! For an efficient use of these tables, first read `HowTo.pdf`.

T2.23A. Integrands of the form $\sqrt{\dfrac{d-x}{(a-x)(b-x)(c-x)}}$, $\sqrt{\dfrac{c-x}{(a-x)(b-x)(d-x)}}$, $\sqrt{\dfrac{b-x}{(a-x)(c-x)(d-x)}}$, and $\sqrt{\dfrac{a-x}{(b-x)(c-x)(d-x)}}$ on the intervals (y,a) and (a,y).

Notation used: $\mu = \arcsin\sqrt{\dfrac{(b-d)(a-y)}{(a-b)(y-d)}}$, $\quad \nu = \arcsin\sqrt{\dfrac{(b-d)(y-a)}{(a-d)(y-b)}}$,

$$q = \sqrt{\frac{(b-c)(a-d)}{(a-c)(b-d)}}, \quad r = \sqrt{\frac{(a-b)(c-d)}{(a-c)(b-d)}}.$$

1. $$\int_y^a \sqrt{\frac{x-d}{(a-x)(x-b)(x-c)}}\,dx = \frac{2(a-d)}{\sqrt{(a-c)(b-d)}}\Pi\left(\mu,\, \frac{b-a}{b-d},\, r\right), \quad a > y \geq b > c > d.$$

2. $$\int_a^y \sqrt{\frac{x-d}{(x-a)(x-b)(x-c)}}\,dx = \frac{2}{\sqrt{(a-c)(b-d)}}\left\{(a-b)\Pi\left(\nu,\, \frac{a-d}{b-d},\, q\right) + (b-d)F(\nu,\, q)\right\},$$
$$y > a > b > c > d.$$

3. $$\int_y^a \sqrt{\frac{x-c}{(a-x)(x-b)(x-d)}}\,dx = \frac{2}{\sqrt{(a-c)(b-d)}}\left[(a-d)\Pi\left(\mu,\, \frac{b-a}{b-d},\, r\right) + (d-c)F(\mu,\, r)\right],$$
$$a > y \geq b > c > d.$$

4. $$\int_a^y \sqrt{\frac{x-c}{(x-a)(x-b)(x-d)}}\,dx = \frac{2}{\sqrt{(a-c)(b-d)}}\left[(a-b)\Pi\left(\nu,\, \frac{a-d}{b-d},\, q\right) + (b-c)F(\nu,\, q)\right],$$
$$y > a > b > c > d.$$

5. $$\int_y^a \sqrt{\frac{x-b}{(a-x)(x-c)(x-d)}}\,dx = \frac{2}{\sqrt{(a-c)(b-d)}}\left[(a-d)\Pi\left(\mu,\, \frac{b-a}{b-d},\, r\right) - (b-d)F(\mu,\, r)\right],$$
$$a > y \geq b > c > d.$$

6. $$\int_a^y \sqrt{\frac{x-b}{(x-a)(x-c)(x-d)}}\,dx = \frac{2(a-b)}{\sqrt{(a-c)(b-d)}}\Pi\left(\nu,\,\frac{a-d}{b-d},\,q\right), \quad y > a > b > c > d.$$

7. $$\int_y^a \sqrt{\frac{a-x}{(x-b)(x-c)(x-d)}}\,dx = \frac{2(d-a)}{\sqrt{(a-c)(b-d)}}\left[\Pi\left(\mu,\,\frac{b-a}{b-d},\,r\right) - F(\mu,\,r)\right],$$
$$a > y \geq b > c > d.$$

8. $$\int_a^y \sqrt{\frac{x-a}{(x-b)(x-c)(x-d)}}\,dx = \frac{2(a-b)}{\sqrt{(a-c)(b-d)}}\left[\Pi\left(\nu,\,\frac{a-d}{b-d},\,q\right) - F(\nu,\,q)\right],$$
$$y > a > b > c > d.$$