! For an efficient use of these tables, first read `HowTo.pdf`.

T3.61A. Integrands involving arctan, arccot and rational functions on the interval $(0, \infty)$.

1. $$\int_0^\infty \operatorname{arccot}(px)\, \operatorname{arccot}(qx)\, dx = \frac{\pi}{2}\left\{\frac{1}{p}\ln\left(1+\frac{p}{q}\right) + \frac{1}{q}\ln\left(1+\frac{q}{p}\right)\right\}, \quad p>0,\ q>0.$$

2. $$\int_0^\infty \frac{\operatorname{arccot} x}{1 \pm x}\, dx = \pm\frac{\pi}{4}\ln 2 + \mathbf{G}.$$

3. $$\int_0^\infty \frac{\arctan x}{1-x^2}\, dx = -\mathbf{G}.$$

4. $$\int_0^\infty \frac{x \arctan x}{1+x^4}\, dx = \frac{\pi^2}{16}.$$

5. $$\int_0^\infty \frac{x \arctan x}{1-x^4}\, dx = -\frac{\pi}{8}\ln 2.$$

6. $$\int_0^\infty \frac{x \operatorname{arccot} x}{1-x^4}\, dx = \frac{\pi}{8}\ln 2.$$

7. $$\int_0^\infty \frac{\operatorname{arccot} x}{x\sqrt{1+x^2}}\, dx = \int_0^\infty \frac{\operatorname{arccot} x}{\sqrt{1+x^2}}\, dx = 2\mathbf{G}.$$

8. $$\int_0^\infty x^p \arctan x\, dx = \frac{\pi}{2(p+1)}\csc\frac{p\pi}{2}, \quad -1>p>-2.$$

9. $$\int_0^\infty x^p \operatorname{arccot} x\, dx = -\frac{\pi}{2(p+1)}\csc\frac{p\pi}{2}, \quad -1<p<0.$$

10. $$\int_0^\infty \left(\frac{x^p}{1+x^{2p}}\right)^{2q} \arctan x \frac{dx}{x} = \frac{\sqrt{\pi^3}}{2^{2q+2}p}\frac{\Gamma(q)}{\Gamma\left(q+\frac{1}{2}\right)}, \quad q>0.$$

11. $\displaystyle\int_0^\infty (1 - x\ \operatorname{arccot} x)\, dx = \frac{\pi}{4}.$

12. $\displaystyle\int_0^\infty (\arctan x)^2 \frac{dx}{x^2\sqrt{1+x^2}} = \int_0^\infty (\operatorname{arccot} x)^2 \frac{x\, dx}{\sqrt{1+x^2}} = -\frac{\pi^2}{4} + 4\mathbf{G}.$

13. $\displaystyle\int_0^\infty \frac{\arctan qx}{(p+x)^2}\, dx = -\frac{q}{1+p^2q^2}\left(\ln pq - \frac{\pi}{2}pq\right), \quad p>0,\ q>0.$

14. $\displaystyle\int_0^\infty \frac{\operatorname{arccot} qx}{(p+x)^2}\, dx = \frac{q}{1+p^2q^2}\left(\ln pq + \frac{\pi}{2pq}\right), \quad p>0,\ q>0.$

15. $\displaystyle\int_0^\infty \frac{x\ \operatorname{arccot} px}{q^2+x^2}\, dx = \frac{\pi}{2}\ln\frac{1+pq}{pq}, \quad p>0,\ q>0.$

16. $\displaystyle\int_0^\infty \frac{x\ \operatorname{arccot} px\, dx}{x^2-q^2} = \frac{\pi}{4}\ln\frac{1+p^2q^2}{p^2q^2}, \quad p>0,\ q>0.$

17. $\displaystyle\int_0^\infty \frac{\arctan px}{x(1+x^2)}\, dx = \frac{\pi}{2}\ln(1+p), \quad p \ge 0.$

18. $\displaystyle\int_0^\infty \frac{\arctan px}{x(1-x^2)}\, dx = \frac{\pi}{4}\ln(1+p^2), \quad p \ge 0.$

19. $\displaystyle\int_0^\infty \arctan qx \frac{dx}{x(p^2+x^2)} = \frac{\pi}{2p^2}\ln(1+pq), \quad p>0,\ q\ge 0.$

20. $\displaystyle\int_0^\infty \arctan qx \frac{dx}{x(1-p^2x^2)} = \frac{\pi}{4}\ln\frac{p^2+q^2}{p^2}, \quad p\ge 0.$

21. $\displaystyle\int_0^\infty \frac{x\arctan qx}{(p^2+x^2)^2}\, dx = \frac{\pi q}{4p(1+pq)}, \quad p>0,\ q\ge 0.$

22. $\displaystyle\int_0^\infty \frac{x\ \operatorname{arccot} qx}{(p^2+x^2)^2}\, dx = \frac{\pi}{4p^2(1+pq)}, \quad p>0,\ q\ge 0.$

23. $\displaystyle\int_0^\infty \arctan qx\ \arcsin x \frac{dx}{x^2} = \frac{1}{2}q\pi\ln\frac{1+\sqrt{1+q^2}}{\sqrt{1+q^2}} + \frac{\pi}{2}\ln\left(q+\sqrt{1+q^2}\right) - \frac{\pi}{2} - \arctan q.$

24. $\displaystyle\int_0^\infty \frac{\arctan px - \arctan qx}{x}\, dx = \frac{\pi}{2}\ln\frac{p}{q}, \quad p>0,\ q>0.$

25. $$\int_0^\infty \frac{\arctan px \arctan qx}{x^2}\, dx = \frac{\pi}{2}\, \ln\, \frac{(p+q)^{p+q}}{p^p q^q}, \quad p>0,\, q>0.$$

26. $$\int_0^\infty \arctan x^2 \frac{dx}{1+x^2} = \int_0^\infty \arctan x^3 \frac{dx}{1+x^2} = \int_0^\infty \operatorname{arccot} x^3 \frac{dx}{1+x^2} = \frac{\pi^2}{8}.$$

27. $$\int_0^\infty \frac{1-x^2}{x^2}\, \arctan x^2\, dx = \frac{\pi}{2}(\sqrt{2}-1).$$

28. $$\int_0^\infty x^{s-1} \arctan(ae^{-x})\, dx = 2^{-s-1}\Gamma(s)\, \Phi\left(-a^2,\, s+1,\, \tfrac{1}{2}\right).$$

29. $$\int_0^\infty \arctan\left(\frac{p\, \sin qx}{1+p\, \cos qx}\right) \frac{x\, dx}{1+x^2} = \frac{\pi}{2}\, \ln\, \left(1+p\, e^{-q}\right), \quad p>-e^q.$$