T1.26. Integrand involving rational functions of $\sin x$, $\cos x$, and $\sqrt{a^2 \sin^2 x - 1}$.

Notation used: $\alpha = \arcsin\left(\dfrac{\sqrt{1-a^2}\,\sin x}{\sqrt{1-a^2\,\sin^2 x}}\right)$, $\beta = \arcsin\sqrt{\dfrac{\sqrt{b^2+c^2} - b\,\sin x - c\,\cos x}{2\sqrt{b^2+c^2}}}$,

$\gamma = \arcsin(\sqrt{2}\,\sin ax)$, $\delta = \arcsin(\sqrt{2}\,\cos ax)$,

$\lambda = \arcsin\sqrt{\dfrac{2\,\sin ax}{1+\sin ax+\cos ax}}$, $r = \sqrt{\dfrac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}}$.

1. $$\int \frac{\sin^2 x\,dx}{\sqrt{a^2\,\sin^2 x - 1}} = \frac{1}{a^2}\left\{\int \frac{dx}{\sqrt{a^2\,\sin^2 x - 1}} + \int \sqrt{a^2\,\sin^2 x - 1}\,dx\right\}$$
$$= -\frac{1}{a}E\left(\alpha,\ \frac{\sqrt{a^2-1}}{a}\right),\quad a^2 > 1.$$

2. $$\int \frac{dx}{\sqrt{(a^2\,\sin^2 x - 1)^5}} = \frac{2a^4(a^2-2)\,\sin^2 x - (3a^2-5)a^2}{3(1-a^2)^2\sqrt{(a^2\,\sin^2 x - 1)^3}}\,\sin x\,\cos x + \frac{1}{3(1-a^2)^2 a}$$
$$\times\left\{(a^2-3)F\left(\alpha,\ \frac{\sqrt{a^2-1}}{a}\right) - 2a^2\left(a^2-2\right)E\left(\alpha,\ \frac{\sqrt{a^2-1}}{a}\right)\right\},\quad a^2 > 1.$$

3. $$\int \frac{dx}{\sin x\,\cos x\sqrt{a^2\,\sin^2 x - 1}} = \frac{1}{2\sqrt{a^2-1}}\,\ln\frac{\sqrt{a^2-1}+\sqrt{a^2\,\sin^2 x - 1}}{\sqrt{a^2-1}-\sqrt{a^2\,\sin^2 x - 1}} - \arcsin\left(\frac{1}{a\,\sin x}\right),$$
$$a^2 > 1.$$

4. $$\int \frac{dx}{\sqrt{1-a^2\,\cos^2 x}} = F\left(\arcsin\left(\frac{\sin x}{\sqrt{1-a^2\,\cos^2 x}}\right), a\right).$$

5. $$\int \sqrt{1-a^2\,\cos^2 x}\,dx = E\left(\arcsin\left(\frac{\sin x}{\sqrt{1-a^2\,\cos^2 x}}\right), a\right) - \frac{a^2\,\sin x\,\cos x}{\sqrt{1-a^2\,\cos^2 x}}.$$

6. $$\int \frac{dx}{\sqrt{1-a^2\,\cos^2 x}} = -\frac{1}{a}\,F\left(\arcsin(a\,\cos x), \frac{1}{a}\right),\quad a > 1.$$

7. $$\int \sqrt{1-a^2\,\cos^2 x}\,dx = \frac{a^2-1}{a}\,F\left(\arcsin(a\,\cos x),\ \frac{1}{a}\right) - a\,E\left(\arcsin(a\,\cos x),\ \frac{1}{a}\right).$$

8. $$\int \frac{dx}{\sqrt{1+a^2\,\cos^2 x}} = \frac{1}{\sqrt{1+a^2}}\,F\left(x,\ \frac{a}{\sqrt{1+a^2}}\right).$$

9. $$\int \sqrt{1+a^2\,\cos^2 x}\,dx = \sqrt{1+a^2}\,E\left(x,\,\frac{a}{\sqrt{1+a^2}}\right).$$

10. $$\int \frac{dx}{\sqrt{a^2\,\cos^2 x-1}} = \frac{1}{a}\,F\left(\arcsin\Big(\frac{a\,\sin x}{\sqrt{a^2-1}}\Big),\,\frac{\sqrt{a^2-1}}{a}\right),\quad a>1.$$

11. $$\int \sqrt{a^2\,\cos^2 x-1}\,dx = a\,E\left(\arcsin\Big(\frac{a\,\sin x}{\sqrt{a^2-1}}\Big),\,\frac{\sqrt{a^2-1}}{a}\right)$$
$$-\frac{1}{a}\,F\left(\arcsin\Big(\frac{a\,\sin x}{\sqrt{a^2-1}}\Big),\,\frac{\sqrt{a^2-1}}{a}\right),\quad a>1.$$

12. $$\int \frac{dx}{\sqrt{(1-a^2\,\sin^2 x)(1-b^2\,\sin^2 x)}} = \frac{1}{\sqrt{1-a^2}}\,F\left(\alpha,\,\sqrt{\frac{b^2-a^2}{1-a^2}}\right),$$
$$0<a^2<b^2<1,\ 0<x\le\frac{\pi}{2}.$$

13. $$\int \frac{\tan^2 x\,dx}{\sqrt{(1-a^2\,\sin^2 x)(1-b^2\,\sin^2 x)}} = \frac{\tan x\sqrt{1-b^2\,\sin^2 x}}{(1-b^2)\sqrt{1-a^2\,\sin^2 x}}$$
$$-\frac{1}{(1-b^2)\sqrt{1-a^2}}\,E\left(\alpha,\,\sqrt{\frac{b^2-a^2}{1-a^2}}\right),\quad 0<a^2<b^2<1,\ 0<x\le\frac{\pi}{2}.$$

14. $$\int \frac{\tan^4 x\,dx}{\sqrt{(1-a^2\,\sin^2 x)(1-b^2\,\sin^2 x)}}$$
$$=\frac{1}{3(1-b^2)^2(1-a^2)^{3/2}}\left[2(2-a^2-b^2)\,E\left(\alpha,\,\sqrt{\frac{b^2-a^2}{1-a^2}}\right)-\big(1-b^2\big)\,F\left(\alpha,\,\sqrt{\frac{b^2-a^2}{1-a^2}}\right)\right]$$
$$+\frac{2a^2+b^2-3+\sin^2 x(4-3a^2-2b^2+a^2b^2)}{3(1-a^2)(1-b^2)^2}\frac{\sin x}{\cos^2 x}\sqrt{\frac{1-b^2\,\sin^2 x}{1-a^2\,\sin^2 x}},$$
$$0<a^2<b^2<1,\ 0<x\le\frac{\pi}{2}.$$

15. $$\int \frac{\sin^2 x\,dx}{\sqrt{(1-a^2\,\sin^2 x)(1-b^2\,\sin^2 x)^3}}$$
$$=\frac{\sqrt{1-a^2}}{(1-b^2)(b^2-a^2)}E\left(\alpha,\,\sqrt{\frac{b^2-a^2}{1-a^2}}\right)-\frac{1}{(b^2-a^2)\sqrt{1-a^2}}\,F\left(\alpha,\,\sqrt{\frac{b^2-a^2}{1-a^2}}\right)$$
$$-\frac{\sin x\,\cos x}{(1-b^2)\sqrt{(1-a^2\,\sin^2 x)(1-b^2\,\sin^2 x)}},\quad 0<a^2<b^2<1,\ 0<x\le\frac{\pi}{2}.$$

16. $$\int \frac{\cos^2 x\,dx}{\sqrt{(1-a^2\sin^2 x)^3(1-b^2\sin^2 x)}}$$
$$= \frac{\sqrt{1-a^2}}{b^2-a^2}E\left(\alpha, \sqrt{\frac{b^2-a^2}{1-a^2}}\right) - \frac{1-b^2}{(b^2-a^2)\sqrt{1-a^2}}F\left(\alpha, \sqrt{\frac{b^2-a^2}{1-a^2}}\right),$$
$$0 < a^2 < b^2 < 1,\ 0 < x \le \frac{\pi}{2}.$$

17. $$\int \frac{\cos^4 x\,dx}{\sqrt{(1-a^2\sin^2 x)^5(1-b^2\sin^2 x)}}$$
$$= \frac{(1-a^2)^{3/2}}{3(b^2-a^2)^2}\left[\frac{(2+a^2-3b^2)(1-b^2)}{(1-a^2)^2}F\left(\alpha, \sqrt{\frac{b^2-a^2}{1-a^2}}\right)\right.$$
$$\left. = +2\frac{2b^2-a^2-1}{1-a^2}E\left(\alpha, \sqrt{\frac{b^2-a^2}{1-a^2}}\right)\right] + \frac{(1-a^2)\sin x\cos x\sqrt{1-b^2\sin^2 x}}{3(b^2-a^2)\sqrt{(1-a^2\sin^2 x)^3}},$$
$$0 < a^2 < b^2 < 1,\ 0 < x \le \frac{\pi}{2}.$$

18. $$\int \frac{dx}{1-a^2\sin^2 x}\sqrt{\frac{1-b^2\sin^2 x}{1-a^2\sin^2 x}} = \frac{1}{\sqrt{1-a^2}}E\left(\alpha, \sqrt{\frac{b^2-a^2}{1-a^2}}\right),$$
$$0 < a^2 < b^2 < 1,\ 0 < x \le \frac{\pi}{2}.$$

19. $$\int \sqrt{\frac{1-a^2\sin^2 x}{(1-b^2\sin^2 x)^3}}\,dx = \frac{\sqrt{1-a^2}}{1-b^2}E\left(\alpha, \sqrt{\frac{b^2-a^2}{1-a^2}}\right) - \frac{b^2-a^2}{1-b^2}\frac{\sin x\cos x}{\sqrt{(1-a^2\sin^2 x)(1-b^2\sin^2 x)}},$$
$$0 < a^2 < b^2 < 1,\ 0 < x \le \frac{\pi}{2}.$$

20. $$\int \frac{dx}{1+(a^2r^2-a^2-r^2)\sin^2 x}\sqrt{\frac{1-a^2\sin^2 x}{1-b^2\sin^2 x}} = \frac{1}{\sqrt{1-a^2}}\Pi\left(\alpha, r^2, \sqrt{\frac{b^2-a^2}{1-a^2}}\right),$$
$$0 < a^2 < b^2 < 1,\ 0 < x \le \frac{\pi}{2}.$$

21. $$\int \frac{dx}{\sqrt{a+b\sin x+c\cos x}}$$
$$= \begin{cases} -\dfrac{2}{\sqrt{a+\sqrt{b^2+c^2}}}F(\beta, r), & 0 < \sqrt{b^2+c^2} < a,\ \arcsin\dfrac{b}{\sqrt{b^2+c^2}} - \pi \le x < \arcsin\dfrac{b}{\sqrt{b^2+c^2}}, \\ -\dfrac{\sqrt{2}}{(b^2+c^2)^{1/4}}F(\beta, r), & \\ \qquad 0 < |a| < \sqrt{b^2+c^2},\ \arcsin\dfrac{b}{\sqrt{b^2+c^2}} - \arccos\left(-\dfrac{a}{\sqrt{b^2+c^2}}\right) \le x < \arcsin\dfrac{b}{\sqrt{b^2+c^2}}. \end{cases}$$

22. $$\int \frac{\sin x\,dx}{\sqrt{a+b\,\sin x+c\,\cos x}} = -\frac{\sqrt{2b}}{\sqrt[4]{(b^2+c^2)^3}}\{2E(\beta,\,r)-F(\beta,\,r)\}+\frac{2c}{b^2+c^2}\sqrt{a+b\,\sin x+c\,\cos x},$$
$$0<|a|<\sqrt{b^2+c^2},\ \arcsin\frac{b}{\sqrt{b^2+c^2}}-\arccos\left(-\frac{a}{\sqrt{b^2+c^2}}\right)\le x<\arcsin\frac{b}{\sqrt{b^2+c^2}}.$$

23. $$\int \frac{(b\,\cos x-c\,\sin x)\,dx}{\sqrt{a+b\,\sin x+c\,\cos x}} = 2\sqrt{a+b\,\sin x+c\,\cos x}\,.$$

24. $$\int \frac{\sqrt{b^2+c^2}+b\,\sin x+c\,\cos x}{\sqrt{a+b\,\sin x+c\,\cos\,x}}\,dx$$
$$= \begin{cases} -2\sqrt{a+\sqrt{b^2+c^2}}\,E(\beta,\,r)+\dfrac{2(a-\sqrt{b^2+c^2})}{\sqrt{a+\sqrt{b^2+c^2}}}\,F(\beta,\,r), \\ \qquad 0<\sqrt{b^2+c^2}<a,\ \arcsin\dfrac{b}{\sqrt{b^2+c^2}}-\pi\le x<\arcsin\dfrac{b}{\sqrt{b^2+c^2}}, \\ -2\sqrt{2}\left(b^2+c^2\right)^{1/4}\,E(\beta,\,r), \\ \qquad 0<|a|<\sqrt{b^2+c^2},\ \arcsin\dfrac{b}{\sqrt{b^2+c^2}}-\arccos\left(-\dfrac{a}{\sqrt{b^2+c^2}}\right)\le x<\arcsin\dfrac{b}{\sqrt{b^2+c^2}}. \end{cases}$$

25. $$\int \sqrt{a+b\,\sin x+c\,\cos x}\,dx$$
$$= \begin{cases} -2\sqrt{a+\sqrt{b^2+c^2}}\,E(\beta,\,r), \\ \qquad 0<\sqrt{b^2+c^2}<a,\ \arcsin\dfrac{b}{\sqrt{b^2+c^2}}-\pi\le x<\arcsin\dfrac{b}{\sqrt{b^2+c^2}}, \\ -2\sqrt{2}\left(b^2+c^2\right)^{1/4}\,E(\beta,\,r)+\dfrac{\sqrt{2}(\sqrt{b^2+c^2}-a)}{\sqrt[4]{b^2+c^2}}F(\beta,\,r), \\ \qquad [0<|a|<\sqrt{b^2+c^2},\ \arcsin\dfrac{b}{\sqrt{b^2+c^2}}-\arccos\left(\dfrac{-a}{\sqrt{b^2+c^2}}\right)\le x<\arcsin\dfrac{b}{\sqrt{b^2+c^2}}. \end{cases}$$

26. $$\int \frac{dx}{\sqrt{\cos 2ax}} = \frac{1}{a\sqrt{2}}F\left(\gamma,\,\frac{1}{\sqrt{2}}\right),\quad 0<ax\le\frac{\pi}{4}.$$

27. $$\int \frac{\cos^2 ax}{\sqrt{\cos 2ax}}\,dx = \frac{1}{a\sqrt{2}}E\left(\gamma,\,\frac{1}{\sqrt{2}}\right),\quad 0<ax\le\frac{\pi}{4}.$$

28. $$\int \frac{dx}{\cos^2 ax\sqrt{\cos 2ax}} = \frac{\sqrt{2}}{a}E\left(\gamma,\,\frac{1}{\sqrt{2}}\right)-\frac{\tan x}{a}\sqrt{\cos 2ax},\quad 0<ax\le\frac{\pi}{4}.$$

29. $$\int \frac{dx}{\cos^4 ax\sqrt{\cos 2ax}} = \frac{2\sqrt{2}}{a}E\left(\gamma,\,\frac{1}{\sqrt{2}}\right)$$
$$-\frac{\sqrt{2}}{3a}F\left(\gamma,\,\frac{1}{\sqrt{2}}\right)-\frac{(6\,\cos^2 ax+1)\,\sin ax}{3a\,\cos^3 ax}\sqrt{\cos 2ax},\quad 0<x\le\frac{\pi}{4}.$$

30. $$\int \frac{\tan^2 ax\,dx}{\sqrt{\cos 2ax}} = \frac{\sqrt{2}}{a}E\left(\gamma,\,\frac{1}{\sqrt{2}}\right)-\frac{1}{a\sqrt{2}}F\left(\gamma,\,\frac{1}{\sqrt{2}}\right)-\frac{1}{a}\,\tan ax\sqrt{\cos 2ax},\quad 0<x\le\frac{\pi}{2}.$$

31. $\displaystyle\int \frac{\tan^4 ax\,dx}{\sqrt{\cos 2ax}} = \frac{1}{3a\sqrt{2}}F\left(\gamma,\,\frac{1}{\sqrt{2}}\right) - \frac{\sin ax}{3a\,\cos^3 ax}\sqrt{\cos 2ax}, \quad 0 < ax \le \frac{\pi}{4}.$

32. $\displaystyle\int \frac{dx}{(1-2r^2\,\sin^2 ax)\sqrt{\cos 2ax}} = \frac{1}{a\sqrt{2}}\Pi\left(\gamma,\,r^2,\,\frac{1}{\sqrt{2}}\right), \quad 0 < ax \le \frac{\pi}{4}.$

33. $\displaystyle\int \frac{dx}{\sqrt{\cos^3 2ax}} = \frac{1}{a\sqrt{2}}F\left(\gamma,\,\frac{1}{\sqrt{2}}\right) - \frac{\sqrt{2}}{a}E\left(\gamma,\,\frac{1}{\sqrt{2}}\right) + \frac{\sin 2ax}{a\sqrt{\cos 2ax}}, \quad 0 < ax \le \frac{\pi}{4}.$

34. $\displaystyle\int \frac{\sin^2 ax\,dx}{\sqrt{\cos^3 2ax}} = \frac{\sin 2ax}{2a\sqrt{\cos 2ax}} - \frac{1}{a\sqrt{2}}E\left(\gamma,\,\frac{1}{\sqrt{2}}\right), \quad 0 < ax \le \frac{\pi}{4}.$

35. $\displaystyle\int \frac{dx}{\sqrt{\cos^5 2ax}} = \frac{1}{3a\sqrt{2}}F\left(\gamma,\,\frac{1}{\sqrt{2}}\right) + \frac{\sin 2ax}{3a\sqrt{\cos^3 2ax}}, \quad 0 < ax \le \frac{\pi}{4}.$

36. $\displaystyle\int \sqrt{\cos 2ax}\,dx = \frac{\sqrt{2}}{a}E\left(\gamma,\,\frac{1}{\sqrt{2}}\right) - \frac{1}{a\sqrt{2}}F\left(\gamma,\,\frac{1}{\sqrt{2}}\right), \quad 0 < ax \le \frac{\pi}{4}.$

37. $\displaystyle\int \frac{\sqrt{\cos 2ax}}{\cos^2 ax}\,dx = \frac{\sqrt{2}}{a}\left\{F\left(\gamma,\,\frac{1}{\sqrt{2}}\right) - E\left(\gamma,\,\frac{1}{\sqrt{2}}\right)\right\} + \frac{1}{a}\,\tan ax\sqrt{\cos 2ax}, \quad 0 < x \le \frac{\pi}{4}.$

38. $\displaystyle\int \frac{dx}{\sqrt{-\cos 2ax}} = -\frac{1}{a\sqrt{2}}F\left(\delta,\,\frac{1}{\sqrt{2}}\right).$

39. $\displaystyle\int \frac{\cos^2 ax\,dx}{\sqrt{-\cos 2ax}} = \frac{1}{a\sqrt{2}}\left[E\left(\delta,\,\frac{1}{\sqrt{2}}\right) - F\left(\delta,\,\frac{1}{\sqrt{2}}\right)\right].$

40. $\displaystyle\int \frac{\cos^4 ax\,dx}{\sqrt{-\cos 2ax}} = \frac{1}{3a\sqrt{2}}\left[3F\left(\delta,\,\frac{1}{\sqrt{2}}\right) - \frac{5}{2}E\left(\delta,\,\frac{1}{\sqrt{2}}\right)\right] - \frac{1}{12a}\,\sin 2ax\sqrt{-\cos 2ax}.$

41. $\displaystyle\int \frac{dx}{\sin^2 ax\sqrt{-\cos 2ax}} = \frac{1}{a}\,\cot ax\sqrt{-\cos 2ax} - \frac{\sqrt{2}}{a}E\left(\delta,\,\frac{1}{\sqrt{2}}\right).$

42. $\displaystyle\int \frac{dx}{\sin^4 ax\sqrt{-\cos 2ax}} = \frac{2}{3a\sqrt{2}}\left[F\left(\delta,\,\frac{1}{\sqrt{2}}\right) - 6E\left(\delta,\,\frac{1}{\sqrt{2}}\right)\right] + \frac{1}{3a}\frac{\cos ax}{\sin^3 ax}(6\,\sin^2 ax + 1)\sqrt{-\cos 2ax}.$

43. $\displaystyle\int \frac{\cot^2 ax\,dx}{\sqrt{-\cos 2ax}} = \frac{1}{a\sqrt{2}}\left[F\left(\delta,\,\frac{1}{\sqrt{2}}\right) - 2E\left(\delta,\,\frac{1}{\sqrt{2}}\right)\right] + \frac{1}{a}\,\cot ax\sqrt{-\cos 2ax}.$

44. $\displaystyle\int \frac{dx}{(1-2r^2\,\cos^2 ax)\sqrt{-\cos 2ax}} = -\frac{1}{a\sqrt{2}}\Pi\left(\delta,\,r^2,\,\frac{1}{\sqrt{2}}\right).$

45. $\displaystyle\int \frac{dx}{\sqrt{-\cos^3 2ax}} = \frac{1}{a\sqrt{2}}\left[F\left(\delta,\,\frac{1}{\sqrt{2}}\right) - 2E\left(\delta,\,\frac{1}{\sqrt{2}}\right)\right] + \frac{\sin 2ax}{a\sqrt{-\cos 2ax}}.$

46. $\displaystyle\int \frac{\cos^2 ax\,dx}{\sqrt{-\cos^3 2ax}} = \frac{\sin 2ax}{2a\sqrt{-\cos 2ax}} - \frac{1}{a\sqrt{2}}E\left(\delta,\,\frac{1}{\sqrt{2}}\right).$

47. $\displaystyle\int \frac{dx}{\sqrt{-\cos^5 2ax}} = -\frac{1}{3a\sqrt{2}}F\left(\delta,\,\frac{1}{\sqrt{2}}\right) - \frac{\sin 2ax}{3a\sqrt{-\cos^3 2ax}}.$

48. $\displaystyle\int \sqrt{-\cos 2ax}\,dx = \frac{1}{a\sqrt{2}}\left[F\left(\delta,\,\frac{1}{\sqrt{2}}\right) - 2E\left(\delta,\,\frac{1}{\sqrt{2}}\right)\right].$

49. $\displaystyle\int \frac{dx}{\sqrt{\sin 2ax}} = \frac{\sqrt{2}}{a}F\left(\lambda,\,\frac{1}{\sqrt{2}}\right).$

50. $\displaystyle\int \frac{\sin ax\,dx}{\sqrt{\sin 2ax}} = \frac{\sqrt{2}}{a}\left\{\frac{1+i}{2}\Pi\left(\lambda,\,\frac{1+i}{2},\,\frac{1}{\sqrt{2}}\right)\right.$
$\displaystyle\left. +\frac{1-i}{2}\Pi\left(\lambda,\,\frac{1-i}{2},\,\frac{1}{\sqrt{2}}\right) + F\left(\lambda,\,\frac{1}{\sqrt{2}}\right) - 2E\left(\lambda,\,\frac{1}{\sqrt{2}}\right)\right\}.$

51. $\displaystyle\int \frac{\sin ax\,dx}{(1+\sin ax+\cos ax)\sqrt{\sin 2ax}} = \frac{\sqrt{2}}{a}\left[F\left(\lambda,\,\frac{1}{\sqrt{2}}\right) - E\left(\lambda,\,\frac{1}{\sqrt{2}}\right)\right].$

52. $\displaystyle\int \frac{\sin ax\,dx}{(1-\sin ax+\cos ax)\sqrt{\sin 2ax}} = \frac{\sqrt{2}}{a}\left\{\sqrt{\tan ax} - E\left(\lambda,\,\frac{1}{\sqrt{2}}\right)\right\}, \quad ax \neq \frac{\pi}{2}.$

53. $\displaystyle\int \frac{(1+\cos ax)\,dx}{(1+\sin ax+\cos ax)\sqrt{\sin 2ax}} = \frac{\sqrt{2}}{a}E\left(\lambda,\,\frac{1}{\sqrt{2}}\right).$

54. $\displaystyle\int \frac{(1+\cos ax)\,dx}{(1-\sin ax+\cos ax)\sqrt{\sin 2ax}} = \frac{\sqrt{2}}{a}\left\{F\left(\lambda,\,\frac{1}{\sqrt{2}}\right) - E\left(\lambda,\,\frac{1}{\sqrt{2}}\right) + \sqrt{\tan ax}\right\}, \quad ax \neq \frac{\pi}{2}.$

55. $\displaystyle\int \frac{(1-\sin ax+\cos ax)\,dx}{(1+\sin ax+\cos ax)\sqrt{\sin\ 2ax}} = \frac{\sqrt{2}}{a}\left\{2E\left(\lambda,\,\frac{1}{\sqrt{2}}\right) - F\left(\lambda,\,\frac{1}{\sqrt{2}}\right)\right\}.$

56. $\displaystyle\int \frac{(1+\sin ax+\cos ax)\,dx}{[1+\cos ax+(1-2r^2)\,\sin\ ax]\sqrt{\sin 2ax}} = \frac{\sqrt{2}}{a}\Pi\left(\lambda,\,r^2,\,\frac{1}{\sqrt{2}}\right).$

57. $\displaystyle\int \frac{dx}{\sqrt{a+b\,\tan^2 cx}} = \begin{cases} \dfrac{1}{c\sqrt{a-b}}\,\arcsin\left(\sqrt{\dfrac{a-b}{a}}\,\sin cx\right), & (4k-1)\pi/2 < x \le (4k+1)\pi/2, \\ \dfrac{-1}{c\sqrt{a-b}}\,\arcsin\left(\sqrt{\dfrac{a-b}{a}}\,\sin cx\right), & (4k+1)\pi/2 < x \le (4k+3)\pi/2;\ k \in \mathrm{N}. \end{cases}$