! For an efficient use of these tables, first read `HowTo.pdf`.

T3.02A. Powers of x, binomials of the form $(a + b\,x)$ and polynomials in x on the interval $(0,\infty)$.

1. $$\int_0^\infty \frac{x^{\mu-1}\,dx}{1+x^\nu} = \frac{\pi}{\nu}\csc\frac{\mu\pi}{\nu} = \frac{1}{\nu}\mathrm{B}\left(\frac{\mu}{\nu}, \frac{\nu-\mu}{\nu}\right), \quad \Re\{\nu\} > \Re\{\mu\} > 0.$$

2. $$\text{p.v.}\int_0^\infty \frac{x^{p-1}\,dx}{1-x^q} = \frac{\pi}{q}\cot\frac{p\pi}{q}, \quad p < q.$$

3. $$\int_0^\infty \frac{x^{\mu-1}\,dx}{(p+qx^\nu)^{n+1}} = \frac{1}{\nu p^{n+1}}\left(\frac{p}{q}\right)^{\mu/\nu}\frac{\Gamma(\mu/\nu)\,\Gamma(1+n-\mu/\nu)}{\Gamma(1+n)}, \quad 0 < \frac{\mu}{\nu} < n+1,\ p \neq 0,\ q \neq 0.$$

4. $$\int_0^\infty \frac{x^{p-1}\,dx}{(1+x^q)^2} = \frac{(p-q)\pi}{q^2}\csc\frac{(p-q)\pi}{q}, \quad p < 2q.$$

5. $$\int_0^\infty \frac{x^{\mu-1}\,dx}{(1+x^{2\nu})(1+x^{3\nu})} = -\frac{\pi}{8\nu}\frac{\csc(\mu\pi/3\nu)}{1-4\cos^2(\mu\pi/3\nu)}, \quad 0 < \Re\{\mu\} < 5\,\Re\{\nu\}.$$

6. $$\int_0^\infty [x^{\nu-\mu} - x^\nu(1+x)^{-\mu}]\,dx = \frac{\nu}{\nu-\mu+1}\,\mathrm{B}(\nu,\,\mu-\nu), \quad \Re\{\mu\} > \Re\{\nu\} > 0.$$

7. $$\int_0^\infty \frac{1-x^q}{1-x^r}x^{p-1}\,dx = \frac{\pi}{r}\sin\frac{q\pi}{r}\csc\frac{p\pi}{r}\csc\frac{(p+q)\pi}{r}, \quad p+q < r,\ p > 0.$$

8. $$\int_0^\infty \frac{(1-x^p)x^{\nu-1}}{1-x^{np}}\,dx = \frac{\pi}{np}\sin\left(\frac{\pi}{n}\right)\csc\frac{(p+\nu)\pi}{np}\csc\frac{\pi\nu}{np}, \quad 0 < \Re\{\nu\} < (n-1)p.$$

9. $$\int_0^\infty \frac{x^{\mu-1}\,dx}{\sqrt{1+x^\nu}} = \frac{1}{\nu}\mathrm{B}\left(\frac{\mu}{\nu}, \frac{1}{2}-\frac{\mu}{\nu}\right), \quad \Re\{\nu\} > \Re\{2\mu\} > 0.$$

10. $$\int_0^\infty \frac{dx}{(1+x^2)^{3/2}\sqrt{1+\frac{4x^2}{3(1+x^2)^2}+\sqrt{1+\frac{4x^2}{3(1+x^2)^2}}}} = \frac{\pi}{2\sqrt{6}}.$$

11. $$\int_0^\infty \frac{dx}{(x^2+a^2)^n} = \frac{(2n-3)!!}{2\cdot(2n-2)!!}\frac{\pi}{a^{2n-1}}.$$

12. $$\int_0^\infty x^{\mu-1}(1+x^2)^{\nu-1}\,dx = \frac{1}{2}\,\mathrm{B}\left(\frac{\mu}{2},\,1-\nu-\frac{\mu}{2}\right), \quad \Re\{\mu\}>0,\ \Re\{\nu+\tfrac{1}{2}\mu\}<1.$$

13. $$\int_0^\infty \frac{x^{2m}\,dx}{(ax^2+c)^n} = \frac{(2m-1)!!(2n-2m-3)!!\,\pi}{2\,(2n-2)!!\,a^m c^{n-m-1}\sqrt{ac}}, \quad a>0,\ c>0,\ n>m+1.$$

14. $$\int_0^\infty \frac{x^{2m+1}\,dx}{(ax^2+c)^n} = \frac{m!(n-m-2)!}{2(n-1)!a^{m+1}c^{n-m-1}}, \quad ac>0,\ n>m+1\geq 1.$$

15. $$\int_0^\infty \frac{x^{\mu+1}}{(1+x^2)^2}\,dx = \frac{\mu\pi}{4\,\sin\frac{\mu\pi}{2}}, \quad -2<\Re\{\mu\}<2.$$

16. $$\int_0^\infty x^{\mu-1}(1+\beta x^p)^{-\nu}\,dx = \frac{1}{p}\beta^{-\mu/p}\mathrm{B}\left(\frac{\mu}{p},\,\nu-\frac{\mu}{p}\right),$$
$$|\arg\beta|<\pi,\ p>0,\ 0<\Re\{\mu\}<p\,\Re\{\nu\}.$$

17. $$\int_0^\infty \frac{dx}{(ax^2+2bx+c)^{n+\frac{3}{2}}} = \frac{(-2)^n}{(2n+1)!!}\frac{\partial^n}{\partial c^n}\left\{\frac{1}{\sqrt{c}(\sqrt{ac}+b)}\right\}, \quad a\geq 0,\ c>0,\ b>-\sqrt{ac}.$$

18. $$\int_0^\infty \frac{x\,dx}{(ax^2+2bx+c)^n} = \begin{cases} \dfrac{(-1)^n}{(n-1)!}\dfrac{\partial^{n-2}}{\partial c^{n-2}}\left\{\dfrac{1}{2(ac-b^2)}-\dfrac{b}{2(ac-b^2)^{\frac{3}{2}}}\ \operatorname{arccot}\dfrac{b}{\sqrt{ac-b^2}}\right\}, & ac>b^2,\\[2ex] \dfrac{(-1)^n}{(n-1)!}\dfrac{\partial^{n-2}}{\partial c^{n-2}}\left\{\dfrac{1}{2(ac-b^2)}+\dfrac{b}{4(b^2-ac)^{\frac{3}{2}}}\ \ln\dfrac{b+\sqrt{b^2-ac}}{b-\sqrt{b^2-ac}}\right\}, & b^2>ac>0,\\[2ex] \dfrac{a^{n-2}}{2(n-1)(2n-1)b^{2n-2}}, \quad \text{for } ac=b^2, & a>0,\ b>0,\ n\geq 2. \end{cases}$$

19. $$\int_0^\infty \frac{x^n\,dx}{(ax^2+2bx+c)^{n+3/2}} = \frac{n!}{(2n+1)!!\sqrt{c}(\sqrt{ac}+b)^{n+1}}, \quad a\geq 0,\ c>0,\ b>-\sqrt{ac}.$$

20. $$\int_0^\infty \frac{x^{n+1}\,dx}{(ax^2+2bx+c)^{n+3/2}} = \frac{n!}{(2n+1)!!\sqrt{a}(\sqrt{ac}+b)^{n+1}}, \quad a>0,\ c\geq 0,\ b>-\sqrt{ac}.$$

21. $$\int_0^\infty \frac{x^{n+1/2}\,dx}{(ax^2+2bx+c)^{n+1}} = \frac{(2n-1)!!\pi}{2^{2n+1/2}(b+\sqrt{ac})^{n+1/2}\,n!\,\sqrt{a}}, \quad a>0,\ c>0,\ b+\sqrt{ac}>0.$$

22. $$\int_0^\infty \frac{x^{\mu-1}\,dx}{(1+2x\cos t+x^2)^\nu} = 2^{\nu-1/2}(\sin t)^{1/2-\nu} t\,\Gamma\left(\nu+1/2\right)\mathrm{B}(\mu,\,2\nu-\mu)P^{(1/2-\nu)}_{\mu-\nu-1/2}(\cos t),$$
$$0<t<\pi,\ 0<\Re\{\mu\}<\Re\{2\nu\}.$$

23. $$\int_0^\infty (1+2\beta x+x^2)^{\mu-1/2}x^{-\nu-1}\,dx$$
$$= \begin{cases} 2^{-\mu}(\beta^2-1)^{\mu/2}\Gamma(1-\mu)\mathrm{B}(\nu-2\mu+1,\,-\nu)P^{(\mu)}_{\nu-\mu}(\beta), & \Re\{\nu\}<0,\ \Re\{2\mu-\nu\}\}<1,\ |\arg(\beta\pm1)|<\pi, \\ -\pi\csc\nu\pi C_\nu^{1/2-\mu}(\beta), & -2<\Re\{\frac{1}{2}-\mu\}<\Re\{\nu\}<0,\ |\arg(\beta\pm1)|<\pi. \end{cases}$$

24. $$\int_0^\infty \frac{x^{\mu-1}\,dx}{x^2+2ax\,\cos t+a^2} = \pi a^{\mu-2}\csc t\csc(\mu\pi)\,\sin[(\mu-1)t],\ a>0,\ 0<|t|<\pi,\ 0<\Re\{\mu\}<2.$$

25. $$\int_0^\infty \frac{x^{\mu-1}\,dx}{(x^2+2ax\,\cos t+a^2)^2} = \frac{\pi a^{\mu-4}}{2}\csc\mu\pi\,\csc^3 t$$
$$\times\left\{(\mu-1)\,\sin t\,\cos[(\mu-2)t]-\sin[(\mu-1)t]\right\}, \quad a>0,\ 0<|t|<\pi;\ 0<\Re\{\mu\}<4.$$

26. $$\int_0^\infty \frac{x^{\mu-1}\,dx}{\sqrt{1+2x\,\cos t+x^2}} = \pi\csc(\mu\pi)\,\mathrm{P}_{\mu-1}(\cos t), \quad -\pi<t<\pi,\ 0<\Re\{\mu\}<1.$$

27. $$\int_0^\infty \frac{dx}{(ax^2+2bx+c)^n} = \frac{(-1)^{n-1}}{(n-1)!}\frac{\partial^{n-1}}{\partial c^{n-1}}\left[\frac{1}{\sqrt{ac-b^2}}\ \mathrm{arccot}\ \frac{b}{\sqrt{ac-b^2}}\right], \quad a>0,\ ac>b^2.$$

28. $$\int_0^\infty \left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-p-1}dx = \begin{cases} \dfrac{\sqrt{\pi}\,\Gamma\left(p+\frac{1}{2}\right)}{2ac^{p+1/2}\Gamma(p+1)}, & \text{if } a>0,\ b<0,\ c>0,\ p>-\frac{1}{2}, \\ \dfrac{1}{2}\,\dfrac{\mathrm{B}\left(p+\frac{1}{2},\frac{1}{2}\right)}{a(4ab+x)^{p+1/2}}, & \text{if } a>0.\ b<0,\ c>-4ab,\ p>-\frac{1}{2}. \end{cases}$$

29. $$\int_0^\infty (\sqrt{x^2+a^2}-x)^n\,dx = \frac{na^{n+1}}{n^2-1}, \quad n\geq 2.$$

30. $$\int_0^\infty \frac{dx}{(x+\sqrt{x^2+a^2})^n} = \frac{n}{a^{n-1}(n^2-1)}, \quad n\geq 2.$$

31. $$\int_0^\infty x^m(\sqrt{x^2+a^2}-x)^n\,dx = \frac{n\cdot m!\,a^{m+n+1}}{(n-m-1)(n-m+1)\dots(m+n+1)},$$
$$a>0,\ 0\leq m\leq n-2.$$

32. $$\int_0^\infty \frac{x^m\,dx}{(x+\sqrt{x^2+a^2})^n} = \frac{n\cdot m!}{(n-m-1)(n-m+1)\dots(m+n+1)\,a^{n-m-1}},\quad a>0,\ 0\le m\le n-2.$$

33. $$\int_0^\infty \frac{x^{-p}\,dx}{1+x^3} = \frac{\pi}{3}\csc\frac{(1-p)\pi}{3},\quad -2<p<1.$$

34. $$\int_0^\infty \frac{x^\nu\,dx}{(x+\gamma)(x^2+\beta^2)} = \frac{\pi}{2(\beta^2+\gamma^2)}\left[\gamma\beta^{\nu-1}\sec\frac{\nu\pi}{2}+\beta^\nu\csc\frac{\nu\pi}{2}-2\gamma^\nu\csc(\nu\pi)\right],\quad \Re\{\beta\}>0,\ |\arg\gamma|<\pi,\ -1<\Re\{\nu\}<2,\ \nu\ne 0.$$

35. $$\int_0^\infty \frac{x^{p-1}\,dx}{(a^2+x^2)(b^2-x^2)} = \frac{\pi}{2}\frac{a^{p-2}+b^{p-2}\cos(p\pi/2)}{a^2+b^2}\csc\frac{p\pi}{2},\quad 0<p<4,\ a>0,\ b>0.$$

36. $$\int_0^\infty \frac{x^{\mu-1}\,dx}{(b+x^2)(c+x^2)} = \frac{\pi}{2}\frac{c^{\mu/2-1}-b^{\mu/2-1}}{b-c}\csc\frac{\mu\pi}{2},\quad |\arg b|<\pi,\ |\arg c|<\pi,\ 0<\Re\{\mu\}<4.$$

37. $$\int_0^\infty x^{\lambda-1}(1+\alpha x^p)^{-\mu}(1+\beta x^p)^{-\nu}\,dx = \frac{1}{p}\alpha^{-\lambda/p}\,\mathrm{B}\left(\frac{\lambda}{p},\,\mu+\nu-\frac{\lambda}{p}\right)\,{}_2F_1\left(\nu,\,\frac{\lambda}{p};\,\mu+\nu;\,1-\frac{\beta}{\alpha}\right),\quad |\arg\alpha|<\pi,\ |\arg\beta|<\pi,\ p>0,\ 0<\Re\{\lambda\}<2\,\Re\{(\mu+\nu\}.$$

38. $$\int_0^\infty \frac{x^2\,dx}{(x^2+a^2)\,(x^2+b^2)\,(x^2+c^2)} = \frac{\pi}{2a\,(b^2-c^2)}\left[\frac{b}{a+b}-\frac{c}{a+c}\right] = \frac{\pi}{2(a+b)(a+c)(b+c)}.$$

39. $$\int_0^\infty \frac{(x^\nu-a^\nu)\,dx}{(x-a)(\beta+x)} = \frac{\pi}{a+\beta}\left\{\beta^\nu\csc(\nu\pi)-a^\nu\cot(\nu\pi)-\frac{a^\nu}{\pi}\ln\frac{\beta}{a}\right\},\quad |\arg\beta|<\pi,\ |\Re\{\nu\}|<1,\ \nu\ne 0.$$

40. $$\int_0^\infty \frac{x^p-x^q}{x-1}\frac{dx}{x+a} = \frac{\pi}{1+a}\left(\frac{a^p-\cos p\pi}{\sin p\pi}-\frac{a^q-\cos q\pi}{\sin q\pi}\right),\quad p^2<1,\ q^2<1,\ a>0.$$

41. $$\int_0^\infty \frac{x^p-a^p}{x-a}\frac{x^p-1}{x-1}\,dx = \frac{\pi}{a-1}\left\{\frac{a^{2p}-1}{\sin(2p\pi)}-\frac{1}{\pi}a^p\ln a\right\}.\quad p^2<\tfrac{1}{4}.$$

42. $$\int_0^\infty \frac{x^p-a^p}{x-a}\frac{x^{-p}-1}{x-1}\,dx = \frac{\pi}{a-1}\left\{2(a^p-1)\cot p\pi-\frac{1}{\pi}(a^p+1)\ln a\right\},\quad p^2<1.$$

43. $$\int_0^\infty \frac{x^p - a^p}{x-a}\frac{1-x^{-p}}{1-x}x^q\,dx = \frac{\pi}{a-1}\left\{\frac{a^{p+q}-1}{\sin[(p+q)\pi]} + \frac{a^p - a^q}{\sin[(q-p)\pi]}\right\}\frac{\sin p\pi}{\sin q\pi},$$
$$(p+q)^2 < 1,\ (p-q)^2 < 1.$$

44. $$\int_0^\infty \left(\frac{x^p - x^{-p}}{1-x}\right)^2 dx = 2(1 - 2p\pi\,\cot 2p\pi), \quad 0 < p^2 < \tfrac{1}{4}.$$

45. $$\int_0^\infty \frac{x^{\mu-1}(1-x)}{1-x^n}\,dx = \frac{\pi}{n}\,\sin\frac{\pi}{n}\csc\frac{\mu\pi}{n}\csc\frac{(\mu+1)\pi}{n}, \quad 0 < \Re\{\mu\} < n-1.$$

46. $$\int_0^\infty \frac{x^q - 1}{x^p - x^{-p}}\frac{dx}{x} = \frac{\pi}{2p}\,\tan\frac{q\pi}{2p}, \quad p > q.$$

47. $$\int_0^\infty \left\{\frac{1}{1+x^{2^n}} - \frac{1}{1+x^{2^m}}\right\}\frac{dx}{x} = 0.$$

48. $$\int_0^\infty \frac{\left[\left(ax + \frac{b}{x}\right)^2 + c\right]^{-p-1}\,dx}{x^2} = \frac{\sqrt{\pi}}{2bc^{p+1/2}}\frac{\Gamma\left(p+\frac{1}{2}\right)}{\Gamma(p+1)}, \quad p > -\tfrac{1}{2}.$$

49. $$\int_0^\infty \left(a + \frac{b}{x^2}\right)\left[\left(ax + \frac{b}{x}\right)^2 + c\right]^{-p-1} dx = \frac{\sqrt{\pi}}{c^{p+1/2}}\frac{\Gamma\left(p+\frac{1}{2}\right)}{\Gamma(p+1)}, \quad p > -\frac{1}{2}.$$

50. $$\int_0^\infty \frac{x^{\mu-1}[\sqrt{1+x^2}+\beta]^\nu}{\sqrt{1+x^2}}\,dx = 2^{\mu/2-1}(\beta^2-1)^{\nu/2+\mu/4}\Gamma\left(\frac{\mu}{2}\right)\Gamma(1-\mu-\nu)\,P_{\mu/2-1}^{(\nu+\mu/2)}(\beta),$$
$$\Re\{\beta\} > -1,\ 0 < \Re\{\mu\} < 1 - \Re\{\nu\}.$$

51. $$\int_0^\infty \frac{x^{\mu-1}[\sqrt{\beta^2+x^2}+x]^\nu}{\sqrt{\beta^2+x^2}}\,dx = \frac{\beta^{\mu+\nu-1}}{2^\mu}\,\mathrm{B}\left(\mu, \frac{1-\mu-\nu}{2}\right),$$
$$\Re\{\beta\} > 0,\ 0 < \Re\{\mu\} < 1 - \Re\{\nu\}.$$

52. $$\int_0^\infty \frac{x^{\mu-1}\left[\cos t \pm i\,\sin t\sqrt{1+x^2}\right]^\nu}{\sqrt{1+x^2}}\,dx = 2^{(\mu-1)/2}\,\frac{\Gamma(\mu/2)\,\Gamma(1-\mu-\nu)}{\Gamma(-\nu)}\,\sin^{(1-\mu)/2} t$$
$$\times\left[\pi^{-1/2}\,Q_{-(\mu+1)/2-\nu}^{((\mu+1)/2)}(\cos t) \mp \frac{i}{2}\pi^{1/2}\,P_{(\mu-1)/2}^{(-(\mu+1)/2-\nu)}(\cos t)\right], \quad \Re\{\mu\} > 0.$$

53. $$\int_0^\infty \frac{x^{\mu-1}[\sqrt{(\beta^2-1)(x^2+1)}+\beta]^\nu}{\sqrt{x^2+1}}\,dx$$
$$= \frac{2^{(\mu-1)/2}}{\sqrt{\pi}}\,e^{-i\pi(\mu-1)/2}\frac{\Gamma\,(\mu/2)\,\Gamma(1-\mu-\nu)}{\Gamma(-\nu)}(\beta^2-1)\frac{1-\mu}{4}Q^{((\mu-1)/2)}_{-\nu-(\mu+1)/2}(\beta),$$
$$\Re\{\beta\}>1,\ \Re\{\nu\}<0,\ \Re\{\mu\}<1-\Re\{\nu\}.$$

54. $$\int_0^\infty \left(\frac{x^p}{1+x^{2p}}\right)^q \frac{dx}{1-x^2} = 0.$$