C4282

! For an efficient use of these tables, first read `HowTo.pdf`.

T2.51B. Integrands involving trigonometric functions and rational polynomials of degree k for $k = 1, 2, 3, 4, n$, on the interval $(0, \pi/2)$.

1. $$\int_0^{\pi/2} \frac{x^m}{\sin x}\,dx = \begin{cases} \left(\frac{\pi}{2}\right)^m \left[\frac{1}{m} + \sum_{k=1}^{\infty} \frac{2^{2k-1}-1}{4^{2k-1}(m+2k)} \zeta(2k)\right], & m \neq 2, \\ = 2\pi\mathbf{G} - \frac{7}{2}\zeta(3), & m = 2. \end{cases}$$

2. $$\int_0^{\pi/2} \frac{x\,dx}{\sin x} = \int_0^{\pi/2} \frac{\left(\frac{\pi}{2} - x\right)\,dx}{\cos x} = 2\mathbf{G}.$$

3. $$\int_0^{\pi/2} x\,\tan x\,dx = \infty.$$

4. $$\int_0^{\pi/2} x\,\cot x\,dx = \frac{\pi}{2}\ln 2.$$

5. $$\int_0^{\pi/2} \left(\frac{\pi}{2} - x\right)\tan x\,dx = \frac{1}{2}\int_0^{\pi} \left(\frac{\pi}{2} - x\right)\tan x\,dx = \frac{\pi}{2}\ln 2.$$

6. $$\int_0^{\pi/2} \frac{x\,\cot x}{\cos 2x}\,dx = \frac{\pi}{4}\ln 2.$$

7. $$\int_0^{\pi/2} x^p\,\cot x\,dx = \left(\frac{\pi}{2}\right)^p \left\{\frac{1}{p} - 2\sum_{k=1}^{\infty} \frac{1}{4^k(p+2k)}\zeta(2k)\right\}.$$
