! For an efficient use of these tables, first read `HowTo.pdf`.

T3.52A. Integrands involving logarithm functions and powers of $(a+b\,x)$ on the interval $(0,\infty)$.

1. $$\int_0^\infty \frac{x^{\mu-1}\ln\, x}{\beta+x}\,dx = \frac{\pi\beta^{\mu-1}}{\sin\mu\pi}(\ln\,\beta-\pi\cot\mu\pi), \quad |\arg\beta|<\pi,\ 0<\Re\{\mu\}<1.$$

2. $$\int_0^\infty \frac{x^{\mu-1}\ln\, x}{a-x}\,dx = \pi a^{\mu-1}\left(\cot\mu\pi\ln\,a-\frac{\pi}{\sin^2\mu\pi}\right), \quad a>0,\ 0<\Re\{\mu\}<1.$$

3. $$\int_0^\infty \frac{x^{\mu-1}\ln\, x}{(x+\beta)(x+\gamma)}\,dx = \frac{\pi}{(\gamma-\beta)\sin\mu\pi}\left[\beta^{\mu-1}\ln\,\beta-\gamma^{\mu-1}\ln\,\gamma-\pi\cot\mu\pi(\beta^{\mu-1}-\gamma^{\mu-1})\right],$$ $$|\arg\beta|<\pi,\ |\arg\gamma|<\pi,\ 0<\Re\{\mu\}<2,\ \mu\neq 1.$$

4. $$\int_0^\infty \frac{x^{\mu-1}\ln\, x\,dx}{(x+\beta)(x-1)} = \frac{\pi}{(\beta+1)\sin^2\mu\pi}\left[\pi-\beta^{\mu-1}(\sin\mu\pi\ln\,\beta-\pi\cos\mu\pi)\right],$$ $$|\arg\beta|<\pi,\ 0<\Re\{\mu\}<2,\ \mu\neq 1.$$

5. $$\int_0^\infty \frac{x^{p-1}\ln\, x}{1-x^2}\,dx = -\frac{\pi^2}{4}\csc^2\frac{p\pi}{2}, \quad 0<p<2.$$

6. $$\int_0^\infty \frac{x^{\mu-1}\ln\, x}{(x+a)^2}\,dx = \frac{(1-\mu)a^{\mu-2}\pi}{\sin\mu\pi}\left(\ln\,a-\pi\cot\mu\pi+\frac{1}{\mu-1}\right),$$ $$|\arg a|<\pi\ 0<\Re\{\mu\}<2\ \ (\mu\neq 1).$$

7. $$\int_0^\infty \ln\,x\frac{dx}{(a+x)^{\mu+1}} = \frac{1}{\mu a^\mu}\left(\ln\,a-\psi(\mu)-\gamma_e\right), \quad \Re\{\{\mu\}>0,\ a\neq 0,\ |\arg a|<\pi.$$

8. $$\int_0^\infty \ln\,x\frac{dx}{(a+x)^{n+1/2}} = \frac{2}{(2n-1)\,a^{n-1/2}}\left(\ln\,a+2\ln\,2-2\sum_{k=1}^{n-1}\frac{1}{2k-1}\right),$$ $$|\arg a|<\pi, n=1,2,\ldots.$$

9. $\displaystyle\int_0^\infty \frac{x^{p-1}\ln x}{1-x^q}\,dx = -\frac{\pi^2}{q^2\sin^2\frac{p\pi}{q}}, \quad 0<p<q.$

10. $\displaystyle\int_0^\infty \frac{\ln x}{x^q-1}\frac{dx}{x^p} = \frac{\pi^2}{q^2\sin^2\frac{p-1}{q}\pi}, \quad p<1,\, p+q>1.$

11. $\displaystyle\int_0^\infty \frac{x^{p-1}\ln x}{1+x^q}\,dx = -\frac{\pi^2}{q^2}\frac{\cos\frac{p\pi}{q}}{\sin^2\frac{p\pi}{q}}, \quad 0<p<q.$

12. $\displaystyle\int_0^\infty \ln x\frac{1-x^p}{1-x^2}\,dx = \frac{\pi^2}{4}\,\tan^2\frac{p\pi}{2}, \quad p<1.$

13. $\displaystyle\int_0^\infty \frac{x^\nu\ln(x/\beta)\,dx}{(x+\beta)(x+\gamma)} = \frac{\pi\left[\gamma^\nu\ln\frac{\gamma}{\beta}+\pi(\beta^\nu-\gamma^\nu)\cot\nu\pi\right]}{\sin\nu\pi(\gamma-\beta)},$

$|\arg\beta|<\pi,\ |\arg\gamma|<\pi,\ |\Re\{\nu\}|<1.$

14. $\displaystyle\int_0^\infty \ln\frac{x}{q}\left(\frac{x^p}{q^{2p}+x^{2p}}\right)\frac{dx}{x} = 0, \quad q>0.$

15. $\displaystyle\int_0^\infty \ln\frac{x}{q}\left(\frac{x^p}{q^{2p}+x^{2p}}\right)^r\frac{dx}{q^2+x^2} = 0, \quad q>0.$

16. $\displaystyle\int_0^\infty \ln x\ln\frac{x}{a}\frac{dx}{(x-1)(x-a)} = \frac{[4\pi^2+(\ln a)^2]\ln a}{6(a-1)}, \quad a>0,\ a=1.$

17. $\displaystyle\int_0^\infty \ln x\ln\frac{x}{a}\frac{x^p\,dx}{(x-1)(x-a)} = \frac{\pi^2[(a^p+1)\ln a-2\pi(a^p-1)\cot p\pi]}{(a-1)\sin^2 p\pi}, \quad p^2<1,\ a>0.$