! For an efficient use of these tables, first read `HowTo.pdf`.

T2.53D. Integrands involving rational functions of $(a+b\,x)$ and trigonometric functions on the interval $(0,2\pi)$.

1. $$\int_0^{2\pi} \frac{x\,\sin x\,dx}{1-2a\,\cos x+a^2} = \begin{cases} \dfrac{2\pi}{a}\,\ln(1-a), & a^2<1, \\[2mm] \dfrac{2\pi}{a}\,\ln\left(1-\dfrac{1}{a}\right), & a^2>1, \end{cases} \qquad a\neq 0.$$

2. $$\int_0^{2\pi} \frac{x\,\sin nx\,dx}{1-2a\,\cos x+a^2} = \frac{2\pi}{1-a^2}\left[(a^{-n}-a^n)\,\ln(1-a)+\sum_{k=1}^{n-1}\frac{a^{-k}-a^k}{n-k}\right], \quad a^2<1,\ a\neq 0.$$

3. $$\int_0^{2\pi} \frac{\sin nx - a\,\sin[(n+1)x]}{1-2a\,\cos x+a^2}x\,dx = -2\pi a^n\left[\ln(1-a)+\sum_{k=1}^{n}\frac{1}{ka^k}\right], \quad |a|<1.$$

4. $$\int_0^{2\pi} \frac{\cos nx - a\,\cos[(n+1)x]}{1-2a\,\cos x+a^2}x\,dx = 2\pi a^n, \quad a^2<1.$$

5. $$\int_0^{2\pi} \frac{x\,\sin nx}{1\pm a\,\cos x}\,dx = \frac{2\pi}{\sqrt{1-a^2}}\left[(\mp 1)^n\frac{(1+\sqrt{1-a^2})^n-(1-\sqrt{1-a^2})^n}{a^n}\right.$$
$$\left.\times\ \ln\frac{2\sqrt{1\pm a}}{\sqrt{1+a}+\sqrt{1-a}}+\sum_{k=0}^{n-1}\frac{(\mp 1)^k}{n-k}\,\frac{(1+\sqrt{1-a^2})^k-(1-\sqrt{1-a^2})^k}{a^k}\right], \quad a^2<1.$$

6. $$\int_0^{2\pi} \frac{x\,\cos nx}{1\pm a\,\cos x}\,dx = \frac{2\pi^2}{\sqrt{1-a^2}}\left(\frac{1-\sqrt{1-a^2}}{\mp a}\right)^n, \quad a^2<1.$$

7. $$\int_0^{2\pi} \frac{x\,\sin x\,dx}{a+b\,\cos x} = \frac{2\pi}{b}\,\ln\frac{a+\sqrt{a^2-b^2}}{2(a+b)}, \quad a>|b|>0.$$