! For an efficient use of these tables, first read `HowTo.pdf`.

T3.45A. Integrands involving trigonometric and hyperbolic functions and powers of $(a + b\,x)$ on the interval $(0, \infty)$.

1. $$\int_0^\infty \frac{\sin ax}{\sinh \beta x}\, x^{2m}\, dx = (-1)^m \frac{\pi}{2\beta} \frac{\partial^{2m}}{\partial a^{2m}} \left(\tanh \frac{a\pi}{2\beta}\right), \quad \Re\{\beta\} > 0.$$

2. $$\int_0^\infty \frac{\cos ax}{\sinh \beta x}\, x^{2m+1}\, dx = (-1)^m \frac{\pi}{2\beta} \frac{\partial^{2m+1}}{\partial a^{2m+1}} \left(\tanh \frac{a\pi}{2\beta}\right), \quad \Re\{\beta\} > 0.$$

3. $$\int_0^\infty \frac{\sin ax}{\cosh \beta x}\, x^{2m+1}\, dx = (-1)^{m+1} \frac{\pi}{2\beta} \frac{\partial^{2m+1}}{\partial a^{2m+1}} \left(\operatorname{sech} \frac{a\pi}{2\beta}\right), \quad \Re\{\beta\} > 0.$$

4. $$\int_0^\infty \frac{\cos ax}{\cosh \beta x}\, x^{2m}\, dx = (-1)^m \frac{\pi}{2\beta} \frac{\partial^{2m}}{\partial a^{2m}} \left(\operatorname{sech} \frac{a\pi}{2\beta}\right), \quad \Re\{\beta\} > 0.$$

5. $$\int_0^\infty x\, \frac{\sin 2ax}{\cosh \beta x}\, dx = \frac{\pi^2}{4\beta^2} \frac{\sinh(a\pi/\beta)}{\cosh^2(a\pi/\beta}, \quad \Re\{\beta\} > 0,\ a > 0.$$

6. $$\int_0^\infty x\, \frac{\cos 2ax}{\cosh \beta x}\, dx = \frac{\pi^2}{4\beta^2} \operatorname{sech}^2(a\pi/\beta), \quad \Re\{\beta\} > 0,\ a > 0.$$

7. $$\int_0^\infty \frac{\sin ax}{\cosh \beta x}\, dx = 2 \arctan\left(\exp \frac{a\pi}{\beta}\right) - \frac{\pi}{2}, \quad \Re\{\beta\} > 0,\ a > 0.$$

8. $$\int_0^\infty \left(x^2 + \beta^2\right) \frac{\cos ax}{\cosh \frac{\pi x}{2\beta}}\, dx = \frac{6\beta^3}{\cosh^3 a\beta}, \quad \Re\{\beta\} > 0,\ a > 0.$$

9. $$\int_0^\infty x\left(x^2 + 4\beta^2\right) \frac{\cos ax}{\sinh \frac{\pi x}{2\beta}}\, dx = \frac{6\beta^4}{\cosh^4 a\beta}, \quad \Re\{\beta\} > 0,\ a > 0.$$

10. $$\int_0^\infty \frac{\sin ax}{\sinh \pi x}\,\frac{dx}{x^2+\beta^2} = -\frac{1}{2\beta^2} - \frac{\pi\, e^{-a\beta}}{\beta\, \sin\pi\beta} + \frac{1}{2\beta^2}\,\left[{}_2F_1\left(1,\, -\beta;\, 1-\beta;\, -e^{-a}\right) + {}_2F_1\left(1,\, \beta;\, 1+\beta;\, -e^{-a}\right)\right] = \frac{1}{2\beta^2} - \frac{\pi\, e^{-a\beta^2}}{2\beta\, \sin\pi\beta} - \sum_{k=1}^{\infty} \frac{(-1)^k\, e^{-ak}}{k^2-\beta^2}, \quad \Re\{\beta\}>0,\ \beta\neq 0,1,2,\dots;\ a>0.$$

11. $$\int_0^\infty \frac{\sin ax}{\sinh \pi x}\,\frac{dx}{x^2+m^2} = \frac{(-1)^m\, a\, e^{-ma}}{2m} + \frac{1}{2m}\sum_{k=1}^{m-1} \frac{(-1)^k\, e^{-ak}}{m-k} + \frac{(-1)^m\, a\, e^{-ma}}{2m}\,\ln\left(1+e^{-a}\right) + \frac{1}{2\,m!}\,\frac{\partial^{m-1}}{\partial z^{m-1}}\left[\frac{(1+z)^{m-1}}{z}\,\ln(1+z)\right]_{z=e^{-a}}, \quad ; a>0.$$

12. $$\int_0^\infty \frac{\sin ax}{\sinh \pi x}\,\frac{dx}{1+x^2} = \frac{1}{2}\int_{-\infty}^\infty \frac{\sin ax}{\sinh \pi x}\,\frac{dx}{1+x^2} = -\frac{a}{2}\,\cosh a + \sinh a\,\ln\left(2\cosh\frac{a}{2}\right).$$

13. $$\int_0^\infty \frac{\sin ax}{\sinh \frac{\pi x}{2}}\,\frac{dx}{1+x^2} = \frac{1}{2}\int_{-\infty}^\infty \frac{\sin ax}{\sinh \frac{\pi x}{2}}\,\frac{dx}{1+x^2} = \frac{\pi}{2}\,\sinh a - \cosh a\,\arctan(\sinh a).$$

14. $$\int_0^\infty \frac{\sin ax}{\sinh \frac{\pi x}{4}}\,\frac{dx}{1+x^2} = -\frac{\pi}{\sqrt{2}}\,e^{-a} + \frac{\sinh a}{\sqrt{2}}\,\ln\frac{2\cosh 2+\sqrt{2}}{2\cosh a-\sqrt{2}} + \sqrt{2}\,\cosh a\,\arctan\left(\frac{\sqrt{2}}{2\sinh a}\right), \quad a>0.$$

15. $$\int_0^\infty \frac{\sin ax}{\cosh \frac{\pi x}{4}}\,\frac{dx}{1+x^2} = \frac{\pi}{\sqrt{2}}\,e^{-a} + \frac{\sinh a}{\sqrt{2}}\,\ln\frac{2\cosh 2+\sqrt{2}}{2\cosh a-\sqrt{2}} - \sqrt{2}\,\cosh a\,\arctan\left(\frac{\sqrt{2}}{2\sinh a}\right), \quad a>0.$$

16. $$\int_0^\infty \frac{\cos ax}{\sinh \pi x}\,\frac{x\,dx}{1+x^2} = -\frac{1}{2} + \frac{a}{2}\,e^{-a} + \cosh a\,\ln\left(1+e^{-a}\right), \quad a>0.$$

17. $$\int_0^\infty \frac{\cos ax}{\sinh \frac{\pi x}{2}}\,\frac{x\,dx}{1+x^2} = \frac{\pi}{2}\,e^{-a} - 1 + 2\sinh a\,\arctan\left(e^{-a}\right), \quad a>0.$$

18. $$\int_0^\infty \frac{\cos ax}{\cosh \pi x}\,\frac{x\,dx}{x^2+\beta^2} = \sum_{k=0}^{\infty}(-1)^k\,\frac{(k+\frac{1}{2})^2\,e^{-a\beta} - \beta\,e^{-(k+1/2)a}}{\beta\,\left[\left(k+\frac{1}{2}\right)^2-\beta^2\right]}, \quad \Re\{\beta\}>0,\ a>0.$$

19. $$\int_0^\infty \frac{\cos ax}{\cosh \pi x}\,\frac{dx}{(m+\frac{1}{2})^2+x^2} = \frac{(-1)^m\,e^{-(m+1/2)\,a}}{2m+1}\,\left[a+\ln\left(1+e^{-a}\right)\right] + \frac{e^{-a/2}}{2m+1}\sum_{k=0}^{m-1}\frac{(-1)^k\,e^{-ak}}{k-m} + \frac{e^{-a/2}}{(2m+1)(m+1)}\,{}_2F_1\left(1,\,m+1;\,m+2;\,-e^{-a}\right) \quad a>0.$$

20. $$\int_0^\infty \frac{\cos ax}{\cosh \pi x}\,\frac{dx}{1+x^2} = 2\,\cosh\frac{a}{2} - \left[e^{a}\,\arctan\left(e^{-a/2}\right) + e^{-a}\,\arctan\left(e^{a/2}\right)\right], \quad a>0.$$

21. $$\int_0^\infty \frac{\cos ax}{\cosh \frac{\pi x}{2}}\,\frac{dx}{1+x^2} = a\,e^{-a} + \cosh a\,\ln\left(1+e^{-2a}\right), \quad a>0.$$

22. $$\int_0^\infty \frac{\cos ax}{\cosh \frac{\pi x}{4}}\,\frac{dx}{1+x^2} = \frac{\pi}{\sqrt{2}}\,e^{-a} + \frac{2\sinh a}{\sqrt{2}}\,\arctan\left(\frac{1}{\sqrt{2}\,\sinh a}\right) - \frac{\cosh a}{\sqrt{2}}\,\ln\frac{2\cosh a+\sqrt{2}}{2\cosh a-\sqrt{2}}, \quad a>0.$$

23. $$\int_0^\infty \frac{\sin ax}{x}\,\frac{\sinh \beta x}{\sinh \gamma x}\,dx = \arctan\left(\tan\frac{\pi\beta}{2\gamma}\,\tanh\frac{\pi a}{2\gamma}\right), \quad |\Re\{\beta\}|<\Re\{\gamma\},\ a>0.$$

24. $$\int_0^\infty \frac{\cos ax}{x}\,\frac{\sinh \beta x}{\sinh \gamma x}\,dx = \frac{1}{2}\,\ln\frac{\cosh\frac{\pi a}{2\gamma}+\sin\frac{\pi\beta}{2\gamma}}{\cosh\frac{\pi a}{2\gamma}-\sin\frac{\pi\beta}{2\gamma}}, \quad |\Re\{\beta\}|<\Re\{\gamma\}.$$

25. $$\int_0^\infty \frac{x\,\sin ax}{x^2+b^2}\,\frac{\sinh \beta x}{\sinh \pi x}\,dx = \frac{\pi}{2}\,\frac{e^{-ab}\,\sin b\beta}{\sin b\pi} + \sum_{k=1}^{\infty}(-1)^k\,\frac{k\,e^{-ak}\,\sin k\beta}{k^2-b^2}, \quad 0<\Re\{\beta\}<\pi,\ a>0,\ b>0.$$

26. $$\int_0^\infty \frac{x\,\sin ax}{1+x^2}\,\frac{\sinh \beta x}{\sinh \pi x}\,dx = \frac{1}{2}\,e^{-a}\,(a\,\sin\beta-\beta\,\cos\beta) - \frac{1}{2}\,\sinh a\,\sin\beta\,\ln\left[1+2\,e^{-a}\,\cos\beta+e^{-2a}\right] + \cosh a\,\cos\beta\,\arctan\frac{\sin\beta}{e^{a}+\cos\beta}, \quad \Re\{\beta\}<\pi,\ a>0.$$

27. $$\int_0^\infty \frac{x\,\sin ax}{1+x^2}\,\frac{\sinh \beta x}{\sinh \frac{\pi x}{2}}\,dx = \frac{\pi}{2}\,e^{-a}\,\sin\beta + \frac{1}{2}\,\cos\beta\,\sinh a\,\ln\frac{\cosh a+\sin\beta}{\cosh a-\sin\beta} - \sin\beta\,\cosh a\,\arctan\left(\frac{\cos\beta}{\sinh a}\right), \quad \Re\{\beta\}<\frac{\pi}{2},\ a>0.$$

28. $$\int_0^\infty \frac{x\,\cos ax}{x^2+b^2}\,\frac{\sinh\beta x}{\sinh\pi x}\,dx = \frac{\pi}{2b}\,\frac{e^{-ab}\,\sin b\beta}{\sin b\pi} + \sum_{k=1}^\infty (-)^k\,\frac{e^{-ak}\,\sin k\beta}{k^2-b^2},$$
$$0<\Re\{\beta\}<\pi,\ a>0,\ b>0.$$

29. $$\int_0^\infty \frac{x\,\cos ax}{1+x^2}\,\frac{\sinh\beta x}{\sinh\pi x}\,dx = \frac{1}{2}\,e^{-a}\,(a\,\sin\beta-\beta\,\cos\beta)+\frac{1}{2}\,\cosh a\,\sin\beta\,\ln\left[1+2\,e^{-a}\,\cos\beta+e^{-2a}\right],$$
$$-\sinh a\,\cos\beta\,\arctan\frac{\sin\beta}{e^a+\cos\beta},\quad |\Re\{\beta\}|<\pi,\ a>0,\ b>0.$$

30. $$\int_0^\infty \frac{x\,\cos ax}{1+x^2}\,\frac{\sinh\beta x}{\sinh\frac{\pi x}{2}}\,dx = \frac{\pi}{2}\,e^{-a}\,\sin\beta-\frac{1}{2}\,\cosh a\,\cos\beta\,\ln\frac{\cosh a+\sin\beta}{\cosh a-\sin\beta}$$
$$+\sinh a\,\sin\beta\,\arctan\frac{\cos\beta}{\sinh a},\quad |\Re\{\beta\}|<\frac{\pi}{2},\ a>0,\ b>0.$$

31. $$\int_0^\infty \frac{\sin ax}{\frac{1}{4}+x^2}\,\frac{\sinh\beta x}{\cosh\pi x}\,dx = e^{-a/2}\left(a\sin\frac{\beta}{2}-\beta\,\cos\frac{\beta}{2}\right)-\sinh\frac{a}{2}\,\sin\frac{\beta}{2}\,\ln\left[1+2\,e^{-a}\,\cos\beta+e^{-2a}\right]$$
$$+\cosh\frac{a}{2}\,\cos\frac{\beta}{2}\,\arctan\frac{\sin\beta}{1+e^{-a}\,\cos\beta},\quad |\Re\{\beta\}|<\pi,\ a>0.$$

32. $$\int_0^\infty \frac{\sin ax}{x^2+\beta^2}\,\frac{\cosh\gamma x}{\sinh\pi x}\,dx = \frac{1}{2\beta^2}-\frac{\pi}{2\beta}\,\frac{e^{-a\beta}\,\cos\beta\gamma}{\sin\beta\pi}+\sum_{k=1}^\infty(-1)^k\,\frac{e^{-ak}\,\cos k\gamma}{k^2-\beta^2},$$
$$0<\Re\{\beta\},\ |\Re\{\gamma\}|<\pi,\ a>0.$$

33. $$\int_0^\infty \frac{\sin ax}{1+x^2}\,\frac{\cosh\beta x}{\sinh\pi x}\,dx = \frac{1}{2}\,e^{-a}\,(a\,\cos\beta+\beta\,\sin\beta)+\frac{1}{2}\,\sinh a\,\cos\beta\,\ln\left[1+2\,e^{-a}\,\cos\beta+e^{-2a}\right]$$
$$+\cosh a\,\sin\beta\,\arctan\frac{\sin\beta}{e^a+\cos\beta},\quad 0<\Re\{\beta\},\ |\Re\{\beta\}|<\pi,\ a>0.$$

34. $$\int_0^\infty \frac{\sin ax}{1+x^2}\,\frac{\cosh\beta x}{\sinh\frac{\pi x}{2}}\,dx = -\frac{\pi}{2}\,e^{-a}\,\cos\beta+\frac{1}{2}\,\sinh a\,\sin\beta\,\ln\frac{\cosh a+\sin\beta}{\cosh a-\sin\beta}$$
$$+\cosh a\,\cos\beta\,\arctan\frac{\cos\beta}{\sinh a},\quad |\Re\{\beta\}|<\frac{\pi}{2},\ a>0.$$

35. $$\int_0^\infty \frac{x\,\cos ax}{x^2+b^2}\,\frac{\cosh\beta x}{\sinh\pi x}\,dx = \frac{\pi}{2}\,\frac{e^{-ab}\,\cos b\beta}{\sin b\pi}+\sum_{k=1}^\infty(-1)^k\,\frac{k\,e^{-ak}\,\cos k\beta}{k^2-b^2},$$
$$|\Re\{\beta\}|<\pi,\ a>0.$$

36. $$\int_0^\infty \frac{x\,\cos ax}{1+x^2}\,\frac{\cosh\beta x}{\sinh \pi x}\,dx = \frac{1}{2}\,e^{-a}\,\left(a\,\cos\beta+\beta\,\sin\beta\right) - \frac{1}{2} + \frac{1}{2}\,\cosh a\,\cos\beta\,\ln\left[1+2\,e^{-a}\,\cos\beta+e^{-2a}\right]$$
$$+\sinh a\,\sin\beta\,\arctan\frac{\sin\beta}{e^a+\cos\beta},\quad |\Re\{\beta\}|<\pi,\ a>0.$$

37. $$\int_0^\infty \frac{x\,\cos ax}{1+x^2}\,\frac{\cosh\beta x}{\sinh\frac{\pi x}{2}}\,dx = -1+\frac{\pi}{2}\,e^{-a}\,\cos\beta+\frac{1}{2}\,\cosh a\,\sin\beta\,\ln\frac{\cosh a+\sin\beta}{\cosh a-\sin\beta}$$
$$+\sinh a\,\cos\beta\,\arctan\frac{\cos\beta}{\sinh a},\quad |\Re\{\beta\}|<\frac{\pi}{2},\ a>0.$$

38. $$\int_0^\infty \frac{\cos ax}{1+x^2}\,\frac{\cosh\beta x}{\cosh\frac{\pi x}{2}}\,dx = a\,e^{-a}\,\cos\beta+\beta\,e^{-a}\,\sin\beta+\sinh a\,\sin\beta\,\arctan\frac{e^{-2a}\,\sin 2\beta}{1+e^{-2a}\,\cos 2\beta}$$
$$+\frac{1}{2}\,\cosh a\,\cos\beta\,\ln\left[1+2e^{-2a}\,\cos 2\beta+e^{-4a}\right],\quad |\Re\{\beta\}|<\frac{\pi}{2},\ a>0.$$

39. $$\int_0^\infty x\,\cos 2ax\,\tanh x\,dx=\infty.$$

40. $$\int_0^\infty \cos ax\,\tanh bx\,\frac{dx}{x}=\ln\,\coth\frac{a\pi}{4b},\quad \Re\{b\}>0,\ a>0.$$

41. $$\int_0^\infty \frac{\sin ax}{1+x^2}\,\tanh\frac{\pi x}{2}\,dx = a\,\cosh a-\sinh a\,\ln\left(2\sinh a\right),\quad a>0$$

42. $$\int_0^\infty \frac{\sin ax}{1+x^2}\,\tanh\frac{\pi x}{4}\,dx = -\frac{\pi}{2}\,e^{a}+\sinh a\,\ln\,\coth\frac{a}{2}+2\cosh a\,\arctan\left(e^a\right).$$

43. $$\int_0^\infty \frac{\sin ax}{1+x^2}\,\coth\pi x\,dx = \frac{a}{2}\,e^{-a}-\sinh a\,\ln\left(1-e^{-a}\right),\quad a>0.$$

44. $$\int_0^\infty \frac{\sin ax}{1+x^2}\,\coth\frac{\pi x}{2}\,dx = \sinh a\,\ln\,\coth\frac{a}{2},\quad a>0.$$

45. $$\int_0^\infty \frac{x\,\cos ax}{1+x^2}\,\tanh\frac{\pi x}{2}\,dx = -a\,e^{-a}-\cosh a\,\ln\left(1-e^{-2a}\right),\quad a>0.$$

46. $$\int_0^\infty \frac{x\,\cos ax}{1+x^2}\,\tanh\frac{\pi x}{4}\,dx = -\frac{\pi}{2}\,e^{a}+\cosh a\,\ln\,\coth\frac{a}{2}+2\sinh a\,\arctan\left(e^a\right),\quad a>0.$$

47. $$\int_0^\infty \frac{x\,\cos ax}{1+x^2}\,\coth\pi x\,dx = -\frac{a}{2}\,e^{-a}-\frac{1}{2}-\cosh a\,\ln\left(1-e^{-a}\right).$$

48. $$\int_0^\infty \frac{x\ \cos ax}{1+x^2}\ \coth\frac{\pi x}{2}\,dx = -1+\cosh a\ \ln\ \coth\frac{a}{2}, \quad a>0.$$

49. $$\int_0^\infty \frac{x\ \cos ax}{1+x^2}\ \tanh\frac{\pi x}{4}\,dx = -2+\frac{\pi}{2}\,e^{-a}+\cosh a\ \ln\ \coth\frac{a}{2}+2\sinh a\ \arctan\left(e^{-a}\right), \quad a>0.$$

50. $$\int_0^\infty \frac{\sin ax}{\cosh^2 x}\,dx = \frac{\pi}{2}\,\frac{1}{\sinh\frac{\pi a}{2}}\ \left(\frac{\pi a}{2}\ \coth\frac{\pi a}{2}-1\right).$$

51. $$\int_0^\infty \frac{1-\cos ax}{\sinh bx}\,\frac{dx}{x} = \ln\left(\coth\frac{a\pi}{2b}\right).$$

52. $$\int_0^\infty \frac{\sin ax-\sin bx}{\cosh\beta x}\,\frac{dx}{x} = 2\arctan\frac{\exp\frac{b\pi}{2\beta}-\exp\frac{b\pi}{2\beta}}{1+\exp\frac{(a+b)\pi}{2\beta}}, \quad \Re\{\beta\}>0.$$

53. $$\int_0^\infty \frac{\cos ax-\cos bx}{\sinh\beta x}\,\frac{dx}{x} = \ln\frac{\cosh\frac{b\pi}{2\beta}}{\cosh\frac{a\pi}{2\beta}}, \quad \Re\{\beta\}>0.$$

54. $$\int_0^\infty \frac{\cos ax\ \sin bx}{\cosh cx}\,\frac{dx}{x} = \arctan\frac{\sinh\frac{b\pi}{2c}}{\cosh\frac{a\pi}{2c}}, \quad \Re\{c\}>|\Im\{a\}|+|\Im\{b\}|.$$

55. $$\int_0^\infty \sin^2 ax\,\frac{\cosh bx}{\sinh x}\,\frac{dx}{x} = \frac{1}{4}\ \ln\ \frac{\cosh 2a\pi+\cos b\pi}{1+\cos b\pi}, \quad |\Re\{b\}>|<1.$$

56. $$\int_0^\infty \frac{\sin x}{\cosh ax+\cos x}\,\frac{x\,dx}{x^2-\pi^2} = \arctan\frac{1}{a}-\frac{1}{a}.$$

57. $$\int_0^\infty \frac{\sin x}{\cosh ax-\cos x}\,\frac{x\,dx}{x^2-\pi^2} = \frac{a}{1+a^2}-\arctan\frac{1}{a}.$$

58. $$\int_0^\infty \frac{\sin 2x}{\cosh 2ax-\cos 2x}\,\frac{x\,dx}{x^2-\pi^2} = \frac{1}{2a}\,\frac{1+2a^2}{1+a^2}-\arctan\frac{1}{a}.$$

59. $$\int_0^\infty \frac{\cosh ax\ \sin x}{\cosh 2ax-\cos 2x}\,\frac{x\,dx}{x^2-\pi^2} = -\frac{1}{2a\,(1+a^2)}.$$

60. $$\int_0^\infty \frac{\cos x}{\cosh \pi x + \cos \pi b} \, \frac{x\,dx}{x^2+c^2} = \frac{\pi\, e^{-ac}}{2c\,(\cos b\pi + \cos c\pi)} + \frac{1}{\sinh b\pi} \sum_{k=0}^{\infty} \left\{ \frac{\exp\left[-(2k+1-b)\,a\right]}{c^2-(2k+1-b)^2} - \frac{\exp\left[-(2k+1+b)\,a\right]}{c^2-(2+1+b)^2} \right\},$$
$$0 < \Re\{b\} < 1,\ \Re\{c\} > 0,\ a > 0.$$

61. $$\int_0^\infty \frac{\sin ax\, \sinh bx}{\cos 2ax + \cosh 2bx}\, x^{p-1}\,dx = \frac{\Gamma(p)}{(a^2+b^2)^{p/2}}\, \sin\left(p \arctan \frac{a}{b}\right) \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^p}, \quad p > 0.$$

62. $$\int_0^\infty \sin ax^2\, \frac{\sin \frac{\pi x}{2}\, \sinh \frac{\pi x}{2}}{\cos \pi x + \cosh \pi x}\, x\,dx = \frac{1}{4} \left[\frac{\partial \vartheta_1\,(z\,|\,q)}{\partial z}\right]_{z=0,\, q=e^{-2a}}, \quad a > 0.$$

63. $$\int_0^\infty \sinh(a\,\sin x)\, \cos(a\,\cos x)\, \sin x\, \sin 2nx\, \frac{dx}{x} = \frac{(-1)^{n-1}\, a^{2n-1}\, \pi}{8\,(2n-1)!} \left[1 + \frac{a^2}{2n(2n+1)}\right].$$

64. $$\int_0^\infty \cosh(a\,\sin x)\, \cos(a\,\cos x)\, \sin x\, \cos(2n-1)x\, \frac{dx}{x} = \frac{(-1)^{n-1}\, a^{2(n-1)}\, \pi}{8\,[2(n-1)!]} \left[1 - \frac{a^2}{2n(2n+1)}\right].$$

65. $$\int_0^\infty \sinh(a\,\sin x)\, \cos(a\,\cos x)\, \cos x\, \cos 2nx\, \frac{dx}{x} = \frac{(-1)^{n-1}\, a^{2n-1}\, \pi}{8\,(2n-1)!} + \frac{3(-1)^n\, a^{2n+1}\, \pi}{8\,(2n+1)!} + \frac{\pi}{2} \sum_{k=n+1}^{\infty} \frac{(-1)^k\, a^{2k+1}}{(2k+1)!}.$$

66. $$\int_0^\infty \sin(a\,\cos bx)\, \sinh(a\,\sin bx)\, \frac{x\,dx}{c^2-x^2} = \frac{\pi}{2}\, \left[\cos(a\,\cos bc)\, \cosh(a\,\sin bc) - 1\right], \quad b > 0.$$

67. $$\int_0^\infty \sin(a\,\cos bx)\, \cosh(a\,\sin bx)\, \frac{x\,dx}{c^2-x^2} = \frac{\pi}{2c}\, \cos(a\,\cos bc)\, \sinh(a\,\sin bc), \quad b > 0,\ c > 0.$$

68. $$\int_0^\infty \cos(a\,\cos bx)\, \sinh(a\,\sin bx)\, \frac{x\,dx}{c^2-x^2} = \frac{\pi}{2}\, \left[a\,\cos bc - \sin(a\,\cos bc)\, \cosh(a\,\sin bc)\right],$$
$$b > 0.$$

69. $$\int_0^\infty \cos(a\,\cos bx)\, \cosh(a\,\sin bx)\, \frac{dx}{c^2-x^2} = -\frac{\pi}{2c}\, \sin(a\,\cos bc)\, \sinh(a\,\sin bc), \quad b > 0.$$