! For an efficient use of these tables, first read `HowTo.pdf`.

T2.58B. Integrands involving product of trigonometric and exponential functions of trigonometric functions on the interval $(0, \pi)$.

1. $\displaystyle\int_0^\pi e^{-p\,\cos x}\,\sin(p\,\sin x)\,dx = -\int_{-\pi}^0 e^{-p\,\cos x}\,\sin(p\,\sin x)\,dx = -2\,\mathrm{Si}\,(p).$

2. $\displaystyle\int_0^\pi e^{p\,\cos x}\,\sin(p\,\sin x)\,\sin mx\,dx = \frac{1}{2}\int_0^{2\pi} e^{p\,\cos x}\,\sin(p\,\sin x)\,\sin mx\,dx = \frac{\pi}{2}\,\frac{p^m}{m!}.$

3. $\displaystyle\int_0^\pi e^{p\,\cos x}\,\cos(p\,\sin x)\,\cos mx\,dx = \frac{1}{2}\int_0^{2\pi} e^{p\,\cos x}\,\cos(p\,\sin x)\,\cos mx\,dx = \frac{\pi}{2}\,\frac{p^m}{m!}.$

4. $\displaystyle\int_0^\pi e^{p\,\cos x}\,\sin(p\,\sin x)\,\csc x\,dx = \pi\,\sinh p.$

5. $\displaystyle\int_0^\pi e^{p\,\cos x}\,\sin(p\,\sin x)\,\tan\frac{x}{2}\,dx = \pi\,(1 - e^p).$

6. $\displaystyle\int_0^\pi e^{p\,\cos x}\,\sin(p\,\sin x)\,\cot\frac{x}{2}\,dx = \pi\,(e^p - 1).$

7. $\displaystyle\int_0^\pi e^{p\,\cos x}\,\cos(p\,\sin x)\frac{\sin 2nx}{\sin x}\,dx = \pi\sum_{k=0}^{n-1}\frac{p^{2k+1}}{(2k+1)!}, \quad p > 0.$

8. $\displaystyle\int_0^\pi e^{\beta\,\cos x}\,\cos(ax + \beta\,\sin x)\,dx = \beta^{-a}\,\sin(a\pi)\,\gamma(a,\,\beta).$

9. $\displaystyle\int_0^\pi e^{r(\cos px + \cos qx)}\,\sin(r\,\sin px)\,\sin(r\,\sin qx)\,dx = \frac{\pi}{2}\sum_{k=1}^\infty \frac{1}{\Gamma(pk+1)\Gamma(qk+1)}\,r^{(p+q)k}.$

10. $$\int_0^\pi e^{t(\cos px+\cos qx)}\,\cos(t\,\sin px)\,\cos(t\,\sin qx)\,dx = \frac{\pi}{2}\left(2+\sum_{k=1}^{\infty}\frac{t^{(p+q)k}}{\Gamma(pk+1)\Gamma(qk+1)}\right).$$

11. $$\int_0^\pi e^{q\,\cos x}\frac{\sin rx}{1-2p^r\,\cos rx+p^{2r}}\,\sin(q\,\sin x)\,dx = \frac{\pi}{2pr}\sum_{k=1}^{\infty}\frac{(pq)^{k\,r}}{\Gamma(kr+1)},\quad r>0,\ 0<p<1.$$

12. $$\int_0^\pi e^{q\,\cos x}\frac{1-p^r\,\cos rx}{1-2p^r\,\cos rx+p^{2r}}\,\cos(q\,\sin x)\,dx = \frac{\pi}{2}\left[2+\sum_{k=1}^{\infty}\frac{(pq)^{k\,r}}{\Gamma(kr+1)}\right],\quad r>0,\ 0<p<1.$$