! For an efficient use of these tables, first read `HowTo.pdf`.

T2.27B. Integrands of the form $\sqrt{\dfrac{x^2 \pm b^2}{(a^2 \pm x^2)^n}}$ for $n = 1, 3$ on the intervals (y, b) and (b, y).

Notation used: $\delta = \arccos \dfrac{y}{b}, \quad \varepsilon = \arccos \dfrac{b}{y},$

$$\zeta = \arcsin \frac{a}{b}\sqrt{\frac{b^2 - y^2}{a^2 - y^2}}, \quad \kappa = \arcsin \frac{a}{y}\sqrt{\frac{y^2 - b^2}{a^2 - b^2}},$$

$$q = \frac{\sqrt{a^2 - b^2}}{a}, \quad r = \frac{b}{\sqrt{a^2 + b^2}}, \quad s = \frac{a}{\sqrt{a^2 + b^2}}, \quad t = \frac{b}{a}.$$

1. $\displaystyle\int_y^b \sqrt{\frac{b^2 - x^2}{(a^2 + x^2)^3}}\, dx = \frac{\sqrt{a^2 + b^2}}{a^2} E(\delta, r) - \frac{1}{\sqrt{a^2 + b^2}} F(\delta, r) - \frac{y}{a^2}\sqrt{\frac{b^2 - y^2}{a^2 + y^2}}, \quad b > y \geq 0.$

2. $\displaystyle\int_b^y \sqrt{\frac{x^2 - b^2}{(a^2 + x^2)^3}}\, dx = \frac{\sqrt{a^2 + b^2}}{a^2} E(\varepsilon, s) - \frac{b^2}{a^2\sqrt{a^2 + b^2}} F(\varepsilon, s) - \frac{1}{y}\sqrt{\frac{y^2 - b^2}{y^2 + a^2}}, \quad y > b > 0.$

3. $\displaystyle\int_y^b \sqrt{\frac{b^2 - x^2}{(a^2 - x^2)^3}}\, dx = \frac{1}{a}\left\{F(\zeta, t) - E(\zeta, t)\right\}, \quad a > b > y \geq 0.$

4. $\displaystyle\int_b^y \sqrt{\frac{x^2 - b^2}{(a^2 - x^2)^3}}\, dx = \frac{1}{y}\sqrt{\frac{y^2 - b^2}{a^2 - y^2}} - \frac{1}{a} E(\kappa, q), \quad a > y > b > 0.$