

! For an efficient use of these tables, first read [HowTo.pdf](#).

**T2.12C.** Integrands of the form  $\frac{x^n}{\sqrt{(a-x)(b-x)(c-x)(d-x)}}$ ,  $n = 0, 1$ ;  
and  $\frac{1}{x\sqrt{(a-x)(b-x)(c-x)(d-x)}}$  and  $\frac{1}{(p-x)\sqrt{(a-x)(b-x)(c-x)(d-x)}}$  on the intervals  
( $y, c$ ) and ( $c, y$ ).

Notation used:  $\gamma = \arcsin \sqrt{\frac{(b-d)(c-y)}{(c-d)(b-y)}}$ ,  $\delta = \arcsin \sqrt{\frac{(b-d)(y-c)}{(b-c)(y-d)}}$ ,

$$q = \sqrt{\frac{(b-c)(a-d)}{(a-c)(b-d)}}, \quad r = \sqrt{\frac{(a-b)(c-d)}{(a-c)(b-d)}}.$$

$$1. \int_y^c \frac{dx}{\sqrt{(a-x)(b-x)(c-x)(x-d)}} = \frac{2}{\sqrt{(a-c)(b-d)}} F(\gamma, r), \quad a > b > c > y \geq d.$$

$$2. \int_c^y \frac{dx}{\sqrt{(a-x)(b-x)(x-c)(x-d)}} = \frac{2}{\sqrt{(a-c)(b-d)}} F(\delta, q), \quad a > b \geq y > c > d.$$

$$3. \int_y^c \frac{x dx}{\sqrt{(a-x)(b-x)(c-x)(x-d)}} = \frac{2}{\sqrt{(a-c)(b-d)}} \left\{ (c-b) \Pi \left( \gamma, \frac{c-d}{b-d}, r \right) + b F(\gamma, r) \right\},$$

$$a > b > c > y \geq d.$$

$$4. \int_c^y \frac{x dx}{\sqrt{(a-x)(b-x)(x-c)(x-d)}} = \frac{2}{\sqrt{(a-c)(b-d)}} \left\{ (c-d) \Pi \left( \delta, \frac{b-c}{b-d}, q \right) + d F(\delta, q) \right\},$$

$$a > b \geq y > c > d.$$

$$5. \int_y^c \frac{dx}{x \sqrt{(a-x)(b-x)(c-x)(x-d)}} = \frac{2}{bc \sqrt{(a-c)(b-d)}} \left\{ (b-c) \Pi \left( \gamma, \frac{b(c-d)}{c(b-d)}, r \right) + c F(\gamma, r) \right\}, \quad a > b > c > y \geq d.$$

$$\begin{aligned}
6. \int_c^y \frac{dx}{x\sqrt{(a-x)(b-x)(x-c)(x-d)}} \\
= \frac{2}{cd\sqrt{(a-c)(b-d)}} \left\{ (d-c)\Pi\left(\delta, \frac{d(b-c)}{c(b-d)}, q\right) + cF(\delta, q) \right\}, \quad a > b \geq y > c > d.
\end{aligned}$$

$$\begin{aligned}
7. \int_y^c \frac{dx}{(p-x)\sqrt{(a-x)(b-x)(c-x)(x-d)}} &= \frac{2}{(p-b)(p-c)\sqrt{(a-c)(b-d)}} \\
&\times \left[ (c-b)\Pi\left(\gamma, \frac{(c-d)(p-b)}{(b-d)(p-c)}, r\right) + (p-c)F(\gamma, r) \right], \quad a > b > c > y \geq d, \quad p \neq c.
\end{aligned}$$

$$\begin{aligned}
8. \int_c^y \frac{dx}{(p-x)\sqrt{(a-x)(b-x)(x-c)(x-d)}} &= \frac{2}{(p-c)(p-d)\sqrt{(a-c)(b-d)}} \\
&\times \left[ (c-d)\Pi\left(\delta, \frac{(b-c)(p-d)}{(b-d)(p-c)}, q\right) + (p-c)F(\delta, q) \right], \quad a > b \geq y > c > d, \quad p \neq c.
\end{aligned}$$


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