! For an efficient use of these tables, first read `HowTo.pdf`.

T2.63B. Integrands involving logarithm functions of complicated arguments, like $\left(1 + a^2/x^2\right)$, $(1 \pm e^{-x})$, $\left(1 + 2e^{-x} \cos t + e^{-2x}\right)$ and others, on the interval $(0, y)$.

1. $\displaystyle\int_0^y \ln \sin x \, dx = L\left(\frac{\pi}{2} - y\right) - L\left(\frac{\pi}{2}\right).$

2. $\displaystyle\int_0^y \ln \cos x \, dx = -L(y).$

3. $\displaystyle\int_0^y \ln\left(1 - \sin^2 \alpha \sin^2 x\right) dx = (\pi - 2\theta) \ln \cot \frac{\alpha}{2} + 2y \ln\left(\frac{1}{2} \sin \alpha\right) - \frac{\pi}{2} \ln 2$
$\displaystyle+L(\theta + y) - L(\theta - y) + L\left(\frac{\pi}{2} - 2y\right), \quad \cot \theta = \cos \alpha \tan y; \ -\pi \le \alpha \le \pi, \ -\frac{\pi}{2} \le y \le \frac{\pi}{2}.$

4. $\displaystyle\int_0^y \ln\left(1 - \frac{\sin^2 x}{\sin^2 a}\right) dx = -y \ln \sin^2 \alpha - L\left(\frac{\pi}{2} - \alpha + y\right) + L\left(\frac{\pi}{2} - \alpha - y\right),$
$\displaystyle-\frac{\pi}{2} \le y \le \frac{\pi}{2}, \ |\sin y| \le |\sin \alpha|.$

5. $\displaystyle\int_0^y \ln \tan x \, dx = L(y) + L\left(\frac{\pi}{2} - y\right) - L\left(\frac{\pi}{2}\right).$

6. $\displaystyle\int_0^y \ln\left(\cos x + \sqrt{\cos^2 x - \cos^2 x}\right) dx = -\left(\frac{\pi}{2} - t - \varphi\right) \ln \cos t + \frac{1}{2} L(y + \varphi) - \frac{1}{2} L(y - \varphi) - L(\varphi),$
$\displaystyle\text{where } \cos \varphi = \frac{\sin y}{\sin t}, \ 0 \le y \le t \le \frac{\pi}{2}.$

7. $\displaystyle\int_0^y \ln\left(\cos x + \sqrt{\cos^2 x - \cos^2 x}\right) dx = -\left(\frac{\pi}{2} - y\right) \ln \cos y.$

8. $$\int_0^y \ln \frac{\sin y + \sin t \, \cos x \sqrt{\sin^2 y - \sin^2 x}}{\sin y - \sin t \, \cos x \sqrt{\sin^2 y - \sin^2 x}} \, dx = \pi \, \ln \left[\tan\left(\frac{t}{2}\right) \, \sin y + \sqrt{\tan^2 \frac{t}{2} \, \sin^2 y + 1} \, \right],$$

$$t > 0, \; y > 0.$$