! For an efficient use of these tables, first read `HowTo.pdf`.

T2.13B. Integrands of the form $\dfrac{x^n}{\sqrt{(a^2 \pm x^2)\,(b^2 \pm x^2)}}$, $n = 0, 2, 4$, on the intervals (y, b) and (b, y).

Notation used: $\delta = \arccos\dfrac{y}{b}$, $\varepsilon = \arccos\dfrac{b}{y}$,

$$\zeta = \arcsin\frac{a}{b}\sqrt{\frac{b^2-y^2}{a^2-y^2}}, \quad \kappa = \arcsin\frac{a}{y}\sqrt{\frac{y^2-b^2}{a^2-b^2}},$$

$$r = \frac{b}{\sqrt{a^2+b^2}}, \quad s = \frac{a}{\sqrt{a^2+b^2}}, \quad t = \frac{b}{a}.$$

1. $$\int_y^b \frac{dx}{\sqrt{(x^2+a^2)(b^2-x^2)}} = \frac{1}{\sqrt{a^2+b^2}}F(\delta,\, r), \quad b > y \geq 0.$$

2. $$\int_b^y \frac{dx}{\sqrt{(x^2+a^2)(x^2-b^2)}} = \frac{1}{\sqrt{a^2+b^2}}F(\varepsilon,\, s), \quad y > b > 0.$$

3. $$\int_y^b \frac{dx}{\sqrt{(a^2-x^2)(b^2-x^2)}} = \frac{1}{a}F(\zeta,\, t), \quad a > b > y \geq 0.$$

4. $$\int_b^y \frac{dx}{\sqrt{(a^2-x^2)(x^2-b^2)}} = \frac{1}{a}F(\kappa,\, q), \quad a \geq y > b > 0.$$

5. $$\int_y^b \frac{x^2\,dx}{\sqrt{(a^2+x^2)(b^2-x^2)}} = \sqrt{a^2+b^2}E(\delta,\, r) - \frac{a^2}{\sqrt{a^2+b^2}}F(\delta,\, r), \quad b > y \geq 0.$$

6. $$\int_b^y \frac{x^2\,dx}{\sqrt{(a^2+x^2)(x^2-b^2)}} = \frac{b^2}{\sqrt{a^2+b^2}}F(\varepsilon,\, s) - \sqrt{a^2+b^2}E(\varepsilon,\, s) + \frac{1}{y}\sqrt{(y^2+a^2)(y^2-b^2)}, \quad y > b > 0.$$

7. $$\int_y^b \frac{x^2\,dx}{\sqrt{(a^2-x^2)(b^2-x^2)}} = a\{F(\zeta,\,t)-E(\zeta,\,t)\}+y\sqrt{\frac{b^2-y^2}{a^2-y^2}},\quad a>b>y\geq 0.$$

8. $$\int_b^y \frac{x^2\,dx}{\sqrt{(a^2-x^2)(x^2-b^2)}} = aE(\kappa,\,q)-\frac{1}{y}\sqrt{(a^2-y^2)(y^2-b^2)},\quad a\geq y>b>0.$$

9. $$\int_y^b \frac{x^4\,dx}{\sqrt{(a^2+x^2)(b^2-x^2)}} = \frac{1}{3\sqrt{a^2+b^2}}\{(2a^2-b^2)a^2F(\delta,\,r)-2(a^4-b^4)E(\delta,\,r)\}$$
$$+\frac{y}{3}\sqrt{(a^2+y^2)(b^2-y^2)},\quad b>y\geq 0.$$

10. $$\int_b^y \frac{x^4\,dx}{\sqrt{(a^2+x^2)(x^2-b^2)}} = \frac{1}{3\sqrt{a^2+b^2}}\{(2b^2-a^2)b^2F(\varepsilon,\,s)+2(a^4-b^4)E(\varepsilon,\,s)\}$$
$$+\frac{2b^2-2a^2+y^2}{3y}\sqrt{(y^2+a^2)(y^2-b^2)},\quad y>b>0.$$

11. $$\int_y^b \frac{x^4\,dx}{\sqrt{(a^2-x^2)(b^2-x^2)}} = \frac{a}{3}\{(2a^2+b^2)F(\zeta,\,t)-2(a^2+b^2)E(\zeta,\,t)\}$$
$$+\frac{y}{3}(y^2+a^2+2b^2)\sqrt{\frac{b^2-y^2}{a^2-y^2}},\quad a>b>y\geq 0.$$

12. $$\int_b^y \frac{x^4\,dx}{\sqrt{(a^2-x^2)(x^2-b^2)}} = \frac{a}{3}\{2(a^2+b^2)E(\kappa,\,q)-b^2F(\kappa,\,q)\}$$
$$-\frac{y^2+2a^2+2b^2}{3y}\sqrt{(a^2-y^2)(y^2-b^2)},\quad a\geq y>b>0.$$