! For an efficient use of these tables, first read `HowTo.pdf`.

T3.24A. Integrands involving exponentials and rational functions on the interval $(0, \infty)$.

1. $$\int_0^\infty x^n e^{-\mu x}\,dx = n!\mu^{-n-1}, \quad \Re\{\mu\} > 0.$$

2. $$\int_0^\infty \frac{e^{-\mu x}\,dx}{x+\beta} = -e^{\beta\mu}\,\mathrm{Ei}\,(-\mu\beta), \quad |\arg\beta| < \pi,\ \Re\{\mu\} > 0.$$

3. $$\int_0^\infty \frac{e^{-\mu x}\,dx}{a-x} = e^{-\mu a}\,\mathrm{Ei}\,(a\mu), \quad a < 0,\ \Re\{\mu\} > 0.$$

4. $$\int_0^\infty \frac{e^{-\mu x}\,dx}{(x+\beta)^n} = \frac{1}{(n-1)!}\sum_{k=1}^{n-1}(k-1)!(-\mu)^{n-k-1}\beta^{-k} - \frac{(-\mu)^{n-1}}{(n-1)!}e^{\beta\mu}\,\mathrm{Ei}\,(-\beta\mu),$$
$$n \geq 2,\ |\arg\beta| < \pi,\ \Re\{\mu\} > 0.$$

5. $$\int_0^\infty \frac{e^{-px}\,dx}{(a+x)^2} = pe^{\alpha p}\,\mathrm{Ei}\,(-ap) + \frac{1}{a}, \quad p > 0,\ a > 0.$$

6. $$\int_0^\infty \frac{x^n e^{-\mu x}}{x+\beta}\,dx = (-1)^{n-1}\beta^n e^{\beta\mu}\,\mathrm{Ei}\,(-\beta\mu) + \sum_{k=1}^{n}(k-1)!(-\beta)^{n-k}\mu^{-k},$$
$$|\arg\beta| < \pi,\ \Re\{\mu\} > 0;\ n \geq 0.$$

7. $$\int_0^\infty \frac{e^{-\mu x}\,dx}{\beta^2+x^2} = \frac{1}{\beta}[\mathrm{Ci}\,(\beta\mu)\,\sin\beta\mu - \mathrm{Si}\,(\beta\mu)\,\cos\beta\mu], \quad \Re\{\beta\} > 0,\ \Re\{\mu\} > 0.$$

8. $$\int_0^\infty \frac{xe^{-\mu x}\,dx}{\beta^2+x^2} = -\,\mathrm{Ci}\,(\beta\mu)\,\cos\beta\mu - \mathrm{Si}\,(\beta\mu)\,\sin\beta\mu, \quad \Re\{\beta\} > 0,\ \Re\{\mu\} > 0.$$

9. $$\int_0^\infty \frac{e^{-\mu x}\,dx}{\beta^2-x^2} = \frac{1}{2\beta}[e^{-\beta\mu}\,\mathrm{Ei}\,(\beta\mu) - e^{\beta\mu}\mathrm{Ei}\,(-\beta\mu)], \quad |\arg(\pm\beta)| < \pi,\ \Re\{\mu\} > 0.$$

10. $$\int_0^\infty \frac{xe^{-\mu x}\,dx}{\beta^2 - x^2} = \frac{1}{2}[e^{-\beta\mu}\operatorname{Ei}(\beta\mu) + e^{\beta\mu}\operatorname{Ei}(-\beta\mu)], \quad |\arg(\pm\beta)| < \pi,\ \Re\{\mu\} > 0;$$

for $\beta > 0$ replace $\operatorname{Ei}(\beta\mu)$ by $\overline{\operatorname{Ei}}(\beta\mu)$.

11. $$\int_0^\infty \frac{e^{-\mu x}\,dx}{(\beta^2 + x^2)^2} = \frac{1}{2\beta^3}\big\{\operatorname{Ci}(\beta\mu)\sin\beta\mu - \operatorname{Si}(\beta\mu)\cos\beta\mu - \beta\mu[\operatorname{Ci}(\beta\mu)\cos\beta\mu + \operatorname{Si}(\beta\mu)\sin\beta\mu]\big\}.$$

12. $$\int_0^\infty \frac{xe^{-\mu x}\,dx}{(\beta^2 + x^2)^2} = \frac{1}{2\beta^2}\left\{-\beta\mu[\operatorname{Ci}(\beta\mu)\sin\beta\mu - \operatorname{Si}(\beta\mu)\cos\beta\mu]\right\}, \quad \Re\{\beta\} > 0,\ \Re\{\mu\} > 0.$$

13. $$\int_0^\infty \frac{e^{-px}\,dx}{(a^2 - x^2)^2} = \frac{1}{4a^3}[(ap-1)e^{ap}\operatorname{Ei}(-ap) + (1+ap)e^{-ap}\operatorname{Ei}(ap)], \quad \Im\{a^2\} \neq 0,\ p > 0.$$

14. $$\int_0^\infty \frac{xe^{-px}\,dx}{(a^2 - x^2)^2} = \frac{1}{4a^2}\left\{-2 + ap[e^{-ap}\operatorname{Ei}(ap) - e^{ap}\operatorname{Ei}(-ap)]\right\}, \quad \Im\{a^2\} \neq 0,\ p > 0.$$

15. $$\int_0^\infty \frac{x^{2n+1}e^{-px}}{a^2 + x^2}\,dx = (-1)^{n-1}a^{2n}[\operatorname{Ci}(ap)\cos ap + \operatorname{Si}(ap)\sin ap] + \frac{1}{p^{2n}}\sum_{k=1}^{n}(2n-2k+1)!(-a^2p^2)^{k-1}, \quad p > 0.$$

16. $$\int_0^\infty \frac{x^{2n}e^{-px}}{a^2 + x^2}\,dx = (-1)^{n}a^{2n-1}[\operatorname{Ci}(ap)\sin ap - \operatorname{Si}(ap)\cos ap] + \frac{1}{p^{2n-1}}\sum_{k=1}^{n}(2n-2k)!(-a^2p^2)^{k-1}, \quad p > 0.$$

17. $$\int_0^\infty \frac{x^{2n+1}e^{-px}}{a^2 - x^2}\,dx = \frac{1}{2}a^{2n}[e^{ap}\operatorname{Ei}(-ap) + e^{-ap}\operatorname{Ei}(ap)] - \frac{1}{p^{2n}}\sum_{k=1}^{n}(2n-2k+1)!(a^2p^2)^{k-1}, \quad p > 0.$$

18. $$\int_0^\infty \frac{x^{2n}e^{-px}}{a^2 - x^2}\,dx = \frac{1}{2}a^{2n-1}[e^{-ap}\operatorname{Ei}(ap) - e^{ap}\operatorname{Ei}(-ap)] - \frac{1}{p^{2n-1}}\sum_{k=1}^{n}(2n-2k)!(a^2p^2)^{k-1}, \quad p > 0.$$

19. $$\int_0^\infty \frac{e^{-\mu x}\,dx}{a^3+a^2x+ax^2+x^3} = \frac{1}{2a^2}\Big\{\operatorname{Ci}(a\mu)(\sin a\mu+\cos a\mu) + \operatorname{Si}(a\mu)(\sin a\mu-\cos a\mu) - e^{a\mu}\operatorname{Ei}(-a\mu)\Big\},\quad \Re\{\mu\}>0,\ a>0.$$

20. $$\int_0^\infty \frac{xe^{-\mu x}\,dx}{a^3+a^2x+ax^2+x^3} = \frac{1}{2a}\Big\{\operatorname{Ci}(a\mu)(\sin a\mu-\cos a\mu) - \operatorname{Si}(a\mu)(\sin a\mu+\cos a\mu) - e^{a\mu}\operatorname{Ei}(-a\mu)\Big\},\quad \Re\{\mu\}>0,\ a>0.$$

21. $$\int_0^\infty \frac{x^2e^{-\mu x}\,dx}{a^3+a^2x+ax^2+x^3} = \frac{1}{2}\Big\{-\operatorname{Ci}(a\mu)(\sin a\mu+\cos a\mu) - \operatorname{Si}(a\mu)(\sin a\mu-\cos a\mu) - e^{a\mu}\operatorname{Ei}(-a\mu)\Big\},\quad \Re\{\mu\}>0,\ a>0.$$

22. $$\int_0^\infty \frac{e^{-\mu x}\,dx}{a^3-a^2x+ax^2-x^3} = \frac{1}{2a^2}\Big\{\operatorname{Ci}(a\mu)(\sin a\mu-\cos a\mu) - \operatorname{Si}(a\mu)(\sin a\mu+\cos a\mu) + e^{-a\mu}\operatorname{Ei}(a\mu)\Big\},\quad \Re\{\mu\}>0,\ a>0.$$

23. $$\int_0^\infty \frac{xe^{-\mu x}\,dx}{a^3-a^2x+ax^2-x^3} = \frac{1}{2a}\Big\{-\operatorname{Ci}(a\mu)(\sin a\mu+\cos a\mu) - \operatorname{Si}(a\mu)(\sin a\mu-\cos a\mu) + e^{-a\mu}\operatorname{Ei}(a\mu)\Big\},\quad \Re\{\mu\}>0,\ a>0.$$

24. $$\int_0^\infty \frac{x^2e^{-\mu x}\,dx}{a^3-a^2x+ax^2-x^3} = \frac{1}{2}\Big\{\operatorname{Ci}(a\mu)(\cos a\mu-\sin a\mu) + \operatorname{Si}(a\mu)(\cos a\mu+\sin a\mu) + e^{-a\mu}\operatorname{Ei}(a\mu)\Big\},\quad \Re\{\mu\}>0,\ a>0.$$

25. $$\int_0^\infty \frac{e^{-px}}{a^4-x^4}\,dx = \frac{1}{4a^3}\Big\{e^{-ap}\operatorname{Ei}(ap) - e^{ap}\operatorname{Ei}(-ap) + 2\operatorname{Ci}(ap)\sin ap - 2\operatorname{Si}(ap)\cos ap\Big\},\quad p>0,\ a>0.$$

26. $$\int_0^\infty \frac{xe^{-px}dx}{a^4-x^4} = \frac{1}{4a^2}\Big\{e^{ap}\operatorname{Ei}(-ap) + e^{-ap}\operatorname{Ei}(ap) - 2\operatorname{Ci}(ap)\cos ap - 2\operatorname{Si}(ap)\sin ap\Big\},\quad p>0,\ a>0.$$

27. $$\int_0^\infty \frac{x^2e^{-px}dx}{a^4-x^4} = \frac{1}{4a}\Big\{e^{-ap}\operatorname{Ei}(ap) - e^{ap}\operatorname{Ei}(-ap) - 2\operatorname{Ci}(ap)\sin ap + 2\operatorname{Si}(ap)\cos ap\Big\},\quad p>0,\ a>0.$$

28. $$\int_0^\infty \frac{x^3 e^{-px} dx}{a^4 - x^4} = \frac{1}{4}\big\{e^{ap}\,\mathrm{Ei}\,(-ap) + e^{-ap}\,\mathrm{Ei}\,(ap) + 2\ \mathrm{Ci}\,(ap)\,\cos ap + 2\ \mathrm{Si}\,(ap)\,\sin ap\big\}, \quad p > 0,\ a > 0.$$

C4282

29. $$\int_0^\infty \frac{x^{4n} e^{-px}}{a^4 - x^4}\,dx = \frac{1}{4}a^{4n-3}[e^{-ap}\,\mathrm{Ei}\,(ap) - e^{ap}\,\mathrm{Ei}\,(-ap) + 2\ \mathrm{Ci}\,(ap)\,\sin ap - 2\ \mathrm{Si}\,(ap)\,\cos ap] - \frac{1}{p^{4n-3}}\sum_{k=1}^{n}(4n-4k)!(a^4p^4)^{k-1}, \quad p > 0,\ a > 0.$$

C4282

30. $$\int_0^\infty \frac{x^{4n+1} e^{-px}}{a^4 - x^4}\,dx = \frac{1}{4}a^{4n-2}[e^{ap}\,\mathrm{Ei}\,(-ap) + e^{-ap}\,\mathrm{Ei}\,(ap) - 2\ \mathrm{Ci}\,(ap)\,\cos ap - 2\ \mathrm{Si}\,(ap)\,\sin ap] - \frac{1}{p^{4n-2}}\sum_{k=1}^{n}(4n-4k+1)!(a^4p^4)^{k-1}, \quad p > 0,\ a > 0.$$

C4282

31. $$\int_0^\infty \frac{x^{4n+2} e^{-px}}{a^4 - x^4}\,dx = \frac{1}{4}a^{4n-1}[e^{-ap}\,\mathrm{Ei}\,(ap) - e^{ap}\,\mathrm{Ei}\,(-ap) - 2\ \mathrm{Ci}\,(ap)\,\sin ap + 2\ \mathrm{Si}\,(ap)\,\cos ap] - \frac{1}{p^{4n-1}}\sum_{k=1}^{n}(4n-4k+2)!(a^4p^4)^{k-1}, \quad p > 0,\ a > 0.$$

C4282

32. $$\int_0^\infty \frac{x^{4n+3} e^{-px}}{a^4 - x^4}\,dx = \frac{1}{4}a^{4n}[e^{ap}\,\mathrm{Ei}\,(-ap) + e^{-ap}\,\mathrm{Ei}\,(ap) + 2\ \mathrm{Ci}\,(ap)\,\cos ap + 2\ \mathrm{Si}\,(ap)\,\sin ap] - \frac{1}{p^{4n}}\sum_{k=1}^{n}(4n-4k+3)!(a^4p^4)^{k-1}, \quad p > 0,\ a > 0.$$

C4282

C4282

C4282

C4282