! For an efficient use of these tables, first read `HowTo.pdf`.

T3.31A. Integrands involving hyperbolic functions and exponentials on the interval $(0, \infty)$.

1. $$\int_0^\infty e^{-\mu x} \sinh^\nu \beta x \, dx = \frac{1}{2^{\nu+1}\beta} \mathrm{B}\left(\frac{\mu}{2\beta} - \frac{\nu}{2}, \nu + 1\right), \quad \Re\{\beta\} > 0, \Re\{\nu\} > -1, \ \Re\{\mu\} > \Re\{\beta\nu\}.$$

2. $$\int_0^\infty e^{-\mu x} \frac{\sinh \beta x}{\sinh bx} \, dx = \frac{1}{2b}\left[\psi\left(\frac{1}{2} + \frac{\mu+\beta}{2b}\right) - \psi\left(\frac{1}{2} + \frac{\mu-\beta}{2b}\right)\right], \quad \Re\{\mu + b \pm \beta\} > 0.$$

3. $$\int_0^\infty e^{-x} \frac{\sinh ax}{\sinh x} \, dx = \frac{1}{a} - \frac{\pi}{2} \cot \frac{a\pi}{2}, \quad 0 < a < 2.$$

4. $$\int_0^\infty \frac{e^{-px} \, dx}{(\cosh px)^{2q+1}} = \frac{2^{2q-2}}{p} \mathrm{B}(q, q) - \frac{1}{2qp}, \quad p > 0, \ q > 0.$$

5. $$\int_0^\infty e^{-\mu x} \frac{dx}{\cosh x} = \beta\left(\frac{\mu+1}{2}\right), \quad \Re\{\mu\}\} > -1.$$

6. $$\int_0^\infty e^{-\mu x} \tanh x \, dx = \beta\left(\frac{\mu}{2}\right) - \frac{1}{\mu}, \quad \Re\{\mu\} > 0.$$

7. $$\int_0^\infty \frac{e^{-\mu x}}{\cosh^2 x} \, dx = \mu \, \beta\left(\frac{\mu}{2}\right) - 1, \quad \Re\{\mu\} > 0.$$

8. $$\int_0^\infty e^{-\mu x} \frac{\sinh \mu x}{\cosh^2 \mu x} \, dx = \frac{1}{\mu}(1 - \ln 2), \quad \Re\{\mu\} > 0.$$

9. $$\int_0^\infty e^{-qx} \frac{\sinh px}{\sinh qx} \, dx = \frac{1}{p} - \frac{\pi}{2q} \cot \frac{p\pi}{2q}, \quad 0 < p < 2q.$$

10. $$\int_0^\infty e^{-\mu x}(\cosh\beta x-1)^\nu\,dx=\frac{1}{2^\nu\beta}\,\mathrm{B}\left(\frac{\mu}{\beta}-\nu,\,2\nu+1\right),$$
$$\Re\{\beta\}>0,\ \Re\{\nu\}>-\tfrac{1}{2},\ \Re\{\mu\}>\Re\{\beta\nu\}.$$

11. $$\int_0^\infty e^{-\mu x}(\cosh x-\cosh u)^{\nu-1}\,dx=-i\sqrt{\frac{2}{\pi}}e^{i\,\pi\nu}\Gamma(\nu)\,\sinh^{\nu-\frac{1}{2}}u\,Q^{(1/2-\nu)}_{\mu-1/2}(\cosh u),$$
$$\Re\{\nu\}>0,\ \Re\{\mu\}>\Re\{\nu\}-1.$$

12. $$\int_0^\infty \frac{e^{-\mu x}}{\cosh x-\cos t}\,dx=2\,\csc t\sum_{k=1}^{\infty}\frac{\sin kt}{\mu+k},\quad \Re\{\mu\}>-1,\ t\neq 2n\pi.$$

13. $$\int_0^\infty \frac{1-e^{-x}\,\cos t}{\cosh x-\cos t}e^{-(\mu-1)x}\,dx=2\sum_{k=0}^{\infty}\frac{\cos kt}{\mu+k},\quad \Re\{\mu\}>0,\ t\neq 2n\pi.$$

14. $$\int_0^\infty \frac{e^{px}-\cos t}{(\cosh px+\cos t)^2}\,dx=\frac{1}{p}\left(t\,\csc t+\frac{1}{1+\cos t}\right),\quad p>0.$$

15. $$\int_0^\infty \frac{\sinh ax}{e^{px}+1}\,dx=\frac{\pi}{2p}\,\csc\frac{a\pi}{p}-\frac{1}{2a},\quad p>a,\ p>0.$$

16. $$\int_0^\infty \frac{\sinh ax}{e^{px}-1}\,dx=\frac{1}{2a}-\frac{\pi}{2p}\,\cot\frac{a\pi}{p},\quad p>a,\ p>0.$$

17. $$\int_0^\infty e^{-\beta\,x^2}\,\sinh ax\,dx=\frac{1}{2}\,\frac{\sqrt{\pi}}{\sqrt{\beta}}\,\exp\frac{a^2}{4\beta}\,\mathrm{erf}\left(\frac{a}{2\sqrt{\beta}}\right),\quad \Re\{\beta\}>0.$$

18. $$\int_0^\infty e^{-\beta\,x^2}\,\cosh ax\,dx=\frac{1}{2}\sqrt{\frac{\pi}{\beta}}\,\exp\frac{a^2}{4\beta},\quad \Re\{\beta\}>0.$$

19. $$\int_0^\infty e^{-\beta\,x^2}\,\sinh^2\,ax\,dx=\frac{1}{4}\sqrt{\frac{\pi}{\beta}}\left(\exp\frac{a^2}{\beta}-1\right),\quad \Re\{\beta\}>0.$$

20. $$\int_0^\infty e^{-\beta\,x^2}\,\cosh^2\,ax\,dx=\frac{1}{4}\sqrt{\frac{\pi}{\beta}}\left(\exp\frac{a^2}{\beta}+1\right),\quad \Re\{\beta\}>0.$$

21. $$\int_0^\infty \exp(-\beta\,\sinh x)\,\sinh\gamma x\,dx=\frac{\pi}{2}\,\cot\frac{\gamma\pi}{2}[J_\gamma(\beta)-\mathbf{J}_\gamma(\beta)]-\frac{\pi}{2}[\mathbf{E}_\gamma(\beta)+Y_\gamma(\beta)]=\gamma\,S_{-1,\,\gamma}(\beta),$$
$$\Re\{\beta\}>0.$$

22. $$\int_0^\infty \exp(-\beta\cosh x)\,\sinh\gamma x\,\sinh x\,dx = \frac{\gamma}{\beta}K_\gamma(\beta).$$

23. $$\int_0^\infty \exp(-\beta\sinh x)\,\cosh\gamma x\,dx = \frac{\pi}{2}\tan\frac{\pi\gamma}{2}[\mathbf{J}_\gamma(\beta) - J_\gamma(\beta)] - \frac{\pi}{2}[\mathbf{E}_\gamma(\beta) + Y_\gamma(\beta)] = S_{0,\gamma}(\beta),$$
$\Re\{\beta\} > 0,\ \gamma$ not an integer.

24. $$\int_0^\infty \exp(-\beta\cosh x)\,\cosh\gamma x\,dx = K_\gamma(\beta), \quad \Re\{\beta\} > 0.$$

25. $$\int_0^\infty \exp(-\beta\sinh x)\,\sinh\gamma x\,\cosh x\,dx = \frac{\gamma}{\beta}S_{0,\gamma}(\beta), \quad \Re\{\beta\} > 0.$$

26. $$\int_0^\infty \exp(-\beta\sinh x)\,\sinh[(2n+1)x]\,\cosh x\,dx = O_{2n+1}(\beta), \quad \Re\{\beta\} > 0.$$

27. $$\int_0^\infty \exp(-\beta\sinh x)\,\cosh\gamma x\,\cosh x\,dx = \frac{1}{\beta}S_{1,\gamma}(\beta), \quad \Re\{\beta\} > 0.$$

28. $$\int_0^\infty \exp(-\beta\sinh x)\,\cosh 2nx\,\cosh x\,dx = O_{2n}(\beta), \quad \Re\{\beta\} > 0.$$

29. $$\int_0^\infty \exp(-\beta\cosh x)\,\sinh^{2\nu} x\,dx = \frac{1}{\sqrt{\pi}}\left(\frac{2}{\beta}\right)^\nu \Gamma\left(\nu+\frac{1}{2}\right)K_\nu(\beta),$$
$\Re\{\beta\} > 0,\ \Re\{\nu\} > -\frac{1}{2}.$

30. $$\int_0^\infty \exp[-2(\beta\coth x + \mu x)]\,\sinh^{2\nu} x\,dx = \frac{1}{4}\beta^{(\nu-1)/2}\Gamma(\mu-\nu)$$
$$\times\left[W_{-\mu+1/2,\,\nu}(4\beta) - (\mu-\nu)W_{-\mu-1/2,\,\nu}(4\beta)\right], \quad \Re\{\beta\} > 0,\ \Re\{\mu\} > \Re\{\nu\}.$$

31. $$\int_0^\infty \exp\left(-\frac{\beta^2}{2}\sinh x\right)\sinh^{\nu-1} x\,\cosh^\nu x\,dx = -\pi D_\nu\left(\beta e^{\frac{i\pi}{4}}\right) D_\nu\left(\beta e^{-\frac{i\pi}{4}}\right),$$
$\Re\{\nu\} > 0,\ |\arg\beta| \le \frac{\pi}{4}.$

32. $$\int_0^\infty \frac{\exp(2\nu x - 2\beta\sinh x)}{\sqrt{\sinh x}}\,dx = \frac{1}{2}\sqrt{\pi^3\beta}\left[J_{\nu+1/4}(\beta)J_{\nu-1/4}(\beta) + Y_{\nu+1/4}(\beta)Y_{\nu-1/4}(\beta)\right],$$
$\Re\{\beta\} > 0.$

33. $$\int_0^\infty \frac{\exp(-2\nu x - 2\beta\,\sinh x)}{\sqrt{\sinh x}}\,dx = \frac{1}{2}\sqrt{\pi^3\beta}\,\left[J_{\nu+1/4}(\beta)Y_{\nu-1/4}(\beta) - J_{\nu-1/4}(\beta)Y_{\nu+1/4}(\beta)\right], \quad \Re\{\beta\} > 0$$

34. $$\int_0^\infty \frac{\exp(-2\beta\,\sinh x)\,\sinh 2\nu x}{\sqrt{\sinh x}}\,dx = \frac{1}{4i}\sqrt{\frac{\pi^3\beta}{2}}\,\big[e^{i\,\nu\pi}\,H^{(1)}_{1/2+\nu}(\beta)\,H^{(2)}_{1/2-\nu}(\beta) - e^{-i\,\nu\pi}\,H^{(1)}_{1/2-\nu}(\beta)\,H^{(2)}_{1/2+\nu}(\beta)\big], \quad \Re\{\beta\} > 0.$$

35. $$\int_0^\infty \frac{\exp(-2\beta\,\sinh x)\,\cosh 2\nu x}{\sqrt{\sinh x}}\,dx = \frac{1}{4}\sqrt{\frac{\pi^3\beta}{2}}\,\big[e^{i\,\nu\pi}\,H^{(1)}_{1/2+\nu}(\beta)\,H^{(2)}_{1/-\nu}(\beta) + e^{-i\,\nu\pi}\,H^{(1)}_{1/2-\nu}(\beta)H^{(2)}_{1/2+\nu}(\beta)\big], \quad \Re\{\beta\} > 0.$$

36. $$\int_0^\infty \frac{\exp(-2\beta\,\cosh x)\,\cosh 2\nu x}{\sqrt{\cosh x}}\,dx = \sqrt{\frac{\beta}{\pi}}\,K_{\nu+1/r4}(\beta)K_{\nu-1/4}(\beta), \quad \Re\{\beta\} > 0.$$

37. $$\int_0^\infty \frac{\exp[-2\beta(\cosh x - 1)]\,\cosh 2\nu x}{\sqrt{\cosh x}}\,dx = \sqrt{\frac{\beta}{\pi}}\,e^{2\beta}K_{\nu+1/4}(\beta)K_{\nu-1/4}(\beta), \quad \Re\{\beta\} > 0.$$

38. $$\int_0^\infty \frac{\cos\left[\left(\nu+\frac{1}{4}\right)\pi\right]\,\exp(-2\nu x - 2\beta\,\sinh x) + \sin\left[\left(\nu+\frac{1}{4}\right)\pi\right]\,\exp(2\nu x - 2\beta\,\sinh x)}{\sqrt{\sinh x}}\,dx$$
$$= \frac{1}{2}\sqrt{\pi^3\beta}\,\left[J_{1/4+\nu}(\beta)J_{1/4-\nu}(\beta) + Y_{1/4+\nu}(\beta)Y_{1/4-\nu}(\beta)\right], \quad \Re\{\beta\} > 0.$$

39. $$\int_0^\infty \frac{\sin\left[\left(\nu+\frac{1}{4}\right)\pi\right]\,\exp(-2\nu x - 2\beta\,\sinh x) - \cos\left[\left(\nu+\frac{1}{4}\right)\pi\right]\,\exp(2\nu x - 2\beta\,\sinh x)}{\sqrt{\sinh x}}\,dx$$
$$= \frac{1}{2}\sqrt{\pi^3\beta}\,\left[J_{1/4+\nu}(\beta)Y_{1/4-\nu}(\beta) - J_{1/4-\nu}(\beta)Y_{1/4+\nu}(\beta)\right], \quad \Re\{\beta\} > 0.$$

40. $$\int_0^\infty \frac{\exp[-\beta(\cosh x - 1)]\,\cosh \nu x\,\sinh x}{\sqrt{\cosh x(\cosh x - 1)}}\,dx = e^{\beta}K_\nu(\beta), \quad \Re\{\beta\} > 0.$$

41. $$\int_0^\infty e^{-\mu x^4}\,\sinh ax^2\,dx = \frac{\pi}{4}\sqrt{\frac{a}{2\mu}}\,\exp\left(\frac{a^2}{8\mu}\right)I_{1/4}\left(\frac{a^2}{8\mu}\right), \quad \Re\{\mu\} > 0,\ a \geq 0.$$

42. $$\int_0^\infty e^{-\mu x^4}\,\cosh ax^2\,dx = \frac{\pi}{4}\sqrt{\frac{a}{2\mu}}\,\exp\left(\frac{a^2}{8\mu}\right)I_{-1/4}\left(\frac{a^2}{8\mu}\right), \quad \Re\{\mu\} > 0,\ a > 0.$$

43. $$\int_0^\infty e^{-\beta x} \sinh[(2n+1) \operatorname{arcsinh} x]\, dx = O_{2n+1}(\beta), \quad \Re\{\beta\} > 0.$$

44. $$\int_0^\infty e^{-\beta x} \cosh(2n \operatorname{arcsinh} x)\, dx = O_{2n}(\beta), \quad \Re\{\beta\} > 0.$$

45. $$\int_0^\infty e^{-\beta x} \sinh(\nu \operatorname{arcsinh} x)\, dx = \frac{\nu}{\beta} S_{0,\,\nu}(\beta), \quad [\Re\{\beta\} > 0].$$

46. $$\int_0^\infty e^{-\beta x} \cosh(\nu \operatorname{arcsinh} x)\, dx = \frac{1}{\beta} S_{1,\,\nu}(\beta), \quad \Re\{\beta\} > 0.$$