! For an efficient use of these tables, first read `HowTo.pdf`.

T2.24A. Integrands of the form $\sqrt{\dfrac{d-x}{(a-x)(b-x)(c-x)^3}}$, $\sqrt{\dfrac{c-x}{(a-x)(b-x)(d-x)^3}}$, $\sqrt{\dfrac{b-x}{(a-x)(c-x)(d-x)^3}}$, and $\sqrt{\dfrac{a-x}{(b-x)(c-x)(d-x)^3}}$, and similar expressions with cubes of one of the factors in the denominator, on the intervals (y,a) and (a,y).

Notation used: $\alpha = \arcsin\sqrt{\dfrac{(a-c)(d-y)}{(a-d)(c-y)}}$, $\mu = \arcsin\sqrt{\dfrac{(b-d)(a-y)}{(a-b)(y-d)}}$, $\nu = \arcsin\sqrt{\dfrac{(b-d)(y-a)}{(a-d)(y-b)}}$,

$q = \sqrt{\dfrac{(b-c)(a-d)}{(a-c)(b-d)}}$, $r = \sqrt{\dfrac{(a-b)(c-d)}{(a-c)(b-d)}}$.

1. $$\int_y^a \sqrt{\frac{x-c}{(a-x)(x-b)(x-d)^3}}\,dx = \frac{2}{a-d}\sqrt{\frac{a-c}{b-d}}E(\mu,\,r), \quad a > y \ge b > c > d.$$

2. $$\int_a^y \sqrt{\frac{x-c}{(x-a)(x-b)(x-d)^3}}\,dx = \frac{2}{a-d}\sqrt{\frac{a-c}{b-d}}[F(\nu,\,q) - E(\nu,\,q)] + \frac{2}{a-d}\sqrt{\frac{(y-a)(y-c)}{(y-b)(y-d)}},$$
$$y > a > b > c > d.$$

3. $$\int_y^a \sqrt{\frac{x-b}{(a-x)(x-c)(x-d)^3}}\,dx = 2\frac{\sqrt{(a-c)(b-d)}}{(a-d)(c-d)}E(\mu,\,r)$$
$$-\frac{2(b-c)}{(c-d)\sqrt{(a-c)(b-d)}}F(\mu,\,r), \quad a > y \ge b > c > d.$$

4. $$\int_a^y \sqrt{\frac{x-b}{(x-a)(x-c)(x-d)^3}}\,dx = \frac{2(b-d)}{(a-d)(c-d)}\sqrt{\frac{(y-a)(y-c)}{(y-b)(y-d)}}$$
$$+\frac{2(a-b)}{(a-d)\sqrt{(a-c)(b-d)}}F(\nu,\,q) + 2\frac{\sqrt{(a-c)(b-d)}}{(a-d)(c-d)}E(\nu,\,q),$$
$$y > a > b > c > d.$$

5. $$\int_y^a \sqrt{\frac{a-x}{(x-b)(x-c)(x-d)^3}}\,dx = \frac{2}{c-d}\sqrt{\frac{a-c}{b-d}}[F(\mu,\,r)-E(\mu,\,r)], \quad a>y\ge b>c>d.$$

6. $$\int_a^y \sqrt{\frac{x-a}{(x-b)(x-c)(x-d)^3}}\,dx = \frac{-2}{c-d}\sqrt{\frac{a-c}{b-d}}E(\nu,\,q)+\frac{2}{c-d}\sqrt{\frac{(y-a)(y-c)}{(y-b)(y-d)}},$$
$$y>a>b>c>d.$$

7. $$\int_y^a \sqrt{\frac{x-d}{(a-x)(x-b)(x-c)^3}}\,dx = \frac{2}{b-c}\sqrt{\frac{b-d}{a-c}}E(\mu,\,r)-\frac{2(c-d)}{(a-c)(b-c)}\sqrt{\frac{(a-y)(y-b)}{(y-c)(y-d)}},$$
$$a>y\ge b>c>d.$$

8. $$\int_a^y \sqrt{\frac{x-d}{(x-a)(x-b)(x-c)^3}}\,dx = \frac{2}{b-c}\sqrt{\frac{b-d}{a-c}}[F(\nu,\,q)-E(\nu,\,q)]+\frac{2}{a-c}\sqrt{\frac{(y-a)(y-d)}{(y-b)(y-c)}},$$
$$y>a>b>c>d.$$

9. $$\int_y^a \sqrt{\frac{b-x}{(a-x)(c-x)^3(d-x)}}\,dx = \frac{2}{c-d}\sqrt{\frac{b-d}{a-c}}E(\alpha,\,q), \quad a>b>c>d>y.$$

10. $$\int_y^a \sqrt{\frac{x-b}{(a-x)(x-c)^3(x-d)}}\,dx = \frac{2}{c-d}\sqrt{\frac{b-d}{a-c}}[F(\mu,\,r)-E(\mu,\,r)]$$
$$+\frac{2}{a-c}\sqrt{\frac{(a-y)(y-b)}{(y-c)(y-d)}}, \quad a>y\ge b>c>d.$$

11. $$\int_a^y \sqrt{\frac{x-b}{(x-a)(x-c)^3(x-d)}}\,dx = \frac{2}{c-d}\sqrt{\frac{b-d}{a-c}}E(\nu,\,q)-\frac{2(b-c)}{(a-c)(c-d)}\sqrt{\frac{(y-a)(y-d)}{(y-b)(y-c)}},$$
$$y>a>b>c>d.$$

12. $$\int_y^a \sqrt{\frac{a-x}{(x-b)(x-c)^3(x-d)}}\,dx = \frac{2\sqrt{(a-c)(b-d)}}{(b-c)(c-d)}E(\mu,\,r)-\frac{2(a-d)}{(c-d)\sqrt{(a-c)(b-d)}}F(\mu,\,r)$$
$$-\frac{2}{b-c}\sqrt{\frac{(a-y)(y-b)}{(y-c)(y-d)}}, \quad a>y\ge b>c>d.$$

13. $$\int_a^y \sqrt{\frac{x-a}{(x-b)(x-c)^3(x-d)}}\,dx = \frac{2\sqrt{(a-c)(b-d)}}{(b-c)(c-d)}E(\nu,\,q) - \frac{2(a-b)}{(b-c)\sqrt{(a-c)(b-d)}}F(\nu,\,q) - \frac{2}{c-d}\sqrt{\frac{(y-a)(y-d)}{(y-b)(y-c)}}, \quad y>a>b>c>d.$$

14. $$\int_y^a \sqrt{\frac{x-d}{(a-x)(x-b)^3(x-c)}}\,dx = \frac{2(a-d)}{(a-b)\sqrt{(a-c)(b-d)}}F(\mu,\,r) - \frac{2\sqrt{(a-c)(b-d)}}{(a-b)(b-c)}E(\mu,\,r) + \frac{2(b-d)}{(a-b)(b-c)}\sqrt{\frac{(a-y)(y-c)}{(y-b)(y-d)}}, \quad a>y>b>c>d.$$

15. $$\int_a^y \sqrt{\frac{x-d}{(x-a)(x-b)^3(x-c)}}\,dx = \frac{2\sqrt{(a-c)(b-d)}}{(a-b)(b-c)}E(\nu,\,q) - \frac{2(c-d)}{(b-c)\sqrt{(a-c)(b-d)}}F(\nu,\,q) \qquad y>a>b>c>d.$$

16. $$\int_y^a \sqrt{\frac{x-c}{(a-x)(x-b)^3(x-d)}}\,dx = \frac{2}{a-b}\sqrt{\frac{a-c}{b-d}}[F(\mu,\,r)-E(\mu,\,r)] + \frac{2}{a-b}\sqrt{\frac{(a-y)(y-c)}{(y-b)(y-d)}}, \qquad a>y\geq b>c>d.$$

17. $$\int_a^y \sqrt{\frac{x-c}{(x-a)(x-b)^3(x-d)}}\,dx = \frac{2}{a-b}\sqrt{\frac{a-c}{b-d}}E(\nu,\,q), \quad y>a>b>c\,d.$$

18. $$\int_y^a \sqrt{\frac{a-x}{(x-b)^3(x-c)(x-d)}}\,dx = \frac{2}{c-b}\sqrt{\frac{a-c}{b-d}}E(\mu,\,r) + \frac{2}{b-c}\sqrt{\frac{(a-y)(y-c)}{(y-b)(y-d)}}, \qquad a>y>b>c>d.$$

19. $$\int_a^y \sqrt{\frac{x-a}{(x-b)^3(x-c)(x-d)}}\,dx = \frac{2}{b-c}\sqrt{\frac{a-c}{b-d}}[F(\nu,\,q)-E(\nu,\,q)], \quad y>a>b>c>d.$$
