! For an efficient use of these tables, first read `HowTo.pdf`.

T3.47A. Integrands involving trigonometric and hyperbolic functions, exponentials and powers of $(a + b\,x)$ on the interval $(0, \infty)$.

1. $$\int_0^\infty x\,e^{-\beta x^2}\,\cosh x\,\sin x\,dx = \frac{1}{4}\sqrt{\frac{\pi}{\beta^3}}\left(\cos\frac{1}{2\beta} + \sin\frac{1}{2\beta}\right), \quad \Re\{\beta\} > 0.$$

2. $$\int_0^\infty x\,e^{-\beta x^2}\,\sinh x\,\cos x\,dx = \frac{1}{4}\sqrt{\frac{\pi}{\beta^3}}\left(\cos\frac{1}{2\beta} - \sin\frac{1}{2\beta}\right), \quad \Re\{\beta\} > 0.$$

3. $$\int_0^\infty x^2 e^{-\beta x^2}\,\cosh x\,\cos x\,dx = \frac{1}{4}\sqrt{\frac{\pi}{\beta^3}}\left(\cos\frac{1}{2\beta} - \frac{1}{\beta}\,\sin\frac{1}{2\beta}\right), \quad \Re\{\beta\} > 0.$$

4. $$\int_0^\infty x^2 e^{-\beta x^2}\,\sinh x\,\sin x\,dx = \frac{1}{4}\sqrt{\frac{\pi}{\beta^3}}\left(\sin\frac{1}{2\beta} + \frac{1}{\beta}\,\cos\frac{1}{2\beta}\right), \quad \Re\{\beta\} > 0.$$

5. $$\int_0^\infty x\,e^{-\beta x^2}(\sinh x + \sin x)\,dx = \frac{1}{2}\sqrt{\frac{\pi}{\beta^3}}\,\cosh\frac{1}{4\beta}, \quad \Re\{\beta\} > 0.$$

6. $$\int_0^\infty x\,e^{-\beta x^2}(\sinh x - \sin x)\,dx = \frac{1}{2}\sqrt{\frac{\pi}{\beta^3}}\,\sinh\frac{1}{4\beta}, \quad \Re\{\beta\} > 0.$$

7. $$\int_0^\infty x^2 e^{-\beta x^2}(\cosh x + \cos x)\,dx = \frac{1}{2}\sqrt{\frac{\pi}{\beta^3}}\left(\cosh\frac{1}{4\beta} + \frac{1}{2\beta}\,\sinh\frac{1}{4\beta}\right), \quad \Re\{\beta\} > 0.$$

8. $$\int_0^\infty x^2 e^{-\beta x^2}(\cosh x - \cos x)\,dx = \frac{1}{2}\sqrt{\frac{\pi}{\beta^3}}\left(\sinh\frac{1}{4\beta} + \frac{1}{2\beta}\,\cosh\frac{1}{4\beta}\right), \quad \Re\{\beta\} > 0.$$

9. $$\int_0^\infty x\,e^{-\beta x^2}(\cosh x\,\sin x + \sinh x\,\cos x)\,dx = \frac{1}{2\beta}\sqrt{\frac{\pi}{\beta}}\,\cos\frac{1}{2\beta}, \quad \Re\{\beta\} > 0.$$

10. $$\int_0^\infty x\,e^{-\beta x^2}(\cosh x\,\sin x - \sinh x\,\cos x)\,dx = \frac{1}{2\beta}\sqrt{\frac{\pi}{\beta}}\,\sin\frac{1}{2\beta}, \quad \Re\{\beta\} > 0.$$

11. $$\int_0^\infty e^{-x^2} \sinh x^2 \, \cos ax \frac{dx}{x^2} = \sqrt{\frac{\pi}{2}} e^{-\frac{a^2}{8}} - \frac{\pi a}{4} \left[1 - \operatorname{erf}\left(\frac{a}{\sqrt{8}} \right) \right], \quad a > 0.$$

12. $$\int_0^\infty x \, e^{-\beta x^2} \cosh(2ax \, \sin t) \, \sin(2ax \, \cos t) \, dx = \frac{a}{2} \sqrt{\frac{\pi}{\beta^3}} \exp\left(-\frac{a^2}{\beta} \cos 2t \right) \cos\left(t - \frac{a^2}{\beta} \sin 2t \right), \quad \Re\{\beta\} > 0.$$

13. $$\int_0^\infty x \, e^{-\beta x^2} \sinh(2ax \, \sin t) \, \cos(2ax \, \cos t) \, dx = \frac{a}{2} \sqrt{\frac{\pi}{\beta^3}} \exp\left(-\frac{a^2}{\beta} \cos 2t \right) \sin\left(t - \frac{a^2}{\beta} \sin 2t \right), \quad \Re\{\beta\} > 0.$$

14. $$\int_0^\infty e^{-\beta x^2} \sinh ax \, \sin bx \, dx = \frac{1}{2} \sqrt{\frac{\pi}{\beta}} \exp\left(\frac{a^2 - b^2}{4\beta} \right) \sin \frac{ab}{2\beta}, \quad \Re\{\beta\} > 0.$$

15. $$\int_0^\infty e^{-\beta x^2} \cosh ax \, \cos bx \, dx = \frac{1}{2} \sqrt{\frac{\pi}{\beta}} \exp\left(\frac{a^2 - b^2}{4\beta} \right) \cos \frac{ab}{2\beta}, \quad \Re\{\beta\} > 0.$$

16. $$\int_0^\infty x \, e^{-\beta x^2} \cosh ax \, \sin ax \, dx = \frac{a}{4\beta} \sqrt{\frac{\pi}{\beta}} \left(\cos \frac{a^2}{2\beta} + \sin \frac{a^2}{2\beta} \right), \quad \Re\{\beta\} > 0.$$

17. $$\int_0^\infty x \, e^{-\beta x^2} \sinh ax \, \cos ax \, dx = \frac{a}{4\beta} \sqrt{\frac{\pi}{\beta}} \left(\cos \frac{a^2}{2\beta} - \sin \frac{a^2}{2\beta} \right), \quad \Re\{\beta\} > 0.$$

18. $$\int_0^\infty x^2 e^{-\beta x^2} \cosh ax \, \sin ax \, dx = \frac{1}{4} \sqrt{\frac{\pi}{\beta^3}} \left(\sin \frac{a^2}{2\beta} + \frac{a^2}{\beta} \cos \frac{a^2}{2\beta} \right), \quad \Re\{\beta\} > 0.$$

19. $$\int_0^\infty x^2 e^{-\beta x^2} \cosh ax \, \cos ax \, dx = \frac{1}{4} \sqrt{\frac{\pi}{\beta^3}} \left(\cos \frac{a^2}{2\beta} - \frac{a^2}{\beta} \sin \frac{a^2}{2\beta} \right), \quad \Re\{\beta\} > 0.$$