s^2\sin^2 x + t^2\cos^2 x}$$
$$= \frac{\pi}{p^2t^2 - s^2r^2}\left\{\frac{p}{r}\ln\left(1+\frac{qr}{p}\right) - \frac{s}{t}\ln\left(1+\frac{qt}{s}\right)\right\},$$
$$q>0,\ p>0,\ r>0,\ s>0,\ t>0.$$

62. $$\int_0^{\pi/2} \ln\left(1-k^2\sin^2 x\right) \frac{dx}{\sqrt{1-k^2\sin^2 x}} = \ln\, k'\mathbf{K}(k).$$

63. $$\int_0^{\pi/2} \ln\left(1-k^2\,\sin^2 x\right) \frac{\sin^2 x\,dx}{\sqrt{1-k^2\sin^2 x}} = \frac{1}{k^2}\{(k^2-2+\ln\, k')\mathbf{K}(k) + (2-\ln\, k')\mathbf{E}(k)\}.$$

64. $$\int_0^{\pi/2} \ln\left(1-k^2\,\sin^2 x\right) \frac{\cos^2 x\,dx}{\sqrt{1-k^2\sin^2 x}} = \frac{1}{k^2}[(1+k'^2-k'^2\ln\, k')\mathbf{K}(k) - (2-\ln\, k')\mathbf{E}(k)].$$

65. $$\int_0^{\pi/2} \ln\left(1-k^2\,\sin^2 x\right) \frac{dx}{\sqrt{(1-k^2\sin^2 x)^3}} = \frac{1}{k'^2}[(k^2-2)\mathbf{K}(k) + (2+\ln\, k')\mathbf{E}(k)].$$

66. $$\int_0^{\pi/2} \ln\left(1-k^2\,\sin^2 x\right) \frac{\sin^2 x\,dx}{\sqrt{(1-k^2\sin^2 x)^3}} = \frac{1}{k^2k'^2}[(2+\ln\, k')\mathbf{E}(k) - (1+k'^2+k'^2\ln\, k')\mathbf{K}(k)].$$

67. $$\int_0^{\pi/2} \ln\left(1-k^2\,\sin^2 x\right) \frac{\cos^2 x\,dx}{\sqrt{(1-k^2\sin^2 x)^3}} = \frac{1}{k^2}[(1+k'^2+\ln\, k')\mathbf{K}(k) - (2+\ln\, k')\mathbf{E}(k)].$$

68. $$\int_0^{\pi/2} \ln\left(1-k^2\sin^2 x\right)\sqrt{1-k^2\sin^2 x}\,dx = (1+k'^2)\mathbf{K}(k) - (2-\ln\, k')\mathbf{E}(k).$$

69. $$\int_0^{\pi/2} \ln\left(1-k^2\sin^2 x\right)\sin^2 x\sqrt{1-k^2\sin^2 x}\,dx = \frac{1}{9k^2}\Big\{(-2+11k^2-6k^4+3k'^2\ln\, k')\mathbf{K}(k)$$
$$+[2-10k^2-3(1-2k^2)\ln\, k']\mathbf{E}(k)\Big\}.$$

70. $$\int_0^{\pi/2} \ln(1-k^2\sin^2 x)\cos^2 x\sqrt{1-k^2\sin^2 x}\,dx = \frac{1}{9k^2}\Big\{(2+7k^2-3k^4-3k'^2\ln k')\mathbf{K}(k) -[2+8k^2-3(1+k^2)\ln k']\mathbf{E}(k)\Big\}.$$

71. $$\int_0^{\pi/2} \ln(1-k^2\sin^2 x)\frac{\sin x\cos x\,dx}{\sqrt{(1-k^2\sin^2 x)^{2n+1}}} = \frac{2}{(2n-1)^2k^2}\{[1+(2n-1)\ln k']k'^{1-2n}-1\}.$$

72. $$\int_0^{\pi/2} \frac{\cos x\ln\left(1+\sqrt{\sin^2\beta-\cos^2\beta\,\tan^2\alpha\sin^2 x}\right)}{1-\sin^2\alpha\cos^2 x}\,dx$$
$$= \csc 2\alpha\{(2\alpha+2\gamma-\pi)\ln\cos\beta+2L(\alpha)-2L(\gamma)+L(\alpha+\gamma)-L(\alpha-\gamma)\},$$
$$\cos\gamma=\frac{\sin\alpha}{\sin\beta};\ 0<\alpha<\beta<\frac{\pi}{2}.$$

73. $$\int_0^{\pi/2} \frac{\cos x\ln\left(1-\sqrt{\sin^2\beta-\cos^2\beta\,\tan^2\alpha\sin^2 x}\right)}{1-\sin^2\alpha\cos^2 x}\,dx$$
$$= \csc 2\alpha\{(\pi+2\alpha-2\gamma)\ln\cos\beta+2L(\alpha)+2L(\gamma)-L(\alpha+\gamma)+L(\alpha-\gamma)\},$$
$$\cos\gamma=\frac{\sin\alpha}{\sin\beta};\ 0<\alpha<\beta<\frac{\pi}{2}.$$

74. $$\int_\beta^{\pi/2} \frac{\ln\left(\sin x+\sqrt{\sin^2 x-\sin^2\beta}\right)}{1-\cos^2\alpha\cos^2 x}\,dx$$
$$= -\csc\alpha\left\{\arctan\left(\frac{\tan\beta}{\sin\alpha}\right)\ln\sin\beta+\frac{\pi}{2}\ln\frac{1+\sin\alpha}{\sin\alpha+\sqrt{1-\cos^2\alpha\cos^2\beta}}\right\},$$
$$0<\alpha<\pi,\ 0<\beta<\frac{\pi}{2}.$$

75. $$\int_0^{\pi/2} \frac{\ln\cos x\,dx}{1-2p\cos 2x+p^2} = \begin{cases} \dfrac{\pi}{2(1-p^2)}\ln\dfrac{1+p}{2}, & p^2<1,\\[2ex] \dfrac{\pi}{2(p^2-1)}\ln\dfrac{p+1}{2p}, & p^2>1.\end{cases}$$