

! For an efficient use of these tables, first read [HowTo.pdf](#).

**T2.12A.** Integrands of the form  $\frac{x^n}{\sqrt{(a-x)(b-x)(c-x)(d-x)}}$ ,  $n = 0, 1$ ; and  $\frac{1}{x\sqrt{(a-x)(b-x)(c-x)(d-x)}}$  and  $\frac{1}{(p-x)\sqrt{(a-x)(b-x)(c-x)(d-x)}}$  on the intervals  $(y, a)$  and  $(a, y)$ .

Notation used:  $\mu = \arcsin \sqrt{\frac{(b-d)(a-y)}{(a-b)(y-d)}}$ ,  $\nu = \arcsin \sqrt{\frac{(b-d)(y-a)}{(a-d)(y-b)}}$ ,

$$q = \sqrt{\frac{(b-c)(a-d)}{(a-c)(b-d)}}, \quad r = \sqrt{\frac{(a-b)(c-d)}{(a-c)(b-d)}}.$$

$$1. \int_y^a \frac{dx}{\sqrt{(a-x)(x-b)(x-c)(x-d)}} = \frac{2}{\sqrt{(a-c)(b-d)}} F(\mu, r), \quad a > y \geq b > c > d.$$

$$2. \int_a^y \frac{dx}{\sqrt{(x-a)(x-b)(x-c)(x-d)}} = \frac{2}{\sqrt{(a-c)(b-d)}} F(\nu, q), \quad y > a > b > c > d.$$

$$3. \int_y^a \frac{x dx}{\sqrt{(a-x)(x-b)(x-c)(x-d)}} = \frac{2}{\sqrt{(a-c)(b-d)}} \left\{ (a-d) \Pi \left( \mu, \frac{b-a}{b-d}, r \right) + dF(\mu, r) \right\},$$

$a > y \geq b > c > d.$

$$4. \int_a^y \frac{x dx}{\sqrt{(x-a)(x-b)(x-c)(x-d)}} = \frac{2}{\sqrt{(a-c)(b-d)}} \left\{ (a-b) \Pi \left( \nu, \frac{a-d}{b-d}, q \right) + bF(\nu, q) \right\}$$

$y > a > b > c > d.$

$$5. \int_y^a \frac{dx}{x\sqrt{(a-x)(x-b)(x-c)(x-d)}} = \frac{2}{ad\sqrt{(a-c)(b-d)}} \left\{ (d-a) \Pi \left( \mu, \frac{d(b-a)}{a(b-d)}, r \right) + aF(\mu, r) \right\}, \quad a > y \geq b > c > d.$$

$$\begin{aligned}
6. \int_a^y \frac{dx}{x\sqrt{(x-a)(x-b)(x-c)(x-d)}} \\
= \frac{2}{ab\sqrt{(a-c)(b-d)}} \left\{ (b-a)\Pi\left(\nu, \frac{b(a-d)}{a(b-d)}, q\right) + aF(\nu, q) \right\}, \quad y > a > b > c > d.
\end{aligned}$$

$$\begin{aligned}
7. \int_y^a \frac{dx}{(p-x)\sqrt{(a-x)(x-b)(x-c)(x-d)}} &= \frac{2}{(p-a)(p-d)\sqrt{(a-c)(b-d)}} \\
&\times \left[ (a-d)\Pi\left(\mu, \frac{(b-a)(p-d)}{(b-d)(p-a)}, r\right) + (p-a)F(\mu, r) \right], \quad a > y \geq b > c > d, \quad p \neq a.
\end{aligned}$$

$$\begin{aligned}
8. \int_a^y \frac{dx}{(p-x)\sqrt{(x-a)(x-b)(x-c)(x-d)}} &= \frac{2}{(p-a)(p-b)\sqrt{(a-c)(b-d)}} \\
&\times \left[ (a-b)\Pi\left(\nu, \frac{(a-d)(p-b)}{(b-d)(p-a)}, q\right) + (p-a)F(\nu, q) \right], \quad y > a > b > c > d, \quad p \neq a.
\end{aligned}$$


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