! For an efficient use of these tables, first read `HowTo.pdf`.

T3.07A. Integrands of the form $\sqrt{\dfrac{\pm(a-x)}{(b-x)^n(c-x)}}$, $\sqrt{\dfrac{\pm(b-x)}{(a-x)^n(c-x)}}$ and $\sqrt{\dfrac{\pm(c-x)}{(a-x)^n(b-x)}}$ for $n=1,3$, on the intervals $(-\infty,y)$ and (y,∞).

Notation used: $\alpha=\arcsin\sqrt{\dfrac{a-c}{a-y}}$, $\nu=\arcsin\sqrt{\dfrac{a-c}{y-c}}$,

$$p=\sqrt{\frac{a-b}{a-c}},\quad q=\sqrt{\frac{b-c}{a-c}}.$$

1. $$\int_{-\infty}^{y}\sqrt{\frac{a-x}{(b-x)(c-x)^3}}\,dx=\frac{2}{\sqrt{a-c}}\,F(\alpha,\,p)-\frac{2\sqrt{a-c}}{b-c}E(\alpha,\,p)+\frac{2(a-c)}{b-c}\sqrt{\frac{b-y}{(a-y)(c-y)}},\quad a>b>c>y.$$

2. $$\int_{y}^{\infty}\sqrt{\frac{x-a}{(x-b)(x-c)^3}}\,dx=\frac{2\sqrt{a-c}}{b-c}\,E(\nu,\,q)-\frac{2(a-b)}{(b-c)\sqrt{a-c}}F(\nu,\,q),\quad y\ge a>b>c.$$

3. $$\int_{-\infty}^{y}\sqrt{\frac{a-x}{(b-x)^3(c-x)}}\,dx=\frac{2\sqrt{a-c}}{b-c}E(\alpha,\,p)-2\frac{a-b}{b-c}\sqrt{\frac{c-y}{(a-y)(b-y)}},\quad a>b>c\ge y.$$

4. $$\int_{y}^{\infty}\sqrt{\frac{x-a}{(x-b)^3(x-c)}}\,dx=\frac{2\sqrt{a-c}}{b-c}\,[F(\nu,\,q)-E(\nu,\,q)]+2\sqrt{\frac{y-a}{(y-b)(y-c)}},\quad y\ge a>b>c.$$

5. $$\int_{-\infty}^{y}\sqrt{\frac{b-x}{(a-x)^3(c-x)}}\,dx=\frac{2}{\sqrt{a-c}}E(\alpha,\,p),\quad a>b>c\ge y.$$

6. $$\int_{y}^{\infty}\sqrt{\frac{x-b}{(x-a)^3(x-c)}}\,dx=\frac{2}{\sqrt{a-c}}\,[F(\nu,\,q)-E(\nu,\,q)]+2\sqrt{\frac{y-b}{(y-a)(y-c)}},\quad y>a>b>c.$$

7. $$\int_{-\infty}^{y} \sqrt{\frac{b-x}{(a-x)(c-x)^3}}\,dx = \frac{2}{\sqrt{a-c}}\,[F(\alpha,\,p)-E(\alpha,\,p)]+2\sqrt{\frac{b-y}{(a-y)(c-y)}}, \quad a>b>c>y.$$

8. $$\int_{y}^{\infty} \sqrt{\frac{x-b}{(x-a)(x-c)^3}}\,dx = \frac{2}{\sqrt{a-c}}\,E(\nu,\,q), \quad y \geq a>b>c.$$

9. $$\int_{-\infty}^{y} \sqrt{\frac{c-x}{(a-x)^3(b-x)}}\,dx = \frac{2\sqrt{a-c}}{a-b}\,E(\alpha,\,p)-\frac{2(b-c)}{(a-b)\sqrt{a-c}}F(\alpha,\,p), \quad a>b>c \geq y.$$

10. $$\int_{y}^{\infty} \sqrt{\frac{x-c}{(x-a)^3(x-b)}}\,dx = \frac{2}{\sqrt{a-c}}\,F(\nu,\,q)-\frac{2\sqrt{a-c}}{a-b}E(\nu,\,q) + \frac{2(a-c)}{a-b}\sqrt{\frac{y-b}{(y-a)(y-c)}}, \quad y>a>b>c.$$

11. $$\int_{-\infty}^{y} \sqrt{\frac{c-x}{(a-x)(b-x)^3}}\,dx = \frac{2\sqrt{a-c}}{a-b}\,[F(\alpha,\,p)-E(\alpha,\,p)] + 2\sqrt{\frac{c-y}{(a-y)(b-y)}}, \quad a>b>c \geq y.$$

12. $$\int_{y}^{\infty} \sqrt{\frac{x-c}{(x-a)(x-b)^3}}\,dx = \frac{2\sqrt{a-c}}{a-b}\,E(\nu,\,q)-2\frac{b-c}{a-b}\sqrt{\frac{y-a}{(y-b)(y-c)}}, \quad y \geq a>b>c.$$