! For an efficient use of these tables, first read `HowTo.pdf`.

T2.23C. Integrands of the form $\sqrt{\dfrac{d-x}{(a-x)(b-x)(c-x)}}$, $\sqrt{\dfrac{c-x}{(a-x)(b-x)(d-x)}}$, $\sqrt{\dfrac{b-x}{(a-x)(c-x)(d-x)}}$, and $\sqrt{\dfrac{a-x}{(b-x)(c-x)(d-x)}}$ on the intervals (y, c) and (c, y).

Notation used: $\gamma = \arcsin\sqrt{\dfrac{(b-d)(c-y)}{(c-d)(b-y)}}$, $\delta = \arcsin\sqrt{\dfrac{(b-d)(y-c)}{(b-c)(y-d)}}$,

$$q = \sqrt{\frac{(b-c)(a-d)}{(a-c)(b-d)}}, \quad r = \sqrt{\frac{(a-b)(c-d)}{(a-c)(b-d)}}.$$

1. $$\int_y^c \sqrt{\frac{x-d}{(a-x)(b-x)(c-x)}}\,dx = \frac{2}{\sqrt{(a-c)(b-d)}}\left\{(c-b)\Pi\left(\gamma,\, \frac{c-d}{b-d},\, r\right) + (b-d)F(\gamma,\, r)\right\},$$
$$a > b > c > y \geq d.$$

2. $$\int_c^y \sqrt{\frac{x-d}{(a-x)(b-x)(x-c)}}\,dx = \frac{2(c-d)}{\sqrt{(a-c)(b-d)}}\Pi\left(\delta,\, \frac{b-c}{b-d},\, q\right), \quad a > b \geq y > c > d.$$

3. $$\int_y^c \sqrt{\frac{c-x}{(a-x)(b-x)(x-d)}}\,dx = \frac{2(b-c)}{\sqrt{(a-c)(b-d)}}\left[\Pi\left(\gamma,\, \frac{c-d}{b-d},\, r\right) - F(\gamma,\, r)\right],$$
$$a > b > c > y \geq d.$$

4. $$\int_c^y \sqrt{\frac{x-c}{(a-x)(b-x)(x-d)}}\,dx = \frac{2(c-d)}{\sqrt{(a-c)(b-d)}}\left[\Pi\left(\delta,\, \frac{b-c}{b-d},\, q\right) - F(\delta,\, q)\right],$$
$$a > b \geq y > c > d.$$

5. $$\int_y^c \sqrt{\frac{b-x}{(a-x)(c-x)(x-d)}}\,dx = \frac{2(b-c)}{\sqrt{(a-c)(b-d)}}\Pi\left(\gamma,\, \frac{c-d}{b-d},\, r\right), \quad a > b > c > y \geq d.$$

6. $$\int_c^y \sqrt{\frac{b-x}{(a-x)(x-c)(x-d)}}\,dx = \frac{2}{\sqrt{(a-c)(b-d)}}\left[(d-c)\Pi\left(\delta,\,\frac{b-c}{b-d},\,q\right)+(b-d)F(\delta,\,q)\right]$$
$$a > b \geq y > c > d.$$

7. $$\int_y^c \sqrt{\frac{a-x}{(b-x)(c-x)(x-d)}}\,dx = \frac{2}{\sqrt{(a-c)(b-d)}}\left[(b-c)\Pi\left(\gamma,\,\frac{c-d}{b-d},\,r\right)+(a-b)F(\gamma,\,r)\right],$$
$$a > b > c > y \geq d.$$

8. $$\int_c^y \sqrt{\frac{a-x}{(b-x)(x-c)(x-d)}}\,dx = \frac{2}{\sqrt{(a-c)(b-d)}}\left[(d-c)\Pi\left(\delta,\,\frac{b-c}{b-d},\,q\right)+(a-d)F(\delta,\,q)\right],$$
$$a > b \geq y > c > d.$$