! For an efficient use of these tables, first read HowTo.pdf.

T3.36A. Integrands involving trigonometric functions and rational polynomials of degree k for $k = 1, 2, 3, 4, n$, on the interval $(0, \infty)$.

1. $\displaystyle\int_0^\infty \frac{\sin(ax)}{x}\,dx = \frac{\pi}{2}\ \mathrm{sgn}\ a.$

2. $\displaystyle\int_0^\infty \frac{\sin(ax)}{x+\beta}\,dx = \mathrm{Ci}(a\beta)\ \sin(a\beta) - \cos(a\beta)\ \mathrm{Si}(a\beta), \quad |\arg\beta| < \pi,\ a > 0.$

3. $\displaystyle\int_0^\infty \frac{\cos(ax)}{x+\beta}\,dx = -\sin(a\beta)\ \mathrm{Si}(a\beta) - \cos(a\beta)\ \mathrm{Ci}(a\beta), \quad |\arg\beta| < \pi,\ a > 0.$

4. $\displaystyle\int_0^\infty \frac{\sin(ax)}{\beta - x}\,dx = \sin(\beta a)\ \mathrm{Ci}(\beta a) - \cos(\beta a)[\mathrm{Si}(\beta a) + \pi], \quad a > 0,$ β not real and positive.

5. $\displaystyle\int_0^\infty \frac{\cos(ax)}{\beta - x}\,dx = -\cos(a\beta)\ \mathrm{Ci}(a\beta) + \sin(a\beta)[\mathrm{Si}(a\beta) + \pi], \quad a > 0,$ β not real and positive.

6. $\displaystyle\int_0^\infty \frac{\sin(ax)}{\beta^2 + x^2}\,dx = \frac{1}{2\beta}[e^{-a\beta}\,\mathrm{Ei}(a\beta) - e^{a\beta}\,\mathrm{Ei}(-a\beta)], \quad a > 0,\ \beta > 0.$

7. $\displaystyle\int_0^\infty \frac{\cos(ax)}{\beta^2 + x^2}\,dx = \frac{\pi}{2\beta}e^{-a\beta}, \quad a \ge 0,\ \Re\{\beta\} > 0.$

8. $\displaystyle\int_0^\infty \frac{x\ \sin(ax)}{\beta^2 + x^2}\,dx = \frac{\pi}{2}e^{-a\beta}, \quad a > 0,\ \Re\{\beta\} > 0.$

9. $\displaystyle\int_0^\infty \frac{x\ \cos(ax)}{\beta^2 + x^2}\,dx = -\frac{1}{2}\left[e^{-a\beta}\,\mathrm{Ei}(a\beta) + e^{a\beta}\,\mathrm{Ei}(-a\beta)\right], \quad a > 0,\ \beta > 0.$

10. $\displaystyle\int_0^\infty \frac{\sin(ax)}{\beta^2 - x^2}\,dx = \frac{1}{\beta}\left[\sin(a\beta)\ \mathrm{Ci}(a\beta) - \cos(a\beta)\left(\mathrm{Si}(a\beta) + \frac{\pi}{2}\right)\right], \quad |\arg\beta| < \pi,\ a > 0.$

11. $$\int_0^\infty \frac{\cos(ax)}{b^2-x^2}\,dx = \frac{\pi}{2b}\,\sin(ab), \quad a>0,\ b>0.$$

12. $$\int_0^\infty \frac{x\,\sin(ax)}{b^2-x^2}\,dx = -\frac{\pi}{2}\,\cos(ab), \quad a>0.$$

13. $$\int_0^\infty \frac{x\,\cos(ax)}{\beta^2+x^2}\,dx = \cos(a\beta)\,\mathrm{Ci}(a\beta)+\sin(a\beta)\left[\mathrm{Si}(a\beta)+\frac{\pi}{2}\right], \quad |\arg\{\beta\}|<\pi,\ a>0.$$

14. $$\int_0^\infty \frac{\sin(ax)\,dx}{x(\beta^2+x^2)} = \frac{\pi}{2\beta^2}(1-e^{-a\beta}), \quad \Re\{\beta\}>0,\ a>0.$$

15. $$\int_0^\infty \frac{\sin(ax)\,dx}{x(b^2-x^2)} = \frac{\pi}{2b^2}(1-\cos(ab)), \quad a>0.$$

16. $$\int_0^\infty \frac{\sin(ax)\,\cos(bx)}{x(x^2+\beta^2)}\,dx = \begin{cases} \dfrac{\pi}{2\beta^2}e^{-\beta b}\,\sinh(a\beta), & 0<a<b, \\ -\dfrac{\pi}{2\beta^2}e^{-a\beta}\,\cosh(b\beta)+\dfrac{\pi}{2\beta^2}, & a>b>0. \end{cases}$$

17. $$\int_0^\infty \frac{x\,\sin(ax)\,dx}{b^3\pm b^2x+bx^2\pm x^3}$$
$$=\pm\frac{1}{4b}\left[e^{-ab}\,\mathrm{Ei}(ab)-e^{ab}\,\mathrm{Ei}(-ab)-2\,\mathrm{Ci}(ab)\,\sin(ab)+2\,\cos(ab)\left(\mathrm{Si}(ab)+\frac{\pi}{2}\right)\right]+\frac{\pi e^{-ab}-\pi\,\cos(ab)}{4b},$$
$a>0,\ b>0;$ the result is the principal value if the minus sign is taken.

18. $$\int_0^\infty \frac{x^2\,\sin(ax)\,dx}{b^3\pm b^2x+bx^2\pm x^3}$$
$$=\frac{1}{4}\left[e^{ab}\,\mathrm{Ei}(-ab)-e^{-ab}\,\mathrm{Ei}(ab)+2\,\mathrm{Ci}(ab)\,\sin(ab)-2\,\cos(ab)\left(\mathrm{Si}(ab)+\frac{\pi}{2}\right)\right]\pm\pi(e^{-ab}+\cos(ab)),$$
$a>0,\ b>0;$ the result is the principal value if the minus sign is taken.

19. $$\int_0^\infty \frac{\cos(ax)\,dx}{b^4+x^4} = \frac{\pi\sqrt{2}}{4b^3}\,\exp\left(-\frac{ab}{\sqrt{2}}\right)\left(\cos\frac{ab}{\sqrt{2}}+\sin\frac{ab}{\sqrt{2}}\right), \quad a>0,\ b>0.$$

20. $$\int_0^\infty \frac{\sin(ax)\,dx}{b^4-x^4} = \frac{1}{4b^3}\left[2\,\sin(ab)\,\mathrm{Ci}(ab)-2\,\cos(ab)\left(\mathrm{Si}(ab)+\frac{\pi}{2}\right)+e^{-ab}\,\mathrm{Ei}(ab)-e^{ab}\,\mathrm{Ei}(-ab)\right],$$
$a>0,\ b>0.$

21. $$\int_0^\infty \frac{\cos(ax)\,dx}{b^4-x^4} = \frac{\pi}{4b^3}\left[e^{-ab}+\sin(ab)\right], \quad a>0,\ b>0.$$

22. $$\int_0^\infty \frac{x\ \sin(ax)\,dx}{b^4+x^4} = \frac{\pi}{2b^2}\ \exp\left(-\frac{ab}{\sqrt{2}}\right)\ \sin\frac{ab}{\sqrt{2}}, \quad a>0,\ b>0.$$

23. $$\int_0^\infty \frac{x\ \sin(ax)}{b^4-x^4}\,dx = \frac{\pi}{4b^2}\left[e^{-ab}-\cos(ab)\right], \quad a>0,\ b>0.$$

24. $$\int_0^\infty \frac{x\ \cos(ax)\,dx}{b^4-x^4} = \frac{1}{4b^2}\Big[2\ \cos(ab)\ \mathrm{Ci}(ab)+2\ \sin(ab)\left(\mathrm{Si}(ab)+\frac{\pi}{2}\right)$$
$$-e^{-ab}\,\mathrm{Ei}(ab)-e^{ab}\,\mathrm{Ei}(-ab)\Big], \quad a>0,\ b>0.$$

25. $$\int_0^\infty \frac{x^2\ \cos(ax)\,dx}{b^4+x^4} = \frac{\pi\sqrt{2}}{4b}\ \exp\left(-\frac{ab}{\sqrt{2}}\right)\left(\cos\frac{ab}{\sqrt{2}}-\sin\frac{ab}{\sqrt{2}}\right), \quad a>0,\ b>0.$$

26. $$\int_0^\infty \frac{x^2\ \sin(ax)\,dx}{b^4-x^4} = \frac{1}{4b}\Big[2\ \sin(ab)\ \mathrm{Ci}(ab)$$
$$-2\ \cos(ab)\ \left(\mathrm{Si}(ab)+\frac{\pi}{2}\right)-e^{-ab}\,\mathrm{Ei}\,(ab)+e^{ab}\,\mathrm{Ei}\,(-ab)\Big], \quad a>0,\ b>0.$$

27. $$\int_0^\infty \frac{x^2\ \cos(ax)\,dx}{b^4-x^4} = \frac{\pi}{4b}\left[\sin(ab)-e^{-ab}\right], \quad a>0,\ b>0.$$

28. $$\int_0^\infty \frac{x^3\ \sin(ax)}{b^4+x^4}\,dx = \frac{\pi}{2}\ \exp\left(-\frac{ab}{\sqrt{2}}\right)\cos\frac{ab}{\sqrt{2}}, \quad a>0,\ b>0.$$

29. $$\int_0^\infty \frac{x^3\ \sin(ax)}{b^4-x^4}\,dx = \frac{-\pi}{4}\left[e^{-ab}-\cos(ab)\right], \quad a>0,\ b>0.$$

30. $$\int_0^\infty \frac{x^3\ \cos(ax)\,dx}{b^4-x^4} = \frac{1}{4}\Big[2\ \cos(ab)\ \mathrm{Ci}(ab)+2\ \sin(ab)\left(\mathrm{Si}(ab)+\frac{\pi}{2}\right)$$
$$+e^{-ab}\,\mathrm{Ei}\,(ab)+e^{ab}\,\mathrm{Ei}(-ab)\Big], \quad a>0,\ b>0.$$

31. $$\int_0^\infty \frac{\cos(ax)\,dx}{(\beta^2+x^2)(\gamma^2+x^2)} = \frac{\pi(\beta e^{-a\gamma}-\gamma e^{-a\beta})}{2\beta\gamma(\beta^2-\gamma^2)}, \quad a>0,\ \Re\{\beta\}>0,\ \Re\{\gamma\}>0.$$

32. $$\int_0^\infty \frac{x\ \sin(ax)\,dx}{(\beta^2+x^2)(\gamma^2+x^2)} = \frac{\pi(e^{-a\beta}-e^{-a\gamma})}{2(\gamma^2-\beta^2)}, \quad a>0,\ \Re\{\beta\}>0,\ \Re\{\gamma\}>0.$$

33. $$\int_0^\infty \frac{x^2\ \cos(ax)\,dx}{(\beta^2+x^2)(\gamma^2+x^2)} = \frac{\pi(\beta e^{-a\beta}-\gamma e^{-a\gamma})}{2(\beta^2-\gamma^2)}, \quad a>0,\ \Re\{\beta\}>0,\ \Re\{\gamma\}>0.$$

34. $$\int_0^\infty \frac{x^3\,\sin(ax)\,dx}{(\beta^2+x^2)(\gamma^2+x^2)} = \frac{\pi(\beta^2 e^{-a\beta}-\gamma^2 e^{-a\gamma})}{2(\beta^2-\gamma^2)}, \quad a>0,\ \Re\{\beta\}>0,\ \Re\{\gamma\}>0.$$

35. $$\int_0^\infty \frac{\cos(ax)\,dx}{(b^2-x^2)(c^2-x^2)} = \frac{\pi(b\,\sin(ac)-c\,\sin(ab))}{2bc(b^2-c^2)}, \quad a>0,\ b>0,\ c>0.$$

36. $$\int_0^\infty \frac{x\,\sin(ax)\,dx}{(b^2-x^2)(c^2-x^2)} = \frac{\pi(\cos(ab)-\cos(ac))}{2(b^2-c^2)}, \quad a>0.$$

37. $$\int_0^\infty \frac{x^2\,\cos(ax)\,dx}{(b^2-x^2)(c^2-x^2)} = \frac{\pi(c\,\sin(ac)-b\,\sin(ab))}{2(b^2-c^2)}, \quad a>0,\ b>0,\ c>0.$$

38. $$\int_0^\infty \frac{x^3\,\sin(ax)\,dx}{(b^2-x^2)(c^2-x^2)} = \frac{\pi(b^2\,\cos(ab)-c^2\,\cos(ac))}{2(b^2-c^2)}, \quad a>0,\ b>0,\ c>0.$$

39. $$\int_0^\infty \frac{\cos(ax)\,dx}{(b^2+x^2)^2} = \frac{\pi}{4b^3}(1+ab)\,e^{-ab}, \quad a>0,\ b>0.$$

40. $$\int_0^\infty \frac{x\,\sin(ax)\,dx}{(b^2+x^2)^2} = \frac{\pi}{4b}\,a\,e^{-ab}, \quad a>0,\ b>0.$$

41. $$\int_0^\infty \cos(px)\,\frac{1-x^2}{(1+x^2)^2}\,dx = \frac{\pi p}{2}e^{-p}.$$

42. $$\int_0^\infty \frac{x^3\,\sin(ax)\,dx}{(b^2+x^2)^2} = \frac{\pi}{4}(2-ab)\,e^{-ab}, \quad a>0,\ b>0.$$

43. $$\int_0^\infty \frac{\cos(ax)\,dx}{(x^2+b^2)^2+c^2} = \frac{\pi}{2c}\,\frac{e^{-aA}\,[B\,\cos(aB)+A\,\sin(aB)]}{\sqrt{b^4+c^2}}, \quad a>0,\ b>0,\ c>0,$$

where $2A^2=\sqrt{b^4+c^2}+b^2, \quad 2B^2=\sqrt{b^4+c^2}-b^2.$

44. $$\int_0^\infty \frac{x\,\sin(ax)\,dx}{(x^2+b^2)^2+c^2} = \frac{\pi}{2c}e^{-aA}\,\sin(aB), \quad a>0,\ b>0,\ c>0,$$

where $2A^2=\sqrt{b^4+c^2}+b^2, \quad 2B^2=\sqrt{b^4+c^2}-b^2.$

45. $$\int_0^\infty \frac{(x^2+b^2)\,\cos(ax)\,dx}{(x^2+b^2)^2+c^2} = \frac{\pi}{2}\,\frac{e^{-aA}\,[A\,\cos(aB)-B\,\sin(aB)]}{\sqrt{b^4+c^2}}, \quad a>0,\ b>0,\ c>0,$$

where $2A^2=\sqrt{b^4+c^2}+b^2, \quad 2B^2=\sqrt{b^4+c^2}-b^2.$

46. $$\int_0^\infty \frac{x(x^2+b^2)\,\sin(ax)\,dx}{(x^2+b^2)^2+c^2} = \frac{\pi}{2}e^{-aA}\,\cos(aB), \quad a>0,\ b>0,\ c>0,$$
where $2A^2 = \sqrt{b^4+c^2}+b^2, \quad 2B^2 = \sqrt{b^4+c^2}-b^2.$

47. $$\int_0^\infty \left[\frac{1}{\beta^2+(\gamma-x)^2} - \frac{1}{\beta^2+(\gamma+x)^2}\right]\sin(ax)\,dx = \frac{\pi}{\beta}e^{-a\beta}\,\sin(a\gamma),$$
$$a>0,\ \Re\{\beta\}>0,\ \gamma+i\beta \text{ is not real.}$$

48. $$\int_0^\infty \left[\frac{1}{\beta^2+(\gamma-x)^2} + \frac{1}{\beta^2+(\gamma+x)^2}\right]\cos(ax)\,dx = \frac{\pi}{\beta}e^{-a\beta}\,\cos(a\gamma),$$
$$a>0,\ |\Im\{\gamma\}|<\Re\{\beta\}.$$

49. $$\int_0^\infty \left[\frac{\gamma+x}{\beta^2+(\gamma+x)^2} - \frac{\gamma-x}{\beta^2+(\gamma-x)^2}\right]\sin(ax)\,dx = \pi e^{-a\beta}\,\cos(a\gamma),$$
$$a>0,\ \Re\{\beta\}>0,\ \gamma+i\beta \text{ is not real.}$$

50. $$\int_0^\infty \left[\frac{\gamma+x}{\beta^2+(\gamma+x)^2} + \frac{\gamma-x}{\beta^2+(\gamma-x)^2}\right]\cos(ax)\,dx = \pi e^{-a\beta}\,\sin(a\gamma),$$
$$a>0,\ |\Im\{a\}|<\Re\{\beta\}.$$

51. $$\int_0^\infty \frac{\cos(ax)\,dx}{x^4+2b^2x^2\,\cos 2t+b^4} = \frac{\pi}{2b^3}\,\exp(-ab\,\cos t)\,\frac{\sin(t+ab\,\sin t)}{\sin 2t},$$
$$a>0,\ b>0,\ |t|<\frac{\pi}{2}.$$

52. $$\int_0^\infty \frac{x\,\sin(ax)\,dx}{x^4+2b^2x^2\,\cos 2t+b^4} = \frac{\pi}{2b^2}\,\exp(-ab\,\cos t)\,\frac{\sin(ab\,\sin t)}{\sin 2t},$$
$$a>0,\ b>0,\ |t|<\frac{\pi}{2}.$$

53. $$\int_0^\infty \frac{x^2\,\cos(ax)\,dx}{x^4+2b^2x^2\,\cos 2t+b^4} = \frac{\pi}{2b}\,\exp(-ab\,\cos t)\,\frac{\sin(t-ab\,\sin t)}{\sin 2t},$$
$$a>0,\ b>0,\ |t|<\frac{\pi}{2}.$$

54. $$\int_0^\infty \frac{x^3\,\sin(ax)\,dx}{x^4+2b^2x^2\,\cos 2t+b^4} = \frac{\pi}{2}\,\exp(-ab\,\cos t)\,\frac{\sin(2t-ab\,\sin t)}{\sin 2t},$$
$$a>0,\ b>0,\ |t|<\frac{\pi}{2}.$$

55. $$\int_0^\infty \frac{\sin(ax)\,dx}{x(x^4+2b^2x^2\,\cos 2t+b^4)} = \frac{\pi}{2b^4}\left[1-\exp(-ab\,\cos t)\,\frac{\sin(2t+ab\,\sin t)}{\sin 2t}\right], \quad a>0,\ b>0,\ |t|<\frac{\pi}{2}.$$

56. $$\int_0^\infty \frac{\sin(ax)\,dx}{x(b^4+x^4)} = \frac{\pi}{2b^4}\left[1-\exp\left(-\frac{ab}{\sqrt{2}}\right)\cos\frac{ab}{\sqrt{2}}\right], \quad a>0,\ b>0.$$

57. $$\int_0^\infty \frac{\sin(ax)\,dx}{x(b^4-x^4)} = \frac{\pi}{4b^4}\left[2-e^{-ab}-\cos(ab)\right], \quad a>0,\ b>0.$$

58. $$\int_0^\infty \frac{\sin(ax)\,dx}{x(b^2+x^2)^2} = \frac{\pi}{2b^4}\left[1-\frac{1}{2}(2+ab)\,e^{-ab}\right], \quad a>0,\ b>0.$$

59. $$\int_0^\infty \frac{\cos(ax)\,dx}{(b^2+x^2)(b^4-x^4)} = \frac{\pi}{8b^5}\left[\sin(ab)+(2+ab)\,e^{-ab}\right], \quad a>0,\ b>0.$$

60. $$\int_0^\infty \frac{x\,\sin(ax)\,dx}{(b^2+x^2)(b^4-x^4)} = \frac{\pi}{8b^4}\left[(1+ab)\,e^{-ab}-\cos(ab)\right], \quad a>0,\ b>0.$$

61. $$\int_0^\infty \frac{x^2\,\cos(ax)\,dx}{(b^2+x^2)(b^4-x^4)} = \frac{\pi}{8b^3}\left[\sin(ab)-ab\,e^{-ab}\right], \quad a>0,\ b>0.$$

62. $$\int_0^\infty \frac{x^3\,\sin(ax)\,dx}{(b^2+x^2)(b^4-x^4)} = \frac{\pi}{8b^2}\left[(1-ab)\,e^{-ab}-\cos(ab)\right], \quad a>0,\ b>0.$$

63. $$\int_0^\infty \frac{x^3\,\sin(ax)\,dx}{(b^2+x^2)(b^4-x^4)} = \frac{\pi}{8b^2}\left[(1-ab)\,e^{-ab}-\cos(ab)\right], \quad a>0,\ b>0.$$

64. $$\int_0^\infty \frac{x^4\,\cos(ax)\,dx}{(b^2+x^2)(b^4-x^4)} = \frac{\pi}{8b}\left[\sin(ab)+(ab-2)\,e^{-ab}\right], \quad a>0,\ b>0.$$

65. $$\int_0^\infty \frac{x^5\,\sin(ax)\,dx}{(b^2+x^2)(b^4-x^4)} = \frac{\pi}{8}\left[(ab-3)\,e^{-ab}-\cos(ab)\right], \quad a>0,\ b>0.$$

66. $$\int_0^\infty \frac{\cos(ax)\,dx}{(b^2+x^2)^n} = \frac{\pi e^{-ab}}{(2b)^{2n-1}(n-1)!}\sum_{k=0}^{n-1}\frac{(2n-k-2)!(2ab)^k}{k!(n-k-1)!}$$
$$= \frac{(-1)^{n-1}\pi}{b^{2n-1}(n-1)!}\left[\frac{d^{n-1}}{dp^{n-1}}\left(\frac{e^{-ab\sqrt{p}}}{\sqrt{p}}\right)\right]_{p=1}$$
$$= \frac{(-1)^{n-1}\pi}{2b^{2n-1}(n-1)!}\left[\frac{d^{n-1}}{dp^{n-1}}\,\frac{e^{-abp}}{(1+p)^n}\right]_{p=1}, \quad a>0,\ b>0.$$

67. $$\int_0^\infty \frac{x\,\sin(ax)\,dx}{(x^2+\beta^2)^{n+1}} = \begin{cases} \dfrac{\pi a e^{-a\beta}}{2^{2n}n!\beta^{2n-1}}\displaystyle\sum_{k=0}^{n-1}\frac{(2n-k-2)!(2a\beta)^k}{k!(n-k-1)!}, & n\neq 0, \\ \dfrac{\pi}{2}\,e^{-\alpha\beta}, & n=0, \end{cases} \quad a>0,\ \Re\{\beta\}>0.$$

68. $$\int_0^\infty \frac{\sin(ax)\,dx}{x(\beta^2+x^2)^{n+1}} = \frac{\pi}{2\beta^{2n+2}}\left[1-\frac{e^{-a\beta}}{2^n n!}F_n(a\beta)\right],$$
$$a>0,\ \Re\{\beta\}>0,\ F_0(z)=1,\ F_1(z)=z+2,\dots,\ F_n(z)=(z+2n)F_{n-1}(z)-zF'_{n-1}(z).$$

69. $$\int_0^\infty \frac{x\,\sin(ax)\,dx}{(b^2+x^2)^3} = \frac{\pi a}{16b^3}(1+ab)\,e^{-ab}, \quad a>0,\ b>0.$$

70. $$\int_0^\infty \frac{x\,\sin(ax)\,dx}{(b^2+x^2)^4} = \frac{\pi a}{96b^5}(3+3ab+a^2b^2)\,e^{-ab}, \quad a>0,\ b>0.$$

71. $$\int_0^\infty \frac{x^{m-1}\,\sin(ax)}{x^{2n}+\beta^{2n}}\,dx = -\frac{\pi\beta^{m-2n}}{2n}\sum_{k=1}^{n}\exp\left[-a\beta\,\sin\frac{(2k-1)\pi}{2n}\right]$$
$$\times\cos\left\{\frac{(2k-1)m\pi}{2n}+a\beta\,\cos\frac{(2k-1)\pi}{2n}\right\},$$
$$m\,\text{even};\ a>0,\ |\arg\beta|<\frac{\pi}{2n},\ 0<m\le 2n.$$

72. $$\int_0^\infty \frac{x^{m-1}\,\cos(ax)}{x^{2n}+\beta^{2n}}\,dx = \frac{\pi\beta^{m-2n}}{2n}\sum_{k=1}^{n}\exp\left[-a\beta\,\sin\frac{(2k-1)\pi}{2n}\right]$$
$$\times\sin\left\{\frac{(2k-1)m\pi}{2n}+a\beta\,\cos\frac{(2k-1)\pi}{2n}\right\},$$
$$m\,\text{odd};\ a>0,\ |\arg\beta|<\frac{\pi}{2n},\ 0<m<2n+1.$$

73. $$\int_0^\infty \frac{\sin(ax)\,dx}{x(x^2+2^2)(x^2+4^2)\dots(x^2+4n^2)} = \frac{\pi(-1)^n}{(2n)!2^{2n+1}}\left[2\sum_{k=0}^{n-1}(-1)^k\binom{2n}{k}e^{2(k-n)a}+(-1)^n\binom{2n}{n}\right], \quad a>0,\ n\geq 0.$$

74. $$\int_0^\infty \frac{\cos(ax)\,dx}{(x^2+1^2)(x^2+3^2)\dots[x^2+(2n+1)^2]} = \begin{cases} \dfrac{(-1)^n}{(2n+1)!}\,\dfrac{\pi}{2^{2n+1}}\displaystyle\sum_{k=0}^{n}(-1)^k\binom{2n+1}{k}e^{(2k-2n-1)a}, & a\geq 0,\ n\geq 0,\\ \dfrac{\pi 2^{-2n-1}}{(2n+1)(n!)^2}, & a=0,\ n\geq 0. \end{cases}$$

75. $$\int_0^\infty \frac{x\,\sin(ax)\,dx}{(x^2+1^2)(x^2+3^2)\dots[x^2+(2n+1)^2]} = \frac{\pi(-1)^n}{(2n+1)!2^{2n+1}}\sum_{k=0}^{n}(-1)^k\binom{2n+1}{k}(2n-2k+1)\,e^{(2k-2n-1)a}, \quad a>0,\ n\geq 0.$$

76. $$\int_0^\infty \frac{\cos ax\,dx}{(x^2+2^2)(x^2+4^2)\dots(x^2+4n^2)} = \frac{\pi 2^{1-2n}}{(2n)!}\sum_{k=1}^{n}(-1)^k k\binom{2n}{n-k}e^{-2ak}, \quad n\geq 1,\ a\geq 0.$$

77. $$\int_0^\infty \frac{\sin(ax)\,\sin(bx)}{x}\,dx = \frac{1}{4}\,\ln\left(\frac{a+b}{a-b}\right)^2, \quad a>0,\ b>0,\ a\neq b.$$

78. $$\int_0^\infty \frac{\sin(ax)\,\cos(bx)}{x}\,dx = \begin{cases} \dfrac{\pi}{2}, & a>b\geq 0,\\ \dfrac{\pi}{4}, & a=b>0,\\ 0, & b>a\geq 0. \end{cases}$$

79. $$\int_0^\infty \frac{\sin(ax)\,\sin(bx)}{x^2}\,dx = \begin{cases} \dfrac{a\pi}{2}, & 0<a\leq b,\\ \dfrac{b\pi}{2}, & 0<b\leq a. \end{cases}$$

80. $$\int_0^\infty \frac{\sin(ax)\,\sin(bx)}{\beta^2+x^2}\,dx = \begin{cases} \dfrac{\pi}{4\beta}\left(e^{-|a-b|\beta} - e^{-(a+b)\beta}\right), & a>0,\ b>0,\ \Re\{\beta\}>0, \\ \dfrac{\pi}{2\beta}\,e^{-\alpha\beta}\,\sinh(b\beta), & \beta>0,\ a \ge b \ge 0, \\ \dfrac{\pi}{2\beta}\,e^{-\alpha\beta}\,\sinh(a\beta), & \beta>0,\ b \ge a \ge 0. \end{cases}$$

81. $$\int_0^\infty \frac{\sin(ax)\,\cos(bx)}{\beta^2+x^2}\,dx = \frac{1}{4\beta}e^{-a\beta}\{e^{b\beta}\,\mathrm{Ei}[\beta(a-b)] + e^{-b\beta}\,\mathrm{Ei}[\beta(a+b)]\} - \frac{1}{4\beta}e^{a\beta}\{e^{b\beta}\,\mathrm{Ei}[-\beta(a+b)] + e^{-b\beta}\,\mathrm{Ei}[\beta(b-a)]\}.$$

82. $$\int_0^\infty \frac{\cos(ax)\,\cos(bx)}{\beta^2+x^2}\,dx = \frac{\pi}{4\beta}[e^{-|a-b|\beta} + e^{-(a+b)\beta}], \quad a>0,\ b>0,\ \Re\{\beta\}>0.$$

83. $$\int_0^\infty \frac{x\,\cos(ax)\,\cos(bx)}{\beta^2+x^2}\,dx = \begin{cases} -\dfrac{1}{4}e^{a\beta}\{e^{b\beta}\,\mathrm{Ei}[-\beta(a+b)] + e^{-b\beta}\,\mathrm{Ei}[\beta(b-a)]\} \\ \qquad -\dfrac{1}{4}e^{-a\beta}\{e^{b\beta}\,\mathrm{Ei}[\beta(a-b)] + e^{-b\beta}\,\mathrm{Ei}[\beta(a+b)]\}, & a \ne b, \\ \infty, & a=b. \end{cases}$$

84. $$\int_0^\infty \frac{x\,\sin(ax)\,\cos(bx)}{x^2+\beta^2}\,dx = \begin{cases} \dfrac{\pi}{2}e^{-a\beta}\,\cosh(b\beta), & 0<b<a, \\ \dfrac{\pi}{4}e^{-2a\beta}, & 0<b=a, \\ -\dfrac{\pi}{2}e^{-b\beta}\,\sinh(a\beta), & 0<a<b. \end{cases}$$

85. $$\int_0^\infty \frac{\sin(ax)\,\sin(bx)}{p^2-x^2}\,dx = \begin{cases} -\dfrac{\pi}{2p}\,\cos(ap)\,\sin(bp), & a>b>0, \\ -\dfrac{\pi}{4p}\,\sin(2ap), & a=b>0, \\ -\dfrac{\pi}{2p}\,\sin(ap)\,\cos(bp), & b>a>0. \end{cases}$$

86. $$\int_0^\infty \frac{\sin(ax)\,\cos(bx)}{p^2-x^2}\,x\,dx = \begin{cases} -\dfrac{\pi}{2}\,\cos(ap)\,\cos(bp), & a>b>0, \\ -\dfrac{\pi}{4}\,\cos(2ap), & a=b>0, \\ \dfrac{\pi}{2}\,\sin(ap)\,\sin(bp), & b>a>0. \end{cases}$$

87. $\displaystyle\int_0^\infty \frac{\cos(ax)\,\cos(bx)}{p^2-x^2}\,dx = \begin{cases} \dfrac{\pi}{2p}\,\sin(ap)\,\cos(bp), & a>b>0, \\ \dfrac{\pi}{4p}\,\sin(2ap), & a=b>0, \\ \dfrac{\pi}{2p}\,\cos(ap)\,\sin(bp), & b>a>0. \end{cases}$

88. $\displaystyle\int_0^\infty \frac{\sin(ax)}{\sin(bx)}\cdot\frac{dx}{x^2+\beta^2} = \frac{\pi}{2\beta}\cdot\frac{\sinh(a\beta)}{\sinh(b\beta)}, \quad 0<a<b,\ \Re\{\beta\}>0.$

89. $\displaystyle\int_0^\infty \frac{\sin(ax)}{\cos(bx)}\cdot\frac{x\,dx}{x^2+\beta^2} = -\frac{\pi}{2}\cdot\frac{\sinh(a\beta)}{\cosh(b\beta)}, \quad 0<a<b,\ \Re\{\beta\}>0.$

90. $\displaystyle\int_0^\infty \frac{\cos(ax)}{\sin(bx)}\cdot\frac{x\,dx}{x^2+\beta^2} = \frac{\pi}{2}\cdot\frac{\cosh(a\beta)}{\sinh(b\beta)}, \quad 0<a<b,\ \Re\{\beta\}>0.$

91. $\displaystyle\int_0^\infty \frac{\cos(ax)}{\cos(bx)}\cdot\frac{dx}{x^2+\beta^2} = \frac{\pi}{2\beta}\cdot\frac{\cosh(a\beta)}{\cosh(b\beta)}, \quad 0<a<b,\ \Re\{\beta\}>0.$

92. p.v. $\displaystyle\int_0^\infty \frac{\sin(ax)}{\sin x}\cdot\frac{dx}{b^2-x^2} = \begin{cases} 0, & 0\le a\le 1, \\ \dfrac{\pi}{b}\,\sin(a-1)b, & 1\le a\le 2, \end{cases} \quad b \text{ real, } b/\pi \text{ not integer.}$

93. $\displaystyle\int_0^\infty \frac{\sin(ax)}{\cos(bx)}\cdot\frac{dx}{x(x^2+\beta^2)} = \frac{\pi}{2\beta^2}\cdot\frac{\sinh(a\beta)}{\cosh(b\beta)}, \quad 0<a<b,\ \Re\{\beta\}>0.$

94. $\displaystyle\int_0^\infty \frac{\sin(ax)}{\cos(bx)}\cdot\frac{dx}{x(c^2-x^2)} = 0, \quad 0<a<b,\ c>0.$

95. $\displaystyle\int_0^\infty \frac{dx}{x^{n+1}}\prod_{k=0}^{n}\sin(a_kx) = \frac{\pi}{2}\prod_{k=1}^{n}a_k, \quad a_0>\sum_{k=1}^{n}a_k,\ a_k>0.$

96. $\displaystyle\int_0^\infty \frac{\sin(ax)}{x^{n+1}}\,dx\prod_{k=1}^{n}\sin(a_kx)\prod_{j=1}^{m}\cos(b_jx) = \frac{\pi}{2}\prod_{k=1}^{n}a_k\,, \quad a>\sum_{k=1}^{n}|a_k|+\sum_{j=1}^{m}|b_j|.$

97. $\displaystyle\int_0^\infty \frac{x\,dx}{(x^2+b^2)\,\sin(ax)} = \frac{\pi}{2\,\sinh(ab)}, \quad b>0.$

98. $\displaystyle\int_0^\infty \tan ax\frac{dx}{x} = \frac{\pi}{2}, \quad a>0.$

99. $\displaystyle\int_0^\infty \frac{1}{\left(x + c^2/a^2\right)^2 + b^2}\, \sin\frac{u\left(x + c^2/a^2\right)}{a\sqrt{x}}\, dx = \frac{\pi}{b}\, e^{-\alpha_1 u}\, \sin\left(\alpha_2 u\right), \quad a, c > 0,\ b, u \geq 0$, and

$$\alpha_{1,2} = \frac{b}{c}\sqrt{\frac{\mp 1 + \sqrt{1 + b^2 a^4/c^4}}{2\,\left(1 + b^2 a^4/c^4\right)}}.$$

100. $\displaystyle\int_0^\infty \frac{x\,\tan(ax)\,dx}{x^2 + b^2} = \frac{\pi}{e^{2ab} + 1}, \quad a > 0,\ b > 0.$

101. $\displaystyle\int_0^\infty \frac{x\,\cot(ax)\,dx}{x^2 + b^2} = \frac{\pi}{e^{2ab} - 1}, \quad a > 0,\ b > 0.$

102. $\displaystyle\int_0^\infty \frac{x\,\tan(ax)\,dx}{b^2 - x^2} = \int_0^\infty \frac{x\,\cot(ax)\,dx}{b^2 - x^2} = \int_0^\infty \frac{x\,\csc(ax)\,dx}{b^2 - x^2} = \infty.$
