

! For an efficient use of these tables, first read [HowTo.pdf](#).

T3.25A. Integrands involving exponentials and algebraic functions on the interval $(0, \infty)$.

$$1. \int_0^\infty \frac{e^{-qx}}{\sqrt{x}} dx = \sqrt{\frac{\pi}{q}}, \quad q > 0.$$

$$2. \int_0^\infty \frac{e^{-\mu x}}{\sqrt{x+\beta}} dx = \sqrt{\frac{\pi}{\mu}} e^{\beta\mu} \left[1 - \operatorname{erf}(\sqrt{\beta\mu}) \right], \quad \Re\{\mu\} > 0, \quad |\arg \beta| < \pi.$$

$$3. \int_0^\infty \frac{e^{-px} dx}{\sqrt{x(x+a)}} = e^{ap/2} K_0\left(\frac{ap}{2}\right), \quad a > 0, \quad p > 0.$$

$$4. \int_0^\infty \frac{(x+\beta)e^{-\mu x} dx}{\sqrt{x^2+2\beta x}} = \beta e^{\beta\mu} K_1(\beta\mu), \quad \Re\{\mu\} > 0, \quad |\arg \beta| < \pi.$$

$$5. \int_0^\infty \frac{xe^{-\mu x} dx}{\sqrt{x^2+\beta^2}} = \frac{\beta\pi}{2} [\mathbf{H}_1(\beta\mu) - Y_1(\beta\mu)] - \beta, \quad |\arg \beta| < \frac{\pi}{2}, \quad \Re\{\mu\} > 0.$$

$$6. \int_0^\infty \frac{e^{-\mu x} dx}{(1+\cos t+x)\sqrt{x^2+2x}} = \frac{\exp\left(2\mu \cos^2 \frac{t}{2}\right)}{\sin t} \left(t - \sin t \int_0^u K_0(v) e^{-v \cos t} dv \right),$$

$$\Re\{\mu\} > 0.$$

$$7. \int_0^\infty \frac{e^{-\mu x} dx}{x + \sqrt{x^2 + \beta^2}} = \frac{\pi}{2\beta\mu} [\mathbf{H}_1(\beta\mu) - Y_1(\beta\mu)] - \frac{1}{\beta^2\mu^2}, \quad |\arg \beta| < \frac{\pi}{2}, \quad \Re\{\mu\} > 0.$$

$$8. \int_0^\infty \frac{e^{-\mu x} dx}{\sqrt{(x+a)^3}} = \frac{2}{\sqrt{a}} - 2\sqrt{\pi\mu} e^{\alpha\mu} (1 - \operatorname{erf}(\sqrt{a\mu})), \quad |\arg a| < \pi, \quad \Re\{\mu\} > 0.$$

$$9. \int_0^\infty x^{n-1/2} e^{-\mu x} dx = \sqrt{\pi} \cdot \frac{1}{2} \cdot \frac{3}{2} \cdots \frac{2n-1}{2} \mu^{-n-1/2}$$

$$= \sqrt{\pi} 2^{-n} \mu^{-n-1/2} (2n-1)!!, \quad n \geq 0; \quad \Re\{\mu\} > 0.$$

$$10. \int_0^\infty x^{n-1/2} (2+x)^{n-1/2} e^{-px} dx = \frac{(2n-1)!!}{p^n} e^p K_n(p), \quad p > 0, \quad n = 0, 1, 2, \dots$$

$$11. \int_0^\infty \left[\left(x + \sqrt{x^2 + \beta^2} \right)^n + \left(x - \sqrt{x^2 + \beta^2} \right)^n \right] e^{-\mu x} dx = 2\beta^{n+1} O_n(\beta\mu), \quad \Re\{\mu\} > 0.$$

$$12. \int_0^\infty \frac{(x + \sqrt{1+x^2})^n}{\sqrt{1+x^2}} e^{-\mu x} dx = \frac{1}{2} [S_n(\mu) - \pi \mathbf{E}_n(\mu) - \pi Y_n(\mu)], \quad \Re\{\mu\} > 0.$$

$$13. \int_0^\infty \frac{(x - \sqrt{1+x^2})^n}{\sqrt{1+x^2}} e^{-\mu x} dx = -\frac{1}{2} [S_n(\mu) + \pi \mathbf{E}_n(\mu) + \pi Y_n(\mu)], \quad \Re\{\mu\} > 0.$$
