T1.22. Integrand involving sines and cosines of multiple angles with linear and other arguments.

1. $\displaystyle\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b).$

2. $\displaystyle\int \cos(ax+b)\,dx = -\frac{1}{a}\sin(ax+b).$

3. $\displaystyle\int \sin(ax+b)\,\sin(cx+d)\,dx = \frac{\sin[(a-c)x+b-d]}{2(a-c)} - \frac{\sin[(a+c)x+b+d]}{2(a+c)}, \quad a^2 \neq c^2.$

4. $\displaystyle\int \sin(ax+b)\,\cos(cx+d)\,dx = -\frac{\cos[(a-c)x+b-d]}{2(a-c)} - \frac{\cos[(a+c)x+b+d]}{2(a+c)}, \quad a^2 \neq c^2.$

5. $\displaystyle\int \cos(ax+b)\,\cos(cx+d)\,dx = \frac{\sin[(a-c)x+b-d]}{2(a-c)} + \frac{\sin[(a+c)x+b+d]}{2(a+c)}, \quad a^2 \neq c^2.$

6. $\displaystyle\int \sin(ax+b)\,\sin(ax+d)\,dx = \frac{x}{2}\cos(b-d) - \frac{\sin(2ax+b+d)}{4a}.$

7. $\displaystyle\int \sin(ax+b)\,\cos(ax+d)\,dx = \frac{x}{2}\sin(b-d) - \frac{\cos(2ax+b+d)}{4a}.$

8. $\displaystyle\int \cos(ax+b)\,\cos(ax+d)\,dx = \frac{x}{2}\cos(b-d) + \frac{\sin(2ax+b+d)}{4a}.$

9. $\displaystyle\int \sin ax\,\cos bx\,dx = -\frac{\cos(a+b)x}{2(a+b)} - \frac{\cos(a-b)x}{2(a-b)}, \quad a^2 \neq b^2.$

10. $$\int \sin ax\,\sin bx\,\sin cx\,dx = -\frac{1}{4}\left\{\frac{\cos(a-b+c)x}{a-b+c} + \frac{\cos(b+c-a)x}{b+c-a} + \frac{\cos(a+b-c)x}{a+b-c} - \frac{\cos(a+b+c)x}{a+b+c}\right\}.$$

11. $$\int \sin ax\,\cos bx\,\cos cx\,dx = -\frac{1}{4}\left\{\frac{\cos(a+b+c)x}{a+b+c} - \frac{\cos(b+c-a)x}{b+c-a} + \frac{\cos(a+b-c)x}{a+b-c} + \frac{\cos(a+c-b)x}{a+c-b}\right\}.$$

12. $$\int \cos ax\,\sin bx\,\sin cx\,dx = \frac{1}{4}\left\{\frac{\sin(a+b-c)x}{a+b-c} + \frac{\sin(a+c-b)x}{a+c-b} - \frac{\sin(a+b+c)x}{a+b+c} - \frac{\sin(b+c-a)x}{b+c-a}\right\}.$$

13. $$\int \cos ax \cos bx \cos cx\,dx = \frac{1}{4}\left\{\frac{\sin(a+b+c)x}{a+b+c} + \frac{\sin(b+c-a)x}{b+c-a} + \frac{\sin(a+c-b)x}{a+c-b} + \frac{\sin(a+b-c)x}{a+b-c}\right\}.$$

14. $$\int \frac{\cos px + i\sin px}{\sin nx}\,dx = -2\int \frac{z^{p+n-1}}{1-z^{2n}}\,dz, \quad z = \cos x + i\sin x.$$

15. $$\int \frac{\cos px + i\sin px}{\cos nx}\,dx = -2\,i\int \frac{z^{p+n-1}}{1-z^{2n}}\,dz, \quad z = \cos x + i\sin x.$$

16. $$\int \sin^p x \sin ax\,dx = \frac{1}{p+a}\left\{-\sin^p x \cos ax + p\int \sin^{p-1} x \cos(a-1)x\,dx\right\}.$$

17. $$\int \sin^p x \sin(2n+1)\,x\,dx$$
$$= (2n+1)\left\{\int \sin^{p+1} x\,dx + \sum_{k=1}^{n}(-1)^k \frac{[(2n+1)^2-1^2][(2n+1)^2-3^2]\dots[(2n+1)^2-(2k-1)^2]}{(2k+1)!}\int \sin^{2k+p+1} x\,dx\right\}$$
$$= \frac{\Gamma(p+1)}{\Gamma\left(\frac{p+3}{2}+n\right)}\left\{\sum_{k=0}^{n-1}\left[\frac{(-1)^{k-1}\Gamma\left(\frac{p+1}{2}+n-2k\right)}{2^{2k+1}\Gamma(p-2k+1)}\sin^{p-2k} x \cos(2n-2k+1)x + (-1)^k\frac{\Gamma\left(\frac{p-1}{2}+n-2k\right)}{2^{2k+2}\Gamma(p-2k)}\sin^{p-2k-1} x \sin(2n-2k)\,x\right] + \frac{(-1)^n\Gamma\left(\frac{p+3}{2}-n\right)}{2^{2n}\Gamma(p-2n+1)}\int \sin^{p-2n+1} x\,dx\right\}.$$

18. $$\int \sin^p x \sin 2nx\,dx$$
$$= 2n\left\{\frac{\sin^{p+2} x}{p+2} + \sum_{k=1}^{n-1}(-1)^k\frac{(4n^2-2^2)(4n^2-4^2)\dots[4n^2-(2k)^2]}{(2k+1)!(2k+p+2)}\sin^{2k+p+2} x\right\}$$
$$= \frac{\Gamma(p+1)}{\Gamma\left(\frac{p}{2}+n+1\right)}\left\{\sum_{k=0}^{n-1}\frac{(-1)^{k-1}\Gamma\left(\frac{p}{2}+n-2k\right)}{2^{2k+1}\Gamma(p-2k+1)}\sin^{p-2k} x \cos(2n-2k)x - \frac{(-1)^k\Gamma\left(\frac{p}{2}+n-2k-1\right)}{2^{2k+2}\Gamma(p-2k)}\sin^{p-2k-1} x \sin(2n-2k-1)\,x\right\}, \quad p \neq -2, -4, \dots, -2n.$$

19. $$\int \sin^p x \cos ax\,dx = \frac{1}{p+1}\left\{\sin^p x \sin ax - p\int \sin^{p-1} x \sin(a-1)\,x\,dx\right\}.$$

20. $\displaystyle\int \sin^p x \cos(2n+1)\,x\,dx$

$$= \frac{\sin^{p+1} x}{p+1} + \sum_{k=1}^{n} (-1)^k \frac{[(2n+1)^2 - 1^2][(2n+1)^2 - 3^2]\dots[(2n+1)^2 - (2k-1)^2]}{(2k)!(2k+p+1)} \sin^{2k+p+1} x$$

$$= \frac{\Gamma(p+1)}{\Gamma\left(\frac{p+3}{2}+n\right)} \left\{ \sum_{k=0}^{n-1} \left[\frac{(-1)^k \Gamma\left(\frac{p+1}{2}+n-2k\right)}{2^{2k+1}\Gamma(p-2k+1)} \sin^{p-2k} x \cos(2n-2k+1)x \right.\right.$$

$$\left. + (-1)^k \frac{\Gamma\left(\frac{p-1}{2}+n-2k\right)}{2^{2k+2}\Gamma(p-2k)} \sin^{p-2k-1} x \sin(2n-2k)\,x \right]$$

$$\left. + \frac{(-1)^n \Gamma\left(\frac{p+3}{2}-n\right)}{2^{2n}\Gamma(p-2n+1)} \int \sin^{p-2n} x \cos x \right\}, \quad p \neq -3, -5, \dots, -(2n+1).$$

21. $\displaystyle\int \sin^p x \cos 2nx\,dx$

$$= \int \sin^p x\,dx + \sum_{k=1}^{n} (-1)^k \frac{4n^2 \cdot (4n^2 - 2^2)\dots[4n^2 - (2k-2)^2]}{(2k)!} \int \sin^{2k+p} x\,dx$$

$$= \frac{\Gamma(p+1)}{\Gamma\left(\frac{p}{2}+n+1\right)} \left\{ \sum_{k=0}^{n-1} \left[\frac{(-1)^k \Gamma\left(\frac{p}{2}+n-2k\right)}{2^{2k+1}\Gamma(p-2k+1)} \sin^{p-2k} x \sin(2n-2k)x \right.\right.$$

$$\left. + \frac{(-1)^k \Gamma\left(\frac{p}{2}+n-2k-1\right)}{2^{2k+2}\Gamma(p-2k)} \sin^{p-2k-1} x \cos(2n-2k-1)x \right]$$

$$\left. + \frac{(-1)^n \Gamma\left(\frac{p}{2}-n+1\right)}{2^{2n}\Gamma(p-2n+1)} \int \sin^{p-2n} x\,dx \right\}.$$

22. $$\int \cos^p x \sin ax\,dx = \frac{1}{p+a} \left\{ -\cos^p x \cos ax + p \int \cos^{p-1} x \sin(a-1)x\,dx \right\}.$$

23. $$\int \cos^p x \sin(2n+1)\,x\,dx = (-1)^{n+1} \left\{ \frac{\cos^{p+1} x}{p+1} \right.$$

$$\left. + \sum_{k=1}^{n} (-1)^k \frac{[(2n+1)^2 - 1^2][(2n+1)^2 - 3^2]\dots[(2n+1)^2 - (2k-1)^2]}{(2k)!(2k+p+1)} \cos^{2k+p+1} x \right\}$$

$$= \frac{\Gamma(p+1)}{\Gamma\left(\frac{p+3}{2}+n\right)} \left\{ - \sum_{k=0}^{n-1} \frac{\Gamma\left(\frac{p+1}{2}+n-k\right)}{2^{2k+1}\Gamma(p-2k+1)} \cos^{p-k} x \cos(2n-k+1)x \right.$$

$$\left. + \frac{\Gamma\left(\frac{p+3}{2}\right)}{2^n \Gamma(p-n+1)} \int \cos^{p-n} x \sin(n+1)x\,dx \right\}, \quad p \neq -3, -5, \dots, -(2n+1).$$

24. $\displaystyle\int \cos^p x\, \sin 2nx\, dx$

$$= (-1)^n \left\{ \frac{\cos^{p+2} x}{p+2} + \sum_{k=1}^{n-1} (-1)^k \frac{(4n^2-2^2)(4n^2-4^2)\dots[4n^2-(2k)^2]}{(2k+1)!(2k+p+2)} \cos^{2k+p+2} x \right\}$$

$$= \frac{\Gamma(p+1)}{\Gamma\left(\frac{p}{2}+n+1\right)} \left\{ -\sum_{k=0}^{n-1} \frac{\Gamma\left(\frac{p}{2}+n-k\right)}{2^{k+1}\Gamma(p-k+1)} \cos^{p-k} x \cos(2n-k)x \right.$$

$$\left. + \frac{\Gamma\left(\frac{p}{2}+1\right)}{2^n \Gamma(p-n+1)} \int \cos^{p-n} x \sin nx\, dx \right\}, \quad p \neq -2, -4, \dots, -2n.$$

25. $\displaystyle\int \cos^p x \cos ax\, dx = \frac{1}{p+a} \left\{ \cos^p x \sin ax + p \int \cos^{p-1} x \cos(a-1)x\, dx \right\}.$

26. $\displaystyle\int \cos^p x \cos(2n+1)x\, dx = (-1)^n (2n+1) \left\{ \int \cos^{p+1} x\, dx \right.$

$$\left. + \sum_{k=1}^{n} (-1)^k \frac{[(2n+1)^2-1^2][(2n+1)^2-3^2]\dots[(2n+1)^2-(2k-1)^2]}{(2k+1)!} \int \cos^{2k+p+1} x\, dx \right\}$$

$$= \frac{\Gamma(p+1)}{\Gamma\left(\frac{p+3}{2}+n\right)} \left\{ \sum_{k=0}^{n-1} \frac{\Gamma\left(\frac{p+1}{2}+n-k\right)}{2^{k+1}\Gamma(p-k+1)} \cos^{p-k} x \sin(2n-k+1)x \right.$$

$$\left. + \frac{\Gamma\left(\frac{p+3}{2}\right)}{2^n \Gamma(p-n+1)} \int \cos^{p-n} x \cos(n+1)x\, dx \right\}.$$

27. $\displaystyle\int \cos^p x \cos 2nx\, dx$

$$= (-1)^n \left\{ \int \cos^p x\, dx + \sum_{k=1}^{n} (-1)^k \frac{4n^2[4n^2-2^2]\dots[4n^2-(2k-2)^2]}{(2k)!} \int \cos^{2k+p} x\, dx \right\}$$

$$= \frac{\Gamma(p+1)}{\Gamma\left(\frac{p}{2}+n+1\right)} \left\{ \sum_{k=0}^{n-1} \frac{\Gamma\left(\frac{p}{2}+n-k\right)}{2^{k+1}\Gamma(p-k+1)} \cos^{p-k} x \sin(2n-k)x \right.$$

$$\left. + \frac{\Gamma\left(\frac{p}{2}+1\right)}{2^n \Gamma(p-n+1)} \int \cos^{p-n} x \cos nx\, dx \right\}.$$

28. $\displaystyle\int \frac{\sin(2n+1)x}{\sin x}\, dx = 2\sum_{k=1}^{n} \frac{\sin 2kx}{2k} + x.$

29. $\displaystyle\int \frac{\sin 2nx}{\sin x}\, dx = 2\sum_{k=1}^{n} \frac{\sin(2k-1)x}{2k-1}.$

30. $\displaystyle\int \frac{\cos(2n+1)x}{\sin x}\, dx = 2\sum_{k=1}^{n} \frac{\cos 2kx}{2k} + \ln \sin x.$

31. $\displaystyle\int \frac{\cos 2nx}{\sin x}\,dx = 2\sum_{k=1}^{n}\frac{\cos(2k-1)x}{2k-1} + \ln\tan\frac{x}{2}.$

32. $\displaystyle\int \frac{\sin(2n+1)x}{\cos x}\,dx = 2\sum_{k=1}^{n}(-1)^{n-k+1}\frac{\cos 2kx}{2k} + (-1)^{n+1}\ln\cos x.$

33. $\displaystyle\int \frac{\sin 2nx}{\cos x}\,dx = 2\sum_{k=1}^{n}(-1)^{n-k+1}\frac{\cos(2k-1)x}{2k-1}.$

34. $\displaystyle\int \frac{\cos(2n+1)x}{\cos x}\,dx = 2\sum_{k=1}^{n}(-1)^{n-k}\frac{\sin 2kx}{2k} + (-1)^{n}x.$

35. $\displaystyle\int \frac{\cos 2nx}{\cos x}\,dx = 2\sum_{k=1}^{n}(-1)^{n-k}\frac{\sin(2k-1)x}{2k-1} + (-1)^{n}\ln\tan\left(\frac{\pi}{4}+\frac{x}{2}\right).$

36. $\displaystyle\int \sin(n+1)x\,\sin^{n-1}x\,dx = \frac{1}{n}\sin^{n}x\,\sin nx.$

37. $\displaystyle\int \sin(n+1)x\,\cos^{n-1}x\,dx = -\frac{1}{n}\cos^{n}x\,\cos nx.$

38. $\displaystyle\int \cos(n+1)x\,\sin^{n-1}x\,dx = \frac{1}{n}\sin^{n}x\,\cos nx.$

39. $\displaystyle\int \cos(n+1)x\,\cos^{n-1}x\,dx = \frac{1}{n}\cos^{n}x\,\sin nx.$

40. $\displaystyle\int \sin\left[(n+1)\left(\frac{\pi}{2}-x\right)\right]\sin^{n-1}x\,dx = \frac{1}{n}\sin^{n}x\cos n\left(\frac{\pi}{2}-x\right).$

41. $\displaystyle\int \cos\left[(n+1)\left(\frac{\pi}{2}-x\right)\right]\sin^{n-1}x\,dx = -\frac{1}{n}\sin^{n}x\,\sin n\left(\frac{\pi}{2}-x\right).$

42. $\displaystyle\int \frac{\sin 2x}{\sin^2 x}\,dx = 2\ln\sin x.$

43. $\displaystyle\int \frac{\sin 2x}{\sin^n x}\,dx = -\frac{2}{(n-2)\sin^{n-2}x}.$

44. $\displaystyle\int \frac{\sin 2x}{\cos^2 x}\,dx = -2\ln\cos x.$

45. $\displaystyle\int \frac{\sin 2x\,dx}{\cos^n x} = \frac{2}{(n-2)\cos^{n-2}x}.$

46. $\displaystyle\int \frac{\cos 2x\,dx}{\sin x} = 2\cos x + \ln\tan\frac{x}{2}.$

47. $\displaystyle\int \frac{\cos 2x\,dx}{\sin^2 x} = -\cot x - 2x.$

48. $\displaystyle\int \frac{\cos 2x\,dx}{\sin^3 x} = -\frac{\cos x}{2\sin^2 x} - \frac{3}{2}\ln\tan\frac{x}{2}.$

49. $\int \frac{\cos 2x\, dx}{\cos x} = 2 \sin x - \ln \tan\left(\frac{\pi}{4} + \frac{x}{2}\right).$

50. $\int \frac{\cos 2x\, dx}{\cos^2 x} = 2x - \tan x.$

51. $\int \frac{\cos 2x\, dx}{\cos^3 x} = -\frac{\sin x}{2 \cos^2 x} + \frac{3}{2} \ln \tan\left(\frac{\pi}{4} + \frac{x}{2}\right).$

52. $\int \frac{\sin 3x\, dx}{\sin x} = x + \sin 2x.$

53. $\int \frac{\sin 3x}{\sin^2 x}\, dx = 3 \ln \tan \frac{x}{2} + 4 \cos x.$

54. $\int \frac{\sin 3x}{\sin^3 x}\, dx = -3 \cot x - 4x.$

55. $\int \frac{\sin 3x}{\cos x}\, dx = 2 \sin^2 x + \ln \cos x.$

56. $\int \frac{\sin 3x}{\cos^3 x}\, dx = -\frac{1}{2 \cos^2 x} - 4 \ln \cos x.$

57. $\int \frac{\sin 3x}{\cos^n x}\, dx = \frac{4}{(n-3) \cos^{n-3} x} - \frac{1}{(n-1) \cos^{n-1} x}.$

58. $\int \frac{\cos 3x}{\sin^n x}\, dx = \frac{4}{(n-3) \sin^{n-3} x} - \frac{1}{(n-1) \sin^{n-1} x}.$

59. $\int \frac{\cos 3x}{\sin x}\, dx = -2 \sin^2 x + \ln \sin x.$

60. $\int \frac{\cos 3x}{\sin^3 x}\, dx = -\frac{1}{2 \sin^2 x} - 4 \ln \sin x.$

61. $\int \frac{\sin nx}{\cos^p x}\, dx = 2\int \frac{\sin (n-1)x\, dx}{\cos^{p-1} x} - \int \frac{\sin (n-2)\, x\, dx}{\cos^p x}.$

62. $\int \frac{\cos 3x}{\cos x}\, dx = \sin 2x - x.$

63. $\int \frac{\cos 3x}{\cos^2 x}\, dx = 4 \sin x - 3 \ln \tan\left(\frac{\pi}{4} + \frac{x}{2}\right).$

64. $\int \frac{\cos 3x}{\cos^3 x}\, dx = 4x - 3 \tan x.$

65. $\int \frac{\sin^m x\, dx}{\sin (2n+1)x} = \frac{1}{2n+1}\sum_{k=0}^{2n}(-1)^{n+k} \cos^m \left[\frac{2k+1}{2(2n+1)}\pi\right] \ln \frac{\sin\left[\frac{(k-n)\pi}{2(2n+1)} + \frac{x}{2}\right]}{\sin\left[\frac{k+n+1}{(2n+1)}\pi - \frac{x}{2}\right]}$

where m is a natural number $\leq 2n$.

66. $$\int \frac{\sin^{2m} x\,dx}{\sin 2nx} = \frac{(-1)^n}{2n} \left\{ \ln \cos x + \sum_{k=1}^{n-1} (-1)^k \cos^{2m} \frac{k\pi}{2n} \ln \left(\cos^2 x - \sin^2 \frac{k\pi}{2n} \right) \right\},$$

where m is a natural number $\leq n$.

67. $$\int \frac{\sin^{2m+1} x}{\sin 2nx}\,dx = \frac{(-1)^n}{2n} \left\{ \ln \tan \left(\frac{\pi}{4} - \frac{x}{2} \right) \right.$$
$$\left. + \sum_{k=1}^{n-1} (-1)^k \cos^{2m+1} \frac{k\pi}{2n} \ln \left[\tan \left(\frac{n+k}{4n}\pi - \frac{x}{2} \right) \tan \left(\frac{n-k}{4n}\pi - \frac{x}{2} \right) \right] \right\},$$

where m is a natural number $< n$.

68. $$\int \frac{\sin^{2m} x\,dx}{\cos(2n+1)x} = \frac{(-1)^{n+1}}{2n+1} \left\{ \ln \tan \left(\frac{\pi}{4} - \frac{x}{2} \right) \right.$$
$$\left. + \sum_{k=1}^{n} (-1)^k \cos^{2m} \frac{k\pi}{2n+1} \ln \left[\tan \left(\frac{2n+2k+1}{4(2n+1)}\pi - \frac{x}{2} \right) \tan \left(\frac{2n-2k+1}{2(2n+1)}\pi - \frac{x}{2} \right) \right] \right\},$$

where m is a natural number $\leq n$.

69. $$\int \frac{\sin^{2m+1} x\,dx}{\cos(2n+1)x} = \frac{(-1)^{n+1}}{2n+1} \left\{ \ln \cos x + \sum_{k=1}^{n} (-1)^k \cos^{2m+1} \frac{k\pi}{2n+1} \ln \left(\cos^2 x - \sin^2 \frac{k\pi}{2n+1} \right) \right\},$$

where $m-$ is a natural number $\leq n$.

70. $$\int \frac{\sin^m x\,dx}{\cos 2nx} = \frac{1}{2n} \sum_{k=0}^{2n-1} (-1)^{n+k} \cos^m \left[\frac{2k+1}{4n}\pi \right] \ln \frac{\sin \left[\frac{2k-2n+1}{8n}\pi + \frac{x}{2} \right]}{\sin \left[\frac{2k+2n+1}{8n}\pi - \frac{x}{2} \right]},$$

where m is a natural number $< 2n$.

71. $$\int \frac{\cos^{2m+1} x\,dx}{\sin(2n+1)x} = \frac{1}{2n+1} \left\{ \ln \sin x + \sum_{k=1}^{n} (-1)^k \cos^{2m+1} \frac{k\pi}{2n+1} \ln \left(\sin^2 x - \sin^2 \frac{k\pi}{2n+1} \right) \right\},$$

where m is a natural number $\leq n$.

72. $$\int \frac{\cos^{2m} x\,dx}{\sin(2n+1)x}$$
$$= \frac{1}{2n+1} \left\{ \ln \tan \frac{x}{2} + \sum_{k=1}^{n} (-1)^k \cos^{2m} \frac{k\pi}{2n+1} \ln \left[\tan \left(\frac{x}{2} + \frac{k\pi}{4n+2} \right) \tan \left(\frac{x}{2} - \frac{k\pi}{4n+2} \right) \right] \right\},$$

where m is a natural number $\leq n$.

73. $$\int \frac{\cos^{2m+1} x}{\sin 2nx}\,dx = \frac{1}{2n} \left\{ \ln \tan \frac{x}{2} + \sum_{k=1}^{n-1} (-1)^k \cos^{2m+1} \frac{k\pi}{2n} \ln \left[\tan \left(\frac{x}{2} + \frac{k\pi}{4} \right) \tan \left(\frac{x}{2} - \frac{k\pi}{4n} \right) \right] \right\} <$$

where m is a natural number $< n$.

74. $$\int \frac{\cos^{2m} x}{\sin 2nx}\,dx = \frac{1}{2n} \left\{ \ln \sin x + \sum_{k=1}^{n-1} (-1)^k \cos^{2m} \frac{k\pi}{2n} \ln \left(\sin^2 x - \sin^2 \frac{k\pi}{2n} \right) \right\},$$

where m is a natural number $\leq n$.

75. $\int \frac{\cos^m x}{\cos nx}\,dx = \frac{1}{n}\sum_{k=0}^{n-1}(-1)^k \cos^m \frac{2k+1}{2n}\pi \ln \frac{\sin\left[\frac{2k+1}{4n}\pi + \frac{x}{2}\right]}{\sin\left[\frac{2k+1}{4n}\pi - \frac{x}{2}\right]}$, m is a natural number $\leq n$.

76. $\int \sin x^2\,dx = \sqrt{\frac{\pi}{2}}\,S(x)$.

77. $\int \cos x^2\,dx = \sqrt{\frac{\pi}{2}}\,C(x)$.

78. $\int \sin(ax^2+2bx+c)\,dx = \sqrt{\frac{\pi}{2a}}\left\{\cos\frac{ac-b^2}{a}S\left(\frac{ax+b}{\sqrt{a}}\right) + \sin\frac{ac-b^2}{a}C\left(\frac{ax+b}{\sqrt{a}}\right)\right\}$.

79. $\int \cos(ax^2+2bx+c)\,dx = \sqrt{\frac{\pi}{2a}}\left\{\cos\frac{ac-b^2}{a}C\left(\frac{ax+b}{\sqrt{a}}\right) - \sin\frac{ac-b^2}{a}S\left(\frac{ax+b}{\sqrt{a}}\right)\right\}$.

80. $\int \sin(\ln ax)\,dx = \frac{x}{2}\left[\sin(\ln ax) - \cos(\ln ax)\right]$.

81. $\int \cos(\ln ax)\,dx = \frac{x}{2}\left[\sin(\ln ax) + \cos(\ln ax)\right]$.