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! For an efficient use of these tables, first read `HowTo.pdf`.

T2.75A. Integrands involving arcsin, arccos and rational functions on the interval $(0, 1)$.

1. $\displaystyle\int_0^1 \frac{\arcsin x}{x}\,dx = \frac{\pi}{2}\ln 2.$

2. $\displaystyle\int_0^1 \frac{\arccos x}{1 \pm x}\,dx = \mp\frac{\pi}{2}\ln 2 + 2\,\mathbf{G}.$

3. $\displaystyle\int_0^1 \arcsin x \frac{x}{1+qx^2}\,dx = \frac{\pi}{2q}\ln\frac{2\sqrt{1+q}}{1+\sqrt{1+q}}, \quad q > -1.$

4. $\displaystyle\int_0^1 \arcsin x \frac{x}{1-p^2x^2}\,dx = \frac{\pi}{2p^2}\ln\frac{1+\sqrt{1-p^2}}{2\sqrt{1-p^2}}, \quad p^2 < 1.$

5. $\displaystyle\int_0^1 \arccos x \frac{dx}{\sin^2\lambda - x^2} = 2\csc\lambda \sum_{k=0}^{\infty}\frac{\sin[(2k+1)\lambda]}{(2k+1)^2}.$

6. $\displaystyle\int_0^1 \arcsin x \frac{dx}{x(1+qx^2)} = \frac{\pi}{2}\ln\frac{1+\sqrt{1+q}}{\sqrt{1+q}}, \quad q > -1.$

7. $\displaystyle\int_0^1 \arcsin x \frac{x}{(1+qx^2)^2}\,dx = \frac{\pi}{4q}\frac{\sqrt{1+q}-1}{1+q}, \quad q > -1.$

8. $\displaystyle\int_0^1 \arccos x \frac{x}{(1+qx^2)^2}\,dx = \frac{\pi}{4q}\frac{\sqrt{1+q}-1}{1+q}, \quad q > -1.$

9. $\displaystyle\int_0^1 x\sqrt{1-k^2x^2}\,\arccos x\,dx = \frac{1}{9k^2}\left[\frac{3}{2}\pi + k'^2\mathbf{K}(k) - 2(1+k'^2)\mathbf{E}(k)\right].$

10. $\displaystyle\int_0^1 x\sqrt{1-k^2x^2}\,\arcsin x\,dx = \frac{1}{9k^2}\left[-\frac{3}{2}\pi k'^3 - k'^2\mathbf{K}(k) + 2(1+k'^2)\mathbf{E}(k)\right].$

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11. $\displaystyle\int_0^1 x\sqrt{k'^2+k^2x^2}\,\arcsin x\,dx = \frac{1}{9k^2}\left[\frac{3}{2}\pi + k'^2\mathbf{K}(k) - 2(1+k'^2)\mathbf{E}(k)\right].$

12. $\displaystyle\int_0^1 \frac{x\,\arcsin x}{\sqrt{1-k^2x^2}}\,dx = \frac{1}{k^2}\left[-\frac{\pi}{2}k' + \mathbf{E}(k)\right].$

13. $\displaystyle\int_0^1 \frac{x\,\arccos x}{\sqrt{1-k^2x^2}}\,dx = \frac{1}{k^2}\left[\frac{\pi}{2} - \mathbf{E}(k)\right].$

14. $\displaystyle\int_0^1 \frac{x\,\arcsin x}{\sqrt{k'^2+k^2x^2}}\,dx = \frac{1}{k^2}\left[\frac{\pi}{2} - \mathbf{E}(k)\right].$

15. $\displaystyle\int_0^1 \frac{x\,\arccos x}{\sqrt{k'^2+k^2x^2}}\,dx = \frac{1}{k^2}\left[-\frac{\pi}{2}k' + \mathbf{E}(k)\right].$

16. $\displaystyle\int_0^1 \frac{x\,\arcsin x\,dx}{(x^2-\cos^2\lambda)\sqrt{1-x^2}} = \frac{2}{\sin\lambda}\sum_{k=0}^{\infty}\frac{\sin[(2k+1)\lambda]}{(2k+1)^2}.$

17. $\displaystyle\int_0^1 \frac{x\,\arcsin kx}{\sqrt{(1-x^2)(1-k^2x^2)}}\,dx = -\frac{\pi}{2k}\ln\,k'.$

18. $\displaystyle\int_0^1 \frac{x\,\arccos kx}{\sqrt{(1-x^2)(1-k^2x^2)}}\,dx = \frac{\pi}{2k}\ln(1+k).$

19. $\displaystyle\int_0^1 x^{2n}\,\arcsin x\,dx = \frac{1}{2n+1}\left[\frac{\pi}{2} - \frac{2^n n!}{(2n+1)!!}\right].$

20. $\displaystyle\int_0^1 x^{2n-1}\,\arcsin x\,dx = \frac{\pi}{4n}\left[1 - \frac{(2n-1)!!}{2^n n!}\right].$

21. $\displaystyle\int_0^1 x^{2n}\,\arccos x\,dx = \frac{2^n n!}{(2n+1)(2n+1)!!}.$

22. $\displaystyle\int_0^1 x^{2n-1}\,\arccos x\,dx = \frac{\pi}{4n}\frac{(2n-1)!!}{2^n n!}.$

23. $\displaystyle\int_0^1 (\arcsin x)^2\frac{dx}{x^2\sqrt{1-x^2}} = \pi\ln\,2.$

24. $\displaystyle\int_0^1 (\arccos x)^2 \frac{dx}{(\sqrt{1-x^2})^3} = \pi \ln 2.$

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