! For an efficient use of these tables, first read `HowTo.pdf`.

T3.38A. Integrands involving rational functions of $(a+b\,x)$ and trigonometric functions on the interval $(0,\infty)$.

1. $\displaystyle\int_0^\infty \frac{\sin x}{x}\,dx = \frac{\pi}{2}.$

2. $\displaystyle\int_0^\infty \frac{\sin(a\,x)}{x}\,dx = \frac{\pi}{2}\ \operatorname{sgn}\,a.$

3. $\displaystyle\int_0^\infty \left(\frac{\sin x}{x} - \frac{1}{1+x}\right)\frac{dx}{x} = 1-\gamma_e.$

4. $\displaystyle\int_0^\infty \left(\cos x - \frac{1}{1+x}\right)\frac{dx}{x} = -\gamma_e.$

5. $\displaystyle\int_0^\infty \frac{1-\cos ax}{x^2}\,dx = \frac{a\pi}{2}, \quad a \ge 0.$

6. $\displaystyle\int_0^\infty \left[\frac{\cos x - 1}{x^2} + \frac{1}{2(1+x)}\right]\frac{dx}{x} = \frac{1}{2}\gamma_e - \frac{3}{4}.$

7. $\displaystyle\int_0^\infty \left(\cos x - \frac{1}{1+x^2}\right)\frac{dx}{x} = -\gamma_e.$

8. $\displaystyle\int_0^\infty \frac{\cos ax - \cos bx}{x}\,dx = \ln\frac{b}{a}, \quad a>0,\ b>0.$

9. $\displaystyle\int_0^\infty \frac{a\ \sin bx - b\ \sin ax}{x^2}\,dx = ab\ \ln\frac{a}{b}, \quad a>0,\ b>0.$

10. $\displaystyle\int_0^\infty \frac{\cos ax - \cos bx}{x^2}\,dx = \frac{(b-a)\pi}{2}, \quad a \ge 0,\, b \ge 0.$

11. $$\int_0^\infty \frac{\sin x - x\,\cos x}{x^2}\,dx = 1.$$

12. $$\int_0^\infty \frac{\cos ax - \cos bx}{x(x+\beta)}\,dx$$
$$= \frac{1}{\beta}\left[\operatorname{Ci}(a\beta)\,\cos a\beta + \operatorname{Si}(a\beta)\,\sin a\beta - \operatorname{Ci}(b\beta)\,\cos b\beta - \operatorname{Si}(b\beta)\,\sin b\beta + \ln\frac{b}{a}\right],$$
$$a > 0,\ b > 0,\ |\arg\beta| < \pi.$$

13. $$\int_0^\infty \frac{\cos ax + x\,\sin ax}{1+x^2}\,dx = \pi e^{-a}, \quad a > 0.$$

14. $$\int_0^\infty \frac{\sin ax - ax\,\cos ax}{x^3}\,dx = \frac{\pi}{4}a^2\,\operatorname{sgn} a.$$

15. $$\int_0^\infty \frac{\cos ax - \cos bx}{x^2(x^2+\beta^2)}\,dx = \frac{\pi\left[(b-a)\beta + e^{-b\beta} - e^{-a\beta}\right]}{2\beta^3}, \quad a > 0,\ b > 0,\ |\arg\beta| < \pi.$$

16. $$\int_0^\infty \frac{1}{x}\sum_{k=1}^n a_k\,\cos b_k\,x\,dx = -\sum_{k=1}^n a_k\,\ln\,b_k, \quad b_k > 0,\ \sum_{k=1}^n a_k = 0.$$

17. $$\int_0^\infty \frac{(1-\cos ax)\,\sin bx}{x^2}\,dx = \frac{b}{2}\,\ln\,\frac{b^2-a^2}{b^2} + \frac{a}{2}\,\ln\,\frac{a+b}{a-b}, \quad a > 0,\ b > 0.$$

18. $$\int_0^\infty \frac{(1-\cos ax)\,\cos bx}{x}\,dx = \ln\,\frac{\sqrt{|a^2-b^2|}}{b}, \quad a > 0,\ b > 0,\ a \neq b.$$

19. $$\int_0^\infty \frac{(1-\cos ax)\,\cos bx}{x^2}\,dx = \begin{cases} \frac{\pi}{2}(a-b), & a < b \le 0, \\ 0, & 0 < a \le b. \end{cases}$$

20. $$\int_0^\infty \frac{(\cos a - \cos nax)\,\sin mx}{x}\,dx = \begin{cases} \frac{\pi}{2}(\cos a - 1), & m > na > 0, \\ \frac{\pi}{2}\,\cos a, & na > m. \end{cases}$$

21. $$\int_0^\infty \frac{\sin^2 ax - \sin^2 bx}{x}\,dx = \frac{1}{2}\,\ln\,\frac{a}{b}, \quad a > 0,\ b > 0.$$

22. $$\int_0^\infty \frac{x^3 - \sin^3 x}{x^5}\,dx = \frac{13}{32}\pi.$$

23. $$\int_0^\infty \frac{(3-4\,\sin^2 ax)\,\sin^2 ax}{x}\,dx = \frac{1}{2}\,\ln\,2, \quad a \text{ real},\ a \neq 0.$$

24. $$\int_0^\infty \frac{\sin x}{1-2a\,\cos x + a^2}\cdot\frac{dx}{x} = \frac{\pi}{4a}\left[\left|\frac{1+a}{1-a}\right| - 1\right], \quad a \text{ real},\ a \neq 0,\ a \neq 1.$$

25. $$\int_0^\infty \frac{\sin bx}{1-2a\,\cos x + a^3}\,\frac{dx}{x} = \begin{cases} \dfrac{\pi}{2}\,\dfrac{1+a-2a^{[b]+1}}{(1-a)^2(1-a)}, & b \neq 0,1,2,\dots, \\ \dfrac{\pi}{2}\,\dfrac{1+a-a^b-a^{b+1}}{(1-a^2)(1-a)}, & b = 0,1,2,\dots]. \end{cases} \quad 0 < a < 1.$$

26. $$\int_0^\infty \frac{\sin x\,\cos bx}{1-2a\,\cos x + a^2}\,\frac{dx}{x} = \begin{cases} \dfrac{\pi}{2(1-a)}a^{[b]}, & b \neq 0,1,2,\dots, \\ \dfrac{\pi}{2(1-a)}a^b + \dfrac{\pi}{4}a^{b-1}, & b = 1,2,3,\dots, \end{cases} \quad 0 < a < 1,\ b > 0.$$

27. $$\int_0^\infty \frac{(1-a\cos x)\sin bx}{1-2a\cos x + a^2}\cdot\frac{dx}{x} = \begin{cases} \dfrac{\pi}{2}\cdot\dfrac{1-a^{[b]+1}}{1-a}, & b \neq 1,2,3,\dots, \\ \dfrac{\pi}{2}\cdot\dfrac{1-a^b}{1-a} + \dfrac{\pi a^b}{4}, & b = 1,2,3,\dots, \end{cases} \quad 0 < a < 1,\ b > 0.$$

28. $$\int_0^\infty \frac{1}{1-2a\,\cos bx + a^2}\,\frac{dx}{\beta^2+x^2} = \frac{\pi}{2\beta(1-a^2)}\,\frac{1+ae^{-b\beta}}{1-ae^{-b\beta}}, \quad a^2 < 1,\ b \geq 0.$$

29. $$\int_0^\infty \frac{1}{1-2a\,\cos bx + a^2}\,\frac{dx}{\beta^2-x^2} = \frac{a\pi}{\beta(1-a^2)}\,\frac{\sin b\beta}{1-2a\,\cos b\beta + a^2}, \quad a^2 < 1,\ b > 0.$$

30. $$\int_0^\infty \frac{\sin bcx}{1-2a\,\cos bx + a^2}\,\frac{x\,dx}{\beta^2+x^2} = \frac{\pi}{2}\,\frac{e^{-\beta bc}-a^c}{(1-ae^{-b\beta})(1-ae^{b\beta})}, \quad a^2 < 1,\ b > 0,\ c > 0.$$

31. $$\int_0^\infty \frac{\sin bx}{1-2a\,\cos bx + a^2}\,\frac{x\,dx}{\beta^2+x^2} = \begin{cases} \dfrac{\pi}{2}\,\dfrac{1}{e^{b\beta}-a}, & a^2 < 1,\ b > 0, \\ \dfrac{\pi}{2a}\,\dfrac{1}{ae^{b\beta}-1}, & a^2 > 1,\ b > 0. \end{cases}$$

32. $$\int_0^\infty \frac{\sin bcx}{1-2a\,\cos bx + a^2}\,\frac{x\,dx}{\beta^2-x^2} = \frac{\pi}{2}\,\frac{a^c - \cos\beta bc}{1-2a\,\cos\beta b + a^2}, \quad a^2 < 1,\ b > 0,\ c > 0.$$

33. $$\int_0^\infty \frac{\cos bcx}{1-2a\,\cos bx + a^2}\,\frac{dx}{\beta^2-x^2} = \frac{\pi}{2\beta(1-a^2)}\,\frac{(1-a^2)\,\sin\beta bc + 2a^{c+1}\,\sin\beta b}{1-2a\,\cos\beta b + a^2},$$
$$a^2 < 1,\ b > 0,\ c > 0.$$

34. $$\int_0^\infty \frac{1-a\,\cos bx}{1-2a\,\cos bx+a^2}\,\frac{dx}{1+x^2}=\frac{\pi}{2}\,\frac{e^b}{e^b-a},\quad a^2<1,\ b>0.$$

35. $$\int_0^\infty \frac{\cos bx}{1-2a\,\cos x+a^2}\cdot\frac{dx}{x^2+\beta^2}=\frac{\pi(e^{\beta-\beta b}+ae^{\beta b})}{2\beta(1-a^2)(e^\beta-a)},\quad 0\le b<1,\ |a|<1,\ \Re\{\beta\}>0.$$

36. $$\int_0^\infty \frac{\sin bx\,\sin x}{1-2a\,\cos x+a^2}\cdot\frac{dx}{x^2+\beta^2}$$
$$=\begin{cases}\dfrac{\pi}{2\beta}\,\dfrac{\sinh b\beta}{e^\beta-a}, & 0\le b<1,\\ \dfrac{\pi}{4\beta(ae^\beta-1)}\left[a^m e^{\beta(m+1-b)}-e^{(1-b)\beta}\right] & \\ \quad-\dfrac{\pi}{4\beta(ae^{-\beta}-1)}\left[a^m e^{-(m+1-b)\beta}-e^{-(1-b)\beta}\right], & m\le b\le m+1,\end{cases}\quad 0<a<1,\ \Re\{\beta\}>0.$$

37. $$\int_0^\infty \frac{(\cos x-a)\,\cos bx}{1-2a\,\cos x+a^2}\cdot\frac{dx}{x^2+\beta^2}=\frac{\pi\,\cosh\beta b}{2\beta(e^\beta-a)},\quad 0\le b<1,\ |a|<1,\ \Re\{\beta\}>0.$$

38. $$\int_0^\infty \frac{\sin x}{(1-2a\,\cos 2x+a^2)^{n+1}}\,\frac{dx}{x}=\int_0^\infty \frac{\tan x}{(1-2a\,\cos 2x+a^2)^{n+1}}\,\frac{dx}{x}$$
$$=\int_0^\infty \frac{\tan x}{(1-2a\,\cos 4x+a^2)^{n+1}}\,\frac{dx}{x}=\frac{\pi}{2(1-a^2)^{2n+1}}\sum_{k=0}^{n}\binom{n}{k}^2 a^{2k}.$$

39. $$\int_0^\infty \frac{\sin x}{a\pm b\,\cos 2x}\cdot\frac{dx}{x}\begin{cases}\dfrac{\pi}{2\sqrt{a^2-b^2}}, & a^2>b^2,\\ 0, & a^2<b^2.\end{cases}$$

40. $$\int_0^\infty \frac{\tan x}{a+b\,\cos 2x}\cdot\frac{dx}{x}=\begin{cases}\dfrac{\pi}{2\sqrt{a^2-b^2}}, & a^2>b^2,\\ 0, & a^2<b^2,\end{cases}\quad a>0.$$

41. $$\int_0^\infty \frac{\tan x}{a+b\,\cos 4x}\cdot\frac{dx}{x}=\begin{cases}\dfrac{\pi}{2\sqrt{a^2-b^2}}, & a^2>b^2,\\ 0, & a^2<b^2,\end{cases}\quad a>0.$$

42. $$\int_0^\infty \frac{x(a\,\cos x+b)\,\sin x\,dx}{\cot^2 t+\cos^2 x}=2a\pi\,\ln\cos\frac{t}{2}+\pi bt\,\tan t.$$

43. $$\int_0^\infty \frac{x\,\sin x\,\cos x}{a-\sin^2 x}\,dx = \pi\left[\ln\left(1+\sqrt{\frac{a-1}{a}}\right)-\ln 2\right],\quad a>1.$$

44. $$\int_0^\infty \frac{1}{\beta^2\,\sin^2 ax+\gamma^2\,\cos^2 ax}\cdot\frac{dx}{x^2+\delta^2} = \frac{\pi\,\sinh(2a\delta)}{4\delta(\beta^2\,\sinh^2(a\delta)-\gamma^2\,\cosh^2(a\delta))}\left[\frac{\beta}{\gamma}-\frac{\gamma}{\beta}-\frac{2}{\sinh(2a\delta)}\right],$$
$$\left|\arg\frac{\beta}{\gamma}\right|<\pi,\ \Re\{\delta\}>0,\ a>0.$$

45. $$\int_0^\infty \frac{\sin x\,dx}{x(a^2\,\sin^2 x+b^2\,\cos^2 x)} = \frac{\pi}{2ab},\quad [ab>0].$$

46. $$\int_0^\infty \frac{\sin^2 x\,dx}{x(a^2\,\cos^2 x+b^2\,\sin^2 x)} = \frac{\pi}{2b(a+b)},\quad a>0,\,b>0.$$

47. $$\int_0^\infty \frac{\sin 2x}{a^2\,\cos^2 x+b^2\,\sin^2 x}\cdot\frac{dx}{x} = \frac{\pi}{a(a+b)},\quad a>0,\ b>0.$$

48. $$\int_0^\infty \frac{\sin 2ax}{\beta^2\,\sin^2 ax+\gamma^2\,\cos^2 ax}\cdot\frac{x\,dx}{x^2+\delta^2} = \frac{\pi}{2(\beta^2\,\sinh^2(a\delta)-\gamma^2\,\cosh^2(a\delta))}\left[\frac{\beta-\gamma}{\beta+\gamma}-e^{-2a\delta}\right],$$
$$a>0,\ \left|\arg\frac{\beta}{\gamma}\right|<\pi,\ \Re\{\delta\}>0.$$

49. $$\int_0^\infty \frac{(1-\cos x)\,\sin x}{a^2\,\cos^2 x+b^2\,\sin^2 x}\cdot\frac{dx}{x} = \frac{\pi}{2b(a+b)},\quad a>0,\ b>0.$$

50. $$\int_0^\infty \frac{\sin x\,\cos^2 x}{a^2\,\cos^2 x+b^2\,\sin^2 x}\cdot\frac{dx}{x} = \frac{\pi}{2a(a+b)},\quad a>0,\ b>0.$$

51. $$\int_0^\infty \frac{\sin^3 x}{a^2\,\cos^2 x+b^2\,\sin^2 x}\cdot\frac{dx}{x} = \frac{\pi}{2b}\cdot\frac{2}{a+b},\quad a>0,\ b>0.$$

52. $$\int_0^\infty \frac{\tan x}{a^2\,\cos^2 x+b^2\,\sin^2 x}\,\frac{dx}{x} = \frac{\pi}{2ab},\quad a>0,\ b>0.$$

53. $$\int_0^\infty \frac{\sin^2 x\,\tan x}{a^2\,\cos^2 x+b^2\,\sin^2 x}\cdot\frac{dx}{x} = \frac{\pi}{2b(a+b)},\quad a>0,\ b>0.$$

54. $$\int_0^\infty \frac{\tan x}{a^2\,\cos^2 2x+b^2\,\sin^2 2x}\cdot\frac{dx}{x} = \frac{\pi}{2ab},\quad a>0,\ b>0.$$

55. $\displaystyle\int_0^\infty \frac{\sin^2 2x\ \tan x}{a^2\ \cos^2 2x + b^2\ \sin^2 2x}\cdot\frac{dx}{x} = \frac{\pi}{2b}\cdot\frac{1}{a+b},\quad a>0,\ b>0.$

56. $\displaystyle\int_0^\infty \frac{\cos^2 2x\ \tan x}{a^2\ \cos^2 2x + b^2\ \sin^2 2x}\cdot\frac{dx}{x} = \frac{\pi}{2a}\cdot\frac{1}{a+b},\quad a>0,\ b>0.$

57. $\displaystyle\int_0^\infty \frac{\sin^2 x\ \cos x}{a^2\ \cos^2 2x + b^2\ \sin^2 2x}\cdot\frac{dx}{x\ \cos 4x} = -\frac{\pi}{8b}\,\frac{a}{a^2+b^2},\quad a>0,\ b>0.$

58. $\displaystyle\int_0^\infty \frac{\sin x}{a^2\ \cos^2 x + b^2\ \sin^2 x}\cdot\frac{dx}{x\ \cos 2x} = \frac{\pi}{2ab}\cdot\frac{b^2-a^2}{b^2+a^2},\quad a>0,\ b>0.$

59. $\displaystyle\int_0^\infty \frac{\sin x\ \cos x}{a^2\ \cos^2 x + b^2\ \sin^2 x}\cdot\frac{dx}{x\ \cos 2x} = \frac{\pi}{2a}\cdot\frac{b}{a^2+b^2},\quad a>0,\ b>0.$

60. $\displaystyle\int_0^\infty \frac{\sin x\ \cos^2 x}{a^2\ \cos^2 x + b^2\ \sin^2 x}\cdot\frac{dx}{x\ \cos 2x} = \frac{\pi}{2ab}\cdot\frac{b^2}{a^2+b^2},\quad a>0,\ b>0.$

61. $\displaystyle\int_0^\infty \frac{\sin^3 x}{a^2\ \cos^2 x + b^2\ \sin^2 x}\cdot\frac{dx}{x\ \cos 2x} = -\frac{\pi}{2b}\cdot\frac{a}{a^2+b^2},\quad a>0,\ b>0.$

62. $\displaystyle\int_0^\infty \frac{1-\cos x}{a^2\ \cos^2 x + b^2\ \sin^2 x}\cdot\frac{dx}{x\ \sin x} = \frac{\pi}{2ab},\quad a>0,\ b>0.$

63. $\displaystyle\int_0^\infty \frac{\sin x}{(a^2\ \cos^2 x + b^2\ \sin^2 x)^2}\cdot\frac{dx}{x} = \frac{\pi}{4}\cdot\frac{a^2+b^2}{a^3b^3},\quad ab>0.$

64. $\displaystyle\int_0^\infty \frac{\sin x\ \cos x}{(a^2\ \cos^2 x + b^2\ \sin^2 x)^2}\cdot\frac{dx}{x} = \frac{\pi}{4a^3b},\quad ab>0.$

65. $\displaystyle\int_0^\infty \frac{\sin^3 x}{(a^2\ \cos^2 x + b^2\ \sin^2 x)^2}\cdot\frac{dx}{x} = \frac{\pi}{4ab^3},\quad ab>0.$

66. $\displaystyle\int_0^\infty \frac{\sin x\ \cos^2 x}{(a^2\ \cos^2 x + b^2\ \sin^2 x)^2}\cdot\frac{dx}{x} = \frac{\pi}{4a^3b},\quad ab>0.$

67. $\displaystyle\int_0^\infty \frac{\tan x}{(a^2\ \cos^2 x + b^2\ \sin^2 x)^2}\cdot\frac{dx}{x} = \frac{\pi}{4}\cdot\frac{a^2+b^2}{a^3b^3},\quad ab>0.$

68. $$\int_0^\infty \frac{\tan x}{(a^2\ \cos^2 2x + b^2\ \sin^2 2x)^2} \cdot \frac{dx}{x} = \frac{\pi}{4}\ \frac{a^2+b^2}{a^3b^3}, \quad ab > 0.$$

69. $$\int_0^\infty \frac{\sin^2 x\ \tan x}{(a^2\ \cos^2 x + b^2\ \sin^2 x)^2} \cdot \frac{dx}{x} = \frac{\pi}{4ab^3}, \quad ab > 0.$$

70. $$\int_0^\infty \frac{\tan x\ \cos^2 2x}{(a^2\ \cos^2 2x + b^2\ \sin^2 2x)^2} \cdot \frac{dx}{x} = \frac{\pi}{4a^3b}, \quad ab > 0.$$

71. $$\int_0^\infty \frac{\sin x}{(a^2\ \cos^2 x + b^2\ \sin^2 x)^3} \cdot \frac{dx}{x} = \frac{\pi}{16} \cdot \frac{3a^4+2a^2b^2+3b^4}{a^5b^5}, \quad ab > 0.$$

72. $$\int_0^\infty \frac{\sin x\ \cos x}{(a^2\ \cos^2 x + b^2\ \sin^2 x)^3} \cdot \frac{dx}{x} = \frac{\pi}{16} \cdot \frac{a^2+3b^2}{a^5b^3}, \quad ab > 0.$$

73. $$\int_0^\infty \frac{\sin x\ \cos^2 x}{(a^2\ \cos^2 x + b^2\ \sin^2 x)^3} \cdot \frac{dx}{x} = \frac{\pi}{16} \cdot \frac{a^2+3b^2}{a^5b^3}, \quad ab > 0.$$

74. $$\int_0^\infty \frac{\sin^3 x}{(a^2\ \cos^2 x + b^2\ \sin^2 x)^3} \cdot \frac{dx}{x} = \frac{\pi}{16} \cdot \frac{3a^2+b^2}{a^3b^5}, \quad ab > 0.$$

75. $$\int_0^\infty \frac{\sin^3\ x\ \cos x}{(a^2\ \cos^2 2x + b^2\ \sin^2 2x)^3} \cdot \frac{dx}{x} = \frac{\pi}{64} \cdot \frac{3a^2+b^2}{a^3b^5}, \quad ab > 0.$$

76. $$\int_0^\infty \frac{\tan x}{(a^2\ \cos^2 x + b^2\ \sin^2 x)^3} \cdot \frac{dx}{x} = \frac{\pi}{16}\ \frac{3a^4+2a^2b^2+3b^4}{a^5b^5}, \quad ab > 0.$$

77. $$\int_0^\infty \frac{\sin^2 x\ \tan x}{(a^2\ \cos^2 x + b^2\ \sin^2 x)^3} \cdot \frac{dx}{x} = \frac{\pi}{16} \cdot \frac{3a^2+b^2}{a^3b^5}, \quad ab > 0.$$

78. $$\int_0^\infty \frac{\tan x}{(a^2\ \cos^2 2x + b^2\ \sin^2 2x)^3} \cdot \frac{dx}{x} = \frac{\pi}{16} \cdot \frac{3a^4+2a^2b^2+3b^4}{a^5b^5}, \quad ab > 0.$$

79. $$\int_0^\infty \frac{\tan x\ \cos^2 2x}{(a^2\ \cos^2 2x + b^2\ \sin^2 2x)^3} \cdot \frac{dx}{x} = \frac{\pi}{16} \cdot \frac{a^2+3b^2}{a^5b^3}, \quad ab > 0.$$

80. $$\int_0^\infty \frac{\sin x}{(a^2\ \cos^2 x + b^2\ \sin^2 x)^4} \cdot \frac{dx}{x} = \frac{\pi}{32} \cdot \frac{5a^6+3a^4b^2+3a^2b^4+5b^6}{a^7b^7}, \quad ab > 0.$$

81. $$\int_0^\infty \frac{\sin x\ \cos x}{(a^2\ \cos^2 x + b^2\ \sin^2 x)^4} \cdot \frac{dx}{x} = \frac{\pi}{32} \cdot \frac{a^4 + 2a^2b^2 + 5b^4}{a^7b^5}, \quad ab > 0.$$

82. $$\int_0^\infty \frac{\sin x\ \cos^2 x}{(a^2\ \cos^2 x + b^2\ \sin^2 x)^4} \cdot \frac{dx}{x} = \frac{\pi}{32} \cdot \frac{a^4 + 2a^2b^2 + 5b^4}{a^7b^5}, \quad ab > 0.$$

83. $$\int_0^\infty \frac{\sin^3 x}{(a^2\ \cos^2 x + b^2\ \sin^2 x)^4} \cdot \frac{dx}{x} = \frac{\pi}{32} \cdot \frac{5a^4 + a^2b^2 + b^4}{a^5b^7}, \quad ab > 0.$$

84. $$\int_0^\infty \frac{\sin^3 x\ \cos x}{(a^2\ \cos^2 x + b^2\ \sin^2 x)^4} \cdot \frac{dx}{x} = \frac{\pi}{32} \cdot \frac{a^2 + b^2}{a^5b^5}, \quad ab > 0.$$

85. $$\int_0^\infty \frac{\sin x\ \cos^3 x}{(a^2\ \cos^2 x + b^2\ \sin^2 x)^4} \cdot \frac{dx}{x} = \frac{\pi}{32} \cdot \frac{a^2 + 5b^2}{a^7b^3}, \quad ab > 0.$$

86. $$\int_0^\infty \frac{\sin^3 x\ \cos^2 x}{(a^2\ \cos^2 x + b^2\ \sin^2 x)^4} \cdot \frac{dx}{x} = \frac{\pi}{32} \cdot \frac{a^2 + b^2}{a^5b^5}, \quad ab > 0].$$

87. $$\int_0^\infty \frac{\sin x\ \cos^4 x}{(a^2\ \cos^2 x + b^2\ \sin^2 x)^4} \cdot \frac{dx}{x} = \frac{\pi}{32} \cdot \frac{a^2 + 5b^2}{a^7b^3}, \quad ab > 0.$$

88. $$\int_0^\infty \frac{\sin^5 x}{(a^2\ \cos^2 x + b^2\ \sin^2 x)^4} \cdot \frac{dx}{x} = \frac{\pi}{32} \cdot \frac{5a^2 + b^2}{a^3b^7}, \quad ab > 0.$$

89. $$\int_0^\infty \frac{\sin^3 x\ \cos x}{(a^2\ \cos^2 2x + b^2\ \sin^2 2x)^4} \cdot \frac{dx}{x} = \frac{\pi}{128} \cdot \frac{5a^4 + 2a^2b^2 + b^4}{a^5b^7}, \quad ab > 0.$$

90. $$\int_0^\infty \frac{\sin^5 x\ \cos^3 x}{(a^2\ \cos^2 2x + b^2\ \sin^2 2x)^4} \cdot \frac{dx}{x} = \frac{\pi}{512} \cdot \frac{5a^2 + b^2}{a^3b^7}, \quad ab > 0.$$

91. $$\int_0^\infty \frac{\sin^2 x\ \tan x}{(a^2\ \cos^2 x + b^2\ \sin^2\ x)^4} \cdot \frac{dx}{x} = \frac{\pi}{32} \cdot \frac{5a^4 + 2a^2b^2 + b^4}{a^5b^7}, \quad ab > 0.$$

92. $$\int_0^\infty \frac{\sin^4\ x\ \tan x}{(a^2\ \cos^2 x + b^2\ \sin^2\ x)^4} \cdot \frac{dx}{x} = \frac{\pi}{32} \cdot \frac{5a^2 + b^2}{a^3b^7}, \quad ab > 0.$$

93. $$\int_0^\infty \frac{\cos^2 2x\ \tan x}{(a^2\ \cos^2 2x + b^2\ \sin^2\ 2x)^4} \cdot \frac{dx}{x} = \frac{\pi}{32} \cdot \frac{a^4 + 2a^2b^2 + 5b^4}{a^7b^5}, \quad ab > 0.$$

94. $$\int_0^\infty \frac{\sin^3\,4x\,\tan x}{(a^2\,\cos^2 2x+b^2\,\sin^2\,2x)^4}\cdot\frac{dx}{x}=\frac{\pi}{8}\cdot\frac{a^2+b^2}{a^5b^5},\quad ab>0.$$

95. $$\int_0^\infty \frac{\cos^4 2x\,\tan x}{(a^2\,\cos^2 2x+b^2\,\sin^2\,2x)^4}\cdot\frac{dx}{x}=\frac{\pi}{32}\cdot\frac{a^2+5b^2}{a^7b^3},\quad ab>0.$$
