! For an efficient use of these tables, first read `HowTo.pdf`.

T3.01A. Powers of x and binomials of the form $(a+b\,x)$ on the interval $(0,\infty)$.

1. $$\int_0^\infty \frac{x^{\mu-1}\,dx}{(1+\beta x)^\nu} = \beta^{-\mu}\mathrm{B}(\mu,\,\nu-\mu), \quad |\arg\beta|<\pi,\ \Re\{\nu\}>\Re\{\mu\}>0.$$

2. $$\int_0^\infty \frac{x^{\mu-1}\,dx}{(1+\beta x)^{n+1}} = (-1)^n\frac{\pi}{\beta^\mu}\binom{\mu-1}{n}\csc(\mu\pi), \quad |\arg\beta|<\pi,\ 0<\Re\{\mu\}<n+1.$$

3. $$\int_0^\infty \frac{x^{\mu-1}\,dx}{(1+\beta x)^2} = \frac{(1-\mu)\pi}{\beta^\mu}\csc\mu\pi, \quad 0<\Re\{\mu\}<2.$$

4. $$\int_0^\infty \frac{x^m\,dx}{(a+bx)^{n+\frac12}} = 2^{m+1}m!\frac{(2n-2m-3)!!}{(2n-1)!!}\frac{a^{m-n+\frac12}}{b^{m+1}}, \quad m<n-\tfrac12,\ a>0,\ b>0.$$

5. $$\int_0^\infty \frac{(1+x)^{p-1}}{(x+a)^{p+1}}\,dx = \frac{1-a^{-p}}{p(a-1)}, \quad p\neq 0,\ a>0,\ a\neq 1.$$

6. $$\int_0^\infty x^{\nu-1}(\beta+x)^{-\mu}(x+\gamma)^{-\rho}\,dx = \beta^{-\mu}\gamma^{\nu-\rho}\mathrm{B}(\nu,\,\mu-\nu+\rho)\,{}_2F_1\left(\mu,\,\nu;\,\mu+\rho;\,1-\frac{\gamma}{\beta}\right),$$
$$|\arg\beta|<\pi,\ |\arg\gamma|<\pi,\ \Re\{\nu\}>0,\ \Re\{\mu\}>\Re\{\nu-\rho\}.$$

7. $$\int_0^\infty x^{\lambda-1}(1+x)^\nu(1+\alpha x)^\mu\,dx = \mathrm{B}(\lambda,\,-\mu-\nu-\lambda)\,{}_2F_1(-\mu,\,\lambda;\,-\mu-\nu;\,1-\alpha),$$
$$|\arg\alpha|<\pi,\ -\Re\{\mu+\nu\}>\Re\{\lambda\}>0.$$

8. $$\int_0^\infty x^{\mu-1/2}(x+a)^{-\mu}(x+b)^{-\mu}\,dx = \sqrt{\pi}(\sqrt{a}+\sqrt{b})^{1-2\mu}\frac{\Gamma\left(\mu-\frac12\right)}{\Gamma(\mu)}, \quad \Re\{\mu\}>0.$$

9. $$\int_0^\infty x^{\lambda-1}(1+x)^{-\mu+\nu}(x+\beta)^{-\nu}\,dx = \mathrm{B}(\mu-\lambda,\,\lambda)\,{}_2F_1(\nu,\,\mu-\lambda;\,\mu;\,1-\beta), \quad \Re\{\mu\}>\Re\{\lambda\}>0.$$

10. $$\int_0^\infty [(1+ax)^{-p}+(1+bx)^{-p}]x^{q-1}\,dx = 2(ab)^{-q/2}\mathrm{B}(q,\,p-q)\cos\left\{q\arccos\left[\frac{a+b}{2\sqrt{ab}}\right]\right\},\quad p>q>0.$$

11. $$\int_0^\infty [(1+ax)^{-p}-(1+bx)^{-p}]x^{q-1}\,dx = -2i(ab)^{-q/2}\mathrm{B}(q,\,p-q)\sin\left\{q\,\arccos\left[\frac{a+b}{2\sqrt{ab}}\right]\right\},\quad p>q>0.$$

12. $$\int_0^\infty \left\{\frac{b^p x^{p-1}}{(1+bx)^p}-\frac{(1+bx)^{p-1}}{b^{p-1}x^p}\right\}dx = \pi\,\cot p\pi,\quad 0<p<1,\ b>0.$$

13. $$\int_0^\infty \frac{x^{2p-1}-(a+x)^{2p-1}}{(a+x)^p x^p}\,dx = \pi\,\cot p\pi,\quad p<1.$$

14. $$\int_0^\infty \left\{\frac{x^\nu}{(x+1)^{\nu+1}}-\frac{x^\mu}{(x+1)^{\mu+1}}\right\}dx = \psi(\mu+1)-\psi(\nu+1),\quad \Re\{\mu\}>-1,\ \Re\{\nu\}>-1.$$

15. $$\int_0^\infty \frac{x^{\mu-1}\,dx}{x+a} = \begin{cases}\pi\csc(\mu\pi)a^{\mu-1} & \text{for } a>0,\\ -\pi\,\cot(\mu\pi)(-a)^{\mu-1} & \text{for } a<0,\end{cases}\qquad 0<\Re\{\mu\}<1.$$

16. $$\int_0^\infty \frac{x^{\mu-1}\,dx}{(\beta+x)(\gamma+x)} = \frac{\pi}{\gamma-\beta}(\beta^{\mu-1}-\gamma^{\mu-1})\csc(\mu\pi),\quad |\arg\beta|<\pi,\ |\arg\gamma|<\pi,\ 0<\Re\{\mu\}<2.$$

17. $$\int_0^\infty \frac{x^{\mu-1}\,dx}{(\beta+x)(\alpha-x)} = \frac{\pi}{\alpha+\beta}[\beta^{\mu-1}\csc(\mu\pi)+\alpha^{\mu-1}\,\cot(\mu\pi)],\quad |\arg\beta|<\pi,\ \alpha>0,\ 0<\Re\{\mu\}<2.$$

18. $$\int_0^\infty \frac{x^{\mu-1}\,dx}{(a-x)(b-x)} = \pi\cot(\mu\pi)\frac{a^{\mu-1}-b^{\mu-1}}{b-a},\quad a>b>0,\ 0<\Re\{\mu\}<2.$$

19. $$\int_0^\infty \frac{x^p\,dx}{(1+x)^3} = \frac{\pi}{2}p(1-p)\csc p\pi,\quad -1<p<2.$$

20. $$\int_0^\infty \frac{(x+\beta)x^{\mu-1}\,dx}{(x+\gamma)(x+\delta)} = \pi\csc(\mu\pi)\left\{\frac{\gamma-\beta}{\gamma-\delta}\gamma^{\mu-1}+\frac{\delta-\beta}{\delta-\gamma}\delta^{\mu-1}\right\},\quad |\arg\gamma|<\pi,\ |\arg\delta|<\pi,\ 0<\Re\{\mu\}<1.$$

21. $$\int_0^\infty \frac{x^{\nu-1}(\beta+x)^{1-\mu}}{\gamma+x}\,dx = \beta^{1-\mu}\gamma^{\nu-1}\mathrm{B}(\nu,\,\mu-\nu)\,{}_2F_1\left(\mu-1,\,\nu;\,\mu;\,1-\frac{\gamma}{\beta}\right),$$
$$|\arg\beta|<\pi,\ |\arg\gamma|<\pi,\ 0<\Re\{\nu\}<\Re\{\mu\}.$$

22. $$\int_0^\infty \frac{x^{-\rho}(\beta-x)^{-\sigma}}{\gamma+x}\,dx = \pi\gamma^{-\rho}(\beta-\gamma)^{-\sigma}\csc(\rho\pi)I_{1-\gamma/\beta}(\sigma,\,\rho),$$
$$|\arg\beta|<\pi,\ |\arg\gamma|<\pi,\ -\Re\{\sigma\}<\Re\{\rho\}<1.$$

23. $$\int_0^\infty \frac{x^{\nu-1}(x+a)^{1-\mu}}{x-c}\,dx = \begin{cases} a^{1-\mu}(-c)^{\nu-1}\mathrm{B}(\mu-\nu,\,\nu)\,{}_2F_1\left(\mu-1,\,\nu;\,\mu;\,1+\dfrac{c}{a}\right), & c<0,\\ \pi c^{\nu-1}(a+c)^{1-\mu}\cot[(\mu-\nu)\pi]-\dfrac{a^{1-\mu-\nu}}{a+c}\mathrm{B}(\mu-\nu-1,\,\nu) & \\ \qquad\times\,{}_2F_1\left(2-\mu,\,1;\,2-\mu+\nu;\,\dfrac{a}{a+c}\right), & c>0, \end{cases}$$
$$a>0,\ 0<\Re\{\nu\}<\Re\{\mu\}.$$

24. $$\int_0^\infty \frac{x^{p-1}-x^{q-1}}{1-x}\,dx = \pi(\cot p\pi-\cot q\pi),\quad p>0,\ q>0.$$

25. $$\int_0^\infty \frac{(c+ax)^{-\mu}-(c+bx)^{-\mu}}{x}\,dx = c^{-\mu}\ln\frac{b}{a},\quad \Re\{\mu\}>-1;\ a>0;\ b>0;\ c>0.$$

26. $$\int_0^\infty \left\{\frac{1}{1+x}-(1+x)^{-\nu}\right\}\frac{dx}{x} = \psi(\nu)+\gamma_e,\quad \Re\{\nu\}>0.$$

27. $$\int_0^\infty \frac{(1+x)^{\mu}-1}{(1+x)^{\nu}}\frac{dx}{x} = \psi(\nu)-\psi(\nu-\mu),\quad \Re\{\nu\}>\Re\{\mu\}>0.$$

28. $$\sum_{n=0}^{\infty}(-1)^{n+1}\int_n^{n+1}\frac{dx}{x+u} = \ln\frac{u\left[\Gamma\left(\frac{u}{2}\right)\right]^2}{2\left[\Gamma\left(\frac{u+1}{2}\right)\right]^2},\quad |\arg u|<\pi.$$