! For an efficient use of these tables, first read `HowTo.pdf`.

T3.27A. Integrands involving exponentials and rational functions of powers of $(a+b\,x)$ on the interval $(0,\infty)$.

1. $$\int_0^\infty \frac{x^{\nu-1}\,dx}{e^{\mu x}-1} = \frac{1}{\mu^\nu}\Gamma(\nu)\,\zeta(\nu), \quad \Re\{\mu\}>0,\ \Re\{\nu\}>1.$$

2. $$\int_0^\infty \frac{x^{2n-1}\,dx}{e^{px}-1} = (-1)^{n-1}\left(\frac{2\pi}{p}\right)^{2n}\frac{B_{2n}}{4n}, \quad n=1,\,2,\ldots.$$

3. $$\int_0^\infty \frac{x^{\nu-1}\,dx}{e^{\mu x}+1} = \frac{1}{\mu^\nu}(1-2^{1-\nu})\Gamma(\nu)\zeta(\nu), \quad \Re\{\mu\}>0,\ \Re\{\nu\}>0.$$

4. $$\int_0^\infty \frac{x^{2n-1}\,dx}{e^{px}+1} = (1-2^{1-2n})\left(\frac{2\pi}{p}\right)^{2n}\frac{|B_{2n}|}{4n}, \quad n=1,\,2,\ldots.$$

5. $$\int_0^\infty \frac{x^{\nu-1}e^{-\mu x}}{1-\beta e^{-x}}\,dx = \Gamma(\nu)\sum_{n=0}^\infty(\mu+n)^{-\nu}\beta^n = \Gamma(\nu)\Phi(\beta,\,\nu,\,\mu),$$
$$\Re\{\mu\}>0 \text{ and either } |\beta|\le 1,\ \beta\ne 1,\ \Re\{\nu\}>0;\ \text{or } \beta=1,\ \Re\{\nu\}>1.$$

6. $$\int_0^\infty \frac{x^{\nu-1}e^{-\mu x}}{1-e^{-\beta x}}\,dx = \frac{1}{\beta^\nu}\Gamma(\nu)\zeta\left(\nu,\,\frac{\mu}{\beta}\right), \quad \Re\{\mu\}>0,\ \Re\{\nu\}>1.$$

7. $$\int_0^\infty \frac{x^{n-1}e^{-px}}{1+e^{x}}\,dx = (n-1)!\sum_{k=1}^\infty\frac{(-1)^{k-1}}{(p+k)^n} = (n-1)!\Phi(-1,n,p+1), \quad p>-1;\ n=1,\,2,\ldots.$$

8. $$\int_0^\infty \frac{xe^{-x}\,dx}{e^x-1} = \frac{\pi^2}{6}-1.$$

9. $$\int_0^\infty \frac{xe^{-2x}\,dx}{e^{-x}+1} = 1-\frac{\pi^2}{12}.$$

10. $\displaystyle\int_0^\infty \frac{xe^{-3x}}{e^{-x}+1}\,dx = \frac{\pi^2}{12} - \frac{3}{4}.$

11. $\displaystyle\int_0^\infty \frac{xe^{-2nx}}{1+e^{x}}\,dx = -\frac{\pi^2}{12} + \sum_{k=1}^{2n-1} \frac{(-1)^{k-1}}{k^2}.$

12. $\displaystyle\int_0^\infty \frac{xe^{-(2n-1)x}}{1+e^{x}}\,dx = \frac{\pi^2}{12} + \sum_{k=1}^{2n} \frac{(-1)^{k}}{k^2}.$

13. $\displaystyle\int_0^\infty \frac{x^2e^{-nx}}{1-e^{-x}}\,dx = 2\sum_{k=n}^{\infty} \frac{1}{k^3} = 2\left(\zeta(3) - \sum_{k=1}^{n-1}\frac{1}{k^3}\right), \quad n = 1,\, 2, \ldots.$

14. $\displaystyle\int_0^\infty \frac{x^2e^{-nx}}{1+e^{-x}}\,dx = 2\sum_{k=n}^{\infty} \frac{(-1)^{n+k}}{k^3} = (-1)^{n+1}\left(\frac{3}{2}\zeta(3) + 2\sum_{k=1}^{n-1}\frac{(-1)^k}{k^3}\right), \quad n = 1,\, 2, \ldots.$

15. $\displaystyle\int_0^\infty \frac{x^3e^{-nx}}{1-e^{-x}}\,dx = \frac{\pi^4}{15} - 6\sum_{k=1}^{n-1} \frac{1}{k^4}.$

16. $\displaystyle\int_0^\infty \frac{x^3e^{-nx}}{1+e^{-x}}\,dx = 6\sum_{k=n}^{\infty} \frac{(-1)^{n+k}}{k^4} = (-1)^{n+1}\left(\frac{7}{120}\pi^4 - 6\sum_{k=1}^{n-1}\frac{(-1)^k}{k^4}\right).$

17. $\displaystyle\int_0^\infty e^{-px}(e^{-x}-1)^n\,\frac{dx}{x} = -\sum_{k=0}^{n}(-1)^k\binom{n}{k}\ln(p+n-k).$

18. $\displaystyle\int_0^\infty e^{-px}(e^{-x}-1)^n\,\frac{dx}{x^2} = \sum_{k=0}^{n}(-1)^k\binom{n}{k}(p+n-k)\ln(p+n-k).$

19. $\displaystyle\int_0^\infty x^{n-1}\frac{1-e^{-mx}}{1-e^{x}}\,dx = (n-1)!\sum_{k=1}^{m}\frac{1}{k^n}.$

20. $\displaystyle\int_0^\infty \frac{x^{p-1}}{e^{rx}-q}\,dx = \frac{1}{qr^p}\Gamma(p)\sum_{k=1}^{\infty}\frac{q^k}{k^p} = \Gamma(p)\,r^{-p}\,\Phi(q,p,1), \quad p>0,\ r>0,\ -1<q<1.$

21. $\displaystyle\int_0^\infty x\frac{1+e^{-x}}{e^{x}-1}\,dx = \frac{\pi^2}{3} - 1.$

22. $$\int_0^\infty x\frac{1-e^{-x}}{1+e^{-3x}}e^{-x}\,dx = \frac{2\pi^2}{27}.$$

23. $$\int_0^\infty \frac{1-e^{-\mu x}}{1+e^{x}}\,\frac{dx}{x} = \ln\left[\frac{\Gamma\left(\frac{\mu}{2}+1\right)}{\Gamma\left(\frac{\mu+1}{2}\right)}\sqrt{\pi}\right], \quad \Re\{\mu\} > -1.$$

24. $$\int_0^\infty \frac{e^{-\nu x}-e^{-\mu x}}{e^{-x}+1}\,\frac{dx}{x} = \ln\frac{\Gamma\left(\frac{\nu}{2}\right)\Gamma\left(\frac{\mu+1}{2}\right)}{\Gamma\left(\frac{\mu}{2}\right)\Gamma\left(\frac{\nu+1}{2}\right)}, \quad \Re\{\mu\} > 0,\ \Re\{\nu\} > 0.$$

25. $$\int_0^\infty \frac{e^{-qx}+e^{(q-p)x}}{1-e^{-px}}x\,dx = \left(\frac{\pi}{p}\csc\frac{q\pi}{p}\right)^2, \quad 0 < q < p.$$

26. $$\int_0^\infty \frac{e^{-px}-e^{(p-q)x}}{e^{-qx}+1}\,\frac{dx}{x} = \ln\cot\frac{p\pi}{2q}, \quad 0 < p < q.$$

27. $$\int_0^\infty \left\{\frac{a+be^{-px}}{ce^{px}+g+he^{-px}} - \frac{a+be^{-qx}}{ce^{qx}+g+he^{-qx}}\right\}\frac{dx}{x} = \frac{a+b}{c+g+h}\,\ln\frac{p}{q}, \quad p > 0,\ q > 0.$$

28. $$\int_0^\infty \frac{\left(1-3^{-\beta x}\right)\left(1-e^{-\gamma x}\right)\,e^{\mu x}}{1-e^{-x}}\,\frac{dx}{x} = \ln\frac{\Gamma(\mu)\Gamma(\beta+\gamma+\mu)}{\Gamma(\mu+\beta)\Gamma(\mu+\gamma)},$$
$$\Re\{\mu\} > 0,\ \Re\{\mu\} > -\Re\{\beta\},\ \Re\{\mu\} > -\Re\{\gamma\},\ \Re\{\mu\} > -\Re\{\beta+\gamma\}.$$

29. $$\int_0^\infty \frac{\left[1-e^{(q-p)x}\right]^2}{e^{qx}-e^{(q-2p)x}}\,\frac{dx}{x} = \ln\csc\frac{q\pi}{2p}, \quad 0 < q < p.$$

30. $$\int_0^\infty \frac{e^{-px}-e^{-qx}}{1+e^{-x}}\,\frac{1+e^{-(2n+1)x}}{x}dx$$
$$= \ln\left\{\frac{q(q+2)(q+4)\ldots(q+2n)(p+1)(p+3)\ldots(p+2n-1)}{p(p+2)(p+4)\ldots(p+2n)(q+1)(q+3)\ldots(q+2n-1)}\right\},$$
$$\Re\{p\} > -2n,\ \Re\{q\} > -2n.$$

31. $$\int_0^\infty \frac{(1-e^{-\beta x})(1-e^{-\gamma x})(1-e^{-\delta x})\,e^{-\mu x}}{1-e^{-x}}\,\frac{dx}{x}$$
$$= \ln\frac{\Gamma(\mu)\Gamma(\mu+\beta+\gamma)\Gamma(\mu+\beta+\delta)\Gamma(\mu+\gamma+\delta)}{\Gamma(\mu+\beta)\Gamma(\mu+\gamma)\Gamma(\mu+\delta)\Gamma(\mu+\beta+\gamma+\delta)},$$
$$2\Re\{\mu\} > |\Re\{\beta\}| + |\Re\{\gamma\}| + |\Re\{\delta\}|.$$

32. $$\int_0^\infty \frac{x\,dx}{(x^2+\beta^2)(e^{\mu x}-1)} = \frac{1}{2}\left[\ln\left(\frac{\beta\mu}{2\pi}\right) - \frac{\pi}{\beta\mu} - \psi\left(\frac{\beta\mu}{2\pi}\right)\right], \quad \Re\{\beta\}>0,\ \Re\{\mu\}>0.$$

33. $$\int_0^\infty \frac{x\,dx}{(x^2+\beta^2)^2(e^{2\pi x}-1)} = -\frac{1}{8\beta^3} - \frac{1}{4\beta^2} + \frac{1}{4\beta}\psi'(\beta) \sim \frac{1}{4\beta^4}\sum_{k=0}^{\infty}\frac{|B_{2k+2}|}{\beta^{2k}}, \quad \Re\{\beta\}>0,$$

(asymptotic expansion for $\Re\{\beta\}>0$).

34. $$\int_0^\infty \frac{x\,dx}{(x^2+\beta^2)(e^{\mu x}-1)} = \frac{1}{2}\left[\psi\left(\frac{\beta\mu}{2\pi}+\frac{1}{2}\right) - \ln\left(\frac{\beta\mu}{2\pi}\right)\right], \quad \Re\{\beta\}>0,\ \Re\{\mu\}>0.$$

35. $$\int_0^\infty \frac{x\,dx}{(x^2+\beta^2)^2(e^{2\pi x}+1)} = \frac{1}{4\beta^2} - \frac{1}{4\beta}\psi'\left(\beta+\frac{1}{2}\right), \quad \Re\{\beta\}>0,\ \Re\{\mu\}>0.$$

36. $$\int_0^\infty \frac{(1+i\,x)^{2n}-(1-i\,x)^{2n}}{i}\,\frac{dx}{e^{2\pi x}-1} = \frac{1}{2}\,\frac{2n-1}{2n+1}, \quad n=1,\,2,\ldots.$$

37. $$\int_0^\infty \frac{(1+ix)^{2n}-(1-i\,x)^{2n}}{i}\,\frac{dx}{e^{\pi x}+1} = \frac{1}{2n+1}, \quad n=1,\,2,\ldots.$$

38. $$\int_0^\infty \frac{(1+i\,x)^{2n-1}-(1-i\,x)^{2n-1}}{i}\,\frac{dx}{e^{\pi x}+1} = \frac{1}{2n}\left[1-2^{2n}B_{2n}\right], \quad n=1,\,2,\ldots.$$

39. $$\int_0^\infty \frac{x\,dx}{e^x+e^{-x}-1} = \frac{1}{3}\left[\psi'\left(\frac{1}{3}\right) - \frac{2}{3}\pi^2\right].$$

40. $$\int_0^\infty \frac{xe^{-x}\,dx}{e^x+e^{-x}-1} = \frac{1}{6}\left[\psi'\left(\frac{1}{3}\right) - \frac{5}{6}\pi^2\right].$$

41. $$\int_0^\infty (e^{-\nu x}-1)^n(e^{-\rho x}-1)^m e^{-\mu x}\frac{dx}{x^2} = \sum_{k=0}^{n}(-1)^k\binom{n}{k}\sum_{j=0}^{m}(-1)^j\binom{m}{j}$$
$$\times\{(m-j)\rho+(n-k)\nu+\mu\}\,\ln\left[(m-j)\rho+(n-k)\nu+\mu\right],$$
$$\Re\{\nu\}>0,\ \Re\{\mu\}>0,\ \Re\{\rho\}>0.$$

42. $$\int_0^\infty (1-e^{-\nu x})^n(1-e^{-\rho x})e^{-x}\frac{dx}{x^3} = \frac{1}{2}\sum_{k=0}^{n}(-1)^k\binom{n}{k}(\rho+k\nu+1)^2 \times \ln(\rho+k\nu+1) + \frac{1}{2}\sum_{k=1}^{n}(-1)^{k-1}\binom{n}{k}(k\nu+1)^2\ln(k\nu+1),$$
$$n \geq 2,\ \Re\{\nu\}>0,\ \Re\{\rho\}>0.$$

43. $$\int_0^\infty (e^{-px}-e^{-qx})(e^{-rx}-e^{-sx})\,e^{-x}\frac{dx}{x} = \ln\frac{(p+s+1)(q+r+1)}{(p+r+1)(q+s+1)},$$
$$p+s>-1,\ p+r>-1,\ q>p.$$

44. $$\int_0^\infty (1-e^{-px})(1-e^{-qx})(1-e^{-rx})\,e^{-x}\frac{dx}{x}$$
$$= (p+q+1)\ln(p+q+1)+(p+r+1)\ln(p+r+1)+(q+r+1)\ln(q+r+1)-(p+1)\ln(p+1) -(q+1)\ln(q+1)-(r+1)\ln(r+1)-(p+q+r)\ln(p+q+r), \quad p>0,\ q>0,\ r>0.$$

45. $$\int_0^\infty \frac{x^{\nu-1}}{(e^x-1)^2}\,dx = \Gamma(\nu)[\zeta(\nu-1)-\zeta(\nu)], \quad \Re\{\nu\}>2.$$

46. $$\int_0^\infty \frac{x^{\nu-1}e^{-\mu x}}{(e^x-1)^2}\,dx = \Gamma(\nu)[\zeta(\nu-1,\,\mu+2)-(\mu+1)\zeta(\nu,\,\mu+2)], \quad \Re\{\mu\}>-2,\ \Re\{\nu\}>2.$$

47. $$\int_0^\infty \frac{x^q e^{-px}\,dx}{(1-ae^{-px})^2} = \Gamma(p+1)p^{-q-1}\Phi(a,q,1), \quad -1\leq a<1,\ q>-1,\ p>0.$$

48. $$\int_0^\infty \frac{x^{\nu-1}e^{-\mu x}}{(1-\beta e^{-x})^2}\,dx = \Gamma(\nu)[\Phi(\beta,\,\nu-1,\,\mu)-(\mu-1)\Phi(\beta,\,\nu,\,\mu)],$$
$$\Re\{\nu\}>0,\ \Re\{\mu\}>0,\ |\arg(1-\beta)|<\pi.$$

49. $$\int_0^\infty \frac{(1+a)e^x-a}{(1-e^x)^2}e^{-ax}x^n\,dx = n!\,\zeta(n,\,a), \quad a>-1, n=1,2,\ldots.$$

50. $$\int_0^\infty \frac{(1+a)e^x+a}{(1+e^x)^2}e^{-ax}x^n\,dx = n!\sum_{k=1}^{\infty}\frac{(-1)^k}{(a+k)^n} = n!\,\Phi(-1,n,a+1), \quad a>-1, n=1,2,\ldots.$$

51. $$\int_0^\infty \frac{e^x-e^{-x}+2}{(e^x-1)^2}x^2\,dx = \frac{2}{3}\pi^2-2.$$

52. $$\int_0^\infty \left(\frac{e^{-x}}{x} + \frac{e^{-\mu x}}{e^{-x} - 1} \right) dx = \psi(\mu), \quad \Re\{\mu\} > 0.$$

53. $$\int_0^\infty \left(\frac{1}{1 - e^{-x}} - \frac{1}{x} \right) e^{-x}\, dx = \gamma_e.$$

54. $$\int_0^\infty \left(\frac{1}{2} - \frac{1}{1 + e^{-x}} \right) \frac{e^{-2x}}{x}\, dx = \frac{1}{2}\, \ln\, \frac{\pi}{4}.$$

55. $$\int_0^\infty \left(\frac{1}{2} - \frac{1}{x} + \frac{1}{e^x - 1} \right) \frac{e^{-\mu x}}{x}\, dx = \ln\, \Gamma(\mu) - \left(\mu - \frac{1}{2} \right) \ln\, \mu + \mu - \frac{1}{2}\, \ln\,(2\pi), \quad \Re\{\mu\} > 0.$$

56. $$\int_0^\infty \left(\frac{1}{2} e^{-2x} - \frac{1}{e^x + 1} \right) \frac{dx}{x} = -\frac{1}{2}\, \ln\, \pi.$$

57. $$\int_0^\infty \left(\frac{e^{\mu x} - 1}{1 - e^{-x}} - \mu \right) \frac{e^{-x}}{x}\, dx = -\ln\, \Gamma(\mu) - \ln\, \sin(\pi\mu) + \ln\, \pi, \quad \Re\{\mu\} < 1.$$

58. $$\int_0^\infty \left(\frac{e^{-\nu x}}{1 - e^{-x}} - \frac{e^{-\mu x}}{x} \right) dx = \ln\, \mu - \psi(\nu).$$

59. $$\int_0^\infty \left(\frac{n}{x} - \frac{e^{-\mu x}}{1 - e^{-x/n}} \right) e^{-x}\, dx = n\psi(n\mu + n) - n\, \ln\, n, \quad \Re\{\mu\} > 0,\ n = 1,\, 2, \ldots.$$

60. $$\int_0^\infty \left(\mu - \frac{1 - e^{-\mu x}}{1 - e^{-x}} \right) \frac{e^{-x}}{x}\, dx = \ln\, \Gamma(\mu + 1), \quad \Re\{\mu\} > -1.$$

61. $$\int_0^\infty \left(\nu e^{-x} - \frac{e^{-\mu x} - e^{-(\mu+\nu)x}}{e^x - 1} \right) \frac{dx}{x} = \ln\, \frac{\Gamma(\mu + \nu + 1)}{\Gamma(\mu + 1)}, \quad \Re\{\mu\} > -1,\ \Re\{\nu\} > 0.$$

62. $$\int_0^\infty \left[(1 - e^x)^{-1} + x^{-1} - 1 \right] e^{-xz}\, dx = \psi(z) - \ln\, z, \quad \Re\{z\} > 0.$$

63. $$\int_0^\infty \left(\nu e^{-\mu x} - \frac{1}{\mu} e^{-x} - \frac{1}{\mu}\, \frac{e^{-1} - e^{-\mu\nu x}}{1 - e^{-x}} \right) \frac{dx}{x} = \frac{1}{\mu}\, \ln\, \Gamma(\mu\nu) - \nu\, \ln\, \mu,$$
$$\Re\{\mu\} > 0,\ \Re\{\nu\} > 0.$$

64. $$\int_0^\infty \left(\frac{n-1}{2} + \frac{n-1}{1 - e^{-x}} + \frac{e^{(1-\mu)x}}{1 - e^{x/n}} + \frac{e^{-n\mu x}}{1 - e^{-x}} \right) e^{-x} \frac{dx}{x} = \frac{n-1}{2}\, \ln\, 2\pi - \left(n\mu + \frac{1}{2} \right) \ln\, n,$$
$$\Re\{\mu\} > 0,\ n = 1,\, 2, \ldots.$$

65. $$\int_0^\infty \left(n\mu - \frac{n-1}{2} - \frac{n}{1-e^{-x}} - \frac{e^{(1-\mu)x}}{1-e^{x/n}}\right)\frac{e^{-x}}{x}\,dx = \sum_{k=0}^{n-1} \ln\,\Gamma\left(\mu - \frac{k}{n} + 1\right),$$
$$\Re\{\mu\} > 0,\ n = 1,\ 2, \ldots.$$

66. $$\int_0^\infty \left(\frac{e^{-\nu x}}{1-e^{x}} - \frac{e^{-\mu\nu x}}{1-e^{\mu x}} - \frac{e^{x}}{1-e^{x}} + \frac{e^{\mu x}}{1-e^{\mu x}}\right)\frac{dx}{x} = \nu\,\ln\,\mu,$$
$$\Re\{\mu\} > 0,\ \Re\{\nu\} > 0.$$

67. $$\int_0^\infty \left[\frac{1}{e^x-1} - \frac{\mu e^{-\mu x}}{1-e^{-\mu x}} + \left(a\mu - \frac{\mu+1}{2}\right)e^{-\mu x} + (1-a\mu)\,e^{-x}\right]\,\frac{dx}{x} = \frac{\mu-1}{2}\,\ln(2\pi) + \left(\frac{1}{2} - a\mu\right)\,\ln\,\mu,$$
$$\Re\{\mu\} > 0.$$

68. $$\int_0^\infty \left[\frac{e^{-\nu x}}{1-e^{-x}} - \frac{e^{-\mu\nu x}}{1-e^{-\mu x}} - \frac{(\mu-1)e^{-\mu x}}{1-e^{-\mu x}} - \frac{\mu-1}{2}e^{-\mu}x\right]\,\frac{dx}{x} = \frac{\mu-1}{2}\,\ln(2\pi) + \left(\frac{1}{2} - \mu\nu\right)\,\ln\,\mu,$$
$$\Re\{\mu\} > 0,\ \Re\{\nu\} > 0.$$

69. $$\int_0^\infty \left[1 - e^{-x} - \frac{(1-e^{-\nu x})(1-e^{-\mu x})}{1-e^{-x}}\right]\,\frac{dx}{x} = \ln\,\mathrm{B}(\mu,\,\nu),$$
$$\Re\{\mu\} > 0,\ \Re\{\nu\} > 0.$$

70. $$\int_0^\infty \left[e^{-x} - (1+x)^{-\mu}\right]\,\frac{dx}{x} = \psi(\mu), \quad \Re\{\mu\} > 0.$$

71. $$\int_0^\infty \left(e^{-\mu x} - 1 + \mu x - \frac{1}{2}\mu^2 x^2\right)x^{\nu-1}\,dx = \frac{-1}{\nu(\nu+1)(\nu+2)\mu^\nu}\Gamma(\nu+3),$$
$$\Re\{\mu\} > 0,\ -2 > \Re\{\nu\} > -3.$$

72. $$\int_0^\infty \left[x^{-1} - \frac{1}{2}x^{-2}(x+2)(1-e^{-x})\right]e^{-px}\,dx = -1 + \left(p + \frac{1}{2}\right)\,\ln\,\left(1 + \frac{1}{p}\right), \quad \Re\{p\} > 0.$$

73. $$\int_0^\infty x^{\nu-1}e^{-mx}(e^{-x}-1)^n\,dx = \Gamma(\nu)\sum_{k=0}^{n}(-1)^k\binom{n}{k}\frac{1}{(n+m-k)^\nu}, \quad n = 0;\ 1, \ldots, \Re\{\nu\} > 0.$$

74. $$\int_0^\infty \left[x^{\nu-1}e^{-x} - e^{-\mu x}(1-e^{-x})^{\nu-1}\right]\,dx = \Gamma(\nu) - \frac{\Gamma(\mu)}{\Gamma(\mu+\nu)}, \quad \Re\{\mu\} > 0,\ \Re\{\nu\} > 0.$$

75. $$\int_0^\infty x^{p-1}\left[e^{-x} + \sum_{k=1}^{n}(-1)^k\frac{x^{k-1}}{(k-1)!}\right]\,dx = \Gamma(p), \quad -n < p < -n+1,\ n = 0,\ 1, \ldots.$$

76. $\displaystyle\int_0^\infty \frac{e^{-\nu x} - e^{-\mu x}}{x^{\rho+1}}\, dx = \frac{\mu^\rho - \nu^\rho}{\rho}\Gamma(1-\rho), \quad \Re\{\mu\} > 0,\ \Re\{\nu\} > 0,\ \Re\{\rho\} < 1.$

77. $\displaystyle\int_0^\infty \frac{e^{-\mu x} - e^{-\nu x}}{x}\, dx = \ln\, \frac{\nu}{\mu}, \quad \Re\{\mu\} > 0,\ \Re\{\nu\} > 0.$

78. $\displaystyle\int_0^\infty \left[(x+1)e^{-x} - e^{-x/2}\right]\, \frac{dx}{x} = 1 - \ln\, 2.$

79. $\displaystyle\int_0^\infty \frac{1 - e^{-\mu x}}{x(x+\beta)}\, dx = \frac{1}{\beta}\, \left[\ln\left(\beta\mu\gamma_e\right) - e^{\beta\mu}\,\mathrm{EI}(-\beta\mu)\right], \quad |\arg\beta| < \pi,\ \Re\{\mu\} > 0.$

80. $\displaystyle\int_0^\infty \left(\frac{1}{1+x} - e^{-x}\right)\, \frac{dx}{x} = \gamma_e.$

81. $\displaystyle\int_0^\infty \left(e^{-\mu x} - \frac{1}{1+ax}\right)\, \frac{dx}{x} = \ln\, \frac{a}{\mu} - \gamma_e, \quad a > 0,\ \Re\{\mu\} > 0.$

82. $\displaystyle\int_0^\infty \left\{\frac{e^{-npx} - e^{-nqx}}{n} - \frac{e^{-mpx} - e^{-mqx}}{m}\right\}\, \frac{dx}{x^2} = (q-p)\, \ln\, \frac{m}{n}, \quad p > 0,\ q > 0.$

83. $\displaystyle\int_0^\infty \left\{pe^{-x} - \frac{1 - e^{-px}}{x}\right\}\, \frac{dx}{x} = p\, \ln\, p - p, \quad p > 0.$

84. $\displaystyle\int_0^\infty \left\{\left(\frac{1}{2} + \frac{1}{x}\right)e^{-x} - \frac{1}{x}e^{-x/2}\right\}\, \frac{dx}{x} = \frac{\ln\, 2 - 1}{2}.$

85. $\displaystyle\int_0^\infty \left\{\frac{p^3}{6}e^{-x} - \frac{p^2}{2x} + \frac{p}{x^2} - \frac{1 - e^{-px}}{x^3}\right\}\, \frac{dx}{x} = \frac{p^3}{6}\, \ln\, p - \frac{11}{36}\, p^3, \quad p > 0.$

86. $\displaystyle\int_0^\infty \left(e^{-x} - e^{-2x} - \frac{1}{x}e^{-2x}\right)\, \frac{dx}{x} = 1 - \ln\, 2.$

87. $\displaystyle\int_0^\infty \left\{\left(p - \frac{1}{2}\right)e^{-x} + \frac{x+2}{2x}\left(e^{-px} - e^{-x/2}\right)\right\}\frac{dx}{x} = \left(p - \frac{1}{2}\right)(\ln\, p - 1), \quad p > 0.$

88. $\displaystyle\int_0^\infty \left\{(p-q)e^{-rx} + \frac{1}{mx}(e^{-mpx} - e^{-mqx})\right\}\, \frac{dx}{x} = p\, \ln\, p - q\, \ln\, q - (p-q)\left(1 + \ln\, \frac{r}{m}\right),$

$p > 0,\ q > 0,\ r > 0.$

89. $$\int_0^\infty \left[(p-r)e^{-qx} + (r-q)e^{-px} + (q-p)e^{-rx}\right] \frac{dx}{x^2} = (r-q)p \ln p + (p-r)q \ln q + (q-p)r \ln r,$$
$$p > 0,\ q > 0,\ r > 0.$$

90. $$\int_0^\infty \left\{1 - \frac{x+2}{2x}(1-e^{-x})\right\} e^{-qx} \frac{dx}{x} = -1 + \left(q + \frac{1}{2}\right) \ln \frac{q+1}{q}, \quad q > 0.$$

91. $$\int_0^\infty \left(\frac{e^{-x}-1}{x} + \frac{1}{1+x}\right) \frac{dx}{x} = \gamma_e - 1.$$

92. $$\int_0^\infty \left(e^{-px} - \frac{1}{1+a^2x^2}\right) \frac{dx}{x} = \ln \frac{a}{p} - \gamma_e, \quad p > 0.$$

93. $$\int_0^\infty \left\{\frac{e^{-x}p^2}{2} - \frac{p}{x} + \frac{1-e^{-px}}{x^2}\right\} \frac{dx}{x} = \frac{p^2}{2} \ln p - \frac{3}{4}p^2, \quad p > 0.$$

94. $$\int_0^\infty \frac{(1-e^{-px})^n\, e^{-qx}}{x^3}\, dx = \frac{1}{2}\sum_{k=2}^{n}(-1)^{k-1}\binom{n}{k}(q+kp)^2 \ln(q+kp),$$
$$n > 2,\ q > 0,\ pn + q > 0.$$

95. $$\int_0^\infty (1-e^{-px})^2 e^{-qx} \frac{dx}{x^2} = (2p+q) \ln(2p+q) - 2(p+q) \ln(p+q) + q \ln q,$$
$$q > 0,\ 2p > -q.$$