! For an efficient use of these tables, first read `HowTo.pdf`.

T3.44A. Integrands involving product and quotient of trigonometric and hyperbolic functions on the interval $(0, \infty)$.

1. $$\int_0^\infty \frac{\sin ax}{\sinh \beta x}\, dx = \frac{\pi}{2\beta}\, \tanh \frac{a\pi}{2\beta}, \quad \Re\{\beta\} > 0,\ a > 0.$$

2. $$\int_0^\infty \frac{\sin ax}{\cosh \beta x}\, dx = -\frac{\pi}{2\beta}\, \tanh \frac{a\pi}{2\beta} - \frac{i}{2\beta}\left[\psi\left(\frac{\beta + i\,a}{4\beta}\right) - \psi\left(\frac{\beta - i\,a}{4\beta}\right)\right], \quad \Re\{\beta\} > 0,\ a > 0.$$

3. $$\int_0^\infty \frac{\cos ax}{\cosh \beta x}\, dx = \frac{\pi}{2\beta}\, \operatorname{sech} \frac{a\pi}{2\beta}, \quad \Re\{\beta\} > 0, \text{ all real } a.$$

4. $$\int_0^\infty \sin ax \frac{\sinh \beta x}{\sinh \gamma x}\, dx = \frac{\pi}{2\gamma}\, \frac{\sinh \frac{a\pi}{\gamma}}{\cosh \frac{a\pi}{\gamma} + \cos \frac{\beta\pi}{\gamma}} + \frac{i}{2\gamma}\left[\psi\left(\frac{\beta + \gamma + i\,a}{2\gamma}\right) - \psi\left(\frac{\beta + \gamma - i\,a}{2\gamma}\right)\right], \quad |\Re\{\beta\}| < \Re\{\gamma\},\ a > 0.$$

5. $$\int_0^\infty \cos ax \frac{\sinh \beta x}{\sinh \gamma x}\, dx = \frac{\pi}{2\gamma}\, \frac{\sin \frac{\pi\beta}{\gamma}}{\cosh \frac{a\pi}{\gamma} + \cos \frac{\beta\pi}{\gamma}}, \quad |\Re\{\beta\}| < \Re\{\gamma\}.$$

6. $$\int_0^\infty \sin ax \frac{\sinh \beta x}{\cosh \gamma x}\, dx = \frac{\pi}{\gamma}\, \frac{\sin \frac{\beta\pi}{2\gamma}\, \sinh \frac{a\pi}{2\gamma}}{\cosh \frac{a\pi}{\gamma} + \cos \frac{\beta\pi}{\gamma}}, \quad |\Re\{\beta\}| < \Re\{\gamma\};\ a > 0.$$

7. $$\int_0^\infty \cos ax \frac{\sinh \beta x}{\cosh \gamma x}\, dx = \frac{1}{4\gamma}\left\{\psi\left(\frac{3\gamma - \beta + ia}{4\gamma}\right) + \psi\left(\frac{3\gamma - \beta - ia}{4\gamma}\right) - \psi\left(\frac{3\gamma + \beta - ia}{4\gamma}\right) - \psi\left(\frac{3\gamma + \beta + ia}{4\gamma}\right) + \frac{2\pi\, \sin \frac{\pi\beta}{\gamma}}{\cos \frac{\pi\beta}{\gamma} + \cosh \frac{\pi a}{\gamma}}\right\}, \quad |\Re\{\beta\}| < \Re\{\gamma\},\ a > 0.$$

8. $$\int_0^\infty \sin ax \frac{\cosh \beta x}{\sinh \gamma x}\, dx = \frac{\pi}{2\gamma}\, \frac{\sinh \frac{\pi a}{\gamma}}{\cosh \frac{\pi a}{\gamma} + \cos \frac{\pi\beta}{\gamma}}, \quad |\Re\{\beta\}| < \Re\{\gamma\},\ a > 0.$$

9. $$\int_0^\infty \sin ax \frac{\cosh \beta x}{\cosh \gamma x}\, dx = \frac{i}{4\gamma}\left[\psi\left(\frac{3\gamma+\beta+i\,a}{4\gamma}\right) - \psi\left(\frac{3\gamma+\beta-i\,a}{4\gamma}\right) + \psi\left(\frac{3\gamma-\beta+i\,a}{4\gamma}\right)\right.$$
$$\left. -\psi\left(\frac{3\gamma-\beta-i\,a}{4\gamma}\right) - \frac{2\pi i \sinh \frac{\pi a}{\gamma}}{\cosh \frac{a\pi}{\gamma} + \cos \frac{\beta\pi}{\gamma}}\right], \quad |\Re\{\beta\}| < \Re\{\gamma\},\ a > 0].$$

10. $$\int_0^\infty \cos ax \frac{\cosh \beta x}{\cosh \gamma x}\, dx = \frac{\pi}{\gamma} \frac{\cos \frac{\beta\pi}{2\gamma} \cosh \frac{a\pi}{2\gamma}}{\cosh \frac{a\pi}{\gamma} + \cos \frac{\beta\pi}{\gamma}}, \quad |\Re\{\beta\}| < \Re\{\gamma\}, \text{ all real } a.$$

11. $$\int_0^\infty \frac{\cos ax}{\cosh^2 \beta x}\, dx = \frac{a\pi}{2\beta^2 \sinh \frac{a\pi}{2\beta}}, \quad \Re\{\beta\} > 0,\ a > 0.$$

12. $$\int_0^\infty \sin ax \frac{\sinh \beta x}{\cosh^2 \gamma x}\, dx = \frac{\pi\left(a \sin \frac{\beta\pi}{2\gamma} \cosh \frac{a\pi}{2\gamma} - \beta \cos \frac{\beta\pi}{2\gamma} \sinh \frac{a\pi}{2\gamma}\right)}{\gamma^2\left(\cosh \frac{a\pi}{\gamma} - \cos \frac{\beta\pi}{\gamma}\right)},$$
$$|\Re\{\beta\}| < 2\,\Re\{\gamma\},\ a > 0.$$

13. $$\int_0^\infty \frac{\sin^2 x \cos ax}{\sinh^2 x} dx = \frac{\pi}{4}\left\{\frac{a+2}{1-e^{-\pi(a+2)}} - \frac{2a}{1-e^{-\pi a}} + \frac{a-2}{1-e^{-\pi(a-2)}}\right\} = f(a),$$
$$\text{where } f(0) = \frac{1}{2}(\pi \coth \pi - 1),\ f(\pm 2) = \frac{1}{4} + \frac{\pi}{2}(\coth 2\pi - \coth \pi).$$

14. $$\int_0^\infty \frac{\cos ax\, dx}{b \cosh \beta x + c} = \begin{cases} \dfrac{\pi \sin\left(\frac{a}{\beta} \operatorname{arccosh} \frac{c}{b}\right)}{\beta\sqrt{c^2-b^2} \sinh \frac{a\pi}{\beta}}, & c > b > 0, \\ \dfrac{\pi \sinh\left(\frac{a}{\beta} \arccos \frac{c}{b}\right)}{\beta\sqrt{b^2-c^2} \sinh \frac{a\pi}{\beta}}, & b > |c| > 0, \end{cases} \qquad \Re\{\beta\} > 0,\ a > 0.$$

15. $$\int_0^\infty \frac{\cos ax\, dx}{\cosh \beta x + \cos \gamma} = \frac{\pi}{\beta} \frac{\sinh \frac{a\gamma}{\beta}}{\sin \gamma \sinh \frac{a\pi}{\beta}}, \quad \pi\,\Re\{\beta\} < \Im\{\beta^* \gamma\},\ a > 0.$$

16. $$\int_0^\infty \frac{\cos ax\, dx}{\cosh x - \cosh b} = -\pi \coth a\pi \frac{\sin ab}{\sinh b}, \quad a > 0,\ b > 0.$$

17. $$\int_0^\infty \frac{\cos ax\,dx}{1+2\,\cosh\left(\sqrt{\frac{2\pi}{3}}\,x\right)} = \frac{\sqrt{\pi/2}}{1+2\,\cosh\left(\sqrt{\frac{2\pi}{3}}\,a\right)}, \quad a>0.$$

18. $$\int_0^\infty \frac{\sin ax\,\sinh\beta x}{\cosh\gamma x+\cos\delta}\,dx = \frac{\pi\left\{\sin\left[\frac{\beta}{\gamma}(\pi-\delta)\right]\,\sinh\left[\frac{a}{\gamma}(\pi+\delta)\right]-\sin\left[\frac{\beta}{\gamma}(\pi+\delta)\right]\,\sinh\left[\frac{a}{\gamma}(\pi-\delta)\right]\right\}}{\gamma\,\sin\delta\left(\cosh\frac{2\pi a}{\gamma}-\cos\frac{2\pi\beta}{\gamma}\right)},$$
$$\pi\,\Re\{\gamma\}>|\Re\{\gamma^*\,\delta\}|,\ |\Re\{\beta\}|<\Re\{\gamma\},\ a>0.$$

19. $$\int_0^\infty \frac{\cos ax\,\cosh\beta x}{\cosh\gamma x+\cos b}\,dx = \frac{\pi\left\{\cos\left[\frac{\beta}{\gamma}(\pi-b)\right]\,\cosh\left[\frac{a}{\gamma}(\pi+b)\right]-\cos\left[\frac{\beta}{\gamma}(\pi+b)\right]\,\cosh\left[\frac{a}{\gamma}(\pi-b)\right]\right\}}{\gamma\,\sin b\left(\cosh\frac{2\pi a}{\gamma}-\cos\frac{2\pi\beta}{\gamma}\right)},$$
$$|\Re\{\beta\}|<\Re\{\gamma\},\ 0<b<\pi,\ a<0.$$

20. $$\int_0^\infty \frac{\cos ax\,dx}{(\beta+\sqrt{\beta^2-1}\,\cosh x)^{\nu+1}} = \Gamma(\nu+1-ai)\,e^{a\pi}\frac{Q_\nu^{(i\,a)}(\beta)}{\Gamma(\nu+1)},$$
$$\Re\{\nu\}>-1,\ |\arg(\beta+1)|<\pi;\ a>0.$$

21. $$\int_0^\infty \frac{\sin ax\,\sinh\frac{x}{2}}{\cosh x+\cos\beta}\,dx = \frac{\sinh a\beta}{2\,\sin\frac{\beta}{2}\cosh a\pi}, \quad Re\,\beta<\pi,\ a>0.$$

22. $$\int_0^\infty \frac{\cos ax\,\cosh\frac{\beta}{2}x}{\cosh\beta x+\cosh\gamma}\,dx = \frac{\pi\,\cos\frac{a\gamma}{\beta}}{2\beta\,\cosh\frac{\gamma}{2}\,\cosh\frac{a\pi}{\beta}}, \quad \pi\,\Re\{\beta\}>|\Im\{(\overline{\beta}\gamma)\}|.$$

23. $$\int_0^\infty \frac{\sin ax\,\sinh\beta x}{\cosh 2\beta x+\cos 2ax}\,dx = \frac{a\pi}{4(a^2+\beta^2)}, \quad a>0;\ \Re\{\beta\}>0.$$

24. $$\int_0^\infty \frac{\cos ax\,\cosh\beta x}{\cosh 2\beta x+\cos 2ax}\,dx = \frac{\beta\pi}{4(a^2+\beta^2)}, \quad \Re\{\beta\}>0,\ a>0.$$

25. $$\int_0^\infty \frac{\sinh^{2\mu-1}x\,\cosh^{2\varrho-2\nu+1}x}{(\cosh^2 x-\beta\,\sinh^2 x)^\varrho}\,dx = \frac{1}{2}\mathrm{B}(\mu,\,\nu-\mu)\,{}_2F_1(\varrho,\,\mu;\,\nu;\,\beta), \quad \Re\{\nu\}>\Re\{\mu\}>0.$$

26. $$\int_0^\infty \frac{\cos ax\,dx}{\cosh^\nu\,\beta x} = \frac{2^{\nu-2}}{\beta\Gamma(\nu)}\Gamma\left(\frac{\nu}{2}+\frac{ai}{2\beta}\right)\Gamma\left(\frac{\nu}{2}-\frac{ai}{2\beta}\right), \quad \Re\{\beta\}>0,\ \Re\{\nu\}>0,\ a>0.$$

27. $$\int_0^\infty \frac{\cos ax\,dx}{\cosh^{2n}\,\beta x} = \frac{4^{n-1}\pi a}{2(2n-1)!\beta^2\,\sinh\frac{a\pi}{2\beta}}\prod_{k=1}^{n-1}\left(\frac{a^2}{4\beta^2}+k^2\right)$$
$$= \frac{\pi a(a^2+2^2\beta^2)(a^2+4^2\beta^2)\dots\left[a^2+(2n-2)^2\beta^2\right]}{2(2n-1)!\beta^{2n}\,\sinh\frac{a\pi}{2\beta}}, \quad n\ge 2,\ a>0.$$

28. $$\int_0^\infty \frac{\cos ax\,dx}{\cosh^{2n+1}\,\beta x} = \frac{\pi 2^{2n-1}}{(2n)!\beta\,\cosh\frac{a\pi}{2\beta}}\prod_{k=1}^{n}\left[\frac{a^2}{4\beta^2}+\left(\frac{2k-1}{2}\right)^2\right]$$
$$= \frac{\pi(a^2+\beta^2)(a^2+3^2\beta^2)\dots\left[a^2+(2n-1)^2\beta^2\right]}{2(2n)!\beta^{2n+1}\,\cosh\frac{a\pi}{2\beta}}, \quad \Re\{\beta\}>0,\ n=0,1,\dots,\ \text{ all real } a.$$

29. $$\int_0^\infty \frac{\sin\beta x\,\sin\gamma x}{\cosh\delta x}\,dx = \frac{\pi}{\delta}\cdot\frac{\sinh\frac{\beta\pi}{2\delta}\,\sinh\frac{\gamma\pi}{2\delta}}{\cosh\frac{\beta}{\delta}\pi+\cosh\frac{\gamma}{\delta}\pi}, \quad |\Im\{(\beta+\gamma)\}|<\Re\{\delta\}.$$

30. $$\int_0^\infty \frac{\sin\alpha x\,\cos\beta x}{\sinh\gamma x}\,dx = \frac{\pi\,\sinh\frac{\pi\alpha}{\gamma}}{2\gamma\left(\cosh\frac{\alpha\pi}{\gamma}+\cosh\frac{\beta\pi}{\gamma}\right)}, \quad |\Im\{(\alpha+\beta)\}|<\Re\{\gamma\}.$$

31. $$\int_0^\infty \frac{\cos\beta x\,\cos\gamma x}{\cosh\delta x}\,dx = \frac{\pi}{\delta}\cdot\frac{\cosh\frac{\beta\pi}{2\delta}\,\cosh\frac{\gamma\pi}{2\delta}}{\cosh\frac{\beta\pi}{\delta}+\cosh\frac{\gamma\pi}{\delta}}, \quad |\Im\{(\beta+\gamma)\}|<\Re\{\delta\}.$$

32. $$\int_0^\infty \frac{\sin^2\beta x}{\sinh^2\pi x}\,dx = \frac{\beta}{\pi(e^{2\beta}-1)}+\frac{\beta-1}{2\pi} = \frac{\beta\,\coth\,\beta-1}{2\pi}, \quad |\Im\{\beta\}|<\pi.$$

33. $$\int_0^\infty \sin ax(1-\tanh\beta x)\,dx = \frac{1}{a}-\frac{\pi}{2\beta\,\sinh\frac{a\pi}{2\beta}}, \quad \Re\{\beta\}>0.$$

34. $$\int_0^\infty \sin ax(\coth\,\beta x-1)\,dx = \frac{\pi}{2\beta}\,\coth\,\frac{a\pi}{2\beta}-\frac{1}{a}, \quad \Re\{\beta\}>0.$$

35. $$\int_0^\infty \frac{\cos ax\,dx}{\sqrt{\cosh x\cos b}} = \frac{\pi P_{-1/2+i\,a}(\cos b)}{\sqrt{2}\,\cosh a\pi}, \quad a>0,\ b>0.$$

36. $$\int_0^\infty \frac{\sin\frac{a^2x^2}{\pi}\,\sin bx}{\sinh ax}\,dx = \frac{\pi}{2a}\,\sin\frac{\pi b^2}{4a^2}\,\operatorname{csch}\,\frac{\pi b}{2a}, \quad a>0,\ b>0.$$

37. $$\int_0^\infty \frac{\cos\frac{a^2x^2}{\pi}\,\sin bx}{\sinh ax}\,dx = \frac{\pi}{2a}\frac{\cosh\frac{\pi b}{a}-\cos\frac{\pi b^2}{4a^2}}{\sinh\frac{\pi b}{2a}}, \quad a>0,\ b>0.$$

38. $$\int_0^\infty \frac{\sin\frac{x^2}{\pi}\,\cos ax}{\cosh x}\,dx = \frac{\pi}{2}\frac{\cos\frac{a^2\pi}{4}-\frac{1}{\sqrt{2}}}{\cosh\frac{a\pi}{2}}.$$

39. $$\int_0^\infty \frac{\cos\frac{x^2}{\pi}\,\cos ax}{\cosh x}\,dx = \frac{\pi}{2}\cdot\frac{\sin\frac{a^2\pi}{4}+\frac{1}{\sqrt{2}}}{\cosh\frac{a\pi}{2}}.$$

40. $$\int_0^\infty \frac{\sin(\pi a x^2)\,\cos bx}{\cosh \pi x}\,dx = -\sum_{k=0}^\infty \exp\left[-\left(k+\frac{1}{2}\right)b\right]\,\sin\left[\left(k+\frac{1}{2}\right)^2\pi a\right]$$
$$+\frac{1}{\sqrt{a}}\sum_{k=0}^\infty \exp\left[-\frac{b\left(k+\frac{1}{2}\right)}{a}\right]\,\sin\left[\frac{\pi}{4}-\frac{b^2}{4\pi a}+\frac{\left(k+\frac{1}{2}\right)^2\pi}{a}\right],\quad a>0,\ b>0.$$

41. $$\int_0^\infty \frac{\cos(\pi a x^2)\,\cos bx}{\cosh \pi x}\,dx = \sum_{k=0}^\infty (-1)^k\,\exp\left[-\left(k+\frac{1}{2}\right)b\right]\,\cos\left[\left(k+\frac{1}{2}\right)^2\pi a\right]$$
$$+\frac{1}{\sqrt{a}}\sum_{k=0}^\infty \exp\left[-\frac{b\left(k+\frac{1}{2}\right)}{a}\right]\,\cos\left[\frac{\pi}{4}-\frac{b^2}{4\pi a}+\frac{\left(k+\frac{1}{2}\right)^2\pi}{a}\right],\quad a>0,\ b>0.$$

42. $$\int_0^\infty \sin\pi x^2\,\sin ax\,\coth\,\pi x\,dx = \frac{1}{2}\,\tanh\frac{a}{2}\,\sin\left(\frac{\pi}{4}+\frac{a^2}{4\pi}\right).$$

43. $$\int_0^\infty \cos\pi x^2\,\sin ax\,\coth\,\pi x\,dx = \frac{1}{2}\,\tanh\frac{a}{2}\left[1\cos\left(\frac{\pi}{4}+\frac{a^2}{4\pi}\right)\right].$$

44. $$\int_0^\infty \frac{\sin\pi x^2\,\cos ax}{1+2\,\cosh\left(\frac{2}{\sqrt{3}}\pi x\right)}\,dx = -\sqrt{3}+\frac{\cos\left(\frac{\pi}{12}-\frac{a^2}{4\pi}\right)}{4\,\cosh\left(a/\sqrt{3}\right)-2}.$$

45. $$\int_0^\infty \frac{\cos\pi x^2\,\cos ax}{1+2\,\cosh\left(\frac{2}{\sqrt{3}}\pi x\right)}\,dx = 1-\frac{\sin\left(\frac{\pi}{12}-\frac{a^2}{4\pi}\right)}{4\,\cosh\left(a/\sqrt{3}\right)-2}.$$

46. $$\int_0^\infty \frac{\sin x^2+\cos x^2}{\cosh(\sqrt{\pi}x)}\,\cos ax\,dx = \frac{\sqrt{\pi}}{2}\cdot\frac{\sin a^2+\cos a^2}{\cosh(\sqrt{\pi}a)}.$$

47. $$\int_0^\infty \frac{\sin(2a\,\cosh x)\,\cos bx}{\sqrt{\cosh x}}\,dx = -\frac{\pi}{4}\sqrt{a\pi}\left[J_{1/4+i\,b/2}(a)Y_{1/4-i\,b/2}(a)+J_{1/4-i\,b/2}(a)Y_{1/4+i\,b/2}(a)\right],$$
$$a>0,\ b>0.$$

48. $$\int_0^\infty \frac{\cos(2a\,\cosh x)\,\cos bx}{\sqrt{\cosh x}}\,dx = -\frac{\pi}{4}\sqrt{a\pi}\left[J_{-1/4+i\,b/2}(a)Y_{-1/4-i\,b/2}(a) + J_{-1/4-i\,b/2}(a)Y_{-1/4+i\,b/2}(a)\right], \quad a>0,\ b>0.$$

49. $$\int_0^\infty \frac{\sin(2a\,\sinh x)\,\sin bx}{\sqrt{\sinh x}}\,dx = -\frac{i}{2}\sqrt{\pi a}\left[I_{1/4-i\,b/2}(a)K_{-1/4+i\,b/2}(a) - I_{1/4+i\,b/2}(a)K_{1/4-i\,b/2}(a)\right], \quad a>0,\ b>0.$$

50. $$\int_0^\infty \frac{\cos(2a\,\sinh x)\,\sin bx}{\sqrt{\sinh x}}\,dx = -\frac{i}{2}\sqrt{\pi a}\left[I_{-1/4-i\,b/2}(a)K_{-1/4+i\,b/2}(a) - I_{-1/4+i\,b/2}(a)K_{-1/4-i\,b/2}(a)\right], \quad a>0,\ b>0.$$

51. $$\int_0^\infty \frac{\sin(2a\,\sinh x)\,\cos bx}{\sqrt{\sinh x}}\,dx = \frac{\sqrt{\pi a}}{2}\left[I_{1/4-i\,b/2}(a)K_{1/4+i\,b/2}(a) + I_{1/4+i\,b/2}(a)K_{1/4-i\,b/2}(a)\right], \quad a>0,\ b>0.$$

52. $$\int_0^\infty \frac{\cos(2a\,\sinh x)\,\cos bx}{\sqrt{\sinh x}}\,dx = \frac{\sqrt{\pi a}}{2}\left[I_{-1/4-i\,b/2}(a)K_{-1/4+i\,b/2}(a) + I_{-1/4+i\,b/2}(a)K_{-1/4-i\,b/2}(a)\right], \quad a>0,\ b>0.$$

53. $$\int_0^\infty \sin(a\,\cosh x)\,\sin(a\,\sinh x)\frac{dx}{\sinh x} = \frac{\pi}{2}\sin a, \quad a>0.$$

54. $$\int_0^\infty \sin(a\,\sinh x)\,\sinh\beta x\,dx = \sin\frac{\beta\pi}{2}K_\beta(a), \quad |\Re\{\beta\}|<1,\ a>0.$$

55. $$\int_0^\infty \cos(a\,\sinh x)\,\cosh\beta x\,dx = \cos\frac{\beta\pi}{2}K_\beta(a), \quad |\beta|<1,\ a>0.$$

56. $$\int_0^\infty \sin\left(a\,\cosh x - \frac{1}{2}\beta\pi\right)\cosh\beta x\,dx = \frac{\pi}{2}J_\beta(a), \quad |\Re\{\beta\}|<1,\ a>0.$$

57. $$\int_0^\infty \cos\left(a\,\cosh x - \frac{1}{2}\beta\pi\right)\cosh\beta x\,dx = -\frac{\pi}{2}Y_\beta(a), \quad |\Re\{\beta\}|<1;\ a>0.$$