

! For an efficient use of these tables, first read [HowTo.pdf](#).

T1.04. Integrand involving the binomials $(a + bx)^k \equiv X_k$, $k = 1, 2, 3, 4, n$.

1. $\int X_1^m dx = \begin{cases} \frac{X_1^{m+1}}{b(m+1)} & \text{for } m \neq -1, \\ \frac{1}{b} \ln X_1 & \text{for } m = -1. \end{cases}$
2. $\int x X_1^m dx = \frac{1}{(m+2)b^2} X_1^{m+2} - \frac{a}{(m+1)b^2} X_1^{m+1} \Big], \quad m \neq -1, -2, \dots$
3. $\int x^2 X_1^m dx = \frac{1}{b^3} \left[\frac{X_1^{m+3}}{m+3} - 2a \frac{X_1^{m+2}}{m+2} + a^2 \frac{X_1^{m+1}}{m+1} \right], \quad m \neq -1, -2, \dots$
4. $\int x^k X_1^m dx = \begin{cases} \frac{x^{k+1} X_1^m}{m+k+1} + \frac{ma}{m+k+1} \int x^k X_1^{m-1} dx, \\ \text{or} \\ \frac{1}{a(m+1)} \left[-x^{k+1} X_1^{m+1} + (m+k+2) \int x^k X_1^{m+1} dx \right], \\ \text{or} \\ \frac{1}{b(m+k+1)} \left[x^k X_1^{m+1} - ka \int x^{k-1} X_1^m dx \right]. \end{cases}$
5. $\int \frac{x^n dx}{X_1^m} = \frac{x^n}{X_1^{m-1}(n+1-m)b} - \frac{na}{(n+1-m)b} \int \frac{x^{n-1} dx}{X_1^m}.$
6. $\int \frac{x^{m-1} dx}{X_1^m} = -\frac{x^{m-1}}{X_1^{m-1}(m-1)b} + \frac{1}{b} \int \frac{x^{m-2} dx}{X_1^{m-1}}.$
7. $\int \frac{x^n dx}{X_1} = \frac{x^n}{nb} - \frac{ax^{n-1}}{(n-1)b^2} + \frac{a^2 x^{n-2}}{(n-2)b^3} - \dots + (-1)^{n-1} \frac{a^{n-1} x}{1 \cdot b^n} + \frac{(-1)^n a^n}{b^{n+1}} \ln X_1.$
8. $\int \frac{x^n dx}{X_1^2} = \sum_{k=1}^{n-1} (-1)^{k-1} \frac{ka^{k-1} x^{n-k}}{(n-k)b^{k+1}} + (-1)^{n-1} \frac{a^n}{b^{n+1} X_1} + (-1)^{n+1} \frac{na^{n-1}}{b^{n+1}} \ln X_1.$
9. $\int \frac{x dx}{X_1} = \frac{x}{b} - \frac{a}{b^2} \ln X_1.$
10. $\int \frac{x^2 dx}{X_1} = \frac{x^2}{2b} - \frac{ax}{b^2} + \frac{a^2}{b^3} \ln X_1.$

$$11. \int \frac{dx}{X_1^2} = -\frac{1}{bX_1}.$$

$$12. \int \frac{x dx}{X_1^2} = -\frac{x}{bX_1} + \frac{1}{b^2} \ln X_1 = \frac{a}{b^2 X_1} + \frac{1}{b^2} \ln X_1.$$

$$13. \int \frac{x^2 dx}{X_1^2} = \frac{x}{b^2} - \frac{a^2}{b^3 X_1} - \frac{2a}{b^3} \ln X_1.$$

$$14. \int \frac{dx}{X_1^3} = -\frac{1}{2bX_1^2}.$$

$$15. \int \frac{x dx}{X_1^3} = -\left[\frac{x}{b} + \frac{a}{2b^2}\right] \frac{1}{X_1^2}.$$

$$16. \int \frac{x^2 dx}{X_1^3} = \left[\frac{2ax}{b^2} + \frac{3a^2}{2b^3}\right] \frac{1}{X_1^2} + \frac{1}{b^3} \ln X_1.$$

$$17. \int \frac{x^3 dx}{X_1^3} = \left[\frac{x^3}{b} + 2\frac{a}{b^2}x^2 - 2\frac{a^2}{b^3}x - \frac{5a^3}{2b^4}\right] \frac{1}{X_1^2} - 3\frac{a}{b^4} \ln X_1.$$

$$18. \int \frac{dx}{X_1^4} = -\frac{1}{3bX_1^3}.$$

$$19. \int \frac{x dx}{X_1^4} = -\left[\frac{x}{2b} + \frac{a}{6b^2}\right] \frac{1}{X_1^3}.$$

$$20. \int \frac{x^2 dx}{X_1^4} = -\left[\frac{x^2}{b} + \frac{ax}{b^2} + \frac{a^2}{3b^3}\right] \frac{1}{X_1^3}.$$

$$21. \int \frac{x^3 dx}{X_1^4} = \left[\frac{3ax^2}{b^2} + \frac{9a^2x}{2b^2} + \frac{11a^3}{6b^4}\right] \frac{1}{X_1^3} + \frac{1}{b^4} \ln X_1.$$

$$22. \int \frac{dx}{X_1^5} = -\frac{1}{4bX_1^4}.$$

$$23. \int \frac{x dx}{X_1^5} = -\left[\frac{x}{3b} + \frac{a}{12b^2}\right] \frac{1}{X_1^4}.$$

$$24. \int \frac{x^2 dx}{X_1^5} = -\left[\frac{x^2}{2b} + \frac{ax}{3b^2} + \frac{a^2}{12b^3}\right] \frac{1}{X_1^4}.$$

$$25. \int \frac{x^3 dx}{X_1^5} = -\left[\frac{x^3}{b} + \frac{3ax^2}{2b^2} + \frac{a^2x}{b^3} + \frac{a^3}{4b^4}\right] \frac{1}{X_1^4}.$$

$$26. \int \frac{dx}{x^n X_1^m} = \frac{-1}{(n-1)ax^{n-1}X_1^{m-1}} + \frac{b(2-n-m)}{a(n-1)} \int \frac{dx}{x^{n-1}X_1^m}.$$

$$27. \int \frac{dx}{X_1^m} = -\frac{1}{(m-1)bX_1^{m-1}}.$$

$$28. \int \frac{dx}{xX_1^m} = \frac{1}{X_1^{m-1}a(m-1)} + \frac{1}{a} \int \frac{dx}{xX_1^{m-1}}.$$

$$29. \int \frac{dx}{x^n X_1} = \sum_{k=1}^{n-1} \frac{(-1)^k b^{k-1}}{(n-k)a^k x^{n-k}} + \frac{(-1)^n b^{n-1}}{a^n} \ln \frac{X_1}{x}.$$

$$30. \int \frac{dx}{x X_1} = -\frac{1}{a} \ln \frac{X_1}{x}.$$

$$31. \int \frac{dx}{x^2 X_1} = -\frac{1}{ax} + \frac{b}{a^2} \ln \frac{X_1}{x}.$$

$$32. \int \frac{dx}{x^3 X_1} = -\frac{1}{2ax^2} + \frac{b}{a^2 x} - \frac{b^2}{a^3} \ln \frac{X_1}{x}.$$

$$33. \int \frac{dx}{x X_1^2} = \frac{1}{a X_1} - \frac{1}{a^2} \ln \frac{X_1}{x}.$$

$$34. \int \frac{dx}{x^2 X_1^2} = -\left[\frac{1}{ax} + \frac{2b}{a^2}\right] \frac{1}{X_1} + \frac{2b}{a^3} \ln \frac{X_1}{x}.$$

$$35. \int \frac{dx}{x^3 X_1^2} = \left[-\frac{1}{2ax^2} + \frac{3b}{2a^2 x} + \frac{3b^2}{a^3}\right] \frac{1}{X_1} - \frac{3b^2}{a^4} \ln \frac{X_1}{x}.$$

$$36. \int \frac{dx}{x X_1^3} = \left[\frac{3}{2a} + \frac{bx}{a^2}\right] \frac{1}{X_1^2} - \frac{1}{a^3} \ln \frac{X_1}{x}.$$

$$37. \int \frac{dx}{x^2 X_1^3} = -\left[\frac{1}{ax} + \frac{9b}{2a^2} + \frac{3b^2 x}{a^3}\right] \frac{1}{X_1^2} + \frac{3b}{a^4} \ln \frac{X_1}{x}.$$

$$38. \int \frac{dx}{x^3 X_1^3} = \left[-\frac{1}{2ax^2} + \frac{2b}{a^2 x} + \frac{9b^2}{a^3} + \frac{6b^3 x}{a^4}\right] \frac{1}{X_1^2} - \frac{6b^2}{a^5} \ln \frac{X_1}{x}.$$

$$39. \int \frac{dx}{x X_1^4} = \left[\frac{11}{6a} + \frac{5bx}{2a^2} + \frac{b^2 x^2}{a^3}\right] \frac{1}{X_1^3} - \frac{1}{a^4} \ln \frac{X_1}{x}.$$

$$40. \int \frac{dx}{x^2 X_1^4} = -\left[\frac{1}{ax} + \frac{22b}{3a^2} + \frac{10b^2 x}{a^3} + \frac{4b^3 x^2}{a^4}\right] \frac{1}{X_1^3} + \frac{4b}{a^5} \ln \frac{X_1}{x}.$$

$$41. \int \frac{dx}{x^3 X_1^4} = \left[-\frac{1}{2ax^2} + \frac{5b}{2a^2 x} + \frac{55b^2}{3a^3} + \frac{25b^3 x}{a^4} + \frac{10b^4 x^2}{a^5}\right] \frac{1}{X_1^3} - \frac{10b^2}{a^6} \ln \frac{X_1}{x}.$$

$$42. \int \frac{dx}{x X_1^5} = \left[\frac{25}{12a} + \frac{13bx}{3a^2} + \frac{7b^2 x^2}{2a^3} + \frac{b^3 x^3}{a^4}\right] \frac{1}{X_1^4} - \frac{1}{a^5} \ln \frac{X_1}{x}.$$

$$43. \int \frac{dx}{x^2 X_1^5} = \left[-\frac{1}{ax} - \frac{125b}{12a^2} - \frac{65b^2 x}{3a^3} - \frac{35b^3 x^2}{2a^4} - \frac{5b^4 x^3}{a^5}\right] \frac{1}{X_1^4} + \frac{5b}{a^6} \ln \frac{X_1}{x}.$$

$$44. \int \frac{dx}{x^3 X_1^5} = \left[-\frac{1}{2ax^2} + \frac{3b}{a^2 x} + \frac{125b^2}{4a^3} + \frac{65b^3 x}{a^4} + \frac{105b^4 x^2}{2a^5} + \frac{15b^5 x^3}{a^6}\right] \frac{1}{X_1^4} - \frac{15b^2}{a^7} \ln \frac{X_1}{x}.$$