! For an efficient use of these tables, first read `HowTo.pdf`.

**T3.35A.** Integrands involving product and division of trigonometric functions by powers of $(a + b\,x)$ on the interval $(0, \infty)$.

1. $$\int_0^\infty x^{\mu-1}\,\sin(ax)\,dx = \frac{\Gamma(\mu)}{a^\mu}\,\sin\frac{\mu\pi}{2} = \frac{\pi\,\sec\frac{\mu\pi}{2}}{2a^\mu\Gamma(1-\mu)}, \quad a>0,\ 0<|\Re\{\mu\}|<1.$$

2. $$\int_0^\infty x^{\mu-1}\,\cos(ax)\,dx = \frac{\Gamma(\mu)}{a^\mu}\,\cos\frac{\mu\pi}{a} = \frac{\pi\,\csc\frac{\mu\pi}{2}}{2a^\mu\Gamma(1-\mu)}, \quad a>0,\ 0<\Re\{\mu\}<1.$$

3. $$\int_0^\infty x^{\mu-1}\,\sin(ax)\,\sin(bx)\,dx = \frac{1}{2}\,\cos\frac{\mu\pi}{2}\Gamma(\mu)\left[|b-a|^{-\mu} - (b+a)^{-\mu}\right],$$
$$a>0,\ b>0,\ a\neq b,\ -2<\Re\{\mu\}<1 \text{ for } \mu=0.$$

4. $$\int_0^\infty x^{\mu-1}\,\sin(ax)\,\cos(bx)\,dx = \frac{1}{2}\,\sin\frac{\mu\pi}{2}\Gamma(\mu)\left[(a+b)^{-\mu} + |a-b|^{-\mu}\,\operatorname{sgn}(a-b)\right],$$
$$a>0,\ b>0,\ |\Re\{\mu\}|<1 \text{ for } \mu=0.$$

5. $$\int_0^\infty x^{\mu-1}\,\cos(ax)\,\cos(bx)\,dx = \frac{1}{2}\,\cos\frac{\mu\pi}{2}\Gamma(\mu)\left[(a+b)^{-\mu} + |a-b|^{-\mu}\right],$$
$$a>0,\ b>0,\ 0<\Re\{\mu\}<1.$$

6. $$\int_0^\infty \frac{\sin(ax)\,\sin(bx)\,\sin(cx)}{x^\nu}\,dx = \frac{1}{4}\,\cos\frac{\nu\pi}{2}\Gamma(1-\nu)\big[(c+a-b)^{\nu-1} - (c+a+b)^{\nu-1}$$
$$-|c-a+b|^{\nu-1}\,\operatorname{sgn}(a-b-c) + |c-a-b|^{\nu-1}\,\operatorname{sgn}(a+b-c)\big],$$
$$c>0,\ 0<\Re\{\nu\}<4,\ \nu\neq 1,2,3,\ a\geq b>0.$$

7. $$\int_0^\infty \frac{\sin(ax)\,\sin(bx)\,\sin(cx)}{x}\,dx = \begin{cases} 0, & c<a-b \text{ and } c>a+b, \\ \dfrac{\pi}{8}, & c=a-b \text{ and } c=a+b, \\ \dfrac{\pi}{4}, & a-b<c<a+b, \end{cases} \qquad a\geq b>0,\ c>0.$$

8. $\displaystyle\int_0^\infty \frac{\sin(ax)\,\sin(bx)\,\sin(cx)}{x^2}\,dx$

$$= \frac{1}{4}(c+a+b)\,\ln(c+a+b) - \frac{1}{4}(c+a-b)\,\ln(c+a-b) - \frac{1}{4}|c-a-b|\,\ln|c-a-b| \times \operatorname{sgn}(a+b-c) + \frac{1}{4}|c-a+b|\,\ln|c-a+b|\,\operatorname{sgn}(a-b-c), \quad a \ge b > 0,\ c > 0.$$

9. $\displaystyle\int_0^\infty \frac{\sin(ax)\,\sin(bx)\,\sin(cx)}{x^3}\,dx$

$$= \begin{cases} \dfrac{\pi bc}{2}, & 0 < c < a-b \text{ and } c > a+b, \\ \dfrac{\pi bc}{2} - \dfrac{\pi(a-b-c)^2}{8}, & a-b < c < a+b, \end{cases} \quad a \ge b > 0,\ c > 0.$$

10. $\displaystyle\int_0^\infty x^p\,\sin(ax+b)\,dx = \frac{1}{a^{p+1}}\Gamma(1+p)\,\cos\left(b + \frac{p\pi}{2}\right), \quad a > 0,\ -1 < p < 0.$

11. $\displaystyle\int_0^\infty x^p\,\cos(ax+b)\,dx = -\frac{1}{a^{p+1}}\Gamma(1+p)\,\sin\left(b + \frac{\pi p}{2}\right), \quad a > 0,\ -1 < p < 0.$

12. $\displaystyle\int_0^\infty \frac{\sin(ax)\,dx}{x^\nu(x+b)}$

$$= \frac{i\,\Gamma(1-\nu)}{2b^\nu}\left[e^{-i\,ab}\Gamma(\nu,\,-i\,ab) - e^{i\,ab}\Gamma(\nu,\,i\,ab)\right], \quad a > 0,\ -1 < \Re\{\nu\} < 2,\ |\arg\{b\}| < \pi.$$

13. $\displaystyle\int_0^\infty \frac{\cos(ax)\,dx}{x^\nu(x+b)}$

$$= \frac{\Gamma(1-\nu)}{2b^\nu}\left[e^{i\,ab}\Gamma(\nu,\,i\,ab) + e^{-i\,ab}\Gamma(\nu,\,-i\,ab)\right], \quad a > 0,\ |\Re\{\nu\}| < 1,\ |\arg\{b\}| < \pi.$$

14. $\displaystyle\int_0^\infty \frac{x^{\mu-1}\,\sin(ax)}{1+x^2}\,dx = \frac{\pi}{2}\,\sec\frac{\mu\pi}{2}\,\sinh a$

$$+\frac{1}{2}\,\sin\frac{\mu\pi}{2}\Gamma(\mu)\big\{\exp[-a+i\pi(1-\mu)]\gamma(1-\mu,\,-a) - e^a\gamma(1-\mu,\,a)\big\}, \quad a > 0,\ -1 < \Re\{\mu\} < 3.$$

15. $\displaystyle\int_0^\infty \frac{x^{\mu-1}\,\cos(ax)}{1+x^2}\,dx = \frac{\pi}{2}\,\csc\frac{\mu\pi}{2}\,\cosh a$

$$+\frac{1}{2}\,\cos\frac{\mu\pi}{2}\Gamma(\mu)\big\{\exp[-a+i\pi(1-\mu)]\gamma(1-\mu,\,-a) - e^a\gamma(1-\mu,\,a)\big\}, \quad a > 0,\ 0 < \Re\{\mu\} < 3.$$

16. $$\int_0^\infty \frac{x^{2\mu+1}\,\sin(ax)\,dx}{x^2+b^2} = -\frac{\pi}{2}b^{2\mu}\,\sec(\mu\pi)\,\sinh(ab) + \frac{\sin(\mu\pi)}{2a^{2\mu}}\Gamma(2\mu)\left[{}_1F_1(1;\,1-2\mu;\,ab) + {}_1F_1(1;\,1-2\mu;\,-ab)\right],\quad a>0,\ -\frac{3}{2}<\Re\{\mu\}<\tfrac{1}{2}.$$

17. $$\int_0^\infty \frac{x^{2\mu+1}\,\cos(ax)\,dx}{x^2+b^2} = -\frac{\pi}{2}b^{2(\mu+\frac{1}{2})}\,\csc[(\mu+\frac{1}{2})\pi]\,\cosh(ab) + \frac{\cos[(\mu+\frac{1}{2})\pi]}{2a^{2(\mu+\frac{1}{2})}}\,\Gamma\left[2(\mu+1/2)\right]\left[{}_1F_1(1;\,1-2(\mu+\frac{1}{2});\,ab) + {}_1F_1(1;\,1-2(\mu+\frac{1}{2});\,-ab)\right],\quad a>0,\ -1<\Re\{\mu\}<\tfrac{1}{2}.$$

18. $$\int_0^\infty \frac{x^{\beta-1}\,\sin\left(ax-\frac{\beta\pi}{2}\right)}{\gamma^2+x^2}\,dx = -\frac{\pi}{2}\gamma^{\beta-2}e^{-a\gamma},\quad a>0,\ \Re\{\gamma\}>0,\ 0<\Re\{\beta\}<2.$$

19. $$\int_0^\infty \frac{x^{\beta}\,\cos\left(ax-\frac{\beta\pi}{2}\right)}{\gamma^2+x^2}\,dx = \frac{\pi}{2}\gamma^{\beta-1}e^{-a\gamma},\quad a>0,\ \Re\{\gamma\}>0,\ |\Re\{\beta\}|<1.$$

20. $$\int_0^\infty \frac{x^{\beta-1}\,\sin\left(ax-\frac{\beta\pi}{2}\right)}{x^2-b^2}\,dx = \frac{\pi}{2}b^{\beta-2}\,\cos\left(ab-\frac{\pi\beta}{2}\right),\quad a>0,\ b>0,\ 0<\Re\{\beta\}<2.$$

21. $$\int_0^\infty \frac{x^{\beta}\,\cos\left(ax-\frac{\beta\pi}{2}\right)}{x^2-b^2}\,dx = -\frac{\pi}{2}b^{\beta-1}\,\sin\left(ab-\frac{\pi\beta}{2}\right),\quad a>0,\ b>0,\ |\beta|<1.$$

22. $$\int_0^\infty \left[(\beta+ix)^{-\nu}-(\beta-ix)^{-\nu}\right]\,\sin(ax)\,dx = -\frac{\pi i\,a^{\nu-1}e^{-a\beta}}{\Gamma(\nu)},\quad a>0,\ \Re\{\beta\}>0,\ \Re\{\nu\}>0.$$

23. $$\int_0^\infty \left[(\beta+ix)^{-\nu}+(\beta-ix)^{-\nu}\right]\,\cos(ax)\,dx = \frac{\pi a^{\nu-1}e^{-a\beta}}{\Gamma(\nu)},\quad a>0,\ \Re\{\beta\}>0,\ \Re\{\nu\}>0.$$

24. $$\int_0^\infty x\left[(\beta+ix)^{-\nu}+(\beta-ix)^{-\nu}\right]\,\sin(ax)\,dx = -\frac{\pi a^{\nu-2}(\nu-1-a\beta)}{\Gamma(\nu)}\,e^{-a\beta},\quad a>0,\ \Re\{\beta\}>0,\ \Re\{\nu\}>0.$$

25. $$\int_0^\infty x^{2n}\left[(\beta-ix)^{-\nu}-(\beta+ix)^{-\nu}\right]\,\sin(ax)\,dx = \frac{i\,(-1)^n}{\Gamma(\nu)}(2n)!\,\pi a^{\nu-2n-1}\,e^{-a\beta}\,L_{2n}^{\nu-2n-1}(a\beta),\quad a>0,\ \Re\{\beta\}>0,\ 0\le 2n<\Re\{\nu\}.$$

26. $$\int_0^\infty x^{2n}\left[(\beta+ix)^{-\nu}+(\beta-ix)^{-\nu}\right]\cos(ax)\,dx=\frac{(-1)^n}{\Gamma(\nu)}(2n)!\pi a^{\nu-2n-1}\,e^{-a\beta}\,L_{2n}^{\nu-2n-1}(a\beta),$$
$$a>0,\ \Re\{\beta\}>0,\ 0\le 2n<\Re\{\nu\}.$$

27. $$\int_0^\infty x^{2n+1}\left[(\beta+ix)^{-\nu}+(\beta-ix)^{-\nu}\right]\sin(ax)\,dx=\frac{(-1)^{n+1}}{\Gamma(\nu)}(2n+1)!\,\pi a^{\nu-2n-2}\,e^{-a\beta}\,L_{2n+1}^{\nu-2n-2}(a\beta),$$
$$a>0,\ \Re\{\beta\}>0,\ -1\le 2n+1<\Re\{\nu\}.$$

28. $$\int_0^\infty x^{2n+1}\left[(\beta+ix)^{-\nu}-(\beta-ix)^{-\nu}\right]\cos(ax)\,dx=\frac{(-1)^{n+1}}{\Gamma(\nu)}(2n+1)!\,\pi a^{\nu-2n-2}\,e^{-a\beta}\,L_{2n+1}^{\nu-2n-2}(a\beta),$$
$$a>0,\ \Re\{\beta\}>0,\ 0\le 2n<\Re\{\nu\}-1.$$

29. $$\int_0^\infty (\beta^2+x^2)^{\nu-1/2}\,\sin(ax)\,dx=\frac{\sqrt{\pi}}{2}\left(\frac{2\beta}{a}\right)^\nu\Gamma\left(\nu+\frac{1}{2}\right)\left[I_{-\nu}(a\beta)-\mathbf{L}_\nu(a\beta)\right],$$
$$a>0,\ \Re\{\beta\}>0,\ \Re\{\nu\}<\tfrac{1}{2},\ \nu\ne-\tfrac{1}{2},-\tfrac{3}{2},-\tfrac{5}{2},\ldots.$$

30. $$\int_0^\infty (\beta^2+x^2)^{\nu-1/2}\,\cos(ax)\,dx=\frac{1}{\sqrt{\pi}}\left(\frac{2\beta}{a}\right)^\nu\cos(\pi\nu)\Gamma\left(\nu+\frac{1}{2}\right)\,K_{-\nu}(a\beta),$$
$$a>0,\ \Re\{\beta\}>0,\ \Re\{\nu\}<\tfrac{1}{2}.$$

31. $$\int_0^\infty x(x^2+\beta^2)^{\nu-1/2}\,\sin(ax)\,dx=\frac{1}{\sqrt{\pi}}\,\beta\left(\frac{2\beta}{a}\right)^\nu\cos\nu\pi\Gamma\left(\nu+\frac{1}{2}\right)\,K_{\nu+1}(a\beta)$$
$$=\sqrt{\pi}\,\beta\left(\frac{2\beta}{a}\right)^\nu\frac{1}{\Gamma\left(\frac{1}{2}-\nu\right)}\,K_{\nu+1}(a\beta),\quad a>0,\ \Re\{\beta\}>0,\ \Re\{\nu\}<0,\ \nu\ne-\tfrac{1}{2},-\tfrac{3}{2},\ldots.$$

32. $$\int_0^\infty (x^2+2bx)^{\nu-1/2}\,\sin(ax)\,dx=\frac{\sqrt{\pi}}{2}\left(\frac{2b}{a}\right)^\nu\Gamma\left(\nu+\frac{1}{2}\right)\left[J_{-\nu}(ab)\,\cos(ab)+Y_{-\nu}(ab)\,\sin(ab)\right],$$
$$a>0,\ |\arg b|<\pi,\ -\tfrac{3}{2}<\Re\{\nu\}<\tfrac{1}{2}.$$

33. $$\int_0^\infty (x^2+2bx)^{\nu-1/2}\,\cos(ax)\,dx=-\frac{\sqrt{\pi}}{2}\left(\frac{2b}{a}\right)^\nu\Gamma\left(\nu+\frac{1}{2}\right)\left[Y_{-\nu}(ab)\,\cos(ab)-J_{-\nu}(ab)\,\sin(ab)\right],$$
$$a>0,\ |\Re\{\nu\}|<\tfrac{1}{2}.$$

34. $$\int_0^\infty \frac{x^{2\nu}}{(x^2+\beta^2)^{\mu+1}}\,\sin(ax)\,dx$$

$$= \frac{1}{2}\beta^{2\nu-2\mu} a\,\mathrm{B}\,(1+\nu,\,\mu-\nu)_1\,{}_1F_2\left(\nu+1;\,\nu+1-\mu;\,\frac{3}{2};\,\frac{\beta^2a^2}{4}\right)$$

$$+\frac{\sqrt{\pi}a^{2u-2\nu+1}}{4^{\mu-\nu+1}}\,\frac{\Gamma(\nu-\mu)}{\Gamma\left(\mu-\nu+\frac{3}{2}\right)}{}_1F_2\left(\mu+1;\,\mu-\nu+\frac{3}{2},\,\mu-\nu+1;\,\frac{\beta^2a^2}{4}\right)$$

$$= \frac{\sqrt{\pi}}{2\Gamma(\mu+1)}\beta^{2\nu-2\mu-1}G_{13}^{21}\left(\frac{a^2\beta^2}{4}\bigg|_{\mu-\nu+1/2,\,1/2,\,0}^{-\nu+1/2}\right),$$

$$a>0,\ \Re\{\beta\}>0,\ -1<\Re\{\nu\}<\Re\{\mu\}+1.$$



35. $$\int_0^\infty \frac{x^{2m+1}\,\sin(ax)}{(z+x^2)^{n+1}}\,dx = \frac{(-1)^{n+m}}{n!}\cdot\frac{\pi}{2}\,\frac{d^n}{dz^n}(z^m e^{-a}\sqrt{z}),\quad a>0,\ 0\le m\le n,\ |\arg z|<\pi.$$

36. $$\int_0^\infty \frac{x^{2m+1}\,\sin(ax)\,dx}{(\beta^2+x^2)^{n+\frac{1}{2}}} = \frac{(-1)^{m+1}\sqrt{\pi}}{2^n\beta^n\Gamma\left(n+\frac{1}{2}\right)}\,\frac{d^{2m+1}}{da^{2m+1}}\left[a^nK_n(a\beta)\right],$$

$$a>0,\ \Re\{\beta\}>0,\ -1\le m\le n.$$



37. $$\int_0^\infty \frac{x^{2\nu}\,\cos(ax)\,dx}{(x^2+\beta^2)^{\mu+1}} = \frac{1}{2}\beta^{2\nu-2\mu-1}\,\mathrm{B}\,\left(\nu+\frac{1}{2},\,\mu-\nu+\frac{1}{2}\right)\,{}_1F_2\left(\nu+\frac{1}{2};\,\nu-\mu+\frac{1}{2},\,\frac{1}{2};\,\frac{\beta^2a^2}{4}\right)$$

$$+\frac{\sqrt{\pi}a^{2\mu-2\nu+1}}{4^{\mu-\nu+1}}\,\frac{\Gamma\left(\nu-\mu-\frac{1}{2}\right)}{\Gamma(\mu-\nu+1)}\,{}_1F_2\left(\mu+1;\,\mu-\nu+1,\,\mu-\nu+\frac{3}{2};\,\frac{\beta^2a^2}{4}\right)$$

$$= \frac{\sqrt{\pi}}{2\Gamma(\mu+1)}\beta^{2\nu-2\mu-1}G_{13}^{21}\left(\frac{a^2\beta^2}{4}\bigg|_{\mu-\nu+1/2,\,0,\,1/2}^{-\nu+1/2}\right)$$

$$a>0,\ \Re\{\beta\}>0,\ -\tfrac{1}{2}<\Re\{\nu\}<\Re\{\mu\}+1.$$



38. $$\int_0^\infty \frac{x^{2m}\,\cos(ax)\,dx}{(z+x^2)^{n+1}} = (-1)^{m+n}\frac{\pi}{2\cdot n!}\cdot\frac{d^n}{dz^n}(z^{m-1/2}e^{-a\sqrt{z}}),$$

$$a>0,\ n+1>m\ge 0,\ |\arg z|<\pi.$$

38. $$\int_0^\infty \frac{x^{2m}\,\cos(ax)\,dx}{(\beta^2+x^2)^{n+\frac{1}{2}}} = \frac{(-1)^m\sqrt{\pi}}{2^n\beta^n\Gamma\left(n+\frac{1}{2}\right)}\cdot\frac{d^{2m}}{da^{2m}}\left\{a^nK_n(a\beta)\right\},$$

$$a>0,\ \Re\{\beta\}>0,\ 0\le m\le n.$$



40. $$\int_0^\infty \frac{\sin(ax)\,dx}{\sqrt{x^2+b^2}(x+\sqrt{x^2+b^2})^\nu} = \frac{\pi}{b^\nu\,\sin(\nu\pi)}\left[\sin\frac{\nu\pi}{2}I_\nu(ab)+\frac{i}{2}\mathbf{J}_\nu(iab)-\frac{i}{2}\mathbf{J}_\nu(-iab)\right],$$

$$a>0,\ b>0,\ \Re\{\nu\}>-1.$$

41. $$\int_0^\infty \frac{\cos(ax)\,dx}{\sqrt{x^2+b^2}(x+\sqrt{x^2+b^2})^\nu} = \frac{\pi}{b^\nu\,\sin(\nu\pi)}\left[\frac{1}{2}\mathbf{J}_\nu(iab)+\frac{1}{2}\mathbf{J}_\nu(-iab)-\cos\frac{\nu\pi}{2}I_\nu(ab)\right],$$
$$a>0,\ b>0,\ \Re\{\nu\}>-1.$$

42. $$\int_0^\infty \frac{(x+\sqrt{x^2+\beta^2})^\nu}{\sqrt{x(x^2+\beta^2)}}\,\sin(ax)\,dx = \sqrt{\frac{a\pi}{2}}\beta^\nu I_{1/4-\nu/2}\left(\frac{a\beta}{2}\right)K_{1/4+\nu/2}\left(\frac{a\beta}{2}\right),$$
$$a>0,\ \Re\{\beta\}>0,\ \Re\{\nu\}<\tfrac{3}{2}.$$

43. $$\int_0^\infty \frac{(\sqrt{x^2+\beta^2}-x)^\nu}{\sqrt{x(x^2+\beta^2)}}\,\cos(ax)\,dx = \sqrt{\frac{a\pi}{2}}\beta^\nu I_{-1/4+\nu/2}\left(\frac{a\beta}{2}\right)K_{-1/4-\nu/2}\left(\frac{a\beta}{2}\right),$$
$$a>0,\ \Re\{\beta\}>0;\ \Re\{\ \nu\}>-\tfrac{3}{2}.$$

44. $$\int_0^\infty \frac{(\beta+\sqrt{x^2+\beta^2})^\nu}{x^{\nu+\frac{1}{2}}\sqrt{x^2+\beta^2}}\,\sin(ax)\,dx = \frac{1}{\beta}\sqrt{\frac{2}{a}}\Gamma\left(\frac{3}{4}-\frac{\nu}{2}\right)W_{\nu/2,\,1/4}(a\beta)M_{-\nu/2,\,1/4}(a\beta),$$
$$a>0,\ \Re\{\beta\}>0,\ \Re\{\nu\}<\tfrac{3}{2}.$$

45. $$\int_0^\infty \frac{(\beta+\sqrt{x^2+\beta^2})^\nu}{x^{\nu+\frac{1}{2}}\sqrt{\beta^2+x^2}}\,\cos(ax)\,dx = \frac{1}{\beta\sqrt{2a}}\Gamma\left(\frac{1}{4}-\frac{\nu}{2}\right)W_{\nu/2,\,-1/4}(a\beta)M_{-\nu/2,\,-1/4}(a\beta),$$
$$a>0,\ \Re\{\beta\}>0,\ \Re\{\nu\}<\tfrac{1}{2}.$$

46. $$\int_0^\infty \frac{(\sqrt{x^2+\beta^2}+x)^\nu-(\sqrt{x^2+\beta^2}-x)^\nu}{\sqrt{x^2+\beta^2}}\,\sin(ax)\,dx = 2\beta^\nu\,\sin\frac{\nu\pi}{2}K_\nu(a\beta),$$
$$a>0,\ \Re\{\beta\}>0,\ |\Re\{\nu\}|<1.$$

47. $$\int_0^\infty \frac{(\sqrt{x^2+\beta^2}+x)^\nu+(\sqrt{x^2+\beta^2}-x)^\nu}{\sqrt{x^2+\beta^2}}\,\cos(ax)\,dx = 2\beta^\nu\,\cos\frac{\nu\pi}{2}K_\nu(a\beta),$$
$$a>0,\ \Re\{\beta\}>0,\ |\Re\{\nu\}|<1.$$

48. $$\int_0^\infty \frac{(x+\beta+\sqrt{x^2+2\beta x})^\nu+(x+\beta-\sqrt{x^2+2\beta x})^\nu}{\sqrt{x^2+2\beta x}}\,\sin(ax)\,dx$$
$$=\pi\beta^\nu\left[Y_\nu(\beta a)\,\sin\left(\beta a-\frac{\nu\pi}{2}\right)+J_\nu(\beta a)\,\cos\left(\beta a-\frac{\nu\pi}{2}\right)\right],$$
$$a>0,\ |\arg\beta|<\pi,\ |\Re\{\nu\}|<1.$$

49. $$\int_0^\infty \frac{(x+\beta+\sqrt{x^2+2\beta x})^\nu + (x+\beta-\sqrt{x^2+2\beta x})^\nu}{\sqrt{x^2+2\beta x}} \cos(ax)\, dx$$
$$= \pi\beta^\nu \left[J_\nu(\beta a) \sin\left(\beta a - \frac{\nu\pi}{2}\right) - Y_\nu(\beta a) \cos\left(\beta a - \frac{\nu\pi}{2}\right)\right],$$
$$a > 0,\ |\arg\beta| < \pi,\ |\Re\{\nu\}| < 1.$$

50. $$\int_0^\infty \frac{a^2(b+x)^2 + p(p+1)}{(b+x)^{p+2}} \sin(ax)\, dx = \frac{a}{b^p}, \quad a > 0,\ b > 0,\ p > 0.$$

51. $$\int_0^\infty \frac{a^2(b+x)^2 + p(p+1)}{(b+x)^{p+2}} \cos(ax)\, dx = \frac{p}{b^{p+1}}, \quad a > 0,\ b > 0,\ p > 0.$$