C4282

! For an efficient use of these tables, first read `HowTo.pdf`.

T2.13C. Integrands of the form $\dfrac{x^n}{\sqrt{(a^2 \pm x^2)\,(b^2 \pm x^2)}}$, $n = 0, 2, 4$, on the interval $(0, y)$ and $(0, 1)$.

Notation used: $\alpha = \arctan \dfrac{y}{b}$, $\gamma = \arcsin \dfrac{y}{b}\sqrt{\dfrac{a^2+b^2}{a^2+y^2}}$, $\eta = \arcsin \dfrac{y}{b}$,

$$q = \frac{\sqrt{a^2-b^2}}{a}, \quad r = \frac{b}{\sqrt{a^2+b^2}}, \quad t = \frac{b}{a}.$$

1. $\displaystyle\int_0^y \frac{dx}{\sqrt{(x^2+a^2)(x^2+b^2)}} = \frac{1}{a}F(\alpha,\, q), \quad a > b > 0.$

2. $\displaystyle\int_0^y \frac{dx}{\sqrt{(x^2+a^2)(b^2-x^2)}} = \frac{1}{\sqrt{a^2+b^2}}F(\gamma,\, r), \quad b \geq y > 0.$

3. $\displaystyle\int_0^y \frac{dx}{\sqrt{(a^2-x^2)(b^2-x^2)}} = \frac{1}{a}F(\eta,\, t), \quad a > b \geq y > 0.$

4. $\displaystyle\int_0^y \frac{x^2\,dx}{\sqrt{(x^2+a^2)(x^2+b^2)}} = y\sqrt{\frac{a^2+y^2}{b^2+y^2}} - aE(\alpha,\, q), \quad y > 0,\ a > b.$

5. $\displaystyle\int_0^y \frac{x^2\,dx}{\sqrt{(a^2+x^2)(b^2-x^2)}} = \sqrt{a^2+b^2}E(\gamma,\, r) - \frac{a^2}{\sqrt{a^2+b^2}}F(\gamma,\, r) - y\sqrt{\frac{b^2-y^2}{a^2+y^2}}, \quad b \geq y > 0.$

6. $\displaystyle\int_0^y \frac{x^2\,dx}{\sqrt{(a^2-x^2)(b^2-x^2)}} = a\{F(\eta,\, t) - E(\eta,\, t)\}, \quad a > b \geq y > 0.$

7. $$\int_0^y \frac{x^4\,dx}{\sqrt{(x^2+a^2)(x^2+b^2)}} = \frac{a}{3}\{2(a^2+b^2)E(\alpha,\,q) - b^2F(\alpha,\,q)\} + \frac{y}{3}(y^2-2a^2-b^2)\sqrt{\frac{a^2+y^2}{b^2+y^2}}, \quad a>b,\,y>0.$$

8. $$\int_0^y \frac{x^4\,dx}{\sqrt{(a^2+x^2)(b^2-x^2)}} = \frac{1}{3\sqrt{a^2+b^2}}\{(2a^2-b^2)a^2F(\gamma,\,r) - 2(a^4-b^4)E(\gamma,\,r)\} - \frac{y}{3}(2b^2-a^2+y^2)\sqrt{\frac{b^2-y^2}{a^2+y^2}}, \quad a \ge y > 0.$$

9. $$\int_0^y \frac{x^4\,dx}{\sqrt{(a^2-x^2)(b^2-x^2)}} = \frac{a}{3}\{(2a^2+b^2)F(\eta,\,t) - 2(a^2+b^2)E(\eta,\,t)\} + \frac{y}{3}\sqrt{(a^2-y^2)(b^2-y^2)}, \quad a>b\ge y>0.$$

10. $$\int_0^1 \frac{x^2\,dx}{\sqrt{(1+x^2)(1+k^2x^2)}} = \frac{1}{k^2}\left\{\sqrt{\frac{1+k^2}{2}} - E\left(\frac{\pi}{4},\,\sqrt{1-k^2}\right)\right\}.$$

C4282