! For an efficient use of these tables, first read `HowTo.pdf`.

T2.02A. Powers of x and binomials of the form $(a + b\,x)$ on the interval $(0, 1)$.

1. $$\int_0^1 x^{\nu-1}(1-x)^{\mu-1}\,dx = \int_0^1 x^{\mu-1}(1-x)^{\nu-1}\,dx = \mathrm{B}(\mu,\,\nu), \quad \Re\{\mu\} > 0,\ \Re\{\nu\} > 0.$$

2. $$\int_0^1 x^j(1-x)^k\,dx = \frac{j!\,k!}{(j+k+1)!}.$$

3. $$\int_0^1 \frac{x^p\,dx}{(1-x)^p} = p\pi\csc p\pi, \quad p^2 < 1.$$

4. $$\int_0^1 \frac{x^p\,dx}{(1-x)^{p+1}} = -\pi\csc p\pi, \quad -1 < p < 0.$$

5. $$\int_0^1 \frac{(1-x)^p}{x^{p+1}}\,dx = -\pi\csc p\pi, \quad -1 < p < 0.$$

6. $$\int_0^1 \frac{x^{n-1}\,dx}{(1+x)^m} = 2^{-n}\sum_{k=0}^{\infty}\binom{m-n-1}{k}\frac{(-2)^{-k}}{n+k}.$$

7. $$\int_0^1 x^{\lambda-1}(1-x)^{\mu-1}(1-\beta x)^{-\nu}\,dx = \mathrm{B}(\lambda,\,\mu)\,{}_2F_1(\nu,\,\lambda;\,\lambda+\mu;\,\beta),$$
$$\Re\{\lambda\} > 0,\ \Re\{\mu\} > 0,\ |\beta| < 1.$$

8. $$\int_0^1 x^{\mu-1}(1-x)^{\nu-1}(1+ax)^{-\mu-\nu}\,dx = (1+a)^{-\mu}\mathrm{B}(\mu,\,\nu), \quad \Re\{\mu\} > 0,\ \Re\{\nu\} > 0,\ a > -1.$$

9. $$\int_0^1 \frac{x^{q-1}\,dx}{(1-x)^q(1+px)} = \frac{\pi}{(1+p)^q}\csc q\pi, \quad 0 < q < 1,\ p > -1.$$

10. $$\int_0^1 \frac{x^{p-1/2}\,dx}{(1-x)^p(1+qx)^p} = \frac{2\Gamma\left(p+\frac{1}{2}\right)\Gamma(1-p)}{\sqrt{\pi}}\ \cos^{2p}(\arctan\sqrt{q})\,\frac{\sin[(2p-1)\ \arctan(\sqrt{q})]}{(2p-1)\ \sin[\arctan(\sqrt{q})]},$$
$$-\tfrac{1}{2} < p < 1,\ q > 0.$$

11. $$\int_0^1 \frac{x^{p-1/2}\,dx}{(1-x)^p(1-qx)^p} = \frac{\Gamma\left(p+\frac{1}{2}\right)\Gamma(1-p)}{\sqrt{\pi}}\,\frac{(1-\sqrt{q})^{1-2p}-(1+\sqrt{q})^{1-2p}}{(2p-1)\sqrt{q}},$$
$$-\tfrac{1}{2} < p < 1,\ 0 < q < 1.$$

12. $$\int_0^1 x^{\mu-1}(1-x)^{\nu-1}[ax+b(1-x)+c]^{-(\mu+\nu)}\,dx = (a+c)^{-\mu}(b+c)^{-\nu}\mathrm{B}(\mu,\,\nu),$$
$$a \ge 0,\ b \ge 0,\ c > 0,\ \Re\{\mu\} > 0,\ \Re\{\nu\} > 0.$$

13. $$\int_0^1 x^{\lambda-1}(1-x)^{\mu-1}(1-ux)^{-\rho}(1-vx)^{-\sigma}\,dx = \mathrm{B}(\mu,\,\lambda)F_1(\lambda,\,\rho,\,\sigma,\,\lambda+\mu;\,u,\,v),$$
$$\Re\{\lambda\} > 0,\ \Re\{\mu\} > 0.$$

14. $$\int_0^1 [(1+x)^{\mu-1}(1-x)^{\nu-1}+(1+x)^{\nu-1}(1-x)^{\mu-1}]\,dx = 2^{\mu+\nu-1}\mathrm{B}(\mu,\,\nu),\quad \Re\{\mu\} > 0,\ \Re\{\nu\} > 0.$$

15. $$\int_0^1 \left\{a^{\mu}x^{\mu-1}(1-ax)^{\nu-1}+(1-a)^{\nu}x^{\nu-1}[1-(1-a)x]^{\mu-1}\right\}\,dx = \mathrm{B}(\mu,\,\nu),$$
$$\Re\{\mu\} > 0,\ \Re\{\nu\} > 0,\ |a| < 1.$$

16. $$\int_0^1 \frac{x^{\mu-1}+x^{\nu-1}}{(1+x)^{\mu+\nu}}\,dx = \mathrm{B}(\mu,\,\nu),\quad \Re\{\mu\} > 0,\ \Re\{\nu\} > 0.$$

17. $$\int_1^\infty \frac{x^{\mu-1}+x^{\nu-1}}{(1+x)^{\mu+\nu}}\,dx = \mathrm{B}(\mu,\,\nu),\quad \Re\{\mu\} > 0,\ \Re\{\nu\} > 0.$$

18. $$\int_0^1 \frac{x^{\mu-1}\,dx}{1+x} = \beta(\mu),\quad \Re\{\mu\} > 0.$$

19. $$\int_0^1 \frac{x^n\,dx}{\sqrt{1-x}} = 2\frac{(2n)!!}{(2n+1)!!}.$$

20. $$\int_0^1 \frac{x^{n-\frac{1}{2}}\,dx}{\sqrt{1-x}} = \frac{(2n-1)!!}{(2n)!!}\pi.$$

21. $$\int_0^1 \frac{(1-x)^{\nu-1}x^{-\nu}}{a-bx}\,dx = \frac{\pi(a-b)^{\nu-1}}{a^\nu}\csc(\nu\pi), \quad 0 < \Re\{\nu\} < 1,\ 0 < b < a.$$

22. $$\int_0^1 \frac{x^{\mu-1}\,dx}{(1-x)^\mu(1+ax)(1+bx)} = \frac{\pi\csc\mu\pi}{a-b}\left[\frac{a}{(1+a)^\mu} - \frac{b}{(1+b)^\mu}\right], \quad 0 < \Re\{\mu\} < 1.$$

23. $$\int_0^1 \frac{x^{p-1}-x^{-p}}{1-x}\,dx = \pi\,\cot p\pi, \quad p^2 < 1.$$

24. $$\int_0^1 \frac{x^{p-1}+x^{-p}}{1+x}\,dx = \pi\csc p\pi, \quad p^2 < 1.$$

25. $$\int_0^1 \frac{x^p-x^{-p}}{x-1}\,dx = \frac{1}{p} - \pi\,\cot p\pi, \quad p^2 < 1.$$

26. $$\int_0^1 \frac{x^p-x^{-p}}{1+x}\,dx = \frac{1}{p} - \pi\csc p\pi, \quad p^2 < 1.$$

27. $$\int_0^1 \frac{x^{\mu-1}-x^{\nu-1}}{1-x}\,dx = \psi(\nu) - \psi(\mu), \quad \Re\{\mu\} > 0,\ \Re\{\nu\} > 0.$$

28. $$\int_0^1 \left(\frac{x^{q-1}}{1-ax} - \frac{x^{-q}}{a-x}\right)dx = \pi a^{-q}\cot q\pi, \quad 0 < q < 1,\ a > 0.$$

29. $$\int_0^1 \left(\frac{x^{q-1}}{1+ax} + \frac{x^{-q}}{a+x}\right)dx = \pi a^{-q}\csc q\pi, \quad 0 < q < 1,\ a > 0.$$

30. $$\int_0^1 \frac{x^{\mu/2}\,dx}{[(1-x)(1-a^2x)]^{(\mu+1)/2}} = \frac{(1-a)^{-\mu}-(1+a)^{-\mu}}{2a\mu\sqrt{\pi}}\Gamma\left(1+\frac{\mu}{2}\right)\Gamma\left(\frac{1-\mu}{2}\right), \quad -2 < \mu < 1,\ |a| < 1.$$