! For an efficient use of these tables, first read `HowTo.pdf`.

T1.03. Integrand involving rational functions.

1. $\displaystyle\int \frac{dx}{a+b\,x} = \frac{1}{b}\,\ln\,|a+b\,x|.$
2. $\displaystyle\int \frac{dx}{(a+b\,x)^2} = -\frac{1}{b(a+b\,x)}.$
3. $\displaystyle\int \frac{dx}{(a+b\,x)^3} = -\frac{1}{2b\,(a+b\,x)^2}.$
4. $\displaystyle\int \frac{x}{a+b\,x}\,dx = \begin{cases} \dfrac{1}{b^2}\,[a+b\,x-a\,\ln\,(a+b\,x)] \\ \text{or} \\ \dfrac{x}{b}-\dfrac{a}{b^2}\,\ln\,(a+b\,x), \end{cases}$
5. $\displaystyle\int \frac{x}{(a+b\,x)^2}\,dx = \frac{1}{b^2}\,\left[\ln\,(a+b\,x)+\frac{a}{a+b\,x}\right].$
6. $\displaystyle\int \frac{x}{(a+b\,x)^n}\,dx = \frac{1}{b^2}\,\left[\frac{a}{(n+1)(a+b\,x)^{n-1}}-\frac{1}{(n-2)(a+b\,x)^{n-2}}\right], \quad n\neq 1,2.$
7. $\displaystyle\int \frac{x^2}{a+b\,x}\,dx = \frac{1}{b^2}\,\left[\frac{1}{2}\,(a+b\,x)^2-2a(a+b\,x)+a^2\,\ln\,(a+b\,x)\right].$
8. $\displaystyle\int \frac{x^2}{(a+b\,x)^2}\,dx = \frac{1}{b^3}\,\left[a+b\,x-2a\,\ln\,(a+b\,x)-\frac{a^2}{a+b\,x}\right].$
9. $\displaystyle\int \frac{x^2}{(a+b\,x)^3}\,dx = \frac{1}{b^3}\,\left[\ln\,(a+b\,x)+\frac{2a}{a+b\,x}-\frac{a^2}{2(a+b\,x)^2}\right].$
10. $\displaystyle\int \frac{x^2}{(a+b\,x)^n}\,dx = \frac{1}{b^3}\,\left[\frac{-1}{(n-3)(a+b\,x)^{n-3}}+\frac{2a}{(n-2)(a+b\,x)^{n-2}}-\frac{a^2}{(n-1)(a+b\,x)^{n-1}}\right], \quad n\neq 1,2,3.$
11. $\displaystyle\int \frac{dx}{x(a+b\,x)} = -\frac{1}{a}\,\ln\,\frac{a+b\,x}{x}.$
12. $\displaystyle\int \frac{dx}{x(a+b\,x)^2} = \frac{1}{a(a+b\,x)}-\frac{1}{a^2}\,\ln\,\frac{a+b\,x}{x}.$

13. $\displaystyle\int \frac{dx}{x(a+b\,x)^3} = \frac{1}{a^3}\left[\frac{1}{2}\left(\frac{2a+b\,x}{a+b\,x}\right)^2 - \ln\frac{a+b\,x}{x}\right].$

14. $\displaystyle\int \frac{dx}{x^2(a+b\,x)} = \frac{b}{a^2}\ln\frac{a+b\,x}{x} - \frac{1}{ax}.$

15. $\displaystyle\int \frac{dx}{x^3(a+b\,x)} = \frac{b^2}{a^3}\ln\frac{x}{a+b\,x} + \frac{2bx-a}{2a^2x^2}.$

16. $\displaystyle\int \frac{dx}{x^2(a+b\,x)^2} = \frac{2b}{a^3}\ln\frac{a+b\,x}{x} - \frac{a+2bx}{a^2x(a+b\,x)}.$

17. $\displaystyle\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}.$

18. $\displaystyle\int \frac{dx}{a^2-x^2} = \begin{cases} \dfrac{1}{a}\tanh^{-1}\dfrac{x}{a} \\ \text{or} \\ \dfrac{1}{2a}\ln\dfrac{a+x}{a-x}, \end{cases} \qquad a^2 > x^2.$

19. $\displaystyle\int \frac{dx}{x^2-a^2} = \begin{cases} -\dfrac{1}{a}\coth^{-1}\dfrac{x}{a} \\ \text{or} \\ \dfrac{1}{2a}\ln\dfrac{x-a}{x+a}, \end{cases} \qquad x^2 > a^2.$

20. $\displaystyle\int \frac{dx}{\sqrt{a^2-x^2}} = \begin{cases} \sin^{-1}\dfrac{x}{|a|} \\ \text{or} \\ -\cos^{-1}\dfrac{x}{|a|}, \end{cases} \qquad a^2 > x^2.$

21. $\displaystyle\int \frac{x}{a^2\pm x^2}\,dx = \pm\frac{1}{2}\ln\left(a^2\pm x^2\right).$

22. $\displaystyle\int \frac{x}{\left(a^2\pm x^2\right)^{n+1}}\,dx = \mp\frac{1}{2n\left(a^2\pm x^2\right)^n}, \quad n\neq 0.$

23. $\displaystyle\int \frac{dx}{\left(a^2\pm x^2\right)^n}\,dx = \frac{1}{2(n-1)\,a^2}\left[\frac{x}{\left(a^2\pm x^2\right)^{n-1}} + (2n-3)\int\frac{dx}{\left(a^2\pm x^2\right)^{n-1}}\right].$

24. $\displaystyle\int \frac{dx}{\left(x^2-a^2\right)^n}\,dx = \frac{1}{2(n-1)\,a^2}\left[-\frac{x}{\left(x^2-a^2\right)^{n-1}} - (2n-3)\int\frac{dx}{\left(x^2-a^2\right)^{n-1}}\right].$

25. $\displaystyle\int \frac{x}{x^2-a^2}\,dx = \frac{1}{2}\ln\left(x^2-a^2\right).$

26. $\displaystyle\int \frac{x}{\left(x^2-a^2\right)^{n+1}}\,dx = -\frac{1}{2n\left(x^2-a^2\right)^n}.$

27. $\displaystyle\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{|a|}\sec^{-1}\frac{x}{a}.$

28. $\displaystyle\int \frac{dx}{x\sqrt{a^2 \pm x^2}} = -\frac{1}{a}\ln\Big(\frac{a+\sqrt{a^2 \pm x^2}}{x}\Big).$

29. $\displaystyle\int \frac{b}{(x-a)^\alpha}\,dx = b\int \frac{d(x-a)}{(x-a)^\alpha} = -\frac{b}{(\alpha-1)(x-a)^{\alpha-1}}.$

30. $\displaystyle\int \frac{b}{x-a}\,dx = b\int \frac{d(x-a)}{x-a} = b\ln|x-a|.$

31. $\displaystyle\int \frac{Px+Q}{(A+2Bx+Cx^2)^p}\,dx = \frac{QB-PA+(QC-PB)x}{2(p-1)(AC-B^2)(A+2Bx+Cx^2)^{p-1}}$
$\displaystyle + \frac{(2p-3)(QC-PB)}{2(p-1)(AC-B^2)}\int \frac{dx}{(A+2Bx+Cx^2)^{p-1}}.$

32. $\displaystyle\int \frac{dx}{A+2Bx+Cx^2} = \begin{cases} \dfrac{1}{\sqrt{AC-B^2}}\arctan\dfrac{Cx+B}{\sqrt{Ac-B^2}} & \text{for } AC > B^2, \\ \dfrac{1}{2\sqrt{B^2-AC}}\ln\left|\dfrac{Cx+B-\sqrt{B^2-AC}}{Cx+B+\sqrt{B^2-AC}}\right| & \text{for } AC < B^2. \end{cases}$

33. $\displaystyle\int \frac{(Px+Q)\,dx}{A+2Bx+Cx^2}$
$\displaystyle = \begin{cases} \dfrac{P}{2C}\ln|A+2Bx+Cx^2| + \dfrac{QC-PB}{C\sqrt{AC-B^2}}\arctan\dfrac{Cx+B}{\sqrt{AC-B^2}}, & AC > B^2, \\ \dfrac{P}{2C}\ln|A+2Bx+Cx^2| + \dfrac{QC-PB}{2C\sqrt{B^2-AC}}\ln\left|\dfrac{Cx+B-\sqrt{B^2-AC}}{Cx+B+\sqrt{B^2-AC}}\right|, & AC < B^2. \end{cases}$

34. $\displaystyle\int \frac{dx}{\sqrt{x^a-b}} = \frac{x}{\sqrt{x^a-b}}\sqrt{1-\frac{x^a}{b}}\ {}_2F_1\Big(\frac{1}{a},\frac{1}{2},1+\frac{1}{a},\frac{x^a}{b}\Big), \quad a>0.$