! For an efficient use of these tables, first read `HowTo.pdf`.

T1.11. Integrand involving $a + bx^2 + cx^4$ and $a + bx^k + cx^{2k}$.

Notation used: $f = \frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac}$, $g = \frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac}$,

$h = \sqrt{b^2 - 4ac}$, $q = \left(\frac{a}{c}\right)^{1/4}$, $\gamma = 2a(n-1)(b^2 - 4ac)$, $\cos\alpha = -\frac{b}{2\sqrt{ac}}$

$R_2 = a + bx^2 + cx^4$ and $R_k = a + bx^k + cx^{2k}$.

$$1.\ \int \frac{dx}{R_2} = \begin{cases} \dfrac{c}{h}\left\{\displaystyle\int \frac{dx}{cx^2 + f} - \int \frac{dx}{cx^2 + g}\right\}, & \text{for } h^2 > 0, \\ \dfrac{1}{4cq^3 \sin\alpha}\left\{\sin\dfrac{\alpha}{2}\ \ln\dfrac{x^2 + 2qx\cos\frac{\alpha}{2} + q^2}{x^2 - 2qx\cos\frac{\alpha}{2} + q^2} + 2\cos\dfrac{\alpha}{2}\arctan\dfrac{x^2 - q^2}{2qx\sin\frac{\alpha}{2}}\right\}, & \text{for } h^2 < 0. \end{cases}$$

$$2.\ \int \frac{dx}{x^m R_2^{n+1}} = -\frac{1}{(m-1)\,a\,x^{m-1} R_2^n} - \frac{n+m-1}{m-1}\,\frac{b}{a}\int \frac{dx}{x^{m-1} R_2^{n+1}} - \frac{2n+m-1}{m-1}\int \frac{dx}{x^{m-2} R_2^{n+1}}.$$

$$3.\ \int \frac{x\,dx}{R_2} = \begin{cases} \dfrac{1}{2h}\ln\dfrac{cx^2 + f}{cx^2 + g}, & \text{for } h^2 > 0, \\ \dfrac{1}{2cq^2\sin\alpha}\arctan\dfrac{x^2 - q^2\cos\alpha}{q^2\sin\alpha}, & \text{for } h^2 < 0. \end{cases}$$

$$4.\ \int \frac{x^2\,dx}{R_2} = \frac{g}{h}\int \frac{dx}{cx^2 + g} - \frac{f}{h}\int \frac{dx}{cx^2 + f} \quad \text{for } h^2 > 0.$$

$$5.\ \int \frac{dx}{R_2^2} = \frac{bcx^3 + (b^2 - 2ac)\,x}{\gamma\,R_2} + \frac{b^2 - 6ac}{\gamma}\int \frac{dx}{R_2} + \frac{bc}{\gamma}\int \frac{x^2\,dx}{R_2}.$$

$$6.\ \int \frac{dx}{R_2^n} = \frac{bcx^3 + (b^2 - 2ac)\,x}{\gamma\,R_2^{n-1}} + \frac{(4n-7)bc}{\gamma}\int \frac{x^2\,dx}{R_2^{n-1}} + \frac{2(n-1)h^2 + 2ac - b^2}{\gamma}\int \frac{dx}{R_2^{n-1}} \quad \text{for } n > 1.$$

$$7.\ \int \frac{dx}{x^m R_2^n} = -\frac{1}{(m-1)\,ax^{m-1}R_2^{n-1}} - \frac{(m+2n-3)\,b}{(m-1)\,a}\int \frac{dx}{x^{m-2}R^n} - \frac{(m+4n-5)\,b}{(m-1)\,a}\int \frac{dx}{x^{m-4}R_2^n}.$$

Reduction formulas for R_k:

8. $$\int x^{m-1}R_k^n\,dx = \frac{x^m R_k^{n+1}}{ma} - \frac{(m+k+nk)b}{ma}\int x^{m+k-1}R_k^n\,dx - \frac{(m+2k+2kn)c}{ma}\int x^{m+2k-1}R_k^n\,dx.$$

9. $$\int x^{m-1}R_k^n\,dx = \frac{x^m R_k^n}{m} - \frac{bkn}{m}\int x^{m+k-1}R_k^{n-1}\,dx - \frac{2ckn}{m}\int x^{m+2k-1}R_k^{n-1}\,dx.$$

10. $$\int x^{m-1}R_k^n\,dx$$
$$= \begin{cases} \dfrac{x^{m-2k}R_k^{n+1}}{(m+2kn)\,c} - \dfrac{(m-2k)a}{(m+2kn)\,c}\displaystyle\int x^{m-2k-1}R_k^n\,dx - \dfrac{(m-k+kn)b}{(m+2kn)\,c}\displaystyle\int x^{m-k-1}R_k^n\,dx \\ \text{or} \\ \dfrac{x^m R_k^n}{m+2kn} + \dfrac{2kna}{m+2kn}\displaystyle\int x^{m-1}R_k^{n-1}\,dx + \dfrac{bkn}{m+2kn}\displaystyle\int x^{m+k-1}R_k^{n-1}\,dx. \end{cases}$$