! For an efficient use of these tables, first read `HowTo.pdf`.

T3.06A. Integrands of the form $\dfrac{1}{\sqrt{1-x^3}}$, $\dfrac{1}{\sqrt{x^3-1}}$, $\dfrac{x}{\sqrt{1-x^3}}$ and $\dfrac{x}{\sqrt{x^3-1}}$ on the intervals $(-\infty, y)$ and (y, ∞).

Notation used: $\alpha = \arccos \dfrac{1-\sqrt{3}-y}{1+\sqrt{3}-y}$, $\delta = \arccos \dfrac{y-1-\sqrt{3}}{y-1+\sqrt{3}}$.

1. $$\int_{-\infty}^{y} \frac{dx}{\sqrt{1-x^3}} = \frac{1}{(3)^{1/4}} F\left(\alpha, \sin\frac{5\pi}{12}\right).$$

2. $$\int_{y}^{\infty} \frac{dx}{\sqrt{x^3-1}} = \frac{1}{(3)^{1/4}} F\left(\delta, \sin\frac{\pi}{12}\right).$$

3. $$\int_{-\infty}^{y} \frac{dx}{(1-x)\sqrt{1-x^3}} = \frac{1}{(27)^{1/4}} \left[F\left(\alpha, \sin\frac{5\pi}{12}\right) - 2E\left(\alpha, \sin\frac{5\pi}{12}\right)\right] + \frac{2}{\sqrt{3}} \frac{\sqrt{1+y+y^2}}{(1+\sqrt{3}-y)\sqrt{1-y}}, \quad y \neq 1.$$

4. $$\int_{y}^{\infty} \frac{dx}{(x-1)\sqrt{x^3-1}} = \frac{1}{(27)^{1/4}} \left[F\left(\delta, \sin\frac{\pi}{12}\right) - 2E\left(\delta, \sin\frac{\pi}{12}\right)\right] + \frac{2}{\sqrt{3}} \frac{\sqrt{1+y+y^2}}{(y-1+\sqrt{3})\sqrt{y-1}}, \quad y \neq 1.$$

5. $$\int_{-\infty}^{y} \frac{(1-x)\,dx}{(1+\sqrt{3}-x)^2\sqrt{1-x^3}} = \frac{2-\sqrt{3}}{(27)^{1/4}} \left[F\left(\alpha, \sin\frac{5\pi}{12}\right) - E\left(\alpha, \sin\frac{5\pi}{12}\right)\right].$$

6. $$\int_{y}^{\infty} \frac{(x-1)\,dx}{(1+\sqrt{3}-x)^2\sqrt{x^3-1}} = \frac{2(2-\sqrt{3})}{\sqrt{3}} \frac{\sqrt{y^3-1}}{y^2-2y-2} - \frac{2-\sqrt{3}}{(27)^{1/4}} E\left(\delta, \sin\frac{\pi}{12}\right).$$

7. $$\int_{-\infty}^{y} \frac{(1-x)\,dx}{(1-\sqrt{3}-x)^2\sqrt{1-x^3}} = \frac{2+\sqrt{3}}{(27)^{1/4}} \left[\frac{2(3)^{1/4}\sqrt{1-y^3}}{y^2-2y-2} - E\left(\alpha, \sin\frac{5\pi}{12}\right)\right].$$

8. $$\int_y^\infty \frac{(x-1)\,dx}{(1-\sqrt{3}-x)^2\sqrt{x^3-1}} = \frac{2+\sqrt{3}}{(27)^{1/4}}\left[F\left(\delta,\,\sin\frac{\pi}{12}\right) - E\left(\delta,\,\sin\frac{\pi}{12}\right)\right].$$

9. $$\int_{-\infty}^y \frac{(x^2+x+1)\,dx}{(1+\sqrt{3}-x)^2\sqrt{1-x^3}} = \frac{1}{(3)^{1/4}}\,E\left(\alpha,\,\sin\frac{5\pi}{12}\right).$$

10. $$\int_y^\infty \frac{(x^2+x+1)\,dx}{(x-1+\sqrt{3})^2\sqrt{x^3-1}} = \frac{1}{(3)^{1/4}}\,E\left(\delta,\,\sin\frac{\pi}{12}\right).$$

11. $$\int_{-\infty}^y \frac{(1+\sqrt{3}-x)^2\,dx}{[(1+\sqrt{3}-x)^2-4\sqrt{3}p^2(1-x)]\sqrt{1-x^3}} = \frac{1}{(3)^{1/4}}\,\Pi\left(\alpha,\,p^2,\,\sin\frac{5\pi}{12}\right).$$

12. $$\int_y^\infty \frac{(1-\sqrt{3}-x)^2\,dx}{[(1-\sqrt{3}-x)^2-4\sqrt{3}p^2(x-1)]\sqrt{x^3-1}} = \frac{1}{(3)^{1/4}}\,\Pi\left(\delta,\,p^2,\,\sin\frac{\pi}{12}\right).$$