

! For an efficient use of these tables, first read [HowTo.pdf](#).

**T2.09A.** Integrands of the form  $\sqrt{\frac{\pm(a-x)}{(b-x)^n(c-x)}}$ ,  $\sqrt{\frac{\pm(b-x)}{(a-x)^n(c-x)}}$  and  $\sqrt{\frac{\pm(c-x)}{(a-x)^n(b-x)}}$ ,  $n = 1, 3$ , on the intervals  $(y, a)$  and  $(a, y)$ .

Notation used:  $\lambda = \arcsin \sqrt{\frac{a-y}{a-b}}$ ,  $\mu = \arcsin \sqrt{\frac{y-a}{y-b}}$ ,  $p = \sqrt{\frac{a-b}{a-c}}$ ,  $q = \sqrt{\frac{b-c}{a-c}}$ .

$$1. \int_y^a \sqrt{\frac{a-x}{(x-b)(x-c)}} dx = 2\sqrt{a-c} [F(\lambda, p) - E(\lambda, p)], \quad a > y \geq b > c.$$

$$2. \int_a^y \sqrt{\frac{x-a}{(x-b)(x-c)}} dx = -2\sqrt{a-c} E(\mu, q) + 2\sqrt{\frac{(y-a)(y-c)}{y-b}}, \quad y > a > b > c.$$

$$3. \int_y^a \sqrt{\frac{x-b}{(a-x)(x-c)}} dx = 2\sqrt{a-c} E(\lambda, p) - \frac{2(b-c)}{\sqrt{a-c}} F(\lambda, p). \quad a > y \geq b > c.$$

$$4. \int_a^y \sqrt{\frac{x-b}{(x-a)(x-c)}} dx = \frac{2(a-b)}{\sqrt{a-c}} F(\mu, q) - 2\sqrt{a-c} E(\mu, q) \\ + 2\sqrt{\frac{(y-a)(y-c)}{y-b}}, \quad y > a > b > c.$$

$$5. \int_y^a \sqrt{\frac{x-c}{(a-x)(x-b)}} dx = 2\sqrt{a-c} E(\lambda, p), \quad a > y \geq b > c.$$

$$6. \int_a^y \sqrt{\frac{x-c}{(x-a)(x-b)}} dx = 2\sqrt{a-c} [F(\mu, q) - E(\mu, q)] + 2\sqrt{\frac{(y-a)(y-c)}{y-b}}, \quad y > a > b > c.$$

$$7. \int_y^a \sqrt{\frac{a-x}{(x-b)(x-c)^3}} dx = \frac{2\sqrt{a-c}}{b-c} E(\lambda, p) - \frac{2}{\sqrt{a-c}} F(\lambda, p) \\ - \frac{2}{b-c} \sqrt{\frac{(a-y)(y-b)}{y-c}}, \quad a > y \geq b > c.$$

$$8. \int_a^y \sqrt{\frac{x-a}{(x-b)(x-c)^3}} dx = \frac{2\sqrt{a-c}}{b-c} E(\mu, q) - \frac{2(a-b)}{(b-c)\sqrt{a-c}} F(\mu, q) \\ - 2\sqrt{\frac{y-a}{(y-b)(y-c)}}, \quad y > a > b > c.$$

$$9. \int_y^a \sqrt{\frac{a-x}{(x-b)^3(x-c)}} dx = \frac{2\sqrt{a-c}}{c-b} E(\lambda, p) + \frac{2}{b-c} \sqrt{\frac{(a-y)(y-c)}{y-b}}, \quad a > y > b > c.$$

$$10. \int_a^y \sqrt{\frac{x-a}{(x-b)^3(x-c)}} dx = \frac{2\sqrt{a-c}}{b-c} [F(\mu, q) - E(\mu, q)], \quad y > a > b > c.$$

$$11. \int_y^a \sqrt{\frac{x-b}{(a-x)(x-c)^3}} dx = \frac{2}{\sqrt{a-c}} [F(\lambda, p) - E(\lambda, p)] \\ + \frac{2}{a-c} \sqrt{\frac{(a-y)(y-b)}{y-c}}, \quad a > y \geq b > c.$$

$$12. \int_a^y \sqrt{\frac{x-b}{(x-a)(x-c)^3}} dx = \frac{2}{\sqrt{a-c}} E(\mu, q) - 2\frac{b-c}{a-c} \sqrt{\frac{y-a}{(y-b)(y-c)}}, \quad y > a > b > c.$$

$$13. \int_y^a \sqrt{\frac{x-c}{(a-x)(x-b)^3}} dx = \frac{2\sqrt{a-c}}{a-b} [F(\lambda, p) - E(\lambda, p)] + \frac{2}{a-b} \sqrt{\frac{(a-y)(y-c)}{y-b}}, \\ a > y > b > c.$$

$$14. \int_a^y \sqrt{\frac{x-c}{(x-a)(x-b)^3}} dx = \frac{2\sqrt{a-c}}{a-b} E(\mu, q), \quad y > a > b > c.$$


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