T1.38. Integrand involving inverse hyperbolic functions.

1. $\displaystyle\int \operatorname{arcsinh}\frac{x}{a}\,dx = x\,\operatorname{arcsinh}\frac{x}{a} - \sqrt{x^2+a^2}.$

2. $\displaystyle\int \operatorname{arccosh}\frac{x}{a}\,dx = \begin{cases} x\,\operatorname{arccosh}\dfrac{x}{a} - \sqrt{x^2-a^2}, & \operatorname{arccosh}\dfrac{x}{a} > 0, \\ x\,\operatorname{arccosh}\dfrac{x}{a} + \sqrt{x^2-a^2}, & \operatorname{arccosh}\dfrac{x}{a} < 0. \end{cases}$

3. $\displaystyle\int \operatorname{arctanh}\frac{x}{a}\,dx = x\,\operatorname{arctanh}\frac{x}{a} + \frac{a}{2}\ln(a^2-x^2).$

4. $\displaystyle\int \operatorname{arccoth}\frac{x}{a}\,dx = x\,\operatorname{arccoth}\frac{x}{a} + \frac{a}{2}\ln(x^2-a^2).$

5. $\displaystyle\int x\,\operatorname{arcsinh}\frac{x}{a}\,dx = \left(\frac{x^2}{2}+\frac{a^2}{4}\right)\operatorname{arcsinh}\frac{x}{a} - \frac{x}{4}\sqrt{x^2+a^2}.$

6. $\displaystyle\int x\,\operatorname{arccosh}\frac{x}{a}\,dx = \begin{cases} \left(\dfrac{x^2}{2}-\dfrac{a^2}{4}\right)\operatorname{arccosh}\dfrac{x}{a} - \dfrac{x}{4}\sqrt{x^2-a^2}, & \operatorname{arccosh}\dfrac{x}{a} > 0, \\ \left(\dfrac{x^2}{2}-\dfrac{a^2}{4}\right)\operatorname{arccosh}\dfrac{x}{a} + \dfrac{x}{4}\sqrt{x^2-a^2}, & \operatorname{arccosh}\dfrac{x}{a} < 0. \end{cases}$

7. $\displaystyle\int x^n\,\operatorname{arcsinh} x\,dx = \frac{x^{n+1}}{n+1}\operatorname{arcsinh} x - \frac{1}{n+1}\int \frac{x^{n+1}}{\sqrt{1+x^2}}\,dx, \quad n \neq -1.$

8. $\displaystyle\int x^n\,\operatorname{arccosh} x\,dx = \frac{x^{n+1}}{n+1}\operatorname{arccosh} x - \frac{1}{n+1}\int \frac{x^{n+1}}{\sqrt{x^2-1}}\,dx, \quad n \neq -1.$

9. $\displaystyle\int x\,\operatorname{arctanh}\frac{x}{a}\,dx = \frac{x^2-a^2}{2}\operatorname{arctanh}\frac{x}{a} + \frac{ax}{2}, \quad \left|\frac{x}{a}\right| < 1.$

10. $\displaystyle\int x^n\,\operatorname{arctanh} x\,dx = \frac{x^{n+1}}{n+1}\operatorname{arctanh} x - \frac{1}{n+1}\int \frac{x^{n+1}}{1-x^2}\,dx, \quad n \neq -1.$

11. $\displaystyle\int x\,\operatorname{arccoth}\frac{x}{a}\,dx = \frac{x^2-a^2}{2}\operatorname{arccoth}\frac{x}{a} + \frac{ax}{2}, \quad \left|\frac{x}{a}\right| > 1.$

12. $\displaystyle\int x^n \operatorname{arccoth} x\,dx = \frac{x^{n+1}}{n+1}\operatorname{arccoth} x + \frac{1}{n+1}\int \frac{x^{n+1}}{x^2-1}\,dx, \quad n \neq -1.$

13. $\displaystyle\int \operatorname{arcsech} x\,dx = x \operatorname{arcsech} x + \arcsin x.$

14. $\displaystyle\int x \operatorname{arcsech} x\,dx = \frac{x^2}{2}\operatorname{arcsech} x - \frac{1}{2}\sqrt{1-x^2}.$

15. $\displaystyle\int x^n \operatorname{arcsech} x\,dx = \frac{x^{n+1}}{n+1}\operatorname{arcsech} x + \frac{1}{n+1}\int \frac{x^n}{\sqrt{1-x^2}}\,dx, \quad n \neq -1.$

16. $\displaystyle\int \operatorname{arccsch} x\,dx = x \operatorname{arccsch} x + \frac{x}{|x|}\operatorname{arcsinh} x.$

17. $\displaystyle\int x \operatorname{arccsch} x\,dx = \frac{x^2}{2}\operatorname{arccsch} x + \frac{1}{2}\frac{x}{|x|}\sqrt{1+x^2}.$

18. $\displaystyle\int x^n \operatorname{arccsch} x\,dx = \frac{x^{n+1}}{n+1}\operatorname{arccsch} x + \frac{1}{n+1}\frac{x}{|x|}\int \frac{x^n}{\sqrt{1+x^2}}\,dx, \quad n \neq -1.$