! For an efficient use of these tables, first read `HowTo.pdf`.

T2.23B. Integrands of the form $\sqrt{\dfrac{d-x}{(a-x)(b-x)(c-x)}}$, $\sqrt{\dfrac{c-x}{(a-x)(b-x)(d-x)}}$, $\sqrt{\dfrac{b-x}{(a-x)(c-x)(d-x)}}$, and $\sqrt{\dfrac{a-x}{(b-x)(c-x)(d-x)}}$ on the intervals (y,b) and (b,y).

Notation used: $\kappa = \arcsin\sqrt{\dfrac{(a-c)(b-y)}{(b-c)(a-y)}}$, $\lambda = \arcsin\sqrt{\dfrac{(a-c)(y-b)}{(a-b)(y-c)}}$,

$$q = \sqrt{\frac{(b-c)(a-d)}{(a-c)(b-d)}}, \quad r = \sqrt{\frac{(a-b)(c-d)}{(a-c)(b-d)}}.$$

1. $$\int_y^b \sqrt{\frac{x-d}{(a-x)(b-x)(x-c)}}\,dx = \frac{2}{\sqrt{(a-c)(b-d)}}\left\{(b-a)\Pi\left(\kappa,\, \frac{b-c}{a-c},\, q\right) + (a-d)F(\kappa,\, q)\right\}$$
$a > b > y \geq c > d.$

2. $$\int_b^y \sqrt{\frac{x-d}{(a-x)(x-b)(x-c)}}\,dx = \frac{2}{\sqrt{(a-c)(b-d)}}\left\{(b-c)\Pi\left(\lambda,\, \frac{a-b}{a-c},\, r\right) + (c-d)F(\lambda,\, r)\right\},$$
$a \geq y > b > c > d.$

3. $$\int_y^b \sqrt{\frac{x-c}{(a-x)(b-x)(x-d)}}\,dx = \frac{2}{\sqrt{(a-c)(b-d)}}\left[(b-a)\Pi\left(\kappa,\, \frac{b-c}{a-c},\, q\right) + (a-c)F(\kappa,\, q)\right],$$
$a > b > y \geq c > d.$

4. $$\int_b^y \sqrt{\frac{x-c}{(a-x)(x-b)(x-d)}}\,dx = \frac{2(b-c)}{\sqrt{(a-c)(b-d)}}\Pi\left(\lambda,\, \frac{a-b}{a-c},\, r\right), \quad a \geq y > b > c > d.$$

5. $$\int_y^b \sqrt{\frac{b-x}{(a-x)(x-c)(x-d)}}\,dx = \frac{2(a-b)}{\sqrt{(a-c)(b-d)}}\left[\Pi\left(\kappa,\, \frac{b-c}{a-c},\, q\right) - F(\kappa,\, q)\right],$$
$a > b > y \geq c > d.$

6. $$\int_b^y \sqrt{\frac{x-b}{(a-x)(x-c)(x-d)}}\,dx = \frac{2(b-c)}{\sqrt{(a-c)(b-d)}}\left[\Pi\left(\lambda,\, \frac{a-b}{a-c},\, r\right) - F(\lambda,\, r)\right],$$
$$a \geq y > b > c > d.$$

7. $$\int_y^b \sqrt{\frac{a-x}{(b-x)(x-c)(x-d)}}\,dx = \frac{2(a-b)}{\sqrt{(a-c)(b-d)}}\Pi\left(\kappa,\, \frac{b-c}{a-c},\, q\right), \quad a > b > y \geq c > d.$$

8. $$\int_b^y \sqrt{\frac{a-x}{(x-b)(x-c)(x-d)}}\,dx = \frac{2}{\sqrt{(a-c)(b-d)}}\left[(c-b)\Pi\left(\lambda,\, \frac{a-b}{a-c},\, r\right) + (a-c)F(\lambda,\, r)\right],$$
$$a \geq y > b > c > d.$$