! For an efficient use of these tables, first read `HowTo.pdf`.

T2.44B. Integrands involving powers of trigonometric functions and rational trigonometric functions on the interval $(0, \pi/2)$.

1. $$\int_0^{\pi/2} \frac{\sin^{p-1} x \cos^{-p} x}{a \cos x + b \sin x}\,dx = \int_0^{\pi/2} \frac{\sin^{-p} x \cos^{p-1} x}{a \sin x + b \cos x}\,dx = \frac{\pi \csc p\pi}{a^{1-p} b^{p}}, \quad ab > 0,\ 0 < p < 1.$$

2. $$\int_0^{\pi/2} \frac{\sin^{1-p} x \cos^{p} x}{(\sin x + \cos x)^3}\,dx = \int_0^{\pi/2} \frac{\sin^{p} x \cos^{1-p} x}{(\sin x + \cos x)^3}\,dx = \frac{(1-p)p}{2}\,\pi \csc p\pi, \quad -1 < p < 2.$$

3. $$\int_0^{\pi/2} \frac{\sin^{2\mu-1} x \cos^{2\nu-1} x\,dx}{(a^2 \sin^2 x + b^2 \cos^2 x)^{\mu+\nu}} = \frac{1}{2a^{2\mu} b^{2\nu}}\,\mathrm{B}\,(\mu, \nu), \quad \Re\{\mu\} > 0, \Re\{\nu\} > 0.$$

4. $$\int_0^{\pi/2} \frac{\sin^{n-1} x \cos^{n-1} x\,dx}{(a^2 \cos^2 x + b^2 \sin^2 x)^{n}} = \frac{\mathrm{B}\,(n/2, n/2)}{2(ab)^n}, \quad ab > 0.$$

5. $$\int_0^{\pi/2} \frac{\sin^{2n} x\,dx}{(a^2 \cos^2 x + b^2 \sin^2 x)^{n+1}} = \frac{1}{2}\int_0^{\pi} \frac{\sin^{2n} x\,dx}{(a^2 \cos^2 x + b^2 \sin^2 x)^{n+1}}$$
$$= \int_0^{\pi/2} \frac{\cos^{2n} x\,dx}{(a^2 \sin^2 x + b^2 \cos^2 x)^{n+1}} = \frac{1}{2}\int_0^{\pi} \frac{\cos^{2n} x\,dx}{(a^2 \sin^2 x + b^2 \cos^2 x)^{n+1}}$$
$$= \frac{(2n-1)!!\pi}{2^{n+1} n! a b^{2n+1}}, \quad ab > 0.$$

6. $$\int_0^{\pi/2} \frac{\cos^{p+2n} x \cos px\,dx}{(a^2 \cos^2 x + b^2 \sin^2 x)^{n+1}} = \pi \sum_{k=0}^{n} \binom{2n-k}{n} \binom{p+k-1}{k} \frac{b^{p-1}}{(2a)^{2n-k+1}(a+b)^{p+k}},$$
$$a > 0,\ b > 0,\ p > -2n-1.$$

7. $$\int_0^{\pi/2} \frac{\cos^{p} x \cos px\,dx}{1 - 2a \cos 2x + a^2} = \frac{\pi}{2^{p+1}}\,\frac{(1+a)^{p-1}}{1-a}, \quad a^2 < 1,\ p > -1.$$

8. $$\int_0^{\pi/2} \frac{\sin^{2n}\ x\ \cos^{\mu}\ x\ \cos\beta x}{(1-2a\ \cos 2x+a^2)^m}\,dx$$
$$= \frac{(-1)^n\pi(1-a)^{2n-2m+1}}{2^{2m-\beta-1}(1+a)^{2m+\beta+1}} \sum_{k=0}^{m-1}\sum_{l=0}^{m-k-1} \binom{\beta}{k}\binom{2n}{l}\binom{2m-k-l-2}{m-l(-2)^l}(a-1)^k,$$
$$a^2<1,\ \beta=2m-2n-\mu-2,\ \mu>-1.$$

9. $$\int_0^{\pi/2} \frac{\cos^n\ x\ \sin nx\ \sin 2x}{1-2a\ \cos 2x+a^2}\,dx = \frac{\pi}{4a}\left[\left(\frac{1+a}{2}\right)^n - \frac{1}{2^n}\right], \quad a^2<1.$$

10. $$\int_0^{\pi/2} \frac{1-a\ \cos 2nx}{1-2a\ \cos 2nx+a^2}\ \cos^m\ x\ \cos mx\,dx = \frac{\pi}{2^{m+2}}\sum_{k=1}^{\infty}\binom{m}{kn}a^k + \frac{\pi}{2^{m+1}}, \quad a^2<1.$$

11. $$\int_0^{\pi/2} \frac{\cos^p\ x\ \cos px\,dx}{a^2\ \sin^2\ x+b^2\ \cos^2\ x} = \frac{\pi}{2b}\,\frac{a^{p-1}}{(a+b)^p}, \quad p>-1,\ a>0,\ b>0.$$

12. $$\int_0^{\pi/2} \frac{\tan^{\pm\mu}\ x\,dx}{1+\cos t\ \sin 2x} = \pi\ \csc t\ \sin\mu t\ \csc(\mu\pi), \quad |\Re\{\mu\}|<1,\ t^2<\pi^2.$$

13. $$\int_0^{\pi/2} \frac{\tan^{\pm\mu}\ x\ \sin 2x\,dx}{1\mp 2a\ \cos 2x+a^2} = \begin{cases} \dfrac{\pi}{4a}\ \csc\dfrac{\mu\pi}{2}\left[1-\left(\dfrac{1-a}{1+a}\right)^{\mu}\right], & a^2<1, \\ \dfrac{\pi}{4a}\ \csc\dfrac{\mu\pi}{2}\left[1+\left(\dfrac{a-1}{a+1}\right)^{\mu}\right], & a^2>1, \end{cases} \quad -2<\Re\{\mu\}<1.$$

14. $$\int_0^{\pi/2} \frac{\tan^{\pm\mu}\ x(1\mp a\ \cos 2x)}{1\mp 2a\ \cos 2x+a^2}\,dx = \begin{cases} \dfrac{\pi}{4}\ \sec\ \dfrac{\mu\pi}{2}\left[1+\left(\dfrac{1-a}{1+a}\right)^{\mu}\right], & a^2<1, \\ \dfrac{\pi}{4}\ \sec\ \dfrac{\mu\pi}{2}\left[1-\left(\dfrac{a-1}{a+1}\right)^{\mu}\right], & a^2>1, \end{cases} \quad |\Re\{\mu\}|<1.$$

15. $$\int_0^{\pi/2} \frac{\tan^{\mu}\ x\,dx}{(\sin x+\cos x)\ \sin x} = \int_0^{\pi/2} \frac{\cot^{\mu}\ x\,dx}{(\sin x+\cos x)\ \cos x} = \pi\ \csc\mu\pi, \quad 0<\Re\{\mu\}<1.$$

16. $$\int_0^{\pi/2} \frac{\tan^{\mu}\ x\,dx}{(\sin x-\cos x)\ \sin x} = \int_0^{\pi/2} \frac{\cot^{\mu}\ x\,dx}{(\cos x-\sin x)\ \cos x} = -\pi\ \cot\mu\pi, \quad 0<\Re\{\mu\}<1.$$

17. $$\int_0^{\pi/2} \frac{\cot^{\mu+1/2}\ x\,dx}{(\sin x+\cos x)\ \cos x} = \int_0^{\pi/2} \frac{\tan^{\mu-1/2}\ x\,dx}{(\sin x+\cos x)\ \cos x} = \pi\ \sec\ \mu\pi, \quad |\Re\{\mu\}|<\tfrac{1}{2}.$$

18. $$\int_0^{\pi/2} \frac{\tan^{1-2\mu}\ x\,dx}{a^2\ \cos^2\ x + b^2\ \sin^2\ x} = \int_0^{\pi/2} \frac{\cot^{1-2\mu}\ x\,dx}{a^2\ \sin^2\ x + b^2\ \cos^2\ x} = \frac{\pi}{2a^{2\mu} b^{2-2\mu}\ \sin\mu\pi}, \quad 0 < \Re\{\mu\} < 1.$$

19. $$\int_0^{\pi/2} \frac{\tan^{\mu}\ x\,dx}{1 - a\ \sin^2\ x} = \int_0^{\pi/2} \frac{\cot^{\mu}\ x\,dx}{1 - a\ \cos^2\ x} = \frac{\pi\ \sec\ \frac{\mu\pi}{2}}{2\sqrt{(1-a)^{\mu+1}}}, \quad |\Re\{\mu\}| < 1,\ a < 1.$$

20. $$\int_0^{\pi/2} \frac{\tan^{\pm\mu}\ x\,dx}{1 - \cos^2\ t\ \sin^2\ 2x} = \frac{\pi}{2}\ \csc t\ \sec\ \frac{\mu\pi}{2}\ \cos\left[\left(\frac{\pi}{2} - t\right)\mu\right], \quad |\Re\{\mu\}| < 1,\ t^2 < \pi^2.$$

21. $$\int_0^{\pi/2} \frac{\tan^{\pm\mu}\ x\ \sin 2x}{1 - \cos^2\ t\ \sin^2\ 2x}\,dx = \pi\ \csc 2t\ \csc\frac{\mu\pi}{2}\ \sin\left[\left(\frac{\pi}{2} - t\right)\mu\right], \quad |\Re\{\mu\}| < 1,\ t^2 < \pi^2.$$

22. $$\int_0^{\pi/2} \frac{\tan^{\mu}\ x\ \sin^2\ x\,dx}{1 - \cos^2\ t\ \sin^2\ 2x} = \int_0^{\pi/2} \frac{\cot^{\mu}\ x\ \cos^2\ x\,dx}{1 - \cos^2\ t\ \sin^2\ 2x} = \frac{\pi}{2}\ \csc\ 2t\ \sec\ \frac{\mu\pi}{2}\ \cos\left[\frac{\mu\pi}{2} - (\mu+1)t\right], \quad |\Re\{\mu\}| < 1,\ t^2 < \pi^2.$$

23. $$\int_0^{\pi/2} \frac{\tan^{\mu}\ x\ \cos^2\ x\,dx}{1 - \cos^2\ t\ \sin^2\ 2x} = \int_0^{\pi/2} \frac{\cot^{\mu}\ x\ \sin^2\ x\,dx}{1 - \cos^2\ t\ \sin^2\ 2x} = \frac{\pi}{2}\ \csc 2t\ \sec\ \frac{\mu\pi}{2}\ \cos\left[\frac{\mu\pi}{2} - (\mu-1)t\right], \quad |\Re\{\mu\}| < 1,\ t^2 < \pi^2.$$

24. $$\int_0^{\pi/2} \tan^{\mu+1}\ x\ \cos^2\ x\,dx\ over (1 + \cos t\ \sin 2x)^2$$
$$= \int_0^{\pi/2} \frac{\cot^{\mu+1}\ x\ \sin^2\ x\,dx}{(1 + \cos t\ \sin 2x)^2} = \frac{\pi(\mu\ \sin t\ \cos\mu t - \cos t\ \sin\ \mu t)}{2\ \sin\mu\pi\ \sin^3\ t}, \quad |\Re\{\mu\}| < 1,\ t^2 < \pi^2.$$

25. $$\int_0^{\pi/2} \frac{\tan^{\pm\mu}\ x\,dx}{(\sin x + \cos x)^2} = \frac{\mu\pi}{\sin\mu\pi}, \quad 0 < \Re\{\mu\} < 1.$$

26. $$\int_0^{\pi/2} \frac{\tan^{\pm(\mu-1)}\ x\,dx}{\cos^2\ x - \sin^2\ x} = \pm\frac{\pi}{2}\ \cot\ \frac{\mu\pi}{2}, \quad 0 < \Re\{\mu\} < 2.$$

27. $$\int_0^{\pi/2} \frac{\tan^{2\mu-1}\ x\,dx}{1 - 2a(\cos\ t_1\ \sin^2\ x + \cos t_2\ \cos^2\ x) + a^2} = \int_0^{\pi/2} \frac{\cot^{2\mu-1}\ x\,dx}{1 - 2a(\cos t_1\ \cos^2\ x + \cos t_2\ \sin^2\ x) + a^2}$$
$$= \frac{\pi\ \csc\mu\pi}{(1 - 2a\ \cos t_2 + a^2)^{\mu}(1 - 2a\ \cos t_1 + a^2)1 - \mu}, \quad 0 < \Re\{\mu\} < 1,\ t_1^2 < \pi^2,\ t_2^2 < \pi^2.$$

28. $$\int_0^{\pi/2} \frac{\tan^{\mu-1} x \cos^2 x \, dx}{1-\sin^2 x \cos^2 x} = \int_0^{\pi/2} \frac{\cot^{\mu-1} x \sin^2 x \, dx}{1-\sin^2 x \cos^2 x} = \frac{\pi}{4\sqrt{3}} \csc \frac{\mu\pi}{6} \csc\left(\frac{2+\mu}{6}\pi\right),$$

$$0 < \Re\{\mu\} < 4.$$