! For an efficient use of these tables, first read `HowTo.pdf`.

T2.47A. Integrands involving different forms of powers of trigonometric functions on the interval $(0, \pi/4)$.

1. $$\int_0^{\pi/4} (\sin^n 2x - 1)\, \tan\left(\frac{\pi}{4} + x\right)\, dx = \int_0^{\pi/4} (\cos^n 2x - 1)\, \cot x\, dx = -\frac{1}{2}\sum_{k=1}^{n} \frac{1}{k}$$
$$= -\frac{1}{2}\left[\gamma_e + \psi(n+1)\right], \quad n \geq 0.$$

2. $$\int_0^{\pi/4} (\sin^\mu 2x - 1)\, \csc^\mu 2x\, \tan\left(\frac{\pi}{4} + x\right)\, dx = \int_0^{\pi/4} (\cos^\mu 2x - 1)\, \sec^\mu 2x\, \cot x\, dx$$
$$= \frac{1}{2}\left[\gamma_e + \psi(1-\mu)\right], \quad \Re\{\mu\} < 1.$$

3. $$\int_0^{\pi/4} (\sin^{2\mu} 2x - 1)\, \csc^\mu 2x\, \tan\left(\frac{\pi}{4} + x\right)\, dx = \int_0^{\pi/4} (\cos^{2\mu} 2x - 1)\, \sec^\mu 2x\, \cot x\, dx$$
$$= -\frac{1}{2\mu} + \frac{\pi}{2}\, \cot \mu\pi.$$

4. $$\int_0^{\pi/4} (1 - \sec^\mu 2x)\, \cot x\, dx = \int_0^{\pi/4} (1 - \csc^\mu 2x)\, \tan\left(\frac{\pi}{4} + x\right)\, dx = \frac{1}{2}[\gamma_e + \psi(1-\mu)], \quad \Re\{\mu\} < 1.$$

5. $$\int_0^{\pi/4} \frac{(\cot^\mu x - 1)\, dx}{(\cos x - \sin x)\, \sin x} = \int_0^{\pi/2} \frac{(\tan^\mu x - 1)\, dx}{(\sin x - \cos x)\, \cos x} = -\gamma_e - \psi(1-\mu), \quad \Re\{\mu\} < 1.$$

6. $$\int_0^{\pi/4} (\sin^{\mu-1} 2x - \sin^{\nu-1} 2x)\, \tan\left(\frac{\pi}{4} + x\right)\, dx = \int_0^{\pi/4} (\cos^{\mu-1} 2x - \cos^{\nu-1} 2x)\, \cot x\, dx$$
$$= \frac{1}{2}[\psi(\nu) - \psi(\mu)], \quad \Re\{\mu\} > 0,\ \Re\{\nu\} > 0.$$

7. $$\int_0^{\pi/4} (\sin^\mu 2x - \csc^\mu 2x)\cot\left(\frac{\pi}{4}+x\right)dx = \int_0^{\pi/4} (\cos^\mu 2x - \sec^\mu 2x)\tan x\,dx$$
$$= \frac{1}{2\mu} - \frac{\pi}{2}\csc\mu\pi, \quad |\Re\{\mu\}| < 1.$$

8. $$\int_0^{\pi/4} (\sin^\mu 2x - \csc^\mu 2x)\tan\left(\frac{\pi}{4}+x\right)dx = \int_0^{\pi/4} (\cos^\mu 2x - \sec^\mu 2x)\cot x\,dx$$
$$= -\frac{1}{2\mu} + \frac{\pi}{2}\cot\mu\pi, \quad |\Re\{\mu\}| < 1.$$

9. $$\int_0^{\pi/4} (\sin^{\mu-1} 2x + \csc^\mu 2x)\cot\left(\frac{\pi}{4}+x\right)dx = \int_0^{\pi/4} (\cos^{\mu-1} 2x + \sec^\mu 2x)\tan x\,dx = \frac{\pi}{4}\csc\mu\pi$$
$$0 < \Re\{\mu\} < 1.$$

10. $$\int_0^{\pi/4} (\sin^{\mu-1} 2x - \csc^\mu 2x)\tan\left(\frac{\pi}{4}+x\right)dx = \int_0^{\pi/4} (\cos^{\mu-1} 2x - \sec^\mu 2x)\cot x\,dx = \frac{\pi}{2}\cot\mu\pi,$$
$$0 < \Re\{\mu\} < 1.$$

11. $$\int_0^{\pi/4} \frac{\tan^\nu x - \tan^\mu x}{\cos x - \sin x}\cdot\frac{dx}{\sin x} = \psi(\mu) - \psi(\nu), \quad \Re\{\mu\} > 0, \Re\{\nu\} > 0.$$

12. $$\int_0^{\pi/4} \frac{\tan^\mu x - \tan^{1-\mu} x}{\cos x - \sin x}\cdot\frac{dx}{\sin x} = \pi\cot\mu\pi, \quad 0 < \Re\{\mu\} < 1.$$

13. $$\int_0^{\pi/4} (\tan^\mu x + \cot^\mu x)\,dx = \frac{\pi}{2}\sec\frac{\mu\pi}{2}, \quad |\Re\{\mu\}| < 1.$$

14. $$\int_0^{\pi/4} (\tan^\mu x - \cot^\mu x)\tan x\,dx = \frac{1}{\mu} - \frac{\pi}{2}\csc\frac{\mu\pi}{2}, \quad 0 < \Re\{\mu\} < 2.$$

15. $$\int_0^{\pi/4} \frac{\tan^{\mu-1} x - \cot^{\mu-1} x}{\cos 2x}\,dx = \frac{\pi}{2}\cot\frac{\mu\pi}{2}, \quad |\Re\{\mu\}| < 2.$$

16. $$\int_0^{\pi/4} \frac{\tan^\mu x - \cot^\mu x}{\cos 2x}\tan x\,dx = -\frac{1}{\mu} + \frac{\pi}{2}\cot\frac{\mu\pi}{2}, \quad -2 < \Re\{\mu\} < 0.$$

17. $$\int_0^{\pi/4} \frac{\tan^\mu x + \cot^\mu x}{1 + \cos t\,\sin 2x}\,dx = \pi\csc t\,\csc\mu\pi\,\sin\mu t, \quad t \neq n\pi, |\Re\{\mu\}| < 1.$$

18. $$\int_0^{\pi/4} \frac{\tan^{\mu-1} x + \cot^{\mu} x}{(\sin x + \cos x)\cos x}\,dx = \pi \csc \mu\pi, \quad 0 < \Re\{\mu\} < 1.$$

19. $$\int_0^{\pi/4} \frac{\tan^{\mu} x - \cot^{\mu} x}{(\sin x + \cos x)\cos x}\,dx = -\pi \csc \mu\pi + \frac{1}{\mu}, \quad 0 < \Re\{\mu\} < 1.$$

20. $$\int_0^{\pi/4} \frac{\tan^{\nu} x - \cot^{\mu} x}{(\cos x - \sin x)\cos x}\,dx = \psi(1-\mu) - \psi(1+\nu), \quad \Re\{\mu\} < 1,\ \Re\{\nu\} > -1.$$

21. $$\int_0^{\pi/4} \frac{\tan^{\mu-1} x - \cot^{\mu} x}{(\cos x - \sin x)\cos x}\,dx = \pi \cot \mu\pi, \quad 0 < \Re\{\mu\} < 1.$$

22. $$\int_0^{\pi/4} \frac{\tan^{\mu} x - \cot^{\mu} x}{(\cos x - \sin x)\cos x}\,dx = \pi \cot \mu\pi - \frac{1}{\mu}, \quad 0 < \Re\{\mu\} < 1.$$

23. $$\int_0^{\pi/4} \frac{1}{\tan^{\mu} x + \cot^{\mu} x} \cdot \frac{dx}{\sin 2x} = \frac{\pi}{8\mu}, \quad \Re\{\mu\} \neq 0.$$

24. $$\int_0^{\pi/4} (\tan^{\mu} x - \cot^{\mu} x)(\tan^{\nu} x - \cot^{\nu} x)\,dx = \frac{2\pi \sin\frac{\mu\pi}{2} \sin\frac{\nu\pi}{2}}{\cos\mu\pi + \cos\nu\pi}, \quad |\Re\{\mu\}| < 1,\ |\Re\{\nu\}| < 1.$$

25. $$\int_0^{\pi/4} (\tan^{\mu} x + \cot^{\mu} x)(\tan^{\nu} x + \cot^{\nu} x)\,dx = \frac{2\pi \cos\frac{\mu\pi}{2} \cos\frac{\nu\pi}{2}}{\cos\mu\pi + \cos\nu\pi}, \quad |\Re\{\mu\}| < 1,\ |\Re\{\nu\}| < 1.$$

26. $$\int_0^{\pi/4} \frac{(\tan^{\mu} x - \cot^{\mu} x)(\tan^{\nu} x + \cot^{\nu} x)}{\cos 2x}\,dx = -\frac{\pi \sin\mu\pi}{\cos\mu\pi + \cos\nu\pi}, \quad |\Re\{\mu\}| < 1, |\Re\{\nu\}| < 1.$$

27. $$\int_0^{\pi/4} \frac{\tan^{\nu} x - \cot^{\nu} x}{\tan^{\mu} x - \cot^{\mu} x} \cdot \frac{dx}{\sin 2x} = \frac{\pi}{4\mu} \tan\frac{\nu\pi}{2\mu}, \quad 0 < \Re\{\nu\} < 1.$$

28. $$\int_0^{\pi/4} \frac{\tan^{\nu} x + \cot^{\nu} x}{\tan^{\mu} x + \cot^{\mu} x} \cdot \frac{dx}{\sin 2x} = \frac{\pi}{4\mu} \sec\frac{\nu\pi}{2\mu}, \quad 0 < \Re\{\nu\} < 1.$$