T1.27. Integrands involving powers of sines and cosines, and $\sqrt{1-k^2\sin^2 x}$.

Notation used: $R=\sqrt{1-k^2\sin^2 x}$, $k'=\sqrt{1-k^2}$.

1. $\displaystyle\int R\,dx = E(x,\,k).$

2. $\displaystyle\int R\,\sin x\,dx = -\frac{R\,\cos x}{2}-\frac{k'^2}{2k}\,\ln\left(k\,\cos x+R\right).$

3. $\displaystyle\int R\,\cos x\,dx = \frac{R\,\sin x}{2}+\frac{1}{2k}\,\arcsin(k\,\sin x).$

4. $\displaystyle\int R\,\sin^2\,x\,dx = -\frac{R}{3}\,\sin x\,\cos x+\frac{k'^2}{3k^2}F(x,\,k)+\frac{2k^2-1}{3k^2}E(x,\,k).$

5. $\displaystyle\int R\,\sin x\,\cos x\,dx = -\frac{R^3}{3k^2}.$

6. $\displaystyle\int R\,\cos^2\,x\,dx = \frac{R}{3}\,\sin x\,\cos x-\frac{k'^2}{3k^2}F(x,\,k)+\frac{k^2+1}{3k^2}E(x,\,k).$

7. $\displaystyle\int R\,\sin^3\,x\,dx = -\frac{2k^2\,\sin^2\,x+3k^2-1}{8k^2}R\,\cos x+\frac{3k^4-2k^2-1}{8k^3}\,\ln\left(k\,\cos x+R\right).$

8. $\displaystyle\int R\,\sin^2\,x\,\cos x\,dx = \frac{2k^2\,\sin^2\,x-1}{8k^2}R\,\sin x+\frac{1}{8k^3}\,\arcsin(k\,\sin x).$

9. $\displaystyle\int R\,\sin x\,\cos^2\,dx = -\frac{2k^2\,\cos^2\,x+k'^2}{8k^2}R\,\cos x+\frac{k'^4}{8k^3}\,\ln\left(k\,\cos x+R\right).$

10. $\displaystyle\int R\,\cos^3\,x\,dx = \frac{2k^2\,\cos^2\,x+2k^2+1}{8k^2}R\,\sin x+\frac{4k^2-1}{8k^3}\,\arcsin(k\,\sin x).$

11. $$\int R\,\sin^4\,x\,dx = -\frac{3k^2\,\sin^2\,x+4k^2-1}{15k^2}R\,\sin x\,\cos x$$
$$-\frac{2(2k^4-k^2-1)}{15k^4}F(x,\,k)+\frac{8k^4-3k^2-2}{15k^4}\,E(x,\,k).$$

12. $\displaystyle\int R\,\sin^3\,x\,\cos x\,dx = \frac{3k^4\,\sin^4\,x-k^2\,\sin^2\,x-2}{15k^4}R.$

13. $\displaystyle\int R\,\sin^2\,x\,\cos^2\,x\,dx = -\frac{3k^2\,\cos^2\,x-2k^2+1}{15k^2}R\,\sin x\,\cos x$

$$-\frac{k'^2(1+k'^2)}{15k^4}F(x,\ k)+\frac{2(k^4-k^2+1)}{15k^4}\,E(x,\ k).$$

14. $\displaystyle\int R\,\sin x\,\cos^3\,x\,dx=-\frac{3k^4\,\sin^4\,x-k^2\,(5k^2+1)\,\sin^2\,x+5k^2-2}{15k^4}R.$

15. $\displaystyle\int R\,\cos^4\,x\,dx=\frac{3k^2\,\cos^2\,x+3k^2+1}{15k^2}R\,\sin x\,\cos x$

$$+\frac{2k'^2(k'^2-2k^2)}{15k^4}F(x,\ k)+\frac{3k^4+7k^2-2}{15k^4}\,E(x,\ k).$$

16. $\displaystyle\int R\,\sin^5\,x\,dx=\frac{-8k^4\,\sin^4\,x-2k^2(5k^2-1)\,\sin^2\,x-15k^4+4k^2+3}{48k^4}R\,\cos x$

$$+\frac{5k^6-3k^4-k^2-1}{16k^5}\,\ln\left(k\,\cos x+R\right).$$

17. $\displaystyle\int R\,\sin^4\,x\,\cos x\,dx=\frac{8k^4\,\sin^4\,x-2k^2\,\sin^2\,x-3}{48k^4}R\,\sin x+\frac{1}{16k^5}\,\arcsin(k\,\sin x).$

18. $\displaystyle\int R\,\sin^3\,x\,\cos^2\,x\,dx=\frac{8k^4\,\sin^4\,x-2k^2\,(k^2+1)\,\sin^2\,x-3k^4+2k^2-3}{48k^4}R\,\cos x$

$$+\frac{k'^4(k^2+1)}{16k^5}\,\ln\left(k\,\cos x+R\right).$$

19. $\displaystyle\int R\,\sin^2\,x\,\cos^3\,x\,dx=\frac{-8k^4\,\sin^4\,x+2k^2\,(6k^2+1)\,\sin^2\,x-6k^2+3}{48k^4}R\,\sin x$

$$+\frac{2k^2-1}{16k^5}\,\arcsin(k\,\sin x).$$

20. $\displaystyle\int R\,\sin x\,\cos^4\,x\,dx=\frac{-8k^4\,\sin^4\,x+2k^2\,(7k^2+1)\,\sin^2\,x-3k^4-8k^2+3}{48k^4}R\,\cos x$

$$-\frac{k'^6}{16k^5}\,\ln\left(k\,\cos x+R\right).$$

21. $\displaystyle\int R\,\cos^5\,x\,dx=\frac{8k^4\,\sin^4\,x-2k^2\,(12k^2+1)\,\sin^2\,x+24k^4+12k^2-3}{48k^4}R\,\sin x$

$$+\frac{8k^4-4k^2+1}{16k^5}\,\arcsin(k\,\sin x).$$

22. $\displaystyle\int R^3\,dx=\frac{2}{3}(1+k'^2)E(x,\ k)-\frac{k'^2}{3}F(x,\ F)+\frac{k^2}{3}R\,\sin x\,\cos x.$

23. $\displaystyle\int R^3\,\sin x\,dx=\frac{2k^2\,\sin^2\,x+3k^2-5}{8}R\,\cos x-\frac{3k'^4}{8k}\,\ln\left(k\,\cos x+R\right).$

24. $\displaystyle\int R^3\,\cos x\,dx=\frac{-2k^2\,\sin^2\,x+5}{8}R\,\sin x+\frac{3}{8k}\,\arcsin(k\,\sin x).$

25. $$\int R^3 \sin^2 x\,dx = \frac{3k^2 \sin^2 x + 4k^2 - 6}{15} R \sin x \cos x + \frac{k'^2(3-4k^2)}{15k^2} F(x,\, k) - \frac{8k^4 - 13k^2 + 3}{15k^2} E(x,\, k).$$

26. $$\int R^3 \sin x \cos dx = -\frac{R^5}{5k^2}.$$

27. $$\int R^3 \cos^2 x\,dx = \frac{-3k^2 \sin^2 x + k^2 + 5}{15} R \sin x \cos x - \frac{k'^2(k^2+3)}{15k^2} F(x,\, k) - \frac{2k^4 - 7k^2 - 3}{15k^2} E(x,\, k).$$

28. $$\int R^3 \sin^3 x\,dx = \frac{8k^4 \sin^4 x + 2k^2(5k^2-7) \sin^2 x + 15k^4 - 22k^2 + 3}{48k^2} R \cos x - \frac{5k^6 - 9k^4 + 3k^2 + 1}{16k^3} \ln(k \cos x + R).$$

29. $$\int R^3 \sin^2 x \cos x\,dx = \frac{-8k^4 \sin^4 x + 14k^2 \sin^2 x - 3}{48k^2} R \sin x + + \frac{1}{16k^3} \arcsin(k \sin x).$$

30. $$\int R^3 \sin x \cos^2 x\,dx = \frac{-8k^4 \sin^4 x + 2k^2(k^2+7) \sin^2 x + 3k^4 - 8k^2 - 3}{48k^2} \times R \cos x + \frac{k'^6}{16k^3} \ln(k \cos x + R).$$

31. $$\int R^3 \cos^3 x\,dx = \frac{8k^4 \sin^4 x - 2k^2(6k^2+7) \sin^2 x + 30k^2 + 3}{48k^2} R \sin x + \frac{6k^2 - 1}{16k^3} \arcsin(k \sin x).$$

32. $$\int R^n\,dx = \frac{n-1}{n}(2-k^2) \int R^{n-2}\,dx - \frac{n-2}{n}(1-k^2) \int R^{n-4}\,dx + \frac{k^2}{n} \sin x \cos x\, R^{n-2}.$$

33. $$\int \frac{\sin x}{\cos^3 x} R\,dx = \frac{R}{2 \cos^2 x} + \frac{k^2}{4k'} \ln \frac{R+k'}{R-k'}.$$

34. $$\int \frac{\cos x}{\sin^3 x} R\,dx = -\frac{R}{2 \sin^2 x} + \frac{k^2}{4} \ln \frac{1+R}{1-R}.$$

35. $$\int \frac{\sin^2 x}{\cos^2 x} R\,dx = \int \tan^2 x R\,dx = R \tan x + F(x,\, k) - 2E(x,\, k).$$

36. $$\int \frac{\cos^2 x}{\sin^2 x} R\,dx = \int \cot^2 x R\,dx = -R \cot x + k'^2 F(x,\, k) - 2E(x,\, k).$$

37. $$\int \frac{\sin^3 x}{\cos x} R\,dx = -\frac{k^2 \sin^2 x + 3k^2 - 1}{3k^2} R + \frac{k'}{2} \ln \frac{R+k'}{R-k'}.$$

38. $$\int \frac{\cos^3 x}{\sin x} R\,dx = -\frac{k^2 \sin^2 x - 3k^2 - 1}{3k^2} R + \frac{1}{2} \ln \frac{1-R}{1+R}.$$

39. $\displaystyle\int \frac{R\,dx}{\sin^5\ x} = \frac{(k^2-3)\ \sin^2\ x + 2}{8\ \sin^4\ x}\ \cos x\, R + \frac{k'^2(k^2+3)}{16}\ \ln\ \frac{R+\cos x}{R-\cos\ x}.$

40. $\displaystyle\int \frac{R\,dx}{\sin^4\ x\ \cos x} = -\frac{(3-k^2)\ \sin^2\ x+1}{3\ \sin^3\ x} R - \frac{k'}{2}\ \ln\ \frac{R-k'\ \sin x}{R+k'\ \sin x}.$

41. $\displaystyle\int \frac{R\,dx}{\sin^3\ x\ \cos^2\ x} = \frac{3\ \sin^2\ x-1}{2\ \sin^2\ x\ \cos x} R + \frac{k^2-3}{4}\ \ln\ \frac{R-\cos x}{R+\cos x}.$

42. $\displaystyle\int \frac{R\,dx}{\sin^2\ x\ \cos^3\ x} = \frac{3\ \sin^2\ x-2}{2\ \sin x\ \cos^2\ x} R - \frac{2k^2-3}{4k'}\ \ln\ \frac{R+k'\ \sin x}{R-k'\ \sin x}.$

43. $\displaystyle\int \frac{R\,dx}{\sin x\ \cos^4\ x} = \frac{(2k^2-3)\ \sin^2\ x - 3k^2+4}{3k'^2\ \cos^3\ x} R + \frac{1}{2}\ \ln\ \frac{R+\cos x}{R-\cos x}.$

44. $\displaystyle\int \frac{R\,dx}{\cos^5\ x} = \frac{(2k^2-3)\ \sin^2\ x - 4k^2+5}{8k'^2\ \cos^4\ x}\ \sin x R - \frac{4k^2-3}{16k'^3}\ \ln\ \frac{R+k'\ \sin x}{R-k'\ \sin x}.$

45. $\displaystyle\int \frac{\sin x}{\cos^4\ x} R\,dx = \frac{-(2k^2+1)k^2\ \sin^2\ x + 3k^4 - k^2 + 1}{3k'^2\ \cos^3\ x} R.$

46. $\displaystyle\int \frac{\cos x}{\sin^4\ x} R\,dx = -\frac{R^3}{3\ \sin^3\ x}.$

47. $\displaystyle\int \frac{\sin^2}{\cos^3\ x} R\,dx = \frac{\sin x}{2\ \cos^2\ x} R + \frac{2k^2-1}{4k'}\ \ln\ \frac{R+k'\ \sin x}{R-k'\ \sin x} - k\ \arcsin(k\ \sin x).$

48. $\displaystyle\int \frac{\cos^2\ x}{\sin^3\ x} R\,dx = -\frac{\cos x}{2\ \sin^2\ x} R - \frac{k^2+1}{4}\ \ln\ \frac{R+\cos x}{R-\cos x} - k\ \ln\,(k\ \cos\ x + R).$

49. $\displaystyle\int \frac{\sin^3\ x}{\cos^2\ x} R\,dx = -\frac{\sin^2\ x - 3}{2\ \cos x} R - \frac{3k^2-1}{2k}\ \ln\,(k\ \cos x + R).$

50. $\displaystyle\int \frac{\cos^3\ x}{\sin^2\ x} R\,dx = -\frac{\sin^2\ x + 2}{2\ \sin x} R - \frac{2k^2+1}{2k}\ \arcsin(k\ \sin x).$

51. $\displaystyle\int \frac{\sin^4\ x}{\cos x} R\,dx = -\frac{2k^2\ \sin^2\ x + 4k^2 - 1}{8k^2}\ \sin x R$

$$+\frac{8k^4-4k^2-1}{8k^3}\ \arcsin(k\ \sin x) + \frac{k'}{2}\ \ln\ \frac{R+k'\ \sin x}{R-k'\ \sin x}.$$

52. $\displaystyle\int \frac{\cos^4\ x}{\sin x} R\,dx = \frac{-2k^2\ \sin^2\ x + 5k^2 + 1}{8k^2}\ \cos x R$

$$+\frac{1}{2}\ \ln\ \frac{R+\cos x}{R-\cos x} + \frac{3k^4+6k^2-1}{8k^3}\ \ln\,(k\ \cos x + R).$$

53. $\displaystyle\int \sin x\ \cos^n\ x R^r\ dx = -\frac{\cos^{n-1}\ x R^{r+2}}{(n+r+1)k^2} - \frac{(n-1)k'^2}{(n+r+1)k^2}\int \cos^{n-2}\ x\ \sin x R^r\ dx.$

54. $\displaystyle\int \sin^m\ x\ \cos x\, R^r\ dx = -\frac{\sin^{m-1}\ x R^{r+2}}{(m+r+1)k^2} + \frac{m-1}{(m+r+1)k^2}\int \sin^{m-2}\ x\ \cos x R^r\ dx.$

55. $$\int \sin^3 x \cos^n x R^r\,dx = \frac{(n+r+1)k^2 \cos^2 x - [(r+2)k^2+n+1]}{(n+r+1)(n+r+3)k^4} \cos^{n-1} x R^{r+2}$$
$$-\frac{[(r+2)k^2+n+1](n-1)k'^2}{(n+r+1)(n+r+3)k^4}\int \cos^{n-2} x \sin x R^r\,dx.$$

56. $$\int \sin^m x \cos^3 x\, R^r\,dx$$
$$\frac{(m+r+1)k^2 \sin^2 x - [(r+2)k^2-(m+1)k'^2]}{(m+r+1)(m+r+3)k^4}$$
$$\times \sin^{m-1} x R^{r+2} + \frac{[(r+2)k^2-(m-1)k'^2](m-1)}{(m+r+1)(m+r+3)k^4}\int \sin^{m-2} x \cos x R^r\,dx..$$

57. $$\int \frac{\sin^m x \cos^n x}{R^5}\,dx = \frac{\sin^{m-1} x \cos^{n-1} x}{3k^2R^3}$$
$$-\frac{m-1}{3k^2}\int \frac{\sin^{m-2} x \cos^n x}{R^3}\,dx + \frac{n-1}{3k^2}\int \frac{\sin^m x \cos^{n-2} x}{R^3}\,dx.$$

58. $$\int \frac{\sin^m x \cos^n x}{R^3}\,dx = \frac{\sin^{m-1} x \cos^{n-1} x}{k^2R}$$
$$-\frac{m-1}{k^2}\int \frac{\sin^{m-2} x \cos^n x}{R}\,dx + \frac{n-1}{k^2}\int \frac{\sin^m x \cos^{n-2} x}{R}\,dx.$$

59. $$\int \sin^m x \cos^n x R^r\,dx$$
$$= \begin{cases} \dfrac{1}{(m+n+r)k^2}\Big\{\sin^{m-3} x \cos^{n+1} x R^{r+2} + [(m+n-2)+(m+r-1)k^2] \\ \quad \times\displaystyle\int \sin^{m-2} x \cos^n x R^r\,dx - (m-3)\int \sin^{m-4} x \cos^n x R^r\,dx\Big\}, \\ \quad \text{or} \\ \dfrac{1}{(m+n+r)k^2}\Big\{\sin^{m+1} x \cos^{n-3} x R^{r+2} + [(n+r-1)k^2-(m+n-2)k'^2] \\ \quad \times\displaystyle\int \sin^m x \cos^{n-2} x R^r\,dx + (n-3)k'^2\int \sin^m x \cos^{n-4} x R^r\,dx\Big\}, \quad m+n+r \neq 0. \end{cases}$$

60. $$\int \frac{dx}{R^{n+1}} = -\frac{k^2 \sin x \cos x}{(n-1)\,k'^2R^{n-1}} + \frac{n-2}{n-1}\frac{2-k^2}{k'^2}\int \frac{dx}{R^{n-1}} - \frac{n-3}{n-1}\frac{1}{k'^2}\int \frac{dx}{R^{n-3}}.$$

61. $$\int \frac{R\,dx}{\sin x} = -\frac{1}{2}\ln \frac{R+\cos x}{R-\cos x} + k \ln k(k\cos x + R).$$

62. $$\int \frac{R\,dx}{\cos x} = \frac{k'}{2}\ln \frac{R+k'\sin x}{R-k'\sin x} + k \arcsin(k \sin x).$$

63. $$\int \frac{R\,dx}{\sin^2 x} = k'^2F(x,k) - E(x,k) - R\cot x.$$

64. $\displaystyle\int \frac{R\,dx}{\sin x\,\cos x} = \frac{1}{2}\,\ln\,\frac{1-R}{1+R} + \frac{k'}{2}\,\ln\,\frac{R+k'}{R-k'}.$

65. $\displaystyle\int \frac{R\,dx}{\cos^2\,x} = F(x,\,k) - E(x,\,k) + R\,\tan x.$

66. $\displaystyle\int \frac{\sin x}{\cos x} R\,dx = \int R\,\tan x\,dx = -R + \frac{k'}{2}\,\ln\,\frac{R+k'}{R-k'}.$

67. $\displaystyle\int \frac{\cos x}{\sin x} R\,dx = \int R\,\cot x\,dx = R + \frac{1}{2}\,\ln\,\frac{1-R}{1+R}.$

68. $\displaystyle\int \frac{R\,dx}{\sin^3\,x} = -\frac{R\,\cos x}{2\,\sin^2\,x} + \frac{k'^2}{4}\,\ln\,\frac{R+\cos x}{R-\cos x}.$

69. $\displaystyle\int \frac{R\,dx}{\sin^2\,x\,\cos x} = \frac{-R}{\sin x} - \frac{1+k^2}{2k'}\,\ln\,\frac{R-k'\,\sin x}{R+k'\,\sin x}.$

70. $\displaystyle\int \frac{R\,dx}{\sin x\,\cos^2\,x} = \frac{R}{\cos x} + \frac{1}{2}\,\ln\,\frac{R+\cos x}{R-\cos x}.$

71. $\displaystyle\int \frac{R\,dx}{\cos^3\,x} = \frac{R\,\sin x}{2\,\cos^2\,x} + \frac{1}{4k'}\,\ln\,\frac{R+k'\,\sin x}{R-k'\,\sin x}.$

72. $\displaystyle\int \frac{R\,\sin x\,dx}{\cos^2\,x} = \frac{R}{\cos x} - k\,\ln\,(k\,\cos x + R).$

73. $\displaystyle\int \frac{R\,\cos x\,dx}{\sin^2\,x} = -\frac{R}{\sin x} - k\,\arcsin\,(k\,\sin x).$

74. $\displaystyle\int \frac{R\,\sin^2\,x\,dx}{\cos x} = -\frac{R\,\sin x}{2} + \frac{2k^2-1}{2k}\,\arcsin(k\,\sin x) + \frac{k'}{2}\,\ln\,\frac{R+k'\,\sin x}{R-k'\,\sin x}.$

75. $\displaystyle\int \frac{R\,\cos^2\,x\,dx}{\sin x} = \frac{R\,\cos x}{2} + \frac{k^2+1}{2k}\,\ln\,(k\,\cos x + R) + \frac{1}{2}\,\ln\,\frac{R+\cos x}{R-\cos x}.$

76. $\displaystyle\int \frac{R\,dx}{\sin^4\,x} = \frac{1}{3}\{-R\,\cot^3\,x + (k^2-3)R\,\cot x + 2k'^2F(x,\,k) + (k^2-2)E(x,\,k)\}.$

77. $\displaystyle\int \frac{R\,dx}{\sin^3\,x\,\cos x} = -\frac{R}{2\,\sin^2\,x} + \frac{k'}{2}\,\ln\,\frac{R+k'}{R-k'} + \frac{k^2-2}{4}\,\ln\,\frac{1+R}{1-R}.$

78. $\displaystyle\int \frac{R\,dx}{\sin^2\,x\,\cos^2\,x} = \left(\frac{1}{k'^2}\,\tan x - \cot x\right) R + 2F(x,\,k) - \frac{1+k'^2}{k'^2}E(x,\,k).$

79. $\displaystyle\int \frac{R\,dx}{\sin x\,\cos^3\,x} = \frac{R}{2\,\cos^2\,x} - \frac{1}{2}\,\ln\,\frac{1+R}{1-R} + \frac{2-k^2}{4k'}\,\ln\,\frac{R+k'}{R-k'}.$

80. $\displaystyle\int \frac{R\,dx}{\cos^4\,x} = \frac{1}{3k'^2}\big\{[k'^2\,\tan^2\,x - (2k^2-3)\,\tan x]\,R + 2k'^2F(x,\,k) + (k^2-2)E(x,\,k)\big\}.$

81. $\displaystyle\int \frac{dx}{R} = F(x,\,k).$

82. $$\int \frac{\sin x\, dx}{R} = \frac{1}{2k} \ln \frac{R - k \cos x}{R + k \cos x} = -\frac{1}{k} \ln (k \cos x + R).$$

83. $$\int \frac{\cos x\, dx}{R} = \frac{1}{k} \arcsin(k \sin x) = \frac{1}{k} \arctan \frac{k \sin x}{R}.$$

84. $$\int \frac{\sin^2 x\, dx}{R} = \frac{1}{k^2} F(x, k) - \frac{1}{k^2} E(x, k).$$

85. $$\int \frac{\sin x \cos x\, dx}{R} = -\frac{R}{k^2}.$$

86. $$\int \frac{\cos^2 x\, dx}{R} = \frac{1}{k^2} E(x, k) - \frac{k'^2}{k^2} F(x, k).$$

87. $$\int \frac{\sin^3 x\, dx}{R} = \frac{\cos x R}{2k^2} - \frac{1 + k^2}{2k^3} \ln (k \cos x + R).$$

88. $$\int \frac{\sin^2 x \cos x\, dx}{R} = -\frac{\sin x R}{2k^2} + \frac{\arcsin(k \sin x)}{2k^3}.$$

89. $$\int \frac{\sin x \cos^2 x\, dx}{R} = -\frac{\cos x R}{2k^2} + \frac{k'^2}{2k^3} \ln (k \cos x + R).$$

90. $$\int \frac{\cos^3 x\, dx}{R} = \frac{\sin x R}{2k^2} + \frac{2k^2 - 1}{2k^3} \arcsin(k \sin x).$$

91. $$\int \frac{\sin^4 x\, dx}{R} = \frac{\sin x \cos x R}{3k^2} + \frac{2 + k^2}{3k^4} F(x, k) - \frac{2(1 + k^2)}{3k^4} E(x, k).$$

92. $$\int \frac{\sin^3 x \cos x\, dx}{R} = -\frac{1}{3k^4} (2 + k^2 \sin^2 x) R.$$

93. $$\int \frac{\sin^2 x \cos^2 x\, dx}{R} = -\frac{\sin x \cos x R}{3k^2} + \frac{2 - k^2}{3k^4} E(x, k) + \frac{2k^2 - 2}{3k^4} F(x, k).$$

94. $$\int \frac{\sin x \cos^3 x\, dx}{R} = -\frac{1}{3k^4} (k^2 \cos^2 x - 2k'^2) R.$$

95. $$\int \frac{\cos^4 x\, dx}{R} = \frac{\sin x \cos x R}{3k^2} + \frac{4k^2 - 2}{3k^4} E(x, k) + \frac{3k^4 - 5k^2 + 2}{3k^4} F(x, k).$$

96. $$\int \frac{\sin^5 x\, dx}{R} = \frac{2k^2 \sin^2 x + 3k^2 + 3}{8k^4} \cos x R - \frac{3 + 2k^2 + 3k^4}{8k^5} \ln (k \cos x + R).$$

97. $$\int \frac{\sin^4 x \cos x\, dx}{R} = -\frac{2k^2 \sin^2 x + 3}{8k^4} \sin x R + \frac{3}{8k^5} \arcsin(k \sin x).$$

98. $$\int \frac{\sin^3 x \cos x\, dx}{R} = \frac{2k^2 \cos^2 x - k^2 - 3}{8k^4} \cos x R - \frac{k^4 + 2k^2 - 3}{8k^5} \ln (k \cos x + R).$$

99. $$\int \frac{\sin^2 x \cos^3 x\, dx}{R} = -\frac{2k^2 \cos^2 x + 2k^2 - 3}{8k^4} \sin x R + \frac{4k^2 - 3}{8k^5} \arcsin(k \sin x).$$

100. $$\int \frac{\sin x\, \cos^4\, x\, dx}{R} = \frac{3 - 5k^2 + 2k^2\, \sin^2\, x}{8k^4}\, \cos x R - \frac{3k^4 - 6k^2 + 3}{8k^5}\, \ln\left(k\, \cos x + R\right).$$

101. $$\int \frac{\cos^5\, x\, dx}{R} = \frac{2k^2\, \cos^2\, x + 6k^2 - 3}{8k^4}\, \sin x R + \frac{8k^4 - 8k^2 + 3}{8k^5}\, \arcsin(k\, \sin x).$$

102. $$\int \frac{\sin^6\, x\, dx}{R} = \frac{3k^2\, \sin^2\, x + 4k^2 + 4}{15k^4}\, \sin x\, \cos x R + \frac{4k^4 + 3k^2 + 8}{15k^6} F(x,\, k) - \frac{8k^4 + 7k^2 + 8}{15k^6} E(x,\, k).$$

103. $$\int \frac{\sin^5\, x\, \cos x\, dx}{R} = -\frac{3k^4\, \sin^4\, x + 4k^2\, \sin^2\, x + 8}{15k^6} R.$$

104. $$\int \frac{\sin^4\, x\, \cos x\, dx}{R} = \frac{3k^2\, \cos^2\, x - 2k^2 - 4}{15k^4}\, \sin x\, \cos x R + \frac{k^4 + 7k^2 - 8}{15k^6} F(x,\, k) - \frac{2k^4 + 3k^2 - 8}{15k^6} E(x,\, k).$$

105. $$\int \frac{\sin^3\, x\, \cos^3\, x\, dx}{R} = \frac{3k^4\, \sin^4\, x - (5k^4 - 4k^2)\, \sin^2\, x - 10k^2 + 8}{15k^6} R.$$

106. $$\int \frac{\sin^2\, x\, \cos^4\, x\, dx}{R} = -\frac{3k^2\, \cos^2\, x + 3k^2 - 4}{15k^4}\, \sin x\, \cos x R + \frac{9k^4 - 17k^2 + 8}{15k^6} F(x,\, k) - \frac{3k^4 - 13k^2 + 8}{15k^6} E(x,\, k).$$

107. $$\int \frac{\sin x\, \cos^5\, x\, dx}{R} = \frac{-3k^4\, \cos^4\, x + 4k^2 k'^2\, \cos^2\, x - 8k^4 + 16k^2 - 8}{15k^6} R.$$

108. $$\int \frac{\cos^6\, x\, dx}{R} = \frac{3k^2\, \cos^2\, x + 8k^2 - 4}{15k^4}\, \sin x\, \cos x R + \frac{15k^6 - 34k^4 + 27k^2 - 8}{15k^6} F(x,\, k) + \frac{23k^4 - 23k^2 + 8}{15k^6} E(x,\, k).$$

109. $$\int \frac{\sin^7\, x\, dx}{R} = \frac{8k^4\, \sin^4\, x + 10k^2(k^2 + 1)\, \sin^2\, x + 15k^4 + 14k^2 + 15}{48k^6}\, \cos x\, R - \frac{(5k^4 - 2k^2 + 5)(k^2 + 1)}{16k^7}\, \ln\left(k\, \cos x + R\right).$$

110. $$\int \frac{\sin^6\, x\, \cos x\, dx}{R} = -\frac{8k^4\, \sin^4\, x + 10k^2\, \sin^2\, x + 15}{48k^6}\, \sin x R + \frac{5}{16k^7}\, \arcsin(k\, \sin x).$$

111. $$\int \frac{\sin^5\, x\, \cos^2\, x\, dx}{R} = \frac{-8k^4\, \sin^4\, x + 2k^2(k^2 - 5)\, \sin^2\, x + 3k^4 + 4k^2 - 15}{48k^6}\, \cos x R - \frac{k^6 + k^4 + 3k^2 - 5}{16k^7}\, \ln\left(k\, \cos x + R\right).$$

112. $$\int \frac{\sin^4 x\, \cos^3 x\, dx}{R} = \frac{8k^4 \sin^4 x - 2k^2(6k^2-5)\sin^2 x - 18k^2 + 15}{48k^6} \sin x R + \frac{6k^2-5}{16k^7} \arcsin(k \sin x).$$

113. $$\int \frac{\sin^3 x\, \cos^4 x\, dx}{R} = \frac{8k^4 \sin^4 x - 2k^2(6k^2-5)\sin^2 x + 3k^4 - 22k^2 + 15}{48k^6} \cos x R - \frac{k^6 + 3k^4 - 9k^2 + 5}{16k^7} \ln(k \cos x + R).$$

114. $$\int \frac{\sin^2 x\, \cos^5 x\, dx}{R} = \frac{-8k^4 \sin^4 x + 2k^2(12k^2-5)\sin^2 x - 24k^4 + 36k^2 - 15}{48k^6} \sin x R + \frac{8k^4 - 12k^2 + 5}{16k^7} \arcsin(k \sin x).$$

115. $$\int \frac{\sin x\, \cos^6 x\, dx}{R} = \frac{-8k^4 \sin^4 x + 2k^2(13k^2-5)\sin^2 x - 33k^4 + 40k^2 - 15}{48k^6} \cos x R + \frac{5k'^6}{16k^7} \ln(k \cos x + R).$$

116. $$\int \frac{\cos^7 x\, dx}{R} = \frac{8k^4 \sin^4 x - 2k^2(18k^2-5)\sin^2 x + 72k^4 - 54k^2 + 15}{48k^6} \sin x R + \frac{16k^6 - 24k^4 + 18k^2 - 5}{16k^7} \arcsin(k \sin x).$$

117. $$\int \frac{dx}{R^3} = \frac{1}{k'^2} E(x,\, k) - \frac{k^2}{k'^2} \frac{\sin x\, \cos x}{R}.$$

118. $$\int \frac{\sin x\, dx}{R^3} = -\frac{\cos x}{k'^2 R}.$$

119. $$\int \frac{\cos x\, dx}{R^3} = \frac{\sin x}{R}.$$

120. $$\int \frac{\sin x\, dx}{R^3} = \frac{1}{k'^2 k^2} E(x,\, k) - \frac{1}{k^2} F(x,\, k) - \frac{1}{k'^2} \frac{\mathrm{Si}\, x\, \cos x}{R}.$$

121. $$\int \frac{\sin x\, \cos x\, dx}{R^3} = \frac{1}{k^2 R}.$$

122. $$\int \frac{\cos^2 x\, dx}{R^3} = \frac{1}{k^2} F(x,\, k) - \frac{1}{k^2} E(x,\, k) + \frac{\sin x\, \cos x}{R}.$$

123. $$\int \frac{\sin^3 x\, dx}{R^3} = -\frac{\cos x}{k^2 k'^2 R} + \frac{1}{k^3} \ln(k \cos x + R).$$

124. $$\int \frac{\sin^2 x\, \cos x\, dx}{R^3} = \frac{\sin x}{k^2 R} - \frac{1}{k^3} \arcsin(k \sin x).$$

125. $\displaystyle\int \frac{\sin x\, \cos^2\, x\, dx}{R^3} = \frac{\cos x}{k^2 R} - \frac{1}{k^3}\, \ln\left(k\, \cos x + R\right).$

126. $\displaystyle\int \frac{\cos^3\, x\, dx}{R^3} = -\frac{k'^2\, \sin x}{k^2 R} + \frac{1}{k^3}\, \arcsin(k\, \sin x).$

127. $\displaystyle\int \frac{\sin^4\, x\, dx}{R^3} = \frac{k'^2+1}{k'^2 k^4} E(x,\, k) - \frac{2}{k^4} F(x,\, k) - \frac{\sin x\, \cos x}{k^2 k'^2 R}.$

128. $\displaystyle\int \frac{\sin^3\, x\, \cos x\, dx}{R^3} = \frac{2-k^2\, \sin^2\, x}{k^4 R}.$

129. $\displaystyle\int \frac{\sin^2\, x\, \cos^2\, x\, dx}{R^3} = \frac{2-k^2}{k^4} F(x,\, k) - \frac{2}{k^4} E(x,\, k) + \frac{\sin x\, \cos x}{k^2 R}.$

130. $\displaystyle\int \frac{\sin x\, \cos^3\, x\, dx}{R^3} = \frac{k^2\, \sin^2\, x + k^2 - 2}{k^4 R}.$

131. $\displaystyle\int \frac{\cos^4\, x\, dx}{R^3} = \frac{k'^2+1}{k^4} E(x,\, k) - \frac{2k'^2}{k^4} F(x,\, k) - \frac{k'^2\, \sin x\, \cos x}{k^2 R}.$

132. $\displaystyle\int \frac{\sin^5\, x\, dx}{R^3} = \frac{k^2 k'^2\, \sin^2\, x + k^3 - 3}{2k^4 k'^2 R}\, \cos x + \frac{k^2+3}{2k^5}\, \ln\left(k\, \cos x + R\right).$

133. $\displaystyle\int \frac{\sin^4\, x\, \cos x\, dx}{R^3} = \frac{-k^2\, \sin^2\, x + 3}{2k^4 R}\, \sin x - \frac{3}{2k^5}\, \arcsin(k\, \sin x).$

134. $\displaystyle\int \frac{\sin^3\, x\, \cos^2\, x\, dx}{R} = \frac{-k^2\, \sin^2\, x + 3}{2k^4 R}\, \cos x + \frac{k^2-3}{2k^5}\, \ln\left(k\, \cos x + R\right).$

135. $\displaystyle\int \frac{\sin^2\, x\, \cos^3\, x\, dx}{R^3} = \frac{k^2\, \sin^2\, x + 2k^2 - 3}{2k^4 R}\, \sin x - \frac{2k^2-3}{2k^5}\, \arcsin(k\, \sin x).$

136. $\displaystyle\int \frac{\sin x\, \cos^4\, x\, dx}{R^3} = \frac{k^2\, \sin^2\, x + 2k^2 - 3}{2k^4 R}\, \cos x + \frac{3k'^2}{2k^5}\, \ln\left(k\, \cos x + R\right).$

137. $\displaystyle\int \frac{\cos^5\, x\, dx}{R^3} = \frac{-k^2\, \sin^2\, x + 2k^4 - 4k^2 + 3}{2k^4 R}\, \sin x + \frac{4k^2-3}{2k^5}\, \arcsin(k\, \sin x).$

138. $\displaystyle\int \frac{dx}{R^5} = \frac{-k^2\, \sin x\, \cos x}{3k'^2 R^3} - \frac{2k^2(k'^2+1)\, \sin x\, \cos x}{3k'^4 R} - \frac{1}{3k'^2} F(x,\, k) + \frac{2(k'^2+1)}{3k'^4} E(x,\, k).$

139. $\displaystyle\int \frac{\sin x\, dx}{R^5} = \frac{2k^2\, \sin^2\, x + k^2 - 3}{3k'^4 R^3}\, \cos x.$

140. $\displaystyle\int \frac{\cos x\, dx}{R^5} = \frac{-2k^2\, \sin^2\, x + 3}{3R^3}\, \sin x.$

141. $\displaystyle\int \frac{\sin^2\, x\, dx}{R^5} = \frac{k^2+1}{3k'^4 k^2} E(x,\, k) - \frac{1}{3k'^2 k^2}\, F(x,\, k) + \frac{k^2(k^2+1)\, \sin^2 x - 2}{3k'^4 R^3}\, \sin x\, \cos x.$

142. $\displaystyle\int \frac{\sin x\, \cos x\, dx}{R^5} = \frac{1}{3k^2 R^3}.$

143. $$\int \frac{\cos^2 x\,dx}{R^5} = \frac{1}{3k^2}F(x,\,k) + \frac{2k^2-1}{3k^2k'^2}E(x,\,k) + \frac{k^2(2k^2-1)\,\sin^2 x - 3k^2+2}{2k'^2R}\,\sin x\,\cos x.$$

144. $$\int \frac{\sin^3 x}{R^5}\,dx = \frac{(3k^2-1)\,\sin^2 x - 2}{3k'^4R^3}\,\cos x.$$

145. $$\int \frac{\sin^2 x\,\cos x}{R^5}\,dx = \frac{\sin^3 x}{3R^3}.$$

146. $$\int \frac{\sin x\,\cos^2 x}{R^5}\,dx = -\frac{\cos^3 x}{3k'^2R^3}.$$

147. $$\int \frac{\cos^3 x\,dx}{R^5} = \frac{-(2k^2+1)\,\sin^2 x + 3}{3R^3}\,\sin x.$$

148. $$\int \frac{\sin^n x}{R}\,dx = \frac{\sin^{n-3} x}{(n-1)k^2}\,\cos x\,R + \frac{n-2}{n-1}\frac{1+k^2}{k^2}\int \frac{\sin^{n-2} x}{R}\,dx - \frac{n-3}{(n-1)k^2}\int \frac{\sin^{n-4} x}{R}\,dx.$$

149. $$\int \frac{\cos^n x}{R}\,dx = \frac{\cos^{n-3} x}{(n-1)\,k^2}\,\sin x\,R + \frac{n-2}{n-1}\frac{2k^2-1}{k^2}\int \frac{\cos^{n-2} x}{R}\,dx + \frac{n-3}{n-1}\frac{k'^2}{k^2}\int \frac{\cos^{n-4} x}{R}\,dx.$$

150. $$\int \frac{dx}{R\,\sin x} = -\frac{1}{2}\,\ln\,\frac{R+\cos x}{R-\cos x}.$$

151. $$\int \frac{dx}{R\,\cos x} = -\frac{1}{2k'}\,\ln\,\frac{R-k'\,\sin x}{R+k'\,\sin x}.$$

152. $$\int \frac{dx}{R\,\sin^2 x} = \int \frac{1+\cot^2 x}{R}\,dx = F(x,k) - E(x,\,k) - R\,\cot x.$$

153. $$\int \frac{dx}{R\,\sin x\,\cos x} = \int (\tan x + \cot x)\frac{dx}{R} = \frac{1}{2}\,\ln\,\frac{1-R}{1+R} + \frac{1}{2k'}\,\ln\,\frac{R+k'}{R-k'}.$$

154. $$\int \frac{dx}{R\,\cos^2 x} = \int (1+\tan^2 x)\frac{dx}{R} = F(x,\,k) - \frac{1}{k'^2}E(x,\,k) + \frac{1}{k'^2}R\,\tan x.$$

155. $$\int \frac{\sin x}{\cos x}\frac{dx}{R} = \int \tan x\frac{dx}{R} = \frac{1}{2k'}\,\ln\,\frac{R+k'}{R-k'}.$$

156. $$\int \frac{\cos x}{\sin x}\frac{dx}{R} = \int \cot x\frac{dx}{R} = \frac{1}{2}\,\ln\,\frac{1-R}{1+R}.$$

157. $$\int \frac{dx}{R\,\sin^3 x} = -\frac{R\,\cos x}{2\,\sin^2 x} - \frac{1+k^2}{4}\,\ln\,\frac{R+\cos x}{R-\cos x}.$$

158. $$\int \frac{dx}{R\,\sin^2 x\,\cos x} = -\frac{R}{\sin x} - \frac{1}{2k'}\,\ln\,\frac{R-k'\sin x}{R+k'\,\sin x}.$$

159. $$\int \frac{dx}{R\,\sin x\,\cos^2 x} = \frac{R}{k'^2\,\cos x} + \frac{1}{2}\,\ln\,\frac{R-\cos x}{R+\cos x}.$$

160. $$\int \frac{dx}{R\,\cos^3 x} = \frac{R\,\sin x}{2k'^2\,\cos^2 x} + \frac{2k^2-1}{4k'^3}\,\ln\,\frac{R-k'\,\sin x}{R+k'\,\sin x}.$$

161. $$\int \frac{\sin x}{\cos^2 x} \frac{dx}{R} = \frac{R}{k'^2 \cos x}.$$

162. $$\int \frac{\cos x}{\sin^2 x} \frac{dx}{R} = -\frac{R}{\sin x}.$$

163. $$\int \frac{\sin^2 x}{\cos x} \frac{dx}{R} = \frac{1}{2k'} \ln \frac{R + k' \sin x}{R - k' \sin x} - \frac{1}{k} \arcsin(k \sin x).$$

164. $$\int \frac{\cos^2 x}{\sin x} \frac{dx}{R} = \frac{1}{2} \ln \frac{R + \cos x}{R - \cos x} + \frac{1}{k} \ln (k \cos x + R).$$

165. $$\int \frac{dx}{R \sin^4 x} = \frac{1}{3}\{-R \cot^3 x - R(2k^2 + 3) \cot x + (k^2 + 2)F(x, k) - 2(k^2 + 1)E(x, k)\}.$$

166. $$\int \frac{dx}{R \sin^3 x \cos x} = \int (\tan x + 2 \cot x + \cot^3 x) \frac{dx}{R} = -\frac{R}{2 \sin^2 x} + \frac{1}{2k'} \ln \frac{R + k'}{R - k'} - \frac{k^2 + 2}{4} \ln \frac{1 + R}{1 - R}.$$

167. $$\int \frac{dx}{R \sin^2 x \cos^2 x} = \int (\tan^2 x + 2 + \cot^2 x) \frac{dx}{R} = \left(\frac{\tan x}{k'^2} - \cot x\right) R + \frac{k^2 - 2}{k'^2} E(x, k) + 2F(x, k).$$

168. $$\int \frac{dx}{R \sin x \cos^3 x} = \int (\cot x + 2 \tan x + \tan^3 x) \frac{dx}{R} = -\frac{R}{2k'^2 \cos^2 x} - \frac{1}{2} \ln \frac{1 + R}{1 - R} + \frac{2 - 3k^2}{4k'^3} \ln \frac{R + k'}{R - k'}.$$

169. $$\int \frac{dx}{R \cos^4 x} = \frac{1}{3k'^2}\left\{R \tan^3 x - \frac{5k^2 - 3}{k'^2} R \tan x - (3k^2 - 2)F(x, k) + \frac{2(2k^2 - 1)}{k'^2} E(x, k)\right\}.$$

170. $$\int \frac{\sin x}{\cos^3 x} \frac{dx}{R} = \int \tan x(1 + \tan^2 x) \frac{dx}{R} = \frac{R}{2k'^2 \cos^2 x} - \frac{k^2}{4k'^3} \ln \frac{R + k'}{R - k'}.$$

171. $$\int \frac{\cos x}{\sin^3 x} \frac{dx}{R} = -\frac{R}{2 \sin^2 x} - \frac{k^2}{4} \ln \frac{1 + R}{1 - R}.$$

172. $$\int \frac{\sin^2 x}{\cos^2 x} \frac{dx}{R} = \int \frac{\tan^2 x}{R}\, dx = \frac{R}{k'^2} \tan x - \frac{1}{k'^2} E(x, k).$$

173. $$\int \frac{\cos^2 x}{\sin^2 x} \frac{dx}{R} = \int \frac{\cot^2 x}{R}\, dx = -R \cot x - E(x, k).$$

174. $$\int \frac{\sin^3 x}{\cos x} \frac{dx}{R} = \frac{R}{k^2} + \frac{1}{2k'} \ln \frac{R + k'}{R - k'}.$$

175. $$\int \frac{\cos^3 x}{\sin x} \frac{dx}{R} = \frac{R}{k^2} - \frac{1}{2} \ln \frac{1 + R}{1 - R}.$$

176. $$\int \frac{dx}{R\,\sin^5\,x} = -\frac{3(1+k^2)\,\sin^2\,x+2}{8\,\sin^2\,x}\,R\,\cos x + \frac{3k^4+2k^2+3}{16}\,\ln\,\frac{R+\cos x}{R-\cos x}.$$

177. $$\int \frac{dx}{R\,\sin^4\,x\,\cos x} = -\frac{(3+2k^2)\,\sin^2\,x+1}{3\,\sin^3\,x}\,R - \frac{1}{2k'}\,\ln\,\frac{R-k'\,\sin x}{R+k'\,\sin x}.$$

178. $$\int \frac{dx}{R\,\sin^3\,x\,\cos^2\,x} = \frac{(3-k^2)\,\sin^2\,x-k'^2}{2k'^2\,\sin^2\,x\,\cos x}R + \frac{k^2+3}{4}\,\ln\,\frac{R-\cos\,x}{R+\cos x}.$$

179. $$\int \frac{dx}{R\,\sin^2\,x\,\cos^3\,x} = \frac{(3-2k^2)\,\sin^2\,x-2k'^2}{2k'^2\,\sin x\,\cos^2\,x}R - \frac{4k^2-3}{4k'^3}\,\ln\,\frac{R+k'\,\sin x}{R-k'\,\sin x}.$$

180. $$\int \frac{dx}{R\,\sin x\,\cos^4\,x} = \frac{(5k^2-3)\,\sin^2\,x-6k^2+4}{3k'^4\,\cos^3\,x}R - \frac{1}{2}\,\ln\,\frac{R+\cos x}{R-\cos x}.$$

181. $$\int \frac{dx}{R\,\cos^5\,x} = \frac{3(2k^2-1)\,\sin^2\,x-8k^2+5}{8k'^4\,\cos^4\,x}R\,\sin x + \frac{8k^4-8k^2+3}{16k'^5}\,\ln\,\frac{R+k'\,\sin x}{R-k'\,\sin x}.$$

182. $$\int \frac{\sin x}{\cos^4\,x}\frac{dx}{R} = -\frac{2k^2\,\cos^2\,x-k'^2}{2k'^4\,\cos^3\,x}R..$$

183. $$\int \frac{\cos x}{\sin^4\,x}\frac{dx}{R} = -\frac{2k^2\,\sin^2\,x+1}{3\,\sin^3\,x}R.$$

184. $$\int \frac{\sin^2\,x}{\cos^3\,x}\frac{dx}{R} = \frac{R\,\sin x}{2k'^2\,\cos^2\,x} - \frac{1}{4k'^3}\,\ln\,\frac{R+k'\,\sin x}{R-k'\,\sin x}.$$

185. $$\int \frac{\cos^3\,x}{\sin^3\,x}\frac{dx}{R} = -\frac{R\,\cos x}{2\,\sin^2\,x} + \frac{k'^2}{4}\,\ln\,\frac{R+\cos x}{R-\cos x}.$$

186. $$\int \frac{\sin^3\,x}{\cos^2\,x}\frac{dx}{R} = \frac{R}{k'^2\,\cos\,x} + \frac{1}{k}\,\ln\,(k\,\cos x+R).$$

187. $$\int \frac{\cos^3\,x}{\sin^2\,x}\frac{dx}{R} = \frac{-R}{\sin x} - \frac{1}{k}\,\arcsin(k\,\sin x).$$

188. $$\int \frac{\sin^4\,x}{\cos x}\frac{dx}{R} = \frac{R\,\sin x}{2k^2} + \frac{1}{2k'}\,\ln\,\frac{R+k'\,\sin x}{R-k'\,\sin x} - \frac{2k^2+1}{2k^3}\,\arcsin(k\,\sin x).$$

189. $$\int \frac{\cos^4\,x}{\sin x}\frac{dx}{R} = \frac{R\,\cos x}{2k^2} + \frac{1}{2}\,\ln\,\frac{R+\cos x}{R-\cos x} + \frac{3k^2-1}{2k^3}\,\ln\,(k\,\cos x+R).$$

190. $$\int \frac{a+\sin x}{R}dx = aF(x,\,k) + \frac{1}{2k}\,\ln\,\frac{R-k\,\cos x}{R+k\,\cos x}.$$

191. $$\int \frac{dx}{(a+\sin x)R} = \frac{1}{a}\Pi\left(x,\,\frac{1}{a^2},\,k\right) - \int \frac{\sin x\,dx}{(a^2-\sin^2\,x)R}.$$

where

192. $$\int \frac{\sin x\,dx}{(a^2-\sin^2\,x)R} = \frac{-1}{2\sqrt{(1-a^2)(1-a^2k^2)}}\,\ln\,\frac{\sqrt{1-a^2}R-\sqrt{1-k^2a^2}\,\cos x}{\sqrt{1-a^2}R+\sqrt{1-k^2a^2}\,\cos x}.$$

193. $$\int \frac{(a+\sin x)^2}{R} dx = \frac{1+k^2a^2}{k^2} F(x,\,k) - \frac{1}{k^2} E(x,\,k) + \frac{a}{k}\,\ln\,\frac{R-k\,\cos x}{R+k\,\cos x}.$$

194. $$\int \frac{(a+\sin x)^{p+3}\,dx}{R} = \frac{1}{(p+2)k^2}\Bigg[(a+\sin x)^p\,\cos x R$$
$$+2(2p+3)ak^2 \int \frac{(a+\sin x)^{p+2}\,dx}{R} + (p+1)(1+k^2-6a^2k^2) \int \frac{(a+\sin x)^{p+1}\,dx}{R}$$
$$-a(2p+1)(1+k^2-2a^2k^2) \int \frac{(a+b\,\sin x)^p\,dx}{R}$$
$$-p(1-a^2)(1-a^2k^2) \int \frac{(a+\sin x)^{p-1}\,dx}{R}\Bigg], \quad p \neq -2,\ a \neq \pm 1,\ a \neq \pm\frac{1}{k}.$$

195. $$\int \frac{dx}{(a+\sin x)^2 R} = \frac{1}{(1-a^2)(1-a^2k^2)}\Bigg[-\frac{\cos x R}{a+\sin x} - a(1+k^2-2a^2k^2) \int \frac{dx}{(a+\sin x)R}$$
$$-2ak^2 \int \frac{(a+\sin x)\,dx}{R} + k^2 \int \frac{(a+\sin x)^2\,dx}{R}\Bigg].$$

196. $$\int \frac{dx}{(a+\sin x)^3 R} = \frac{1}{2(1-a^2)(1-a^2k^2)}\Bigg[-\frac{\cos x R}{(a+\sin x)^2} - 3a(1+k^2-2a^2k^2) \int \frac{dx}{(a+\sin x)^2 R}$$
$$-(6a^2k^2-k^2-1) \int \frac{dx}{(a+\sin x)R} + 2ak^2 F(x,\,k)\Bigg].$$

197. $$\int \frac{dx}{(1\pm\sin x)R} = \frac{\mp\,\cos x R}{k'^2(1\pm\sin x)} + F(x,\,k) - \frac{1}{k'^2} E(x,\,k).$$

198. $$\int \frac{dx}{(1\pm\sin x)^2 R} = \frac{1}{3k'^4}\Bigg[\mp\frac{k'^2\,\cos x R}{(1\pm\sin x)^2} \mp \frac{(1-5k^2)\,\cos x R}{1\pm\sin x}$$
$$+(1-3k^2)k'^2 F(x,\,k) - (1-5k^2)E(x,\,k)\Bigg].$$

199. $$\int \frac{dx}{(1\pm\sin x)^n R} = \frac{1}{(2n-1)k'^2}\Bigg[\mp\frac{\cos x R}{(1\pm\sin x)^n} + (n-1)(1-5k^2) \int \frac{dx}{(1\pm\sin x)^{n-1} R}$$
$$+2(2n-3)k^2 \int \frac{dx}{(1\pm\sin x)^{n-2} R} - (n-2)k^2 \int \frac{dx}{(1\pm\sin x)^{n-3} R}\Bigg].$$

200. $$\int \frac{dx}{(a+\sin x)^n R} = \frac{1}{(n-1)(1-a^2)(1-a^2k^2)}\Bigg[-\frac{\cos x R}{(a+\sin x)^{n-1}}$$
$$-(2n-3)(1+k^2-2a^2k^2)a \int \frac{dx}{(a+\sin x)^{n-1} R} - (n-2)(6a^2k^2-k^2-1) \int \frac{dx}{(a+\sin x)^{n-2} R}$$
$$-(10-4n)ak^2 \int \frac{dx}{(a+\sin x)^{n-3} R} = (n-3)k^2 \int \frac{dx}{(a+\sin x)^{n-4} R}\Bigg],$$
$$n \neq 1,\ a \neq \pm 1,\ a \neq \pm\frac{1}{k}.$$

201. $$\int \frac{dx}{(1\pm k\,\sin x)^n R} = \frac{1}{(2n-1)k'^2}\left[\pm\frac{k\,\cos x R}{(1\pm k\,\sin x)^n} + (n-1)(5-k^2)\int\frac{dx}{(1\pm k\,\sin x)^{n-1}R}\right.$$
$$\left. -2(2n-3)\int\frac{dx}{(1\pm k\,\sin x)^{n-2}R} + (n-2)\int\frac{dx}{(1\pm k\,\sin x)^{n-3}R}\right].$$

202. $$\int \frac{dx}{(1\pm k\,\sin x)R} = \pm\frac{k\,\cos x R}{k'^2(1\pm k\,\sin x)} + \frac{1}{k'^2}E(x,\,k).$$

203. $$\int \frac{dx}{(1\pm k\,\sin x)^2 R} = \frac{1}{3k'^4}\left[\pm\frac{kk'^2\,\cos x R}{(1\pm k\,\sin x)^2} \pm \frac{k(5-k^2)\,\cos x R}{1\pm k\,\sin x} - 2k'^2F(x,\,k) + (5-k^2)E(x,\,k)\right].$$

204. $$\int \frac{b+\cos x}{R}\,dx = bF(x,\,k) + \frac{1}{k}\,\arcsin(k\,\sin x).$$

205. $$\int \frac{(b+\cos x)^2}{R}\,dx = \frac{b^2k^2-k'^2}{k^2}F(x,\,k) + \frac{1}{k^2}E(x,\,k) + \frac{2b}{k}\,\arcsin(k\,\sin x).$$

206. $$\int \frac{(b+\cos x)^{p+3}\,dx}{R} = \frac{1}{(p+2)k^2}\left[(b+\cos x)^p\,\sin x R + 2(2p+3)bk^2\int\frac{(b+\cos x)^{p+2}\,dx}{R}\right.$$
$$-(p+1)(k'^2-k^2+6b^2k^2)\int\frac{(b+\cos x)^{p+1}\,dx}{R}$$
$$+(2p+1)b(k'^2-k^2+b^2k^2)\int\frac{(b+\cos x)^{p}\,dx}{R}$$
$$\left.+p(1-b^2)(k'^2+k^2b^2)\int\frac{(b+\cos x)^{p-1}\,dx}{R}\right],\quad p\neq -2,\ b\neq\pm1,\ b\neq\frac{ik'}{k}.$$

207. $$\int \frac{dx}{(b+\cos x)R} = \frac{b}{b^2-1}\Pi\left(x,\,\frac{1}{b^2-1},\,k\right) + \int\frac{\cos x\,dx}{(1-b^2-\sin^2\,x)R},$$

where

208. $$\int \frac{\cos x\,dx}{(1-b^2-\sin^2\,x)R} = \frac{1}{2\sqrt{(1-b^2)(k'^2+k^2b^2)}}\,\ln\,\frac{\sqrt{1-b^2}R + k\sqrt{k'^2+k^2b^2}\,\sin x}{\sqrt{1-b^2}R - k\sqrt{k'^2+k^2b^2}\,\sin x}.$$

209. $$\int \frac{dx}{(b+\cos x)^2 R} = \frac{1}{(1-b^2)(k'^2+b^2k^2)}\left[\frac{-k'^2\,\sin x R}{b+\cos x} - (1-2k^2+2b^2k^2)\,b\int\frac{dx}{(b+\cos x)R}\right.$$
$$\left. +2bk^2\int\frac{b+\cos x}{R}dx - k^2\int\frac{(b+\cos x)^2}{R}\,dx\right].$$

210. $$\int \frac{dx}{(b+\cos x)^3 R} = \frac{1}{2(1-b^2)(k'^2+b^2k^2)}\left[\frac{-k'^2\,\sin x R}{(b+\cos x)^2} - 3b(1-2k^2+2k^2b^2)\int\frac{dx}{(b+\cos x)^2 R}\right].$$

211. $$\int \frac{dx}{(b+\cos x)^n R} = \frac{1}{(n-1)(1-b^2)(k'^2+b^2k^2)} \left[\frac{-k'^2 \, \sin x R}{(b+\cos x)^{-1}} \right.$$
$$-(2n-3)(1-2k^2+2b^2k^2)b \int \frac{dx}{(b+\cos x)^{n-1} R}$$
$$-(n-2)(2k^2-1-6b^2k^2) \int \frac{dx}{(b+\cos x)^{n-2} R}$$
$$\left. -(4n-10)bk^2 \int \frac{dx}{(b+\cos x)^{n-3} R} + (n-3)k^2 \int \frac{dx}{(b+\cos x)^{n-4} R} \right],$$
$$n \neq 1, \; b \neq \pm 1, \; b \neq \pm \frac{ik'}{k}.$$

212. $$\int \frac{c+\tan x}{R}\, dx = cF(x,\, k) + \frac{1}{2k'} \, \ln \, \frac{R+k'}{R-k'}.$$

213. $$\int \frac{(c+\tan x)^2}{R}\, dx = \frac{1}{k'^2} \, \tan x R + c^2 F(x,\, k) - \frac{1}{k'^2} E(x,\, k) + \frac{c}{k'} \, \ln \, \frac{R+k'}{R-k'}.$$

214. $$\int \frac{(c+\tan x)^{p+3}\, dx}{R} = \frac{1}{(p+2)k'^2} \left[\frac{(c+\tan x)^p R}{\cos^2 \, x} + 2(2n+3)ck'^2 \int \frac{(c+\tan x)^{p+2}\, dx}{R} \right.$$
$$-(p+1)(1+k'^2+6c^2k'^2) \int \frac{(c+\tan x)^{p+1}\, dx}{R}$$
$$+(2p+1)c(1+k'^2+2c^2k'^2) \int \frac{(c+\tan x)^{p}\, dx}{R}$$
$$\left. -p(1+c^2)(1+k'^2c^2) \int \frac{(c+\tan x)^{p-1}\, dx}{R} \right], \quad p \neq -2.$$

215. $$\int \frac{dx}{(c+\tan x)R} = \frac{c}{1+c^2} F(x,\, k) + \frac{1}{c(1+c^2)} \Pi \left(x,\, -\frac{1+c^2}{c^2},\, k \right) - \int \frac{\sin x \, \cos x \, dx}{[c^2-(1+c^2)\, \sin^2 \, x]R}.$$

where
$$\int \frac{\sin x \, \cos x \, dx}{[c^2-(1+c^2)\, \sin^2 \, x]R} = \frac{1}{2\sqrt{(1+c^2)(1+c^2k'^2)}} \, \ln \, \frac{\sqrt{1+c^2k'^2}+\sqrt{1+c^2}R}{\sqrt{1+c^2k'^2}-\sqrt{1+c^2}R}.$$

216. $$\int \frac{dx}{(c+\tan x)^2 R} = \frac{1}{(1+c^2)(1+k'^2c^2)} \left[\frac{-R}{(c+\tan x)\, \cos^2 \, x} \right.$$
$$\left. +c(1+k'^2+2c^2k'^2) \int \frac{dx}{(c+\tan x)R} - 2ck'^2 \int \frac{c+\tan x}{R} dx + k'^2 \int \frac{(c+\tan x)^2}{R}\, dx \right].$$

217. $$\int \frac{dx}{(c+\tan x)^3 R} = \frac{1}{2(1+c^2)(1+k'^2c^2)} \left[\frac{-R}{(c+\tan x)^2 \, \cos^2 \, x} \right.$$
$$\left. +3c(1+k'^2+2c^2k'^2) \int \frac{dx}{(c+\tan \, x)^2 R} - (1+k'^2+6c^2k'^2) \int \frac{dx}{(c+\tan x)R} + 2ck'^2 F(x,\, k) \right].$$

218. $$\int \frac{dx}{(c+\tan x)^n R} = \frac{1}{(n-1)(1+c^2)(1+k'^2c^2)}\left[-\frac{R}{(c+\tan x)^{n-1}\ \cos^2\ x}\right.$$
$$+(2n-3)c(1+k'^2+2c^2k'^2)\int\frac{dx}{(c+\tan x)^{n-1}R}$$
$$-(n-2)(1+k'^2+6c^2k'^2)\int\frac{dx}{(c+\tan\ x)^{n-2}R}$$
$$\left.+(4n-10)ck'^2\int\frac{dx}{(c+\tan x)^{n-3}R} - (n-3)k'^2\int\frac{dx}{(c+\tan x)^{n-4}R}\right].$$

219. $$\int\frac{\tan^n\ x}{R}\,dx = \frac{\tan^{n-3}\ x}{(n-1)\,k'^2}\,\frac{R}{\cos^2\ x} - \frac{(n-2)(2-k^2)}{(n-1)k'^2}\int\frac{\tan^{n-2}\ x}{R}\,dx - \frac{n-3}{(n-1)k'^2}\int\frac{\tan^{n-4}\ x}{R}\,dx.$$

220. $$\int\frac{\cot^n\ x}{R}\,dx = -\frac{\cot^{n-1}\ x}{n-1}\,\frac{R}{\cos^2\ x} - \frac{n-2}{n-1}(2-k^2)\int\frac{\cot^{n-2}\ x}{R}\,dx - \frac{n-3}{n-1}k'^2\int\frac{\cot^{n-4}\ x}{R}\,dx.$$