! For an efficient use of these tables, first read `HowTo.pdf`.

T2.25C. Integrands of the form $\sqrt{\dfrac{x^2+a^2}{x^2\pm b^2}}$ and $\sqrt{\dfrac{b^2-x^2}{a^2\pm x^2}}$ on the interval $(0, y)$.

Notation used: $\alpha = \arctan\dfrac{y}{b}$, $\gamma = \arcsin\dfrac{y}{b}\sqrt{\dfrac{a^2+b^2}{a^2+y^2}}$, $\eta = \arcsin\dfrac{y}{b}$,

$$q = \frac{\sqrt{a^2-b^2}}{a}, \quad r = \frac{b}{\sqrt{a^2+b^2}}, \quad t = \frac{b}{a}.$$

Then

1. $\displaystyle\int_0^y \sqrt{\frac{x^2+a^2}{x^2+b^2}}\,dx = a\,\{F(\alpha,\, q) - E(\alpha,\, q)\} + y\sqrt{\frac{a^2+y^2}{b^2+y^2}} \quad a > b,\ y > 0.$

2. $\displaystyle\int_0^y \sqrt{\frac{x^2+b^2}{x^2+a^2}}\,dx = \frac{b^2}{a}F(\alpha,\, q) - aE(\alpha,\, q) + y\sqrt{\frac{a^2+y^2}{b^2+y^2}} \quad a > b,\ y > 0.$

3. $\displaystyle\int_0^y \sqrt{\frac{x^2+a^2}{b^2-x^2}}\,dx = \sqrt{a^2+b^2}\,E(\gamma,\, r) - y\sqrt{\frac{b^2-y^2}{a^2+y^2}}, \quad b \ge y > 0.$

4. $\displaystyle\int_0^y \sqrt{\frac{b^2-x^2}{a^2+x^2}}\,dx = \sqrt{a^2+b^2}\,\{F(\gamma,\, r) - E(\gamma,\, r)\} + y\sqrt{\frac{b^2-y^2}{a^2+y^2}}, \quad b \ge y > 0.$

5. $\displaystyle\int_0^y \sqrt{\frac{b^2-x^2}{a^2-x^2}}\,dx = aE(\eta,\, t) - \frac{a^2-b^2}{a}F(\eta,\, t), \quad a > b \ge y > 0.$

6. $\displaystyle\int_0^y \sqrt{\frac{a^2-x^2}{b^2-x^2}}\,dx = aE(\eta,\, t), \quad a > b \ge y > 0.$
