! For an efficient use of these tables, first read `HowTo.pdf`.

T2.24C. Integrands of the form $\sqrt{\dfrac{d-x}{(a-x)(b-x)(c-x)^3}}$, $\sqrt{\dfrac{c-x}{(a-x)(b-x)(d-x)^3}}$, $\sqrt{\dfrac{b-x}{(a-x)(c-x)(d-x)^3}}$, and $\sqrt{\dfrac{a-x}{(b-x)(c-x)(d-x)^3}}$, and similar expressions with cubes of one of the factors in the denominator, on the intervals (y,c) and (c,y).

Notation used: $\gamma = \arcsin\sqrt{\dfrac{(b-d)(c-y)}{(c-d)(b-y)}}$, $\quad \delta = \arcsin\sqrt{\dfrac{(b-d)(y-c)}{(b-c)(y-d)}}$,

$$q = \sqrt{\frac{(b-c)(a-d)}{(a-c)(b-d)}}, \quad r = \sqrt{\frac{(a-b)(c-d)}{(a-c)(b-d)}}.$$

1. $$\int_y^c \sqrt{\frac{c-x}{(a-x)(b-x)(x-d)^3}}\,dx = \frac{2}{d-a}\left[\sqrt{\frac{a-c}{b-d}}E(\gamma,\,r) - \sqrt{\frac{(a-y)(c-y)}{(b-y)(y-d)}}\right],$$
$$a > b > c > y > d.$$

2. $$\int_c^y \sqrt{\frac{x-c}{(a-x)(b-x)(x-d)^3}}\,dx = \frac{2}{a-d}\sqrt{\frac{a-c}{b-d}}[F(\delta,\,q) - E(\delta,\,q)], \quad a > b \geq y > c > d.$$

3. $$\int_y^c \sqrt{\frac{b-x}{(a-x)(c-x)(x-d)^3}}\,dx = \frac{2}{(a-d)(c-d)\sqrt{(a-c)(b-d)}}$$
$$\times[(b-c)(a-d)F(\gamma,\,r) - (a-c)(b-d)E(\gamma,\,r)] + \frac{2(b-d)}{(a-d)(c-d)}\sqrt{\frac{(a-y)(c-y)}{(b-y)(y-d)}},$$
$$a > b > c > y > d.$$

4. $$\int_c^y \sqrt{\frac{b-x}{(a-x)(x-c)(x-d)^3}}\,dx = \frac{2}{(a-d)(c-d)\sqrt{(a-c)(b-d)}}$$
$$\times[(a-c)(b-d)E(\delta,\,q) - (a-b)(c-d)F(\delta,\,q)], \quad a > b \geq y > c > d.$$

5. $$\int_y^c \sqrt{\frac{a-x}{(b-x)(c-x)(x-d)^3}}\,dx = \frac{2}{c-d}\sqrt{\frac{a-c}{b-d}}[F(\gamma,\, r) - E(\gamma,\, r)] + \frac{2}{c-d}\sqrt{\frac{(a-y)(c-y)}{(b-y)(y-d)}}, \quad a > b > c > y > d.$$

6. $$\int_c^y \sqrt{\frac{a-x}{(b-x)(x-c)(x-d)^3}}\,dx = \frac{2}{c-d}\sqrt{\frac{a-c}{b-d}}E(\delta,\, q), \quad a > b \geq y > c > d.$$

7. $$\int_y^c \sqrt{\frac{x-d}{(a-x)(b-x)^3(c-x)}}\,dx = \frac{2\sqrt{(a-c)(b-d)}}{(a-b)(b-c)}E(\gamma,\, r) - \frac{2(a-d)}{(a-b)\sqrt{(a-c)(b-d)}}F(\gamma,\, r), \quad a > b > c > y \geq d.$$

8. $$\int_c^y \sqrt{\frac{x-d}{(a-x)(b-x)^3(x-c)}}\,dx = \frac{2(c-d)}{(b-c)\sqrt{(a-c)(b-d)}}F(\delta,\, q) - \frac{2\sqrt{(a-c)(b-d)}}{(a-b)(b-c)}E(\delta,\, q) + \frac{2(b-d)}{(a-b)(b-c)}\sqrt{\frac{(a-y)(y-c)}{(b-y)(y-d)}}, \quad a > b > y > c > d.$$

9. $$\int_y^c \sqrt{\frac{c-x}{(a-x)(b-x)^3(x-d)}}\,dx = \frac{2}{a-b}\sqrt{\frac{a-c}{b-d}}[F(\gamma,\, r) - E(\gamma,\, r)], \quad a > b > c > y \geq d.$$

10. $$\int_c^y \sqrt{\frac{x-c}{(a-x)(b-x)^3(x-d)}}\,dx = \frac{2}{b-a}\sqrt{\frac{a-c}{b-d}}E(\delta,\, q) + \frac{2}{a-b}\sqrt{\frac{(a-y)(y-c)}{(b-y)(y-d)}}, \quad a > b \geq y > c > d.$$

11. $$\int_y^c \sqrt{\frac{a-x}{(b-x)^3(c-x)(x-d)}}\,dx = \frac{2}{b-c}\sqrt{\frac{a-c}{b-d}}E(\gamma,\, r), \quad a > b > cy \geq d.$$

12. $$\int_c^y \sqrt{\frac{a-x}{(b-x)^3(x-c)(x-d)}}\,dx = \frac{2}{b-c}\sqrt{\frac{a-c}{b-d}}[F(\delta,\, q) - E(\delta,\, q)] + \frac{2}{b-c}\sqrt{\frac{(a-y)(y-c)}{(b-y)(y-d)}}, \quad a > b > y > c > d.$$

13. $$\int_y^c \sqrt{\frac{x-d}{(a-x)^3(b-x)(c-x)}}\,dx = \frac{2}{a-b}\sqrt{\frac{b-d}{a-c}}[F(\gamma,\, r) - E(\gamma,\, r)] + \frac{2}{a-c}\sqrt{\frac{(c-y)(y-d)}{(a-y)(b-y)}}, \quad a > b > c > y \geq d.$$

14. $$\int_c^y \sqrt{\frac{x-d}{(a-x)^3(b-x)(x-c)}}\,dx = \frac{2}{a-b}\sqrt{\frac{b-d}{a-c}}E(\delta,\,q) - \frac{2(a-d)}{(a-b)(a-c)}\sqrt{\frac{(b-y)(y-c)}{(a-y)(y-d)}},$$
$$a > b \geq y > c > d.$$

15. $$\int_y^c \sqrt{\frac{c-x}{(a-x)^3(b-x)(x-d)}}\,dx = \frac{2\sqrt{(a-c)(b-d)}}{(a-b)(a-d)}E(\gamma,\,r) - \frac{2(b-c)}{(a-b)\sqrt{(a-c)(b-d)}}F(\gamma,\,r)$$
$$-\frac{2}{a-d}\sqrt{\frac{(c-y)(y-d)}{(a-y)(b-y)}}, \quad a > b > c > y \geq d.$$

16. $$\int_c^y \sqrt{\frac{x-c}{(a-x)^3(b-x)(x-d)}}\,dx = \frac{2\sqrt{(a-c)(b-d)}}{(a-b)(a-d)}E(\delta,\,q) - \frac{2(c-d)}{(a-d)\sqrt{(a-c)(b-d)}}F(\delta,\,q)$$
$$-\frac{2}{a-b}\sqrt{\frac{(b-y)(y-c)}{(a-y)(y-d)}}, \quad a > b \geq y > c > d.$$

17. $$\int_y^c \sqrt{\frac{b-x}{(a-x)^3(c-x)(x-d)}}\,dx = \frac{2}{a-d}\sqrt{\frac{b-d}{a-c}}E(\gamma,\,r) - \frac{2(a-b)}{(a-c)(a-d)}\sqrt{\frac{(c-y)(y-d)}{(a-y)(b-y)}},$$
$$a > b > c > y \geq d.$$

18. $$\int_c^y \sqrt{\frac{b-x}{(a-x)^3(x-c)(x-d)}}\,dx = \frac{2}{a-d}\sqrt{\frac{b-d}{a-c}}[F(\delta,\,q) - E(\delta,\,q)] + \frac{2}{a-c}\sqrt{\frac{(b-y)(y-c)}{(a-y)(y-d)}},$$
$$a > b \geq y > c > d.$$
