

! For an efficient use of these tables, first read [HowTo.pdf](#).

T1.12. Integrand involving $a + bx + cx^2$ and powers of x .

Notation used: $R = a + bx + cx^2$, $\Delta = 4ac - b^2$.

$$\begin{aligned}
 1. \int \frac{dx}{R} &= \begin{cases} \frac{1}{\sqrt{-\Delta}} \ln \frac{\sqrt{-\Delta} - (b + 2cx)}{(b + 2cx) + \sqrt{-\Delta}} = \frac{-2}{\sqrt{-\Delta}} \operatorname{arctanh} \frac{b + 2cx}{\sqrt{-\Delta}}, & \text{for } \Delta < 0, \\ \frac{1}{\sqrt{-\Delta}} \ln \frac{\sqrt{-\Delta} - (b + 2cx)}{(b + 2cx) + \sqrt{-\Delta}} = \frac{-2}{b + 2cx}, & \text{for } \Delta = 0, \\ \frac{1}{\sqrt{-\Delta}} \ln \frac{\sqrt{-\Delta} - (b + 2cx)}{(b + 2cx) + \sqrt{-\Delta}} = \frac{2}{\sqrt{\Delta}} \arctan \frac{b + 2cx}{\sqrt{\Delta}}, & \text{for } \Delta > 0. \end{cases} \\
 2. \int \frac{dx}{R^2} &= \frac{b + 2cx}{\Delta R} + \frac{2c}{\Delta} \int \frac{dx}{R}. \\
 3. \int \frac{dx}{R^3} &= \frac{b + 2cx}{\Delta} \left\{ \frac{1}{2R^2} + \frac{3c}{\Delta R} \right\} + \frac{6c^2}{\Delta^2} \int \frac{dx}{R}. \\
 4. \int \frac{dx}{R^{n+1}} &= \frac{b + 2cx}{n\Delta R^n} + \frac{(4n - 2)c}{n\Delta} \int \frac{dx}{R^n}. \\
 5. \int \frac{dx}{R^{n+1}} &= \frac{(2cx + b)}{2n + 1} \sum_{k=0}^{n-1} \frac{2k(2n + 1)(2n - 1)(2n - 3) \dots (2n - 2k + 1)c^k}{n(n - 1) \dots (n - k)\Delta^{k+1}R^{n-k}} \\
 &\quad + \int \frac{dx}{R^{n+1}} + 2^n \frac{(2n - 1)!!c^n}{n!\Delta^n} \int \frac{dx}{R}. \\
 6. \int \frac{x dx}{R} &= \frac{1}{2c} \ln \{ R \} - \frac{b}{2c} \int \frac{dx}{R}. \\
 7. \int \frac{x dx}{R^2} &= -\frac{2a + bx}{\Delta R} - \frac{b}{\Delta} \int \frac{dx}{R}. \\
 8. \int \frac{x dx}{R^3} &= -\frac{2a + bx}{2\Delta R^2} - \frac{3b(b + 2cx)}{2\Delta^2 R} - \frac{3bc}{\Delta^2} \int \frac{dx}{R}. \\
 9. \int \frac{x^2 dx}{R} &= \frac{x}{c} - \frac{b}{2c^2} \ln R + \frac{b^2 - 2ac}{2c^2} \int \frac{dx}{R}. \\
 10. \int \frac{x^2 dx}{R^2} &= \frac{ab + (b^2 - 2ac)x}{c\Delta R} + \frac{2a}{\Delta} \int \frac{dx}{R}.
 \end{aligned}$$

11. $\int \frac{x^2 dx}{R^3} = \frac{ab + (b^2 - 2ac)x}{2c \Delta R^2} + \frac{(2ac + b^2)(b + 2cx)}{2c \Delta^2 R} + \frac{2ac + b^2}{\Delta^2} \int \frac{dx}{R}.$
12. $\int \frac{x^3 dx}{R} = \frac{x^2}{2c} - \frac{bx}{c^2} + \frac{b^2 - ac}{2c^3} \ln R - \frac{b(b^2 - 3ac)}{2c^3} \int \frac{dx}{R}.$
13. $\int \frac{x^3 dx}{R^2} = \frac{1}{2c^2} \ln R + \frac{a(2ac - b^2) + b(3ac - b^2)x}{c^2 \Delta R} - \frac{b(6ac - b^2)}{2c^2 \Delta} \int \frac{dx}{R}.$
14. $\int \frac{x^3 dx}{R^3} = - \left(\frac{x^2}{c} + \frac{abx}{c\Delta} + \frac{2a^2}{c\Delta} \right) \frac{1}{2R^2} - \frac{3ab}{2c\Delta} \int \frac{dx}{R^2}.$
15. $\int \frac{x^m dx}{R^n} = - \frac{x^{m-1}}{(2n - m - 1)cR^{n-1}} - \frac{(n - m)b}{(2n - m - 1)c} \int \frac{x^{m-1} dx}{R^n} + \frac{(m - 1)a}{(2n - m - 1)c} \int \frac{x^{m-2} dx}{R^n},$
 $m \neq 2n - 1.$
16. $\int \frac{x^{2n-1} dx}{R^n} = \frac{1}{c} \int \frac{x^{2n-3} dx}{R^{n-1}} - \frac{a}{c} \int \frac{x^{2n-3} dx}{R^n} - \frac{b}{c} \int \frac{x^{2n-2} dx}{R^n}.$
17. $\int \frac{dx}{xR} = \frac{1}{2a} \ln \frac{x^2}{R} - \frac{b}{2a} \int \frac{dx}{R}.$
18. $\int \frac{dx}{xR^2} = \frac{1}{2a^2} \ln \frac{x^2}{R} + \frac{1}{2aR} \left\{ 1 - \frac{b(b + 2cx)}{\Delta} \right\} - \frac{b}{2a^2} \left(1 + \frac{2ac}{\Delta} \right) \int \frac{dx}{R}.$
19. $\int \frac{dx}{xR^3} = \frac{1}{4aR^2} + \frac{1}{2a^2R} + \frac{1}{2a^3} \ln \frac{x^2}{R} - \frac{b}{2a} \int \frac{dx}{R^3} - \frac{b}{2a^2} \int \frac{dx}{R^2} - \frac{b}{2a^3} \int \frac{dx}{R}.$
20. $\int \frac{dx}{x^2R} = - \frac{b}{2a^2} \ln \frac{x^2}{R} - \frac{1}{ax} + \frac{b^2 - 2ac}{2a^2} \int \frac{dx}{R}.$
21. $\int \frac{dx}{x^2R^2} = - \frac{b}{a^3} \ln \frac{x^2}{R} - \frac{a + bx}{a^2xR} + \frac{(b^2 - 3ac)(b + 2cx)}{a^2\Delta R} - \frac{1}{\Delta} \left(\frac{b^4}{a^3} - \frac{6b^2c}{a^2} + \frac{6c^2}{a} \right) \int \frac{dx}{R}.$
22. $\int \frac{dx}{x^2R^3} = - \frac{1}{axR^2} - \frac{3b}{a} \int \frac{dx}{xR^3} - \frac{5c}{a} \int \frac{dx}{R^3}.$
23. $\int \frac{dx}{x^3R} = - \frac{ac - b^2}{2a^3} \ln \frac{x^2}{R} + \frac{b}{a^2x} - \frac{1}{2ax^2} + \frac{b(3ac - b^2)}{2a^3} \int \frac{dx}{R}.$
24. $\int \frac{dx}{x^3R^2} = \left(- \frac{1}{2ax^2} + \frac{3b}{2a^2x} \right) \frac{1}{R} + \left(\frac{3b^2}{a^2} - \frac{2c}{a} \right) \int \frac{dx}{xR^2} + \frac{9bc}{2a^2} \int \frac{dx}{R^2}.$
25. $\int \frac{dx}{x^3R^3} = \left(\frac{-1}{2ax^2} + \frac{2b}{a^2x} \right) \frac{1}{R^2} + \left(\frac{6b^2}{a^2} - \frac{3c}{a} \right) \int \frac{dx}{xR^3} + \frac{10bc}{a^2} \int \frac{dx}{R^3}.$
26. $\int \frac{dx}{x^m R^n} = \frac{-1}{(m - 1)ax^{m-1}R^{n-1}} - \frac{b(m + n - 2)}{a(m - 1)} \int \frac{dx}{x^{m-1}R^n} - \frac{c(m + 2n - 3)}{a(m - 1)} \int \frac{dx}{x^{m-2}R^n}.$
27. $\int x^{m+1} R^n dx = \frac{x^m R^{n+1}}{c(m + 2n + 2)} - \frac{am}{c(m + 2n + 2)} \int x^{m-1} R^n dx - \frac{b(m + n + 1)}{c(m + 2n + 2)} \int x^m R^n dx.$

$$28. \int \frac{R^n dx}{x^{m+1}} = -\frac{R^{n+1}}{amx^m} + \frac{b(n-m+1)}{am} \int \frac{R^n dx}{x^m} + \frac{c(2n-m+2)}{am} \int \frac{R^n dx}{x^{m-1}}.$$

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