! For an efficient use of these tables, first read `HowTo.pdf`.

T3.27B. Integrands involving exponentials and rational functions of powers of $(a+b\,x)$ on the interval $(-\infty,\infty)$.

1. $$\int_{-\infty}^{\infty} \frac{x^2 e^{-\mu x}}{1+e^{-x}}\,dx = \pi^3 \csc^3\, \mu\pi(2-\sin^2\, \mu\pi), \quad 0<\Re\{\mu\}<1.$$

2. $$\int_{-\infty}^{\infty} \frac{x e^{\mu x}\,dx}{\beta+e^x} = \pi\beta^{\mu-1}\csc(\mu\pi)[\ln\,\beta-\pi\,\cot(\mu\pi)], \quad |\arg\beta|<\pi,\ 0<\Re\{\mu\}<1.$$

3. $$\int_{-\infty}^{\infty} \frac{x e^{\mu x}}{e^{\nu x}-1}\,dx = \left(\frac{\pi}{\nu}\csc\frac{\mu\pi}{\nu}\right)^2, \quad \Re\{\nu\}>\Re\{\mu\}>0.$$

4. $$\int_{-\infty}^{\infty} \frac{e^{px}-e^{qx}}{1+e^{rx}}\,\frac{dx}{x} = \ln\left[\tan\frac{p\pi}{2r}\,\cot\frac{q\pi}{2r}\right], \quad |r|>|p|,\ |r|>|q|,\ rp>0,\ rq>0.$$

5. $$\int_{-\infty}^{\infty} \frac{e^{px}-e^{qx}}{1-e^{rx}}\,\frac{dx}{x} = \ln\left[\sin\frac{p\pi}{r}\,\csc\frac{q\pi}{r}\right], \quad |r|>|p|,\ |r|>|q|,\ rp>0,\ rq>0.$$

6. $$\int_{-\infty}^{\infty} \frac{x\,dx}{a^2e^x+b^2e^{-x}} = \frac{\pi}{2ab}\,\ln\,\frac{b}{a}, \quad ab>0.$$

7. $$\int_{-\infty}^{\infty} \frac{x\,dx}{a^2e^x-b^2e^{-x}} = \frac{\pi^2}{4ab}.$$

8. $$\int_{-\infty}^{\infty} \frac{x\,dx}{(\beta+e^x)(1+e^{-x})} = \frac{(\ln\,\beta)^2}{2(\beta-1)}, \quad |\arg\beta|<\pi.$$

9. $$\int_{-\infty}^{\infty} \frac{x\,dx}{(\beta+e^x)(1-e^{-x})} = \frac{\pi^2+(\ln\,\beta)^2}{2(\beta+1)}, \quad |\arg\beta|<\pi.$$

10. $$\int_{-\infty}^{\infty} \frac{x^2\,dx}{(\beta+e^x)(1-e^{-x})} = \frac{[\pi^2+(\ln\,\beta)^2]\,\ln\,\beta}{3(\beta+1)}, \quad |\arg\beta|<\pi.$$

11. $$\int_{-\infty}^{\infty} \frac{x^3\,dx}{(\beta+e^x)(1-e^{-x})} = \frac{[\pi^2+(\ln\,\beta)^2]^2}{4(\beta+1)}, \quad |\arg\beta|<\pi.$$

12. $$\int_{-\infty}^{\infty} \frac{x^4\,dx}{(\beta+e^x)(1-e^{-x})} = \frac{[\pi^2+(\ln\,\beta)^2]^2}{15(\beta+1)}[7\pi^2+3(\ln\,\beta)^2]\,\ln\,\beta.$$

13. $$\int_{-\infty}^{\infty} \frac{x^5\,dx}{(\beta+e^x)(1-e^{-x})} = \frac{[\pi^2+(\ln\,\beta)^2]^2}{6(\beta+1)}[3\pi^2+(\ln\,\beta)^2]^2.$$

14. $$\int_{-\infty}^{\infty} \frac{(x-\ln\,\beta)x\,dx}{(\beta-e^x)(1-e^{-x})} = -\frac{[4\pi^2+(\ln\,\beta)^2]\,\ln\,\beta}{6(\beta-1)}, \quad |\arg\beta|<\pi.$$

15. $$\int_{-\infty}^{\infty} \frac{xe^{-\mu x}\,dx}{(\beta+e^{-x})(\gamma+e^{-x})} = \frac{\pi(\beta^{\mu-1}\,\ln\,\beta-\gamma^{\mu-1}\,\ln\,\gamma)}{(\beta-\gamma)\,\sin\mu\pi} + \frac{\pi^2(\beta^{\mu-1}-\gamma^{\mu-1})\,\cos\mu\pi}{(\gamma-\beta)\,\sin^2\,\mu\pi},$$
$$|\arg\beta|<\pi,\ |\arg\gamma|<\pi,\ \beta\neq\gamma,\ 0<\Re\{\mu\}<2.$$

16. $$\int_{-\infty}^{\infty} \frac{x(x-a)e^{\mu x}\,dx}{(\beta-e^x)(1-e^{-x})} = \frac{-\pi^2}{e^a-1}\,\csc^2\,\mu\pi\,\left[(e^{\alpha\mu}+1)\,\ln\,\mu-2\pi\,\cot\mu\pi\,(e^{\alpha\mu}-1)\right]$$
$$a>0,\ |\arg\beta|<\pi,\ \Re\{\mu\}<1.$$

17. $$\int_{-\infty}^{\infty} \frac{xe^x\,dx}{(\beta+e^x)^2} = \frac{1}{\beta}\,\ln\,\beta, \quad |\arg\beta|<\pi.$$

18. $$\int_{-\infty}^{\infty} \frac{a^2e^x+b^2e^{-x}}{(a^2e^x-b^2e^{-x})^2}x^2\,dx = \frac{\pi^2}{2ab}, \quad ab>0.$$

19. $$\int_{-\infty}^{\infty} \frac{a^2e^x-b^2e^{-x}}{(a^2e^x+b^2e^{-x})^2}x^2\,dx = \frac{\pi}{ab}\,\ln\,\frac{b}{a}, \quad ab>0.$$

20. $$\int_{-\infty}^{\infty} \frac{xe^x\,dx}{(a^2+b^2e^{2x})^n} = \frac{\sqrt{\pi}\,\Gamma\left(n-\frac{1}{2}\right)}{4a^{2n-1}b\Gamma(n)}\left[2\,\ln\,\frac{a}{2b}-\gamma_e-\psi\left(n-\frac{1}{2}\right)\right],$$
$$ab>0,\ n>0.$$

21. $$\int_{-\infty}^{\infty} \frac{(a^2e^x-e^{-x})x^2\,dx}{(a^2e^x+e^{-x})^{p+1}} = -\frac{1}{a^{p+1}}\,\mathrm{B}\left(\frac{p}{2},\frac{p}{2}\right)\,\ln\,a, \quad a>0,\ p>0.$$

22. $$\int_{-\infty}^{\infty} \frac{(e^x-ae^{-x})x^2\,dx}{(a+e^x)^2(1+e^{-x})^2} = \frac{(\ln\,a)^2}{a-1}.$$

23. $\displaystyle\int_{-\infty}^{\infty} \frac{(e^x - ae^{-x})x^2\,dx}{(a+e^x)^2(1-e^{-x})^2} = \frac{\pi^2 + (\ln\ a)^2}{a+1}.$