**!** For an efficient use of these tables, first read `HowTo.pdf`.

**T3.57A.** Integrands involving logarithm and hyperbolic functions on the interval $(0, \infty)$.

1. $\displaystyle\int_0^\infty \frac{\ln x}{\cosh x}\, dx = \pi \ln \left[ \frac{\sqrt{2\pi}\,\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{1}{4}\right)} \right].$

2. $\displaystyle\int_0^\infty \frac{\ln x\, dx}{\cosh x + \cos t} = \frac{\pi}{\sin t} \ln \frac{(2\pi)^{t/\pi}\,\Gamma\left(\frac{\pi+t}{2\pi}\right)}{\Gamma\left(\frac{\pi-t}{2\pi}\right)}, \quad t^2 < \pi^2.$

3. $\displaystyle\int_0^\infty \frac{\ln x\, dx}{\cosh^2 x} = \psi\left(\frac{1}{2}\right) + \ln \pi = \ln \pi - 2\ln 2 - \gamma_e.$

4. $\displaystyle\int_0^\infty \frac{\ln(a^2 + x^2)}{\cosh bx}\, dx = \frac{\pi}{b}\left[ 2\ln \frac{2\Gamma\left(\frac{2ab+3\pi}{4\pi}\right)}{\Gamma\left(\frac{2ab+\pi}{4\pi}\right)} - \ln \frac{2b}{\pi} \right], \quad b > 0,\ a > -\frac{\pi}{2b}.$

5. $\displaystyle\int_0^\infty \ln(1 + x^2) \frac{dx}{\cosh \frac{\pi x}{2}} = 2\ln \frac{4}{\pi}.$

6. $\displaystyle\int_0^\infty \ln(a^2 + x^2) \frac{\sinh\left(\frac{2}{3}\pi x\right)}{\sinh \pi x}\, dx = 2\sin\frac{\pi}{3}\, \ln \frac{6\Gamma\left(\frac{a+4}{6}\right)\Gamma\left(\frac{a+5}{6}\right)}{\Gamma\left(\frac{a+1}{6}\right)\Gamma\left(\frac{a+2}{6}\right)}, \quad a > -1.$

7. $\displaystyle\int_0^\infty \ln(1 + x^2) \frac{dx}{\sinh^2 ax} = \frac{2}{a}\left[ \ln\frac{a}{\pi} + \frac{\pi}{2a} - \psi\left(\frac{\pi + a}{\pi}\right) \right], \quad a > 0.$

8. $\displaystyle\int_0^\infty \ln(1 + x^2) \frac{\cosh \frac{\pi}{2}x}{\sinh^2 \frac{\pi}{2}x}\, dx = \frac{2\pi - 4}{\pi}.$

9. $\displaystyle\int_0^\infty \ln(1 + x^2) \frac{\cosh \frac{\pi}{4}x}{\sinh^2 \frac{\pi}{4}x}\, dx = 4\sqrt{2} - \frac{16}{\pi} + \frac{8\sqrt{2}}{\pi} \ln(\sqrt{2} + 1).$

10. $\displaystyle\int_0^\infty \ln(\cos^2 t + e^{-2x}\, \sin^2 t) \frac{dx}{\sinh x} = -2t^2.$

11. $$\int_0^\infty \ln\left(a+be^{-2x}\right)\frac{dx}{\cosh^2 x} = \frac{2}{(b-a)}\left[\frac{a+b}{2}\ln(a+b) - a\ln a - b\ln 2\right], \quad a>0,\ a+b>0.$$

12. $$\int_0^\infty \ln\cosh\frac{x}{2}\frac{dx}{\cosh x} = \mathbf{G}+\frac{\pi}{4}\ln 2.$$

13. $$\int_0^\infty \ln\coth x\,\frac{dx}{\cosh x} = \frac{\pi}{2}\ln 2.$$

14. $$\int_0^\infty \frac{\ln x}{\sqrt{x}\,\cosh x}\,dx = 2\sqrt{\pi}\sum_{k=0}^{\infty}\frac{(-1)^{k+1}}{\sqrt{2k+1}}\left\{\ln(2k+1)+2\ln 2+\gamma_e\right\}.$$

15. $$\int_0^\infty \ln x\,\frac{(\mu+1)\cosh x - x\sinh x}{\cosh^2 x}x^{\mu}\,dx = 2\Gamma(\mu+1)\sum_{k=0}^{\infty}\frac{(-1)^{k+1}}{(2k+1)^{\mu+1}}, \quad \Re\{\mu\}>-1.$$

16. $$\int_0^\infty \ln x\,\frac{(n+1)\cosh x - x\sinh x}{\cosh^2 x}x^{n}\,dx = \frac{(-1)^n}{2^n}\beta^{(n)}\left(\tfrac{1}{2}\right).$$

17. $$\int_0^\infty \ln 2x\,\frac{n\sinh 2ax - ax}{\sinh^2 ax}x^{2n-1}\,dx = -\frac{1}{n}\left(\frac{\pi}{a}\right)^{2n}|B_{2n}|, \quad n=1,2,\ldots.$$

18. $$\int_0^\infty \ln x\,\frac{ax\cosh ax - (2n+1)\sinh ax}{\sinh^2 ax}x^{2n}\,dx = 2\frac{2^{2n+1}-1}{(2a)^{2n+1}}(2n)!\,\zeta(2n+1).$$

19. $$\int_0^\infty \ln x\,\frac{ax\cosh ax - 2n\sinh ax}{\sinh^2 ax}x^{2n-1}\,dx = \frac{2^{2n-1}-1}{2n}|B_{2n}|\left(\frac{\pi}{a}\right)^{2n}, \quad n=1,2,\ldots;a>0.$$

20. $$\int_0^\infty \ln\,\frac{(2n+1)\cosh ax - ax\sinh ax}{\cosh^2 ax}x^{2n}\,dx = -\left(\frac{\pi}{2a}\right)^{2n+1}|E_{2n}|, \quad a>0.$$

21. $$\int_0^\infty \ln x\,\frac{2ax\sinh ax - (2n+1)\cosh ax}{\cosh^3 ax}x^{2n}\,dx = \begin{cases}\dfrac{2}{a}(2^{2n-1}-1)\left(\dfrac{\pi}{2a}\right)^{2n}|B_{2n}|, & n=1,2,\ldots,\\ \dfrac{1}{a}, & n=0.\end{cases}$$

22. $$\int_0^\infty \ln x\,\frac{2ax\cosh ax - (2n+1)\sinh ax}{\sinh^3 ax}x^{2n}\,dx = \frac{1}{a}\left(\frac{\pi}{a}\right)^{2n}|B_{2n}|, \quad a>0,\ n=1,2,\ldots.$$

23. $$\int_0^\infty \ln x\,\frac{x\sinh x - 6\sinh^2\left(\frac{x}{2}\right) - 6\cos^2\frac{t}{2}}{(\cosh x+\cos t)^2}x^2\,dx = \frac{(\pi-t^2)t}{3\sin t}, \quad 0<t<\pi.$$

24. $$\int_0^\infty \ln\left(1+x^2\right) \frac{\cosh \pi x + \pi x \, \sinh \pi x}{\cosh^2 \pi x} \, \frac{dx}{x^2} = 4 - \pi.$$

25. $$\int_0^\infty \ln\left(1+4x^2\right) \frac{\cosh \pi x + \pi x \, \sinh \pi x}{\cosh^2 \pi x} \, \frac{dx}{x^2} = 4 \ln 2.$$

26. $$\int_0^\infty \ln 2x \, \frac{ax - n\left(1 - e^{-2ax}\right)}{\sinh^2 ax} \, x^{2n-1} \, dx = \frac{1}{2n} \left(\frac{\pi}{a}\right)^{2n} |B_{2n}|, \quad n = 1, 2, \ldots.$$