! For an efficient use of these tables, first read `HowTo.pdf`.

T3.29A. Integrands involving hyperbolic functions on the interval $(0, \infty)$.

1. $\displaystyle\int_0^\infty \frac{dx}{\cosh ax} = \frac{\pi}{2a}, \quad a > 0.$

2. $\displaystyle\int_0^\infty \frac{\sinh ax}{\sinh bx}\, dx = \frac{\pi}{2b} \tan \frac{a\pi}{2b}, \quad b > |a|.$

3. $\displaystyle\int_0^\infty \frac{\sinh ax}{\cosh bx}\, dx = \frac{\pi}{2b} \sec \frac{a\pi}{2b} - \frac{1}{b}\beta\left(\frac{a+b}{2b}\right), \quad b > |a|.$

4. $\displaystyle\int_0^\infty \frac{\cosh ax}{\cosh bx}\, dx = \frac{\pi}{2b} \sec \frac{a\pi}{2b}, \quad b > |a|.$

5. $\displaystyle\int_0^\infty \frac{\sinh ax \cosh bx}{\sinh cx}\, dx = \frac{\pi}{2c} \frac{\sin \frac{a\pi}{c}}{\cos \frac{a\pi}{c} + \cos \frac{b\pi}{c}}, \quad c > |a| + |b|.$

6. $\displaystyle\int_0^\infty \frac{\cosh ax \cosh bx}{\cosh cx}\, dx = \frac{\pi}{c} \frac{\cos \frac{a\pi}{2c} \cos \frac{b\pi}{2c}}{\cos \frac{a\pi}{c} + \cos \frac{b\pi}{c}}, \quad c > |a| + |b|.$

7. $\displaystyle\int_0^\infty \frac{\sinh ax \sinh bx}{\cosh cx}\, dx = \frac{\pi}{c} \frac{\sin \frac{a\pi}{2c} \sin \frac{b\pi}{2c}}{\cos \frac{a\pi}{c} + \cos \frac{b\pi}{c}}, \quad c > |a| + |b|.$

8. $\displaystyle\int_0^\infty \frac{dx}{\cosh x^2} = \sqrt{\pi} \sum_{k=0}^{\infty} \frac{(-1)^k}{\sqrt{2k+1}}.$

0. $\displaystyle\int_0^\infty \frac{\sinh ax \sinh bx}{\cosh^2 bx}\, dx = \frac{a\pi}{2b^2} \sec \frac{a\pi}{2b}, \quad b > |a|.$

10. $\displaystyle\int_0^\infty \frac{\cosh 2\beta x}{\cosh^{2\nu} ax}\, dx = \frac{4^{\nu-1}}{a} \mathrm{B}\left(\nu + \frac{\beta}{a}, \nu - \frac{\beta}{a}\right), \quad \Re\{\nu \pm \beta\} > 0,\ a > 0,\ \beta > 0.$

11. $$\int_0^\infty \frac{\sinh^\mu x}{\cosh^\nu x}\,dx = \frac{1}{2}\,\mathrm{B}\left(\frac{\mu+1}{2}, \frac{\nu-\mu}{2}\right), \quad \Re\{\mu\} > -1,\ \Re\{\mu-\nu\} < 0.$$

12. $$\int_0^\infty \frac{dx}{a+b\,\sinh x} = \frac{1}{\sqrt{a^2+b^2}}\,\ln\frac{a+b+\sqrt{a^2+b^2}}{a+b-\sqrt{a^2+b^2}}, \quad ab \neq 0.$$

13. $$\int_0^\infty \frac{dx}{a+b\,\cosh x} = \begin{cases} \dfrac{2}{\sqrt{b^2-a^2}}\,\arctan\dfrac{\sqrt{b^2-a^2}}{a+b}, & b^2 > a^2, \\[2ex] \dfrac{1}{\sqrt{a^2-b^2}}\,\ln\dfrac{a+b+\sqrt{a^2-b^2}}{a+b-\sqrt{a^2-b^2}}, & b^2 < a^2. \end{cases}$$

14. $$\int_0^\infty \frac{dx}{a\,\sinh x + b\,\cosh x} = \begin{cases} \dfrac{2}{\sqrt{b^2-a^2}}\,\arctan\dfrac{\sqrt{b^2-a^2}}{a+b}, & b^2 > a^2, \\[2ex] \dfrac{1}{\sqrt{a^2-b^2}}\,\ln\dfrac{a+b+\sqrt{a^2-b^2}}{a+b-\sqrt{a^2-b^2}}, & a^2 > b^2. \end{cases}$$

15. $$\int_0^\infty \frac{dx}{a+b\,\cosh x + c\,\sinh x}$$
$$= \begin{cases} \dfrac{2}{\sqrt{b^2-a^2-c^2}}\left[\arctan\dfrac{\sqrt{b^2-a^2-c^2}}{a+b+c} + \epsilon\,\pi\right] & \text{for } b^2 > a^2+c^2, \\[2ex] \qquad\qquad \text{and} \begin{cases} \epsilon = 0, & \text{for } (b-a)(a+b+c) > 0, \\ |\epsilon| = 1, & \text{for } (b-a)(a+b+c) < 0, \\ \epsilon = 1, & \text{for } a < b+c, \\ \epsilon = -1, & \text{for } a > b+c; \end{cases} & \\[2ex] \dfrac{1}{\sqrt{a^2-b^2+c^2}}\,\ln\dfrac{a+b+c+\sqrt{a^2-b^2+c^2}}{a+b+c-\sqrt{a^2-b^2+c^2}} & \text{for } b^2 < a^2+c^2,\ a^2 \neq b^2, \\[2ex] \dfrac{1}{c}\,\ln\dfrac{a+c}{a} & \text{for } a = b \neq 0,\ c \neq 0 \\[2ex] \dfrac{2(a-b)}{c(a-b-c)} & \text{for } b^2 = a^2+c^2,\ c(a-b-c) < 0. \end{cases}$$

16. $$\int_0^\infty \frac{dx}{\cosh ax + \cos t} = \frac{t}{a}\,\csc t, \quad 0 < t < \pi,\ a > 0.$$

17. $$\int_0^\infty \frac{\cosh ax - \cos t_1}{\cosh bx - \cos t_2}\,dx = \frac{\pi}{b}\,\frac{\sin\frac{a(\pi-t_2)}{b}}{\sin t_2\,\sin\frac{a}{b}\pi} - \frac{\pi-t_2}{b\,\sin t_2}\,\cos t_1, \quad 0 < |a| < b,\ 0 < t_2 < \pi.$$

18. $$\int_0^\infty \frac{\cosh ax\,dx}{(\cosh x + \cos t)^2} = \frac{\pi(-\cos t\,\sin at + a\,\sin t\,\cos at)}{\sin^3 t\,\sin a\pi}, \quad 0 < a^2 < 1,\ 0 < t < \pi.$$

19. $$\int_0^\infty \frac{\sinh ax\,\sinh bx}{(\cosh ax+\cos t)^2}\,dx = \frac{b\pi}{a^2}\,\csc t\,\csc\frac{b\pi}{a}\,\sin\frac{bt}{a}, \quad 0<|b|<a,\ 0<t<\pi.$$

20. $$\int_0^\infty \frac{dx}{\left(z+\sqrt{z^2-1}\,\cosh x\right)^\mu} = \frac{1}{2}\int_{-\infty}^\infty \frac{dx}{\left(z+\sqrt{z^2-1}\,\cosh x\right)^\mu} = Q_{\mu-1}(z), \quad \Re\{\mu\}>-1.$$

For a suitable choice of a single-valued branch of the integrand, this formula is valid for arbitrary values of z in the z-plane cut from -1 to $+1$, provided $\mu<0$. But if $\mu>0$, this formula is not valid at the singularities of the integrand.

21. $$\int_0^\infty \frac{dx}{(\beta+\sqrt{\beta^2-1}\,\cosh x)^{n+1}} = Q_n(\beta).$$

22. $$\int_0^\infty \frac{\cosh\gamma x\,dx}{(\beta+\sqrt{\beta^2-1}\,\cosh x)^{\nu+1}} = \frac{e^{-i\,\gamma\pi}\,\Gamma(\nu-\gamma+1)\,Q_\nu^{(\gamma)}(\beta)}{\Gamma(\nu+1)}, \quad \Re\{\nu\pm\gamma\}>-1,\ \nu\neq-1,-2,-3,\ldots.$$

23. $$\int_0^\infty \frac{\sinh^{2\mu}\,x\,dx}{(\beta+\sqrt{\beta^2-1}\,\cosh x)^{\nu+1}} = \frac{2^\mu e^{-i\,\mu\pi}\Gamma(\nu-2\mu+1)\Gamma\left(\mu+\frac{1}{2}\right)}{\sqrt{\pi}\,(\beta^2-1)^{\mu/2}\Gamma(\nu+1)}\,Q_{\nu-\mu}^{(\mu)}(\beta),$$
$$\Re\{\nu-2\mu+1\}>0,\ \Re\{\nu+1\}>0.$$

24. $$\int_0^\infty \frac{\cosh\left(\gamma+\frac{1}{2}\right)x\,dx}{(\beta+\cosh x)^{\nu+1/2}} = \sqrt{\frac{\pi}{2}}(\beta^2-1)^{-\nu/2}\frac{\Gamma(\nu+\gamma+1)\Gamma(\nu-\gamma)\,P_\gamma^{(-\nu)}(\beta)}{\Gamma\left(\nu+\frac{1}{2}\right)},$$
$$\Re\{\nu-\gamma\}>0,\ \Re\{\nu+\gamma+1\}>0.$$

25. $$\int_0^\infty \frac{\sinh^{2\mu}\,x\,dx}{(\cosh a+\sinh a\,\cosh x)^{\nu+1}} = \frac{2^\mu e^{-i\,\mu\pi}}{\sqrt{\pi}\,\sinh^\mu\,a}\,\frac{\Gamma(\nu-2\mu+1)\,\Gamma\left(\mu+\frac{1}{2}\right)}{\Gamma(\nu+1)}Q_{\nu-\mu}^{(\mu)}(\cosh a),$$
$$\Re\{\nu+1\}>0,\ \Re\{\nu-2\mu+1\}>0,\ a>0.$$

26. $$\int_0^\infty \frac{\sinh^{2\mu+1}\,x\,dx}{(\beta+\cosh x)^{\nu+1}} = 2^\mu(\beta^2-1)^{(\mu-\nu)/2}\Gamma(\nu-2\mu)\Gamma(\mu+1)\,P_\mu^{(\mu-\nu)}(\beta),$$
$$\Re\{\nu-\mu\}>\Re\{\mu>-1\};\ \beta \text{ does not lie on the ray } (-\infty,+1).$$

27. $$\int_0^\infty \frac{\sinh^{2\mu-1}\,x\,\cosh x\,dx}{(1+a\,\sinh^2\,x)^\nu} = \frac{1}{2}a^{-\mu}\,\mathrm{B}(\mu,\,\nu-\mu), \quad \Re\{\nu\}>\Re\{\mu\}>0,\ a>0.$$

28. $$\int_0^\infty \frac{\sinh^{\mu-1} x(\cosh x+1)^{\nu-1}\,dx}{(\beta+\cosh x)^\rho} = 2^{\mu+\nu-\rho-2}\,\mathrm{B}\left(\frac{1}{2}\mu,\ \rho+2-\mu-\nu\right) \times {}_2F_1\left(\rho,\ \rho+2-\mu-\nu; \rho+2-\frac{1}{2}\mu-2;\ \frac{1}{2}-\frac{1}{2}\beta\right),$$
$$\Re\{\mu\}>0,\ \Re\{\rho-\mu-\nu\}>-2,\ |\arg(1+\beta)|<\pi.$$

29. $$\int_0^\infty \frac{\sinh^{\mu-1} x(\cosh x-1)^{\nu-1}\,dx}{(\beta+\cosh x)^\rho} = 2^{-(2-\mu-\nu+\rho)}\,{}_2F_1\left(\rho,\ 2-\mu-\nu+\rho;\ 1+\rho-\frac{\mu}{2};\ \frac{1-\beta}{2}\right) \times \mathrm{B}\left(2-\mu-\nu+\rho,\ -1+\nu+\frac{\mu}{2}\right),\quad \beta\notin(-\infty,\ -1),\ \Re\{2+\rho\}>\Re\{\mu+\nu\},\ \Re\{2\nu+\mu\}>2.$$

30. $$\int_0^\infty \frac{\sinh^{\mu-1} x\ \cosh^{\nu-1} x}{(\cosh^2 x-\beta)^\rho}\,dx = \frac{1}{2}{}_2F_1\left(\rho,\ 1+\rho-\frac{\mu+\nu}{2};\ 1+\rho-\frac{\nu}{2};\ \beta\right)\mathrm{B}\left(\frac{\mu}{2},\ 1+\rho-\frac{\mu+\nu}{2}\right),$$
$$\beta\notin(1,\ \infty),\ \Re\{\mu\}>0,\ 2\,\Re\{1+\rho\}>\Re\{\mu+\nu\}.$$