

T1.30. Integrand involving hyperbolic functions, and their powers and products.

$$1. \int \sinh ax \, dx = \frac{1}{a} \cosh ax.$$

$$2. \int \sinh^2 ax \, dx = \frac{1}{4a} \sinh 2ax - \frac{x}{2}.$$

$$3. \int \sinh^3 x \, dx = -\frac{3}{4} \cosh x + \frac{1}{12} \cosh 3x = \frac{1}{3} \cosh^3 x - \cosh x.$$

$$4. \int \sinh^4 x \, dx = \frac{3}{8}x - \frac{1}{4} \sinh 2x + \frac{1}{32} \sinh 4x = \frac{3}{8}x - \frac{3}{8} \sinh x \cosh x + \frac{1}{4} \sinh^3 x \cosh x.$$

$$5. \int \sinh^5 x \, dx = \begin{cases} \frac{5}{8} \cosh x - \frac{5}{48} \cosh 3x + \frac{1}{80} \cosh 5x, \\ \text{or} \\ \frac{4}{5} \cosh x + \frac{1}{5} \sinh^4 x \cosh x - \frac{4}{15} \cosh^3 x. \end{cases}$$

$$6. \int \sinh^6 x \, dx = \begin{cases} -\frac{5}{16}x + \frac{15}{64} \sinh 2x - \frac{3}{64} \sinh 4x + \frac{1}{192} \sinh 6x, \\ \text{or} \\ -\frac{5}{16}x + \frac{1}{6} \sinh^5 x \cosh x - \frac{5}{24} \sinh^3 x \cosh x + \frac{5}{16} \sinh x \cosh x. \end{cases}$$

$$7. \int \sinh^7 x \, dx = \begin{cases} -\frac{35}{64} \cosh x + \frac{7}{64} \cosh 3x - \frac{7}{320} \cosh 5x + \frac{1}{448} \cosh 7x, \\ \text{or} \\ = -\frac{24}{35} \cosh x + \frac{8}{35} \cosh^3 x - \frac{6}{35} \cosh x \sinh^4 x + \frac{1}{7} \cosh x \sinh^6 x. \end{cases}$$

$$8. \int \cosh ax \, dx = \frac{1}{a} \sinh ax.$$

$$9. \int \cosh^2 ax \, dx = \frac{x}{2} + \frac{1}{4a} \sinh 2ax.$$

$$10. \int \cosh^3 x \, dx = \frac{3}{4} \sinh x + \frac{1}{12} \sinh 3x = \sinh x + \frac{1}{3} \sinh^3 x.$$

$$11. \int \cosh^4 x \, dx = \frac{3}{8}x + \frac{1}{4} \sinh 2x + \frac{1}{32} \sinh 4x = \frac{3}{8}x + \frac{3}{8} \sinh x \cosh x + \frac{1}{4} \sinh x \cosh^3 x.$$

$$12. \int \cosh^5 x \, dx = \begin{cases} \frac{5}{8} \sinh x + \frac{5}{48} \sinh 3x + \frac{1}{80} \sinh 5x, \\ \text{or} \\ = \frac{4}{5} \sinh x + \frac{1}{5} \cosh^4 x \sinh x + \frac{4}{15} \sinh^3 x. \end{cases}$$

$$13. \int \cosh^6 x \, dx = \begin{cases} \frac{5}{16}x + \frac{15}{64} \sinh 2x + \frac{3}{64} \sinh 4x + \frac{1}{192} \sinh 6x, \\ \text{or} \\ = \frac{5}{16}x + \frac{5}{16} \sinh x \cosh x + \frac{5}{24} \sinh x \cosh^3 x + \frac{1}{6} \sinh x \cosh^5 x. \end{cases}$$

$$14. \int \cosh^7 x \, dx = \begin{cases} \frac{35}{64} \sinh x + \frac{7}{64} \sinh 3x + \frac{7}{320} \sinh 5x + \frac{1}{448} \sinh 7x, \\ \text{or} \\ = \frac{24}{35} \sinh x + \frac{8}{35} \sinh^3 x + \frac{6}{35} \sinh x \cosh^4 x + \frac{1}{7} \sinh x \cosh^6 x. \end{cases}$$

$$15. \int \sinh^{2m} x \, dx = (-1)^m \binom{2m}{m} \frac{x}{2^{2m}} + \frac{1}{2^{2m-1}} \sum_{k=0}^{m-1} (-1)^k \binom{2m}{k} \frac{\sinh(2m-2k)x}{2m-2k}.$$

$$16. \int \sinh^{2m+1} x \, dx = \begin{cases} \frac{1}{2^{2m}} \sum_{k=0}^m (-1)^k \binom{2m+1}{k} \frac{\cosh(2m-2k+1)x}{2m-2k+1}, \\ \text{or} \\ = (-1)^n \sum_{k=0}^m (-1)^k \binom{m}{k} \frac{\cosh^{2k+1} x}{2k+1}. \end{cases}$$

$$17. \int \sinh^p x \cosh^{2n} x \, dx = \frac{\sinh^{p+1} x}{2n+p} \left\{ \cosh^{2n-1} x + \sum_{k=1}^{n-1} \frac{(2n-1)(2n-3) \dots (2n-2k+1)}{(2n+p-2)(2n+p-4) \dots (2n+p-2k)} \cosh^{2n-2k-1} x \right\} \\ + \frac{(2n-1)!!}{(2n+p)(2n+p-2) \dots (p+2)} \int \sinh^p x \, dx,$$

where p is real and $p \neq -2, -4, \dots, -2n$.

$$18. \int \sinh^p x \cosh^{2n+1} x \, dx = \frac{\sinh^{p+1} x}{2n+p+1} \left\{ \cosh^{2n} x + \sum_{k=1}^n \frac{2^k n(n-1) \dots (n-k+1) \cosh^{2n-2k} x}{(2n+p-1)(2n+p-3) \dots (2n+p-2k+1)} \right\},$$

where p is real and $p \neq -1, -3, \dots, -(2n+1)$.

$$19. \int \cosh^{2m} x \, dx = \binom{2m}{m} \frac{x}{2^{2m}} + \frac{1}{2^{2m-1}} \sum_{k=0}^{m-1} \binom{2m}{k} \frac{\sinh(2m-2k)x}{2m-2k}.$$

$$20. \int \cosh^{2m+1} x \, dx = \begin{cases} \frac{1}{2^{2m}} \sum_{k=0}^m \binom{2m+1}{k} \frac{\sinh(2m-2k+1)x}{2m-2k+1}, \\ \text{or} \\ \sum_{k=0}^m \binom{m}{k} \frac{\sinh^{2k+1} x}{2k+1}. \end{cases}$$

$$\begin{aligned}
21. \int \cosh^p x \sinh^{2n} x \, dx &= \frac{\cosh^{p+1} x}{2n+p} \left\{ \sinh^{2n-1} x \right. \\
&\quad \left. + \sum_{k=1}^{n-1} (-1)^k \frac{(2n-1)(2n-3) \dots (2n-2k+1) \sinh^{2n-2k-1} x}{(2n+p-2)(2n+p-4) \dots (2n+p-2k)} \right\} \\
&\quad + (-1)^n \frac{(2n-1)!!}{(2n+p)(2n+p-2) \dots (p+2)} \int \cosh^p x \, dx,
\end{aligned}$$

where p is real and $p \neq -2, -4, \dots, -2n$.

$$\begin{aligned}
22. \int \cosh^p x \sinh^{2n+1} x \, dx &= \frac{\cosh^{p+1} x}{2n+p+1} \left\{ \sinh^{2n} x \right. \\
&\quad \left. + \sum_{k=1}^n (-1)^k \frac{2^k n(n-1) \dots (n-k+1) \sinh^{2n-2k} x}{(2n+p-1)(2n+p-3) \dots (2n+p-2k+1)} \right\},
\end{aligned}$$

where p is real and $p \neq -1, -3, \dots, -(2n+1)$.

$$23. \int \sinh^p x \cosh^q x \, dx = \begin{cases} \frac{\sinh^{p+1} x \cosh^{q-1} x}{p+q} + \frac{q-1}{p+q} \int \sinh^p x \cosh^{q-2} x \, dx, \\ \text{or} \\ \frac{\sinh^{p-1} x \cosh^{q+1} x}{p+q} - \frac{p-1}{p+q} \int \sinh^{p-2} x \cosh^q x \, dx, \\ \text{or} \\ \frac{\sinh^{p-1} x \cosh^{q+1} x}{q+1} - \frac{p-1}{q+1} \int \sinh^{p-2} x \cosh^{q+2} x \, dx, \\ \frac{\sinh^{p+1} x \cosh^{q-1} x}{p+1} - \frac{q-1}{p+1} \int \sinh^{p+2} x \cosh^{q-2} x \, dx, \\ \text{or} \\ \frac{\sinh^{p+1} x \cosh^{q+1} x}{p+1} - \frac{p+q+2}{p+1} \int \sinh^{p+2} x \cosh^q x \, dx, \\ \text{or} \\ -\frac{\sinh^{p+1} x \cosh^{q+1} x}{q+1} + \frac{p+q+2}{q+1} \int \sinh^p x \cosh^{q+2} x \, dx. \end{cases}$$

$$24. \int \sinh ax \cosh bx \, dx = \frac{\cosh(a+b)x}{2(a+b)} + \frac{\cosh(a-b)x}{2(a-b)}.$$

$$25. \int \sinh ax \cosh ax \, dx = \frac{1}{4a} \cosh 2ax.$$

$$26. \int \sinh^2 x \cosh x \, dx = \frac{1}{3} \sinh^3 x.$$

$$27. \int \sinh^3 x \cosh x \, dx = \frac{1}{4} \sinh^4 x.$$

$$28. \int \sinh^4 x \cosh x \, dx = \frac{1}{5} \sinh^5 x.$$

$$29. \int \sinh x \cosh^2 x \, dx = \frac{1}{3} \cosh^3 x.$$

$$30. \int \sinh^2 x \cosh^2 x \, dx = -\frac{x}{8} + \frac{1}{32} \sinh 4x.$$

$$31. \int \sinh^3 x \cosh^2 x \, dx = \frac{1}{5} \left(\sinh^2 x - \frac{2}{3} \right) \cosh^3 x.$$

$$32. \int \sinh^4 x \cosh^2 x \, dx = \frac{x}{16} - \frac{1}{64} \sinh 2x - \frac{1}{64} \sinh 4x + \frac{1}{192} \sinh 6x.$$

$$33. \int \sinh x \cosh^3 x \, dx = \frac{1}{4} \cosh^4 x.$$

$$34. \int \sinh^2 x \cosh^3 x \, dx = \frac{1}{5} \left(\cosh^2 x + \frac{2}{3} \right) \sinh^3 x.$$

$$35. \int \sinh^3 x \cosh^3 x \, dx = \begin{cases} -\frac{3}{64} \cosh 2x + \frac{1}{192} \cosh 6x = \frac{1}{48} \cosh^3 2x - \frac{1}{16} \cosh 2x, \\ \text{or} \\ \frac{\sinh^6 x}{6} + \frac{\sinh^4 x}{4}, \\ \text{or} \\ \frac{\cosh^6 x}{6} - \frac{\cosh^4 x}{4}. \end{cases}$$

$$36. \int \sinh^4 x \cosh^3 x \, dx = \begin{cases} \frac{1}{7} \sinh^3 x \left(\cosh^4 x - \frac{3}{5} \cosh^2 x - \frac{2}{5} \right), \\ \text{or} \\ \frac{1}{7} \left(\cosh^2 x + \frac{2}{5} \right) \sinh^5 x. \end{cases}$$

$$37. \int \sinh x \cosh^4 x \, dx = \frac{1}{5} \cosh^5 x.$$

$$38. \int \sinh^2 x \cosh^4 x \, dx = -\frac{x}{16} - \frac{1}{64} \sinh 2x + \frac{1}{64} \sinh 4x + \frac{1}{192} \sinh 6x.$$

$$39. \int \sinh^3 x \cosh^4 x \, dx = \begin{cases} \frac{1}{7} \cosh^3 x \left(\sinh^4 x + \frac{3}{5} \sinh^2 x - \frac{2}{5} \right), \\ \text{or} \\ \frac{1}{7} \left(\sinh^2 x - \frac{2}{5} \right) \cosh^5 x. \end{cases}$$

$$40. \int \sinh^4 x \cosh^4 x \, dx = \frac{3x}{128} - \frac{1}{128} \sinh 4x + \frac{1}{1024} \sinh 8x.$$

41. $\int \frac{dx}{\sinh x} = \ln \tanh \frac{x}{2} = \frac{1}{2} \ln \frac{\cosh x - 1}{\cosh x + 1}.$
42. $\int \frac{dx}{\sinh^2 x} = -\coth x.$
43. $\int \frac{dx}{\sinh^3 x} = -\frac{\cosh x}{2 \sinh^2 x} - \frac{1}{2} \ln \tanh \frac{x}{2}.$
44. $\int \frac{dx}{\sinh^4 x} = -\frac{\cosh x}{3 \sinh^3 x} + \frac{2}{3} \coth x = -\frac{1}{3} \coth^3 x + \coth x.$
45. $\int \frac{dx}{\sinh^5 x} = -\frac{\cosh x}{4 \sinh^4 x} + \frac{3 \cosh x}{8 \sinh^2 x} + \frac{3}{8} \ln \tanh \frac{x}{2}.$
46. $\int \frac{dx}{\sinh^6 x} = -\frac{\cosh x}{5 \sinh^5 x} + \frac{4}{15} \coth^3 x - \frac{4}{5} \coth x = -\frac{1}{5} \coth^5 x + \frac{2}{3} \coth^3 x - \coth x.$
47. $\int \frac{dx}{\sinh^7 x} = -\frac{\cosh x}{6 \sinh^6 x} \left(\frac{1}{\sinh^4 x} - \frac{5}{4 \sinh^2 x} + \frac{15}{8} \right) - \frac{5}{16} \ln \tanh \frac{x}{2}.$
48. $\int \frac{dx}{\sinh^8 x} = \coth x - \coth^3 x + \frac{3}{5} \coth^5 x - \frac{1}{7} \coth^7 x.$
49. $\int \frac{dx}{\cosh x} = \arctan(\sinh x) = 2 \arctan(e^x) = \arcsin(\tanh x).$
50. $\int \frac{dx}{\cosh^2 x} = \tanh x.$
51. $\int \frac{dx}{\cosh^3 x} = \frac{\sinh x}{2 \cosh^2 x} + \frac{1}{2} \arctan(\sinh x).$
52. $\int \frac{dx}{\cosh^4 x} = \frac{\sinh x}{3 \cosh^3 x} + \frac{2}{3} \tanh x = -\frac{1}{3} \tanh^3 x + \tanh x.$
53. $\int \frac{dx}{\cosh^5 x} = \frac{\sinh x}{4 \cosh^4 x} + \frac{3 \sinh x}{8 \cosh^2 x} + \frac{3}{8} \arctan(\sinh x).$
54. $\int \frac{dx}{\cosh^6 x} = \frac{\sinh x}{5 \cosh^5 x} - \frac{4}{15} \tanh^3 x + \frac{4}{5} \tanh x = \frac{1}{5} \tanh^5 x - \frac{2}{3} \tanh^3 x + \tanh x.$
55. $\int \frac{dx}{\cosh^7 x} = \frac{\sinh x}{6 \cosh^6 x} \left(\frac{1}{\cosh^4 x} + \frac{5}{4 \cosh^2 x} + \frac{15}{8} \right) + \frac{5}{16} \arctan(\sinh x).$
56. $\int \frac{dx}{\cosh^8 x} = -\frac{1}{7} \tanh^7 x + \frac{3}{5} \tanh^5 x - \tanh^3 x + \tanh x.$
57. $\int \frac{\sinh x}{\cosh x} dx = \ln \cosh x.$
58. $\int \frac{\sinh^2 x}{\cosh x} dx = \sinh x - \arctan(\sinh x).$
59. $\int \frac{\sinh^3 x}{\cosh x} dx = \frac{1}{2} \sinh^2 x - \ln \cosh x = \frac{1}{2} \cosh^2 x - \ln \cosh x.$

$$60. \int \frac{\sinh^4 x}{\cosh x} dx = \frac{1}{3} \sinh^3 x - \sinh x + \arctan(\sinh x).$$

$$61. \int \frac{\sinh x}{\cosh^2 x} dx = -\frac{1}{\cosh x}.$$

$$62. \int \frac{\sinh^2 x}{\cosh^2 x} dx = x - \tanh x.$$

$$63. \int \frac{\sinh^3 x}{\cosh^2 x} dx = \cosh x + \frac{1}{\cosh x}.$$

$$64. \int \frac{\sinh^4 x}{\cosh^2 x} dx = -\frac{3}{2}x + \frac{1}{4} \sinh 2x + \tanh x.$$

$$65. \int \frac{\sinh x}{\cosh^3 x} dx = -\frac{1}{2 \cosh^2 x} = \frac{1}{2} \tanh^2 x.$$

$$66. \int \frac{\sinh^2 x}{\cosh^3 x} dx = -\frac{\sinh x}{2 \cosh^2 x} + \frac{1}{2} \arctan(\sinh x).$$

$$67. \int \frac{\sinh^3 x}{\cosh^3 x} dx = -\frac{1}{2} \tanh^2 x + \ln \cosh x = \frac{1}{2 \cosh^2 x} + \ln \cosh x.$$

$$68. \int \frac{\sinh^4 x}{\cosh^3 x} dx = \frac{\sinh x}{2 \cosh x} + \sinh x - \frac{3}{2} \arctan(\sinh x).$$

$$69. \int \frac{\sinh x}{\cosh^4 x} dx = -\frac{1}{3 \cosh^3 x}.$$

$$70. \int \frac{\sinh^2 x}{\cosh^4 x} dx = \frac{1}{3} \tanh^3 x.$$

$$71. \int \frac{\sinh^3 x}{\cosh^4 x} dx = -\frac{1}{\cosh x} + \frac{1}{3 \cosh^3 x}.$$

$$72. \int \frac{\sinh^4 x}{\cosh^4 x} dx = -\frac{1}{3} \tanh^3 x - \tanh x + x.$$

$$73. \int \frac{\cosh x}{\sinh x} dx = \ln \sinh x.$$

$$74. \int \frac{\cosh^2 x}{\sinh x} dx = \cosh x + \ln \tanh \frac{x}{2}.$$

$$75. \int \frac{\cosh^3 x}{\sinh x} dx = \frac{1}{2} \cosh^2 x + \ln \sinh x.$$

$$76. \int \frac{\cosh^4 x}{\sinh x} dx = \frac{1}{3} \cosh^3 x + \cosh x + \ln \tanh \frac{x}{2}.$$

$$77. \int \frac{\cosh x}{\sinh^2 x} dx = -\frac{1}{\sinh x}.$$

$$78. \int \frac{\cosh^2 x}{\sinh^2 x} dx = x - \coth x.$$

$$79. \int \frac{\cosh^3 x}{\sinh^2 x} dx = \sinh x - \frac{1}{\sinh x}.$$

$$80. \int \frac{\cosh^4 x}{\sinh^2 x} dx = \frac{3}{2}x + \frac{1}{4} \sinh 2x - \coth x.$$

$$81. \int \frac{\cosh x}{\sinh^3 x} dx = -\frac{1}{2 \sinh^2 x} = -\frac{1}{2} \coth^2 x.$$

$$82. \int \frac{\cosh^2 x}{\sinh^3 x} dx = -\frac{\cosh x}{2 \sinh^2 x} + \ln \tanh \frac{x}{2}.$$

$$83. \int \frac{\cosh^3 x}{\sinh^3 x} dx = -\frac{1}{2 \sinh^2 x} + \ln \sinh x = -\frac{1}{2} \coth^2 x + \ln \sinh x.$$

$$84. \int \frac{\cosh^4 x}{\sinh^3 x} dx = -\frac{\cosh x}{2 \sinh^2 x} + \cosh x + \frac{3}{2} \ln \tanh \frac{x}{2}.$$

$$85. \int \frac{\cosh x}{\sinh^4 x} dx = -\frac{1}{3 \sinh^3 x}.$$

$$86. \int \frac{\cosh^2 x}{\sinh^4 x} dx = -\frac{1}{3} \coth^3 x.$$

$$87. \int \frac{\cosh^3 x}{\sinh^4 x} dx = -\frac{1}{\sinh x} - \frac{1}{3 \sinh^3 x}.$$

$$88. \int \frac{\cosh^4 x}{\sinh^4 x} dx = -\frac{1}{3} \coth^3 x - \coth x + x.$$

$$89. \int \frac{dx}{\sinh x \cosh x} = \ln \tanh x.$$

$$90. \int \frac{dx}{\sinh x \cosh^2 x} = \frac{1}{\cosh x} + \ln \tanh \frac{x}{2}.$$

$$91. \int \frac{dx}{\sinh x \cosh^3 x} = \frac{1}{2 \cosh^2 x} + \ln \tanh x = -\frac{1}{2} \tanh^2 x + \ln \tanh x.$$

$$92. \int \frac{dx}{\sinh x \cosh^4 x} = \frac{1}{\cosh x} + \frac{1}{3 \cosh^3 x} + \ln \tanh \frac{x}{2}.$$

$$93. \int \frac{dx}{\sinh^2 x \cosh x} = -\frac{1}{\sinh x} - \arctan(\sinh x).$$

$$94. \int \frac{dx}{\sinh^2 x \cosh^2 x} = -2 \coth 2x.$$

$$95. \int \frac{dx}{\sinh^2 x \cosh^3 x} = -\frac{\sinh x}{2 \cosh^2 x} - \frac{1}{\sinh x} - \frac{3}{2} \arctan(\sinh x).$$

96. $\int \frac{dx}{\sinh^2 x \cosh^4 x} = \frac{1}{3 \sinh x \cosh^3 x} - \frac{8}{3} \coth 2x.$
97. $\int \frac{dx}{\sinh^3 x \cosh x} = -\frac{1}{2 \sinh^2 x} - \ln \tanh x = -\frac{1}{2} \coth^2 x + \ln \coth x.$
98. $\int \frac{dx}{\sinh^3 x \cosh^2 x} = -\frac{1}{\cosh x} - \frac{\cosh x}{2 \sinh^2 x} - \frac{3}{2} \ln \tanh \frac{x}{2}.$
99. $\int \frac{dx}{\sinh^3 x \cosh^3 x} = -\frac{2 \cosh 2x}{\sinh^2 2x} - 2 \ln \tanh x = \frac{1}{2} \tanh^2 x - \frac{1}{2} \coth^2 x - 2 \ln \tanh x.$
100. $\int \frac{dx}{\sinh^3 x \cosh^4 x} = -\frac{2}{\cosh x} - \frac{1}{3 \cosh^2 x} - \frac{\cosh x}{2 \sinh^2 x} - \frac{5}{2} \ln \tanh \frac{x}{2}.$
101. $\int \frac{dx}{\sinh^4 x \cosh x} = \frac{1}{\sinh x} - \frac{1}{3 \sinh^3 x} + \arctan \sinh x.$
102. $\int \frac{dx}{\sinh^4 x \cosh^2 x} = -\frac{1}{3 \cosh x \sinh^3 x} + \frac{8}{3} \coth 2x.$
103. $\int \frac{dx}{\sinh^4 x \cosh^3 x} = \frac{2}{\sinh x} - \frac{1}{3 \sinh^3 x} + \frac{\sinh x}{2 \cosh^2 x} + \frac{5}{2} \arctan(\sinh x).$
104. $\int \frac{dx}{\sinh^4 x \cosh^4 x} = 8 \coth 2x - \frac{8}{3} \coth^3 2x.$
105. $\int \frac{\sinh(2n+1)x}{\sinh x} dx = 2 \sum_{k=0}^{n-1} \frac{\sinh(2n-2k)x}{2n-2k} + x.$
106. $\int \frac{\sinh 2nx}{\sinh x} dx = 2 \sum_{k=0}^{n-1} \frac{\sinh(2n-2k-1)x}{2n-2k-1}.$
107. $\int \frac{\cosh(2n+1)x}{\sinh x} dx = 2 \sum_{k=0}^{n-1} \frac{\cosh(2n-2k)x}{2n-2k} + \ln \sinh x.$
108. $\int \frac{\cosh 2nx}{\sinh x} dx = 2 \sum_{k=0}^{n-1} \frac{\cosh(2n-2k-1)x}{2n-2k-1} + \ln \tanh \frac{x}{2}.$
109. $\int \frac{\sinh(2n+1)x}{\cosh x} dx = 2 \sum_{k=0}^{n-1} (-1)^k \frac{\cosh(2n-2k)x}{2n-2k} + (-1)^n \ln \cosh x.$
110. $\int \frac{\sinh 2nx}{\cosh x} dx = 2 \sum_{k=0}^{n-1} (-1)^k \frac{\cosh(2n-2k-1)x}{2n-2k-1}.$
111. $\int \frac{\cosh(2n+1)x}{\cosh x} dx = 2 \sum_{k=0}^{n-1} (-1)^k \frac{\sinh(2n-2k)x}{2n-2k} + (-1)^n x.$

$$112. \int \frac{\cosh 2nx}{\cosh x} dx = 2 \sum_{k=0}^{n-1} (-1)^k \frac{\sinh(2n-2k-1)x}{2n-2k-1} + (-1)^n \arcsin(\tanh x).$$

$$113. \int \frac{\sinh 2x}{\sinh^n x} dx = -\frac{2}{(n-2) \sinh^{n-2} x}.$$

$$114. \int \frac{\sinh 2x}{\sinh^2 x} dx = 2 \ln \sinh x.$$

$$115. \int \frac{\sinh 2x dx}{\cosh^n x} = \frac{2}{(2-n) \cosh^{n-2} x}.$$

$$116. \int \frac{\sinh 2x}{\cosh^2 x} dx = 2 \ln \cosh x.$$

$$117. \int \frac{\cosh 2x}{\sinh x} dx = 2 \cosh x + \ln \tanh \frac{x}{2}.$$

$$118. \int \frac{\cosh 2x}{\sinh^2 x} dx = -\coth x + 2x.$$

$$119. \int \frac{\cosh 2x}{\sinh^3 x} dx = -\frac{\cosh x}{2 \sinh^2 x} + \frac{3}{2} \ln \tanh \frac{x}{2}.$$

$$120. \int \frac{\cosh 2x}{\cosh x} dx = 2 \sinh x - \arcsin(\tanh x).$$

$$121. \int \frac{\cosh 2x}{\cosh^2 x} dx = -\tanh x + 2x.$$

$$122. \int \frac{\cosh 2x}{\cosh^3 x} dx = -\frac{\sinh x}{2 \cosh^2 x} + \frac{3}{2} \arcsin(\tanh x).$$

$$123. \int \frac{\sinh 3x}{\sinh x} dx = x + \sinh 2x.$$

$$124. \int \frac{\sinh 3x}{\sinh^2 x} dx = 3 \ln \tanh \frac{x}{2} + 4 \cosh x.$$

$$125. \int \frac{\sinh 3x}{\sinh^3 x} dx = -3 \coth x + 4x.$$

$$126. \int \frac{\sinh 3x}{\cosh^n x} dx = \frac{4}{(3-n) \cosh^{n-3} x} - \frac{1}{(1-n) \cosh^{n-1} x}.$$

$$127. \int \frac{\sinh 3x}{\cosh x} dx = 2 \sinh^2 x - \ln \cosh x.$$

$$128. \int \frac{\sinh 3x}{\cosh^3 x} dx = \frac{1}{2 \cosh^2 x} + 4 \ln \cosh x.$$

$$129. \int \frac{\cosh 3x}{\sinh^n x} dx = \frac{4}{(3-n) \sinh^{n-3} x} + \frac{1}{(1-n) \sinh^{n-1} x}.$$

$$130. \int \frac{\cosh 3x}{\sinh x} dx = 2 \sinh^2 x + \ln \sinh x.$$

$$131. \int \frac{\cosh 3x}{\sinh^3 x} dx = -\frac{1}{2 \sinh^2 x} + 4 \ln \sinh x.$$

$$132. \int \frac{\cosh 3x}{\cosh x} dx = \sinh 2x - x.$$

$$133. \int \frac{\cosh 3x}{\cosh^2 x} dx = 4 \sinh x - 3 \arcsin(\tanh x).$$

$$134. \int \frac{\cosh 3x}{\cosh^3 x} dx = 4x - 3 \tanh x.$$

$$135. \int \frac{dx}{\sinh^{2m} x} = \frac{\cosh x}{2m-1} \left\{ -\operatorname{csch}^{2m-1} x \right. \\ \left. + \sum_{k=1}^{m-1} (-1)^{k-1} \frac{2^k (m-1)(m-2) \dots (m-k)}{(2m-3)(2m-5) \dots (2m-2k-1)} \operatorname{csch}^{2m-2k-1} x \right\}.$$

$$136. \int \frac{dx}{\sinh^{2m+1} x} = \frac{\cosh x}{2m} \left\{ -\operatorname{csch}^{2m} x \right. \\ \left. + \sum_{k=1}^{m-1} (-1)^{k-1} \frac{(2m-1)(2m-3) \dots (2m-2k+1)}{2^k (m-1)(m-2) \dots (m-k)} \operatorname{csch}^{2m-2k} x \right\} \\ + (-1)^m \frac{(2m-1)!!}{(2m)!!} \ln \tanh \frac{x}{2}.$$

$$137. \int \frac{\sinh^p x}{\cosh^{2n} x} dx = \frac{\sinh^{p+1} x}{2n-1} \left\{ \operatorname{sech}^{2n-1} x \right. \\ \left. + \sum_{k=1}^{n-1} \frac{(2n-p-2)(2n-p-4) \dots (2n-p-2k)}{(2n-3)(2n-5) \dots (2n-2k-1)} \operatorname{sech}^{2n-2k-1} x \right\} \\ + \frac{(2n-p-2)(2n-p-4) \dots (-p+2)(-p)}{(2n-1)!!} \int \sinh^p x dx, \quad p \text{ real.}$$

$$138. \int \frac{\sinh^p x}{\cosh^{2n+1} x} dx = \frac{\sinh^{p+1} x}{2n} \left\{ \operatorname{sech}^{2n} x \right. \\ \left. + \sum_{k=1}^{n-1} \frac{(2n-p-1)(2n-p-3) \dots (2n-p-2k+1)}{2^k (n-1)(n-2) \dots (n-k)} \operatorname{sech}^{2n-2k} x \right\} \\ + \frac{(2n-p-1)(2n-p-3) \dots (3-p)(1-p)}{2^n n!} \int \frac{\sinh^p x}{\cosh x} dx, \quad p \text{ real.}$$

$$139. \int \frac{\sinh^{2m+1} x}{\cosh x} dx = \begin{cases} \sum_{k=1}^m \frac{(-1)^{m+k}}{2k} \sinh^{2k} x + (-1)^m \ln \cosh x, \\ \text{or} \\ \sum_{k=1}^m \frac{(-1)^{m+k}}{2k} \binom{m}{k} \cosh^{2k} x + (-1)^m \ln \cosh x, \quad m \geq 1. \end{cases}$$

$$140. \int \frac{\sinh^{2m} x}{\cosh x} dx = \sum_{k=1}^m \frac{(-1)^{m+k}}{2k-1} \sinh^{2k-1} x + (-1)^m \arctan(\sinh x), \quad m \geq 1.$$

$$141. \int \frac{dx}{\sinh^{2m+1} x \cosh x} = \sum_{k=1}^m \frac{(-1)^k \operatorname{csch}^{2m-2k+2} x}{2m-2k+2} + (-1)^m \ln \tanh x.$$

$$142. \int \frac{dx}{\sinh^{2m} x \cosh x} = \sum_{k=1}^m \frac{(-1)^k \operatorname{csch}^{2m-2k+2} x}{2m-2k+1} + (-1)^m \arctan \sinh x.$$

$$143. \int \frac{dx}{\cosh^{2m} x} = \frac{\sinh x}{2m-1} \left\{ \operatorname{sech}^{2m-1} x + \sum_{k=1}^{m-1} \frac{2^k (m-1)(m-2) \dots (m-k)}{(2m-3)(2m-5) \dots (2m-2k-1)} \operatorname{sech}^{2m-2k-1} x \right\}.$$

$$144. \int \frac{dx}{\cosh^{2m+1} x} = \frac{\sinh x}{2m} \left\{ \operatorname{sech}^{2m} x + \sum_{k=1}^{m-1} \frac{(2m-1)(2m-3) \dots (2m-2k+1)}{2^k (m-1)(m-2) \dots (m-k)} \operatorname{sech}^{2m-2k} x \right\} \\ + \frac{(2m-1)!!}{(2m)!!} \arctan(\sinh x).$$

$$145. \int \frac{\cosh^p x}{\sinh^{2n+1} x} dx = -\frac{\cosh^{p+1} x}{2n} \left\{ \operatorname{csch}^{2n} x \right. \\ \left. + \sum_{k=1}^{n-1} \frac{(-1)^k (2n-p-1)(2n-p-3) \dots (2n-p-2k+1)}{2^k (n-1)(n-2) \dots (n-k)} \operatorname{csch}^{2n-2k} x \right\} \\ + \frac{(-1)^n (2n-p-1)(2n-p-3) \dots (3-p)(1-p)}{2^n n!} \int \frac{\cosh^p x}{\sinh x} dx, \quad p \text{ real.}$$

$$146. \int \frac{\cosh^{2m} x}{\sinh x} dx = \sum_{k=1}^m \frac{\cosh^{2k-1} x}{2k-1} + \ln \tanh \frac{x}{2}.$$

$$147. \int \frac{\cosh^{2m+1} x}{\sinh x} dx = \sum_{k=1}^m \frac{\cosh^{2k} x}{2k} + \ln \sinh x = \sum_{k=1}^m \binom{m}{k} \frac{\sinh^{2k} x}{2k} + \ln \sinh x.$$

$$148. \int \frac{\cosh^p x}{\sinh^{2n} x} dx = -\frac{\cosh^{p+1} x}{2n-1} \left\{ \operatorname{csch}^{2n-1} x \right. \\ \left. + \sum_{k=1}^{n-1} \frac{(-1)^k (2n-p-2)(2n-p-4) \dots (2n-p-2k)}{(2n-3)(2n-5) \dots (2n-2k-1)} \operatorname{csch}^{2n-2k-1} x \right\} \\ + \frac{(-1)^n (2n-p-2)(2n-p-4) \dots (-p+2)(-p)}{(2n-1)!!} \int \cosh^p x dx, \quad p \text{ real.}$$

$$149. \int \frac{dx}{\sinh x \cosh^{2m} x} = \sum_{k=1}^m \frac{\operatorname{sech}^{2m-2k+1} x}{2m-2k+1} + \ln \tanh \frac{x}{2}.$$

$$150. \int \frac{dx}{\sinh x \cosh^{2m+1} x} = \sum_{k=1}^m \frac{\operatorname{sech}^{2m-2k+2} x}{2m-2k+2} + \ln \tanh x.$$

$$151. \int \frac{\sinh^{2n+1} x}{\cosh^m x} dx = \sum_{\substack{k=0 \\ k \neq (m-1)/2}}^n (-1)^{n+k} \binom{n}{k} \frac{\cosh^{2k-m+1} x}{2k-1+1} + s(-1)^{n+(m-1)/2} \binom{n}{(m-1)/2} \ln \cosh x,$$

where $s = 1$ for m odd and $m < 2n+1$, and $s = 0$ otherwise.

$$152. \int \frac{\cosh^{2n+1} x}{\sinh^m x} dx = \sum_{\substack{k=0 \\ k \neq (m-1)/2}}^n \binom{n}{k} \frac{\sinh^{2k-m+1} x}{2k-m+1} + s \binom{n}{(m-1)/2} \ln \sinh x,$$

where $s = 1$ for m odd and $m < 2n+1$, and $s = 0$ otherwise.

$$153. \int \frac{dx}{\sinh^{2m} x \cosh^{2n} x} = \sum_{k=0}^{m+n-1} \frac{(-1)^{k+1}}{2m-2k-1} \binom{m+n-1}{k} \tanh^{2k-2m+1} x.$$

$$154. \int \frac{dx}{\sinh^{2m+1} x \cosh^{2n+1} x} = \sum_{\substack{k=0 \\ k \neq m}}^n \frac{(-1)^{k+1}}{2m-2k} \binom{m+n}{k} \tanh^{2k-2m} x + (-1)^m \binom{m+n}{m} \ln \tanh x.$$

$$155. \int \tanh^p x dx = -\frac{\tanh^{p-1} x}{p-1} + \int \tanh^{p-2} x dx, \quad p \neq 1.$$

$$156. \int \tanh^{2n+1} x dx = \sum_{k=1}^n \frac{(-1)^{k-1}}{2k} \binom{n}{k} \frac{1}{\cosh^{2k} x} + \ln \cosh x \\ = -\sum_{k=1}^n \frac{\tanh^{2n-2k+2} x}{2n-2k+2} + \ln \cosh x.$$

$$157. \int \tanh^{2n} x dx = -\sum_{k=1}^n \frac{\tanh^{2n-2k+1} x}{2n-2k+1} + x.$$

$$158. \int \coth^p x dx = -\frac{\coth^{p-1} x}{p-1} + \int \coth^{p-2} x dx, \quad p \neq 1.$$

- $$159. \int \coth^{2n+1} x \, dx = \begin{cases} -\sum_{k=1}^n \frac{1}{2n} \binom{n}{k} \frac{1}{\sinh^{2k} x} + \ln \sinh x, \\ \text{or} \\ -\sum_{k=1}^n \frac{\coth^{2n-2k+2} x}{2n-2k+2} + \ln \sinh x. \end{cases}$$
- $$160. \int \coth^{2n} x \, dx = -\sum_{k=1}^n \frac{\coth^{2n-2k+1} x}{2n-2k+1} + x.$$
- $$161. \int \sinh(ax+b) \sinh(cx+d) \, dx = \frac{1}{2(a+c)} \sinh[(a+c)x+b+d] \\ - \frac{1}{2(a-c)} \sinh[(a-c)x+b-d], \quad a^2 \neq c^2.$$
- $$162. \int \sinh(ax+b) \cosh(cx+d) \, dx = \frac{1}{2(a+c)} \cosh[(a+c)x+b+d] \\ + \frac{1}{2(a-c)} \cosh[(a-c)x+b-d], \quad a^2 \neq c^2.$$
- $$163. \int \cosh(ax+b) \cosh(cx+d) \, dx = \frac{1}{2(a+c)} \sinh[(a+c)x+b+d] \\ + \frac{1}{2(a-c)} \sinh[(a-c)x+b-d], \quad a^2 \neq c^2.$$
- $$164. \int \sinh(ax+b) \sinh(ax+d) \, dx = -\frac{x}{2} \cosh(b-d) + \frac{1}{4a} \sinh(2ax+b+d).$$
- $$165. \int \sinh(ax+b) \cosh(ax+d) \, dx = \frac{x}{2} \sinh(b-d) + \frac{1}{4a} \cosh(2ax+b+d).$$
- $$166. \int \cosh(ax+b) \cosh(ax+d) \, dx = \frac{x}{2} \cosh(b-d) + \frac{1}{4a} \sinh(2ax+b+d).$$
- $$167. \int \sinh ax \sinh bx \sinh cx \, dx = \frac{\cosh(a+b+c)x}{4(a+b+c)} - \frac{\cosh(-a+b+c)x}{4(-a+b+c)} \\ - \frac{\cosh(a-b+c)x}{4(a-b+c)} - \frac{\cosh(a+b-c)x}{4(a+b-c)}.$$
- $$168. \int \sinh ax \sinh bx \cosh cx \, dx = \frac{\sinh(a+b+c)x}{4(a+b+c)} - \frac{\sinh(-a+b+c)x}{4(-a+b+c)} \\ - \frac{\sinh(a-b+c)x}{4(a-b+c)} + \frac{\sinh(a+b-c)x}{4(a+b-c)}.$$
- $$169. \int \sinh ax \cosh bx \cosh cx \, dx = \frac{\cosh(a+b+c)x}{4(a+b+c)} - \frac{\cosh(-a+b+c)x}{4(-a+b+c)} \\ + \frac{\cosh(a-b+c)x}{4(a-b+c)} + \frac{\cosh(a+b-c)x}{4(a+b-c)}.$$

$$170. \int \cosh ax \cosh bx \cosh cx \, dx = \frac{\sinh(a+b+c)x}{4(a+b+c)} + \frac{\sinh(-a+b+c)x}{4(-a+b+c)} \\ + \frac{\sinh(a-b+c)x}{4(a-b+c)} + \frac{\sinh(a+b-c)x}{4(a+b-c)}.$$

$$171. \int \sinh^p x \sinh ax \, dx = \frac{1}{p+a} \left\{ \sinh^p x \cosh ax - p \int \sinh^{p-1} x \cosh(a-1)x \, dx \right\}.$$

$$172. \int \sinh^p x \sinh(2n+1)x \, dx = \frac{\Gamma(p+1)}{\Gamma\left(\frac{p+3}{2}+n\right)} \\ \times \left\{ \sum_{k=0}^{n-1} \left[\frac{\Gamma\left(\frac{p+1}{2}+n-2k\right)}{2^{2k+1}\Gamma(p-2k+1)} \sinh^{p-2k} x \cosh(2n-2k+1)x \right. \right. \\ \left. \left. - \frac{\Gamma\left(\frac{p-1}{2}+n-2k\right)}{2^{2k+2}\Gamma(p-2k)} \sinh^{p-2k-1} x \sinh(2n-2k)x \right] \right. \\ \left. + \frac{\Gamma\left(\frac{p+3}{2}-n\right)}{2^{2n}\Gamma(p+1-2n)} \int \sinh^{p-2n} x \sinh x \, dx \right\}, \quad p \geq 0 \text{ integer}.$$

$$173. \int \sinh^p x \sinh 2nx \, dx = \frac{\Gamma(p+1)}{\Gamma\left(\frac{p}{2}+n+1\right)} \\ \times \sum_{k=0}^{n-1} \left[\frac{\Gamma\left(\frac{p}{2}+n-2k\right)}{2^{2k+1}\Gamma(p-2k+1)} \sinh^{p-2k} x \cosh(2n-2k)x \right. \\ \left. - \frac{\Gamma\left(\frac{p}{2}+n-2k-1\right)}{2^{2k+2}\Gamma(p-2k)} \sinh^{p-2k-1} x \sinh(2n-2k-1)x \right], \quad p \geq 0 \text{ integer}.$$

$$174. \int \sinh^p x \cosh ax \, dx = \frac{1}{p+a} \left\{ \sinh^p x \sinh ax - p \int \sinh^{p-1} x \sinh(a-1)x \, dx \right\}.$$

$$175. \int \sinh^p x \cosh(2n+1)x \, dx = \frac{\Gamma(p+1)}{\Gamma\left(\frac{p+3}{2}+n\right)} \\ \times \left\{ \sum_{k=0}^{n-1} \left[\frac{\Gamma\left(\frac{p+1}{2}+n-2k\right)}{2^{2k+1}\Gamma(p-2k+1)} \sinh^{p-2k} x \sinh(2n-2k+1)x \right. \right. \\ \left. \left. - \frac{\Gamma\left(\frac{p-1}{2}+n-2k\right)}{2^{2k+2}\Gamma(p-2k)} \sinh^{p-2k-1} x \cosh(2n-2k)x \right] \right. \\ \left. + \frac{\Gamma\left(\frac{p+3}{2}-n\right)}{2^{2n}\Gamma(p+1-2n)} \int \sinh^{p-2n} x \cosh x \, dx \right\}, \quad p \geq 0 \text{ integer}.$$

176. $\int \sinh^p x \cosh 2nx \, dx = \frac{\Gamma(p+1)}{\Gamma(\frac{p}{2} + n + 1)}$
- $$\times \left\{ \sum_{k=0}^{n-1} \left[\frac{\Gamma(\frac{p}{2} + n - 2k)}{2^{2k+1} \Gamma(p - 2k + 1)} \sinh^{p-2k} x \sinh(2n - 2k)x \right. \right.$$
- $$\left. - \frac{\Gamma(\frac{p}{2} + n - 2k - 1)}{2^{2k+2} \Gamma(p - 2k)} \sinh^{p-2k-1} x \cosh(2n - 2k - 1)x \right]$$
- $$\left. + \frac{\Gamma(\frac{p}{2} - n + 1)}{2^{2n} \Gamma(p + 1 - 2n)} \int \sinh^{p-2n} x \, dx \right\}, \quad p \geq 0 \text{ integer.}$$
177. $\int \cosh^p x \sinh ax \, dx = \frac{1}{p+a} \left\{ \cosh^p x \cosh ax + p \int \cosh^{p-1} x \sinh(a-1)x \, dx \right\}.$
178. $\int \cosh^p x \sinh(2n+1)x \, dx$
- $$= \frac{\Gamma(p+1)}{\Gamma(\frac{p+3}{2} + n)} \left\{ \sum_{k=0}^{n-1} \frac{\Gamma(\frac{p+1}{2} + n - k)}{2^{k+1} \Gamma(p - k + 1)} \cosh^{p-k} x \cosh(2n - k + 1)x \right.$$
- $$\left. + \frac{\Gamma(\frac{p+3}{2})}{2^n \Gamma(p - n + 1)} \int \cosh^{p-n} x \sinh(n+1)x \, dx \right\}, \quad p \geq 0 \text{ integer.}$$
179. $\int \cosh^p x \sinh 2nx \, dx$
- $$= \frac{\Gamma(p+1)}{\Gamma(\frac{p}{2} + n + 1)} \left\{ \sum_{k=0}^{n-1} \frac{\Gamma(\frac{p}{2} + n - k)}{2^{k+1} \Gamma(p - k + 1)} \cosh^{p-k} x \cosh(2n - k)x \right.$$
- $$\left. + \frac{\Gamma(\frac{p}{2} + 1)}{2^n \Gamma(p - n + 1)} \int \cosh^{p-n} x \sinh nx \, dx \right\}, \quad p \geq 0 \text{ integer.}$$
180. $\int \cosh^p x \cosh ax \, dx = \frac{1}{p+a} \left\{ \cosh^p x \sinh ax + p \int \cosh^{p-1} x \cosh(a-1)x \, dx \right\}.$
181. $\int \cosh^p x \cosh(2n+1)x \, dx$
- $$= \frac{\Gamma(p+1)}{\Gamma(\frac{p+3}{2} + n)} \left\{ \sum_{k=0}^{n-1} \frac{\Gamma(\frac{p+1}{2} + n - k)}{2^{k+1} \Gamma(p - k + 1)} \cosh^{p-k} x \sinh(2n - k + 1)x \right.$$
- $$\left. + \frac{\Gamma(\frac{p+3}{2})}{2^n \Gamma(p - n + 1)} \int \cosh^{p-n} x \cosh(n+1)x \, dx \right\}, \quad p \geq 0 \text{ integer.}$$

$$182. \int \cosh^p x \cosh 2nx \, dx = \frac{\Gamma(p+1)}{\Gamma(\frac{p}{2}+n+1)} \left\{ \sum_{k=0}^{n-1} \frac{\Gamma(\frac{p}{2}+n-k)}{2^{k+1}\Gamma(p-k+1)} \cosh^{p-k} x \sinh(2n-k)x \right. \\ \left. + \frac{\Gamma(\frac{p}{2}+1)}{2^n\Gamma(p-n+1)} \cosh^{p-n} x \cosh nx \, dx \right\}, \quad p \geq 0 \text{ integer.}$$

$$183. \int \sinh(n+1)x \sinh^{n-1} x \, dx = \frac{1}{n} \sinh^n x \sinh nx.$$

$$184. \int \sinh(n+1)x \cosh^{n-1} x \, dx = \frac{1}{n} \cosh^n x \cosh nx.$$

$$185. \int \cosh(n+1)x \sinh^{n-1} x \, dx = \frac{1}{n} \sinh^n x \cosh nx.$$

$$186. \int \cosh(n+1)x \cosh^{n-1} x \, dx = \frac{1}{n} \cosh^n x \sinh nx.$$
