

! For an efficient use of these tables, first read [HowTo.pdf](#).

**T1.16.** Integrand involving  $\sqrt{a+bx}$  and  $\alpha + \beta x$ .

Notation used:  $X = a + bx$ ,  $T = \alpha + \beta x$ ,  $\Delta = a\beta - b\alpha$ .

1.  $\int \frac{T dx}{\sqrt{X}} = \frac{2\alpha\sqrt{X}}{b} + \beta \left( \frac{X}{3} - a \right) \frac{2\sqrt{X}}{b^2}.$
2.  $\int \frac{T^2 dx}{\sqrt{X}} = \frac{2\alpha^2\sqrt{X}}{b} + 2\alpha\beta \left( \frac{X}{3} - a \right) \frac{2\sqrt{X}}{b^2} + \beta^2 \left( \frac{X^2}{5} - \frac{2}{3}Xa + a^2 \right) \frac{2\sqrt{X}}{b^3}.$
3.  $\int \frac{T^3 dx}{\sqrt{X}} = \frac{2\alpha^3\sqrt{X}}{b} + 3\alpha^2\beta \left( \frac{X}{3} - a \right) \frac{2\sqrt{X}}{b^2} + 3\alpha\beta^2 \left( \frac{X^2}{5} - \frac{2}{3}Xa + a^2 \right) \frac{2\sqrt{X}}{b^3}$   
 $+ \beta^3 \left( \frac{X^3}{7} - \frac{3X^2a}{5} + Xa^2 - a^3 \right) \frac{2\sqrt{X}}{b^4}.$
4.  $\int \frac{TX dx}{\sqrt{X}} = \frac{2\alpha\sqrt{X^3}}{3b} + \beta \left( \frac{X}{5} - \frac{a}{3} \right) \frac{2\sqrt{X^3}}{b^2}.$
5.  $\int \frac{T^2X dx}{\sqrt{X}} = \frac{2\alpha^2\sqrt{X^3}}{3b} + 2\alpha\beta \left( \frac{X}{5} - \frac{a}{3} \right) \frac{2\sqrt{X^3}}{b^2} + \beta^2 \left( \frac{X^2}{7} - \frac{2Xa}{5} + \frac{a^2}{3} \right) \frac{2\sqrt{X^3}}{b^3}.$
6.  $\int \frac{T^3X dx}{\sqrt{X}} = \frac{2\alpha^3\sqrt{X^3}}{3b} + 3\alpha^2\beta \left( \frac{X}{5} - \frac{a}{3} \right) \frac{2\sqrt{X^3}}{b^2} + 3\alpha\beta^2 \left( \frac{X^2}{7} - \frac{2Xa}{5} + \frac{a^2}{3} \right) \frac{2\sqrt{X^3}}{b^3}$   
 $+ \beta^3 \left( \frac{X^3}{9} - \frac{3X^2a}{7} + \frac{3Xa^2}{5} - \frac{a^3}{3} \right) \frac{2\sqrt{X^3}}{b^4}.$
7.  $\int \frac{TX^2 dx}{\sqrt{X}} = \frac{2\alpha\sqrt{X^5}}{5b} + \beta \left( \frac{X}{7} - \frac{a}{5} \right) \frac{2\sqrt{X^5}}{b^2}.$
8.  $\int \frac{T^2X^2 dx}{\sqrt{X}} = \frac{2\alpha^2\sqrt{X^5}}{5b} + 2\alpha\beta \left( \frac{X}{7} - \frac{a}{5} \right) \frac{2\sqrt{X^5}}{b^2} + \beta^2 \left( \frac{X^2}{9} - \frac{2Xa}{7} + \frac{a^2}{5} \right) \frac{2\sqrt{X^5}}{b^3}.$
9.  $\int \frac{T^3X^2 dx}{\sqrt{X}} = \frac{2\alpha^3\sqrt{X^5}}{5b} + 3\alpha^2\beta \left( \frac{X}{7} - \frac{a}{5} \right) \frac{2\sqrt{X^5}}{b^2} + 3\alpha\beta^2 \left( \frac{X^2}{9} - \frac{2Xa}{7} + \frac{a^2}{5} \right) \frac{2\sqrt{X^5}}{b^3}$   
 $+ \beta^3 \left( \frac{X^3}{11} - \frac{3X^2a}{9} + \frac{3Xa^2}{7} - \frac{a^3}{5} \right) \frac{2\sqrt{X^5}}{b^4}.$

- $$10. \int \frac{TX^3 dx}{\sqrt{X}} = \frac{2\alpha\sqrt{X^7}}{7b} + \beta \left( \frac{X}{9} - \frac{a}{7} \right) \frac{2\sqrt{X^7}}{b^2}.$$
- $$11. \int \frac{T^2 X^3 dx}{\sqrt{X}} = \frac{2\alpha^2\sqrt{X^7}}{7b} + 2\alpha\beta \left( \frac{X}{9} - \frac{a}{7} \right) \frac{2\sqrt{X^7}}{b^2} + \beta^2 \left( \frac{X^2}{11} - \frac{2Xa}{9} + \frac{a^2}{7} \right) \frac{2\sqrt{X^7}}{b^3}.$$
- $$12. \int \frac{T^3 X^3 dx}{\sqrt{X}} = \frac{2\alpha^3\sqrt{X^7}}{7b} + 3\alpha^2\beta \left( \frac{X}{9} - \frac{a}{7} \right) \frac{2\sqrt{X^7}}{b^2} + 3\alpha\beta^2 \left( \frac{X^2}{11} - \frac{2Xa}{9} + \frac{a^2}{7} \right) \frac{2\sqrt{X^7}}{b^3} \\ + \beta^3 \left( \frac{X^3}{13} - \frac{3X^2a}{11} + \frac{3Xa^2}{9} - \frac{a^3}{7} \right) \frac{2\sqrt{X^7}}{b^4}.$$
- $$13. \int \frac{X^m T^n dx}{\sqrt{X}} = \frac{2}{(2n+2m+1)\beta} T^{n+1} X^{m-1} \sqrt{X} + \frac{(2m-1)\Delta}{(2n+2m+1)\beta} \int \frac{X^{m-1} T^n dx}{\sqrt{X}}.$$
- $$14. \int \frac{T^n X^m dx}{\sqrt{X}} = 2\sqrt{X^{2m+1}} \sum_{k=0}^n \binom{n}{k} \frac{\alpha^{n-k} \beta^k}{b^{k+1}} \sum_{p=0}^k (-1)^p \binom{k}{p} \frac{X^{k-p} a^p}{2k-2p+2m+1}.$$
- $$15. \int \frac{T^n dx}{X^m \sqrt{X}} = \begin{cases} \frac{2}{(2m-1)\Delta} \frac{T^{n+1}}{X^m} \sqrt{X} - \frac{(2n-2m+3)\beta}{(2m-1)\Delta} \int \frac{T^n dx}{X^{m-1} \sqrt{X}}, \\ \text{or} \\ -\frac{2}{(2m-1)b} \frac{T^n}{X^m} \sqrt{X} + \frac{2n\beta}{(2m-1)b} \int \frac{T^{n-1} dx}{X^{m-1} \sqrt{X}}, \\ \text{or} \\ \frac{2}{\sqrt{X^{2m-1}}} \sum_{k=0}^n \binom{n}{k} \frac{\alpha^{n-k} \beta^k}{b^{k+1}} \sum_{p=0}^k (-1)^p \binom{k}{p} \frac{X^{k-p} a^p}{2k-2p-2m+1}. \end{cases}$$
- $$16. \int \frac{T dx}{X \sqrt{X}} = -\frac{2a}{b\sqrt{X}} + \frac{2\beta(X+a)}{b^2\sqrt{X}}.$$
- $$17. \int \frac{T^2 dx}{X \sqrt{X}} = -\frac{2\alpha^2}{b\sqrt{X}} + \frac{4\alpha\beta(X+a)}{b^2\sqrt{X}} + \frac{2\beta^2(X^2/3 - 2Xa - a^2)}{b^3\sqrt{X}}.$$
- $$18. \int \frac{T^3 dx}{X \sqrt{X}} = -\frac{2\alpha^3}{b\sqrt{X}} + \frac{6\alpha^2\beta(X+a)}{b^2\sqrt{X}} + \frac{6\alpha\beta^2(X^2/3 - 2Xa - a^2)}{b^3\sqrt{X}} + \frac{2\beta^3(X^3/5 - X^2a + 3Xa^2 + a^3)}{b^4\sqrt{X}}.$$
- $$19. \int \frac{T dx}{X^2 \sqrt{X}} = -\frac{2a}{3b\sqrt{X^3}} - \frac{2\beta(X-a/3)}{b^2\sqrt{X^3}}.$$
- $$20. \int \frac{T^2 dx}{X^2 \sqrt{X}} = -\frac{2a^2}{3b\sqrt{X^3}} - \frac{4\alpha\beta(X-a/3)}{b^2\sqrt{X^3}} + \frac{2\beta^2(X^2+2aX-a^2/3)}{b^3\sqrt{X^3}}.$$
- $$21. \int \frac{T^3 dx}{X^2 \sqrt{X}} = -\frac{2\alpha^3}{3b\sqrt{X^3}} - \frac{6\alpha^2\beta(X-a/3)}{b^2\sqrt{X^3}} + \frac{6\alpha\beta^2(X^2+2aX-a^2/3)}{b^3\sqrt{X^3}} \\ + \frac{2\beta^3(X^3/3 - 3X^2a - 3Xa^2 + a^3/3)}{b^4\sqrt{X^3}}.$$

$$\begin{aligned}
22. \int \frac{T dx}{X^3 \sqrt{X}} &= -\frac{2\alpha}{5b\sqrt{X^5}} - \frac{2\beta(X/3 - a/5)}{b^2\sqrt{X^5}}. \\
23. \int \frac{T^2 dx}{X^3 \sqrt{X}} &= -\frac{2\alpha^2}{5b\sqrt{X^5}} - \frac{4\alpha\beta(X/3 - a/5)}{b^2\sqrt{X^5}} - \frac{2\beta^2(X^2 - 2Xa/3 + a^2/5)}{b^3\sqrt{X^5}}. \\
24. \int \frac{T^3 dx}{X^3 \sqrt{X}} &= -\frac{2\alpha^3}{5b\sqrt{X^5}} - \frac{6\alpha^2\beta(X/3 - a/5)}{b^2\sqrt{X^5}} - \frac{6\alpha\beta^2(X^2 - 2Xa/3 + a^2/5)}{b^3\sqrt{X^5}} \\
&\quad + \frac{2\beta^3(X^3 + 3X^2a - Xa^2 + a^3/5)}{b^4\sqrt{X^5}}.
\end{aligned}$$

$$25. \int \frac{X^m dx}{T^n \sqrt{X}} = \begin{cases} -\frac{2}{(2n-2m-1)\beta} \frac{X^{m-1}}{T^{n-1}} \sqrt{X} - \frac{(2m-1)\Delta}{(2n-2m-1)\beta}, \\ \text{or} \\ -\frac{1}{(n-1)\beta} \frac{X^{m-1}}{T^{n-1}} \sqrt{X} + \frac{(2m-1)b}{2(n-1)\beta} \int \frac{X^{m-1}}{T^{n-1} \sqrt{X}} dx, \\ \text{or} \\ -\frac{1}{(n-1)\Delta} \frac{X^m}{T^{n-1}} \sqrt{X} - \frac{(2n-2m-3)b}{2(n-1)\Delta} \int \frac{X^m dx}{T^{n-1} \sqrt{X}}. \end{cases}$$

$$\begin{aligned}
26. \int \frac{X^m dx}{T^n \sqrt{X}} &= -X^m \sqrt{X} \left\{ \frac{1}{(n-1)\Delta T^{n-1}} \right. \\
&\quad + \sum_{k=2}^{n-1} \frac{(2n-2m-3)(2n-2m-5) \dots (2n-2m-2k+1)b^{k-1}}{2^{k-1}(n-1)(n-2) \dots (n-k)\Delta^k} \frac{1}{T^{n-k}} \Big\} \\
&\quad - \frac{(2n-2m-3)(2n-2m-5) \dots (-2m+3)(-2m+1)b^{n-1}}{2^{n-1} \cdot (n-1)!\Delta^n} \int \frac{X^m dx}{T \sqrt{X}}.
\end{aligned}$$

$$27. \int \frac{X^m dx}{T \sqrt{X}} = \begin{cases} \frac{2}{(2m-1)\beta} \frac{X^m}{\sqrt{X}} + \frac{\Delta}{\beta} \int \frac{X^{m-1} dx}{T \sqrt{X}}, \\ \text{or} \\ 2 \sum_{k=0}^{m-1} \frac{\Delta^k}{(2m-2k-1)\beta^{k+1}} \frac{X^{m-k}}{\sqrt{X}} + \frac{\Delta^m}{\beta^m} \int \frac{dx}{T \sqrt{X}}. \end{cases}$$

$$28. \int \frac{dx}{T \sqrt{X}} = \begin{cases} \frac{1}{\sqrt{\beta\Delta}} \ln \frac{\beta\sqrt{X} - \sqrt{\beta\Delta}}{\beta\sqrt{X} + \sqrt{\beta\Delta}}, & \beta\Delta > 0, \\ \frac{2}{\sqrt{-\beta\Delta}} \arctan \frac{\beta\sqrt{X}}{\sqrt{-\beta\Delta}}, & \beta\Delta < 0, \\ -\frac{2\sqrt{X}}{bT}, & \Delta = 0. \end{cases}$$

$$29. \int \frac{dx}{T X^m \sqrt{X}} = \frac{2}{X^{m-1} \sqrt{X}} + \sum_{k=1}^m \frac{\beta^{k-1} X^k}{\Delta^k (2m-2k+1)} + \frac{\beta^m}{\Delta^m} \int \frac{dx}{T \sqrt{X}}.$$

$$30. \int \frac{dx}{T X \sqrt{X}} = \frac{2}{\Delta \sqrt{X}} + \frac{\beta}{\Delta} \int \frac{dx}{T \sqrt{X}}.$$

31.  $\int \frac{dx}{TX^2\sqrt{X}} = \frac{2}{3\Delta X\sqrt{X}} + \frac{2\beta}{\Delta^2\sqrt{X}} + \frac{\beta^2}{\Delta^2} \int \frac{dx}{T\sqrt{X}}.$
32.  $\int \frac{dx}{TX^3\sqrt{X}} = \frac{2}{5\Delta X^2\sqrt{X}} + \frac{2\beta}{3\Delta^2 X\sqrt{X}} + \frac{2\beta^2}{\Delta^3\sqrt{X}} + \frac{\beta^3}{\Delta^3} \int \frac{dx}{T\sqrt{X}}.$
33.  $\int \frac{dx}{T^2\sqrt{X}} = -\frac{\sqrt{X}}{\Delta T} - \frac{b}{2\Delta} \int \frac{dx}{T\sqrt{X}}.$
34.  $\int \frac{dx}{T^2 X\sqrt{X}} = -\frac{1}{\Delta t\sqrt{X}} - \frac{3b}{\Delta^2\sqrt{X}} - \frac{3b\beta}{2\Delta^2} \int \frac{dx}{T\sqrt{X}}.$
35.  $\int \frac{dx}{T^2 X^2\sqrt{X}} = -\frac{1}{\Delta T X^2\sqrt{X}} - \frac{5b}{3\Delta^2 X\sqrt{X}} - \frac{5b\beta}{\Delta^3\sqrt{X}} - \frac{5b\beta^2}{2\Delta^3} \int \frac{dx}{T\sqrt{X}}.$
36.  $\int \frac{dx}{T^2 X^3\sqrt{X}} = -\frac{1}{\Delta T X^2\sqrt{X}} - \frac{7b}{5\Delta^2 X^2\sqrt{X}} - \frac{7b\beta}{3\Delta^3 X\sqrt{X}} - \frac{7b\beta^2}{\Delta^4\sqrt{X}} - \frac{7b\beta^3}{2\Delta^4} \int \frac{dx}{T\sqrt{X}}.$
37.  $\int \frac{dx}{T^3\sqrt{X}} = -\frac{\sqrt{X}}{2\Delta t^2} + \frac{3b\sqrt{X}}{4\Delta^2 T} + \frac{3b^2}{8\Delta^2} \int \frac{dx}{T\sqrt{X}}.$
38.  $\int \frac{dx}{T^3 X\sqrt{X}} = -\frac{1}{2\Delta T^2\sqrt{X}} + \frac{5b}{4\Delta^2 T\sqrt{X}} + \frac{15b^2}{4\Delta^3\sqrt{X}} + \frac{15b^2\beta}{8\Delta^3} \int \frac{dx}{T\sqrt{X}}.$
39.  $\int \frac{dx}{T^3 X^2\sqrt{X}} = -\frac{1}{2\Delta T^2 X\sqrt{X}} + \frac{7b\sqrt{X}}{4\Delta^2 T X\sqrt{X}} + \frac{35b^2}{12\Delta^2 X\sqrt{X}} + \frac{35b^2\beta}{4\Delta^4\sqrt{X}} + \frac{35b^2\beta^2}{8\Delta^4} \int \frac{dx}{T\sqrt{X}}.$
40.  $\int \frac{dx}{T^3 X^3\sqrt{X}} = -\frac{1}{2\Delta T^2 X^2\sqrt{X}} + \frac{9b}{4\Delta^2 T X^2\sqrt{X}} + \frac{63b^2}{20\Delta^3 X^2\sqrt{X}} + \frac{21b^2\beta}{4\Delta^4 X\sqrt{X}}$   
 $+ \frac{63b^2\beta^2}{4\Delta^5\sqrt{X}} + \frac{63b^2\beta^3}{8\Delta^5} \int \frac{dx}{T\sqrt{X}}.$
41.  $\int \frac{X dx}{T\sqrt{X}} = \frac{2\sqrt{X}}{\beta} + \frac{\Delta}{\beta} \int \frac{dx}{T\sqrt{X}}.$
42.  $\int \frac{X^2 dx}{T\sqrt{X}} = \frac{2X\sqrt{X}}{3\beta} + \frac{2\Delta\sqrt{X}}{\beta^2} + \frac{\Delta^2}{\beta^2} \int \frac{dx}{T\sqrt{X}}.$
43.  $\int \frac{X^3 dx}{T\sqrt{X}} = \frac{2X^2\sqrt{X}}{5\beta} + \frac{2\Delta X\sqrt{X}}{3\beta^2} + \frac{2\Delta^2\sqrt{X}}{\beta^3} + \frac{\Delta^3}{\beta^3} \int \frac{dx}{T\sqrt{X}}.$
44.  $\int \frac{X dx}{T^2\sqrt{X}} = -\frac{X\sqrt{X}}{\Delta T} + \frac{b\sqrt{X}}{\beta\Delta} + \frac{b}{2\beta} \int \frac{dx}{T\sqrt{X}}.$
45.  $\int \frac{X^2 dx}{T^2\sqrt{X}} = -\frac{X^2\sqrt{X}}{\Delta T} + \frac{bX\sqrt{X}}{\beta\Delta} + \frac{3b\sqrt{X}}{\beta^2} + \frac{3b\Delta}{2\beta^2} \int \frac{dx}{T\sqrt{X}}.$
46.  $\int \frac{X^3 dx}{T^2\sqrt{X}} = -\frac{X^3\sqrt{X}}{\Delta T} + \frac{bX^2\sqrt{X}}{\beta\Delta} + \frac{5bX\sqrt{X}}{3\beta^2} + \frac{5b\Delta\sqrt{X}}{\beta^3} + \frac{5\Delta^2 b}{2\beta^3} \int \frac{dx}{T\sqrt{X}}.$

$$47. \int \frac{X dx}{T^3 \sqrt{X}} = -\frac{X \sqrt{X}}{2 \Delta T^2} + \frac{b X \sqrt{X}}{4 \Delta^2 T} - \frac{b^2 \sqrt{X}}{4 \beta \Delta^2} + \frac{b^2}{8 \beta \Delta} \int \frac{dx}{T \sqrt{X}}.$$

$$48. \int \frac{X^2 dx}{T^3 \sqrt{X}} = -\frac{X^2 \sqrt{X}}{2 \Delta T^2} + \frac{b X^2 \sqrt{X}}{4 \Delta^2 T} + \frac{b^2 X \sqrt{X}}{4 \beta \Delta^2} + \frac{3 b^2 \sqrt{X}}{4 \beta^2 \Delta} + \frac{3 b^2}{8 \beta^2} \int \frac{dx}{T \sqrt{X}}.$$

$$49. \int \frac{X^3 dx}{T^3 \sqrt{X}} = -\frac{X^3 \sqrt{X}}{2 \Delta T^2} + \frac{3 b X^3 \sqrt{X}}{\Delta^2 T} + \frac{3 b^2 X^2 \sqrt{X}}{4 \beta \Delta^2} + \frac{5 b^2 X \sqrt{X}}{4 \beta^2 \Delta} \\ + \frac{15 b^2 \sqrt{X}}{4 \beta^3} + \frac{15 b^2 \Delta}{8 \beta^3} \int \frac{dx}{T \sqrt{X}}.$$

$$50. \int \frac{dx}{X^m T^n \sqrt{X}} = \begin{cases} \frac{2}{(2m-1)\Delta} \frac{\sqrt{X}}{T^{n-1} X^m} + \frac{(2n+2m-3)\beta}{(2m-1)\Delta} \int \frac{dx}{T^n X^{m-1} \sqrt{X}}, \\ \text{or} \\ -\frac{1}{(n-1)\Delta} \frac{\sqrt{X}}{X^m T^{n-1}} - \frac{(2n+2m-3)b}{2(n-1)\Delta} \int \frac{dx}{T^{n-1} X^m \sqrt{X}}, \\ \text{or} \\ \frac{\sqrt{X}}{X^m} \left\{ \frac{-1}{(n-1)\Delta} \cdot \frac{1}{T^{n-1}}, \right. \\ \left. + \sum_{k=2}^{n-1} (-1)^k \frac{(2n+2m-3)(2n+2m-5) \dots (2n+2m-2k+1) b^{k-1}}{2^{k-1}(n-1)(n-2) \dots (n-k) \Delta^k} \cdot \frac{1}{T^{n-k}} \right\}, \\ \left. + (-1)^{n-1} \frac{(2n+2m-3)(2n+2m-5) \dots (-2m+3)(-2m+1) b^{n-1}}{2^{n-1}(n-1)! \Delta^{n-1}} \int \frac{dx}{T X^m \sqrt{X}}. \right\}$$

$$51. \int \frac{dx}{X^m T \sqrt{X}} = \frac{2}{(2m-1)\Delta} \frac{1}{X^{m-1} \sqrt{X}} + \frac{\beta}{\Delta} \int \frac{dx}{T X^{m-1} \sqrt{X}}.$$


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