! For an efficient use of these tables, first read `HowTo.pdf`.

T2.28A. Integrands of the form $\dfrac{1}{(1+c\,x)^n}\sqrt{\dfrac{1\pm x+x^2}{x\,(1\pm x)}}$ and $\dfrac{1}{(1\pm x+x^2)^n}\sqrt{\dfrac{x\,(1\pm x)}{1\pm x+x^2}}$ for $n=1,3$ on the interval $(0,y)$.

Notation ysed: $\alpha=\arccos\dfrac{1+(1-\sqrt{3})y}{1+(1+\sqrt{3})y}$, $\beta=\arccos\dfrac{1-(1+\sqrt{3})y}{1+(\sqrt{3}-1)y}$,

$$p=\frac{\sqrt{2+\sqrt{3}}}{2},\quad q=\frac{\sqrt{2-\sqrt{3}}}{2}.$$

1. $$\int_0^y \frac{dx}{[1+(1+\sqrt{3})x]^2}\sqrt{\frac{1-x+x^2}{x(1+x)}}=\frac{1}{(3)^{1/4}}\,E(\alpha,\,p),\quad y>0.$$

2. $$\int_0^y \frac{dx}{[1+(\sqrt{3}-1)x]^2}\sqrt{\frac{1+x+x^2}{x(1-x)}}=\frac{1}{(3)^{1/4}}\,E(\beta,\,q),\quad 1\ge y>0.$$

3. $$\int_0^y \frac{dx}{1-x+x^2}\sqrt{\frac{x(1+x)}{1-x+x^2}}=\frac{1}{(27)^{1/4}}\,E(\alpha,\,p)+-\frac{2-\sqrt{3}}{(27)^{1/4}}\,F(\alpha,\,p)$$
$$-\frac{2(2+\sqrt{3})}{\sqrt{3}}\,\frac{1+(1-\sqrt{3})y}{1+(1+\sqrt{3})y}\sqrt{\frac{y(1+y)}{1-y+y^2}},\quad y>0.$$

4. $$\int_0^y \frac{dx}{1+x+x^2}\sqrt{\frac{x(1-x)}{1+x+x^2}}=\frac{4}{(27)^{1/4}}\,E(\beta,\,q)-\frac{2+\sqrt{3}}{(27)^{1/4}}\,F(\beta,\,q)$$
$$-\frac{2(2-\sqrt{3})}{\sqrt{3}}\,\frac{1-(1+\sqrt{3})y}{1+(\sqrt{3}-1)y}\sqrt{\frac{y(1-y)}{1+y+y^2}},\quad 1\ge y>0.$$

5. $$\int_0^y \frac{dx}{1+x}\sqrt{\frac{x}{1+x^3}}=\frac{1}{(27)^{1/4}}\,[F(\alpha,\,p)-2E(\alpha,\,p)]+\frac{2}{\sqrt{3}}\,\frac{\sqrt{y(1-y+y^2)}}{\sqrt{1+y}\,[1+(1+\sqrt{3})y]},\quad y>0.$$

6. $$\int_0^y \frac{dx}{1-x}\sqrt{\frac{x}{1-x^3}} = \frac{1}{(27)^{1/4}}\left[F(\beta,\, q) - 2E(\beta,\, q)\right] + \frac{2}{\sqrt{3}}\frac{\sqrt{y(1+y+y^2)}}{\sqrt{1-y}\,[1+(\sqrt{3}-1)y]}, \quad 0 < y < 1.$$