! For an efficient use of these tables, first read `HowTo.pdf`.

T3.40A. Integrands involving product of trigonometric functions of linear and quadratic arguments and sum of powers and square roots of $(a + b\,x^n)$ on the interval $(0, \infty)$.

1. $$\int_0^\infty \sin(ax^2)\,\cos(bx)\frac{dx}{x^2} = \frac{b\pi}{2}\left\{S\left(\frac{b}{2\sqrt{a}}\right) - C\left(\frac{b}{2\sqrt{a}}\right) + \sqrt{a\pi}\,\sin\left(\frac{b^2}{4a}+\frac{\pi}{4}\right)\right\},$$
$$a>0,\ b>0.$$

2. $$\int_0^\infty \frac{\sin(ax^2)}{x^2}\,dx = \sqrt{\frac{a\pi}{2}}, \quad a \ge 0.$$

3. $$\int_0^\infty \sin(ax^2)\,\cos(bx^2)\frac{dx}{x^2} = \begin{cases} \dfrac{1}{2}\sqrt{\dfrac{\pi}{2}}(\sqrt{a+b}+\sqrt{a-b}), & a>b>0, \\ \dfrac{1}{2}\sqrt{\pi a}, & b=a\ge 0, \\ \dfrac{1}{2}\sqrt{\dfrac{\pi}{2}}(\sqrt{a+b}-\sqrt{b-a}), & b>a>0. \end{cases}$$

4. $$\int_0^\infty \frac{\sin^2(a^2x^2)}{x^4}\,dx = \frac{2\sqrt{\pi}}{3}a^3, \quad a\ge 0.$$

5. $$\int_0^\infty \frac{\sin^3(a^2x^2)}{x^2}\,dx = \frac{3-\sqrt{3}}{8}\sqrt{\pi}a, \quad a\ge 0.$$

6. $$\int_0^\infty (\sin x^2 - x^2\,\cos x^2)\frac{dx}{x^4} = \frac{1}{3}\sqrt{\frac{\pi}{2}}.$$

7. $$\int_0^\infty \left\{\cos x^2 - \frac{1}{1+x^2}\right\}\frac{dx}{x} = -\frac{1}{2}\gamma_e.$$

8. $$\int_0^\infty \frac{\sin(ax^2)}{\beta^2+x^2}\,dx = \frac{\pi}{2\beta}\left[\sqrt{2}\,\sin\left(a\beta^2+\frac{\pi}{4}\right)C\left(\sqrt{a}\beta\right) - \sqrt{2}\,\cos\left(a\beta^2+\frac{\pi}{4}\right)S\left(\sqrt{a}\beta\right) - \sin\left(a\beta^2\right)\right],$$
$$a>0,\ \Re\{\beta\}>0.$$

9. $$\int_0^\infty \frac{\cos(ax^2)}{\beta^2+x^2}\,dx = \frac{\pi}{2\beta}\left[\cos(a\beta^2) - \sqrt{2}\,\cos\left(a\beta^2+\frac{\pi}{4}\right)C\left(\sqrt{a}\beta\right)\sqrt{2}\,\sin\left(a\beta^2+\frac{\pi}{4}\right)S\left(\sqrt{a}\beta\right)\right],$$
$$a>0,\ \Re\{\beta\}>0.$$

10. $$\int_0^\infty \frac{x^2\,\sin(ax^2)}{\beta^2+x^2}dx = \frac{\beta\pi}{2}\left[\sin(a\beta^2)-\sqrt{2}\sin\left(a\beta^2+\frac{\pi}{4}\right)C\left(\sqrt{a}\beta\right)+\sqrt{2}\cos\left(a\beta^2+\frac{\pi}{4}\right)S(\sqrt{a}\beta)\right]$$
$$-\frac{1}{2}\sqrt{\frac{\pi}{2a}},\quad a>0,\ \Re\{\beta\}>0.$$

11. $$\int_0^\infty \frac{x^2\,\cos(ax^2)}{\beta^2+x^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{2a}} - \frac{\beta\pi}{2}\left[\cos(a\beta^2)-\sqrt{2}\,\cos\left(a\beta^2+\frac{\pi}{4}\right)C\left(\sqrt{a}\beta\right)\right.$$
$$\left.-\sqrt{2}\,\sin\left(a\beta^2+\frac{\pi}{4}\right)S\left(\sqrt{a}\beta\right)\right],\quad a>0,\ \Re\{\beta\}>0.$$

12. $$\int_0^\infty (\cos(ax^2)-\sin(ax^2))\frac{dx}{x^4+b^4} = \frac{\pi e^{-ab^2}}{2b^3\sqrt{2}},\quad a>0,\ b>0.$$

13. $$\int_0^\infty (\cos(ax^2)+\sin(ax^2))\frac{x^2\,dx}{x^4+b^4} = \frac{\pi e^{-ab^2}}{2b\sqrt{2}},\quad a>0, b>0.$$

14. $$\int_0^\infty (\cos(ax^2)+\sin(ax^2))\frac{x^2\,dx}{(x^4+b^4)^2} = \frac{\pi e^{-ab^2}}{4\sqrt{2}b^3}\left(a+\frac{1}{2b^2}\right),\quad a>0,\ b>0.$$

15. $$\int_0^\infty (\cos(ax^2)-\sin(ax^2))\frac{x^4\,dx}{(x^4+b^4)^2} = \frac{\pi e^{-ab^2}}{4\sqrt{2}b}\left(\frac{1}{2b^2}-a\right),\quad a>0,\ b>0.$$

16. $$\int_0^\infty \frac{\sin(ax^2)}{\sqrt{\beta^2+x^4}}\,dx = \frac{1}{2}\sqrt{\frac{a\pi}{2}}\,I_{1/4}\left(\frac{a\beta}{2}\right)K_{1/4}\left(\frac{a\beta}{2}\right),\quad a>0,\ \Re\{\beta\}>0.$$

17. $$\int_0^\infty \frac{\cos(ax^2)}{\sqrt{\beta^2+x^4}}\,dx = \frac{1}{2}\sqrt{\frac{a\pi}{2}}\,I_{-1/4}\left(\frac{a\beta}{2}\right)K_{1/4}\left(\frac{a\beta}{2}\right),\quad a>0,\ \Re\{\beta\}>0.$$

18. $$\int_0^u \frac{\cos(a^2x^2)}{\sqrt{u^4-x^4}}\,dx = \frac{a}{4}\sqrt{\frac{\pi^3}{2}}\left[J_{-1/4}\left(\frac{a^2u^2}{2}\right)\right]^2.$$

19. $$\int_0^\infty \frac{(\sqrt{\beta^4+x^4}+x^2)^\nu}{\sqrt{\beta^4+x^4}}\,\sin(a^2x^2)\,dx = \frac{a}{2}\sqrt{\frac{\pi}{2}}\,\beta^{2\nu} I_{1/4-\nu/2}\left(\frac{a^2\beta^2}{2}\right) K_{1/4+\nu/2}\left(\frac{a^2\beta^2}{2}\right),$$
$$\Re\{\nu\} < \tfrac{3}{2},\ |\arg\beta| < \frac{\pi}{4}.$$

20. $$\int_0^\infty \frac{(\sqrt{\beta^4+x^4}+x^2)^\nu}{\sqrt{\beta^4+x^4}}\,\cos(a^2x^2)\,dx = \frac{a}{2}\sqrt{\frac{\pi}{2}}\,\beta^{2\nu} I_{-1/4-\nu/2}\left(\frac{a^2\beta^2}{2}\right) K_{-1/4+\nu/2}\left(\frac{a^2\beta^2}{2}\right),$$
$$\Re\{\nu\} < \tfrac{3}{2},\ |\arg\beta| < \frac{\pi}{4}.$$

21. $$\int_0^\infty \frac{(\sqrt{\beta^4+x^4}-x^2)^\nu}{\sqrt{\beta^4+x^4}}\,\cos(a^2x^2)\,dx = \frac{a}{2}\sqrt{\frac{\pi}{2}}\,\beta^{2\nu} I_{-1/4+\nu/2}\left(\frac{a^2\beta^2}{2}\right) K_{-1/4-\nu/2}\left(\frac{a^2\beta^2}{2}\right),$$
$$\Re\{\nu\} > -\tfrac{3}{2},\ |\arg\beta| < \frac{\pi}{4}.$$

22. $$\int_0^\infty \frac{\sin\left(a^2x^2\right)dx}{\sqrt{\beta^4+x^4}\sqrt{x^2+\sqrt{\beta^4+x^4}}} = \frac{\sinh\left(a^2\beta^2/2\right)}{\sqrt{2}\beta^2} K_0\left(\frac{a^2\beta^2}{2}\right), \quad |\arg\beta| < \frac{\pi}{4}.$$

23. $$\int_0^\infty \frac{\cos\left(a^2x^2\right)dx}{\sqrt{\beta^4+x^4}\sqrt{(x^2+\sqrt{\beta^4+x^4})^3}} = \frac{\sinh\left(a^2\beta^2/2\right)}{2\sqrt{2}\beta^4} K_1\left(\frac{a^2\beta^2}{2}\right), \quad \arg\beta| < \frac{\pi}{4}.$$

24. $$\int_0^\infty \frac{\sqrt{\sqrt{\beta^4+x^4}+x^2}}{\sqrt{\beta^4+x^4}}\,\sin(a^2x^2)\,dx = \frac{\pi}{2\sqrt{2}}\,e^{-a^2\beta^2/2}\,I_0\left(\frac{a^2\beta^2}{2}\right), \quad |\arg\beta| < \frac{\pi}{4}.$$

25. $$\int_0^\infty \frac{x^2}{R_1R_2}\sqrt{\frac{R_2-R_1}{R_2+R_1}}\,\sin(ax^2)\,dx = \frac{1}{2\sqrt{b}}\,K_0(ac)\,\sin ab,$$
$$R_1 = \sqrt{c^2+(b-x^2)^2},\ R_2 = \sqrt{c^2+(b+x^2)^2},\ \ a>0,\ c>0.$$

26. $$\int_0^\infty \frac{x^2}{R_1R_2}\sqrt{\frac{R_2+R_1}{R_2-R_1}}\,\cos(ax^2)\,dx = \frac{1}{2\sqrt{b}}\,K_0(ac)\,\cos ab,$$
$$R_1 = \sqrt{c^2+(b-x^2)^2},\ R_2 = \sqrt{c^2+(b+x^2)^2},\ \ a>0,\ c>0.$$

27. $$\int_0^\infty \left[\cos(x^k) - \frac{1}{1+x^{2k}}\right]\frac{dx}{x} = -\frac{1}{k}\,\gamma_e, \quad \text{where } k = 2^n.$$

28. $$\int_0^\infty \sin^{2n+1}(ax^2)\frac{dx}{x^{2m}} = \pm\frac{\sqrt{\pi}a^{m-\frac{1}{2}}}{2^{2n-m+\frac{1}{2}}(2m-1)!!}\sum_{k=1}^{n+1}(-1)^{k-1}\binom{2n+1}{n+k}(2k-1)^{m-1/2},$$

choose $+$ sign when $m \equiv 0 \pmod 4$ or $m \equiv 1 \pmod 4$;

choose $-$ sign when $m \equiv 2 \pmod 4$ or $m \equiv 3 \pmod 4$.

29. $$\int_0^\infty \sin^{2n}(ax^2)\frac{dx}{x^{2m}} = \pm\frac{\sqrt{\pi}a^{m-\frac{1}{2}}}{2^{2n-2m+1}(2m-1)!!}\sum_{k=1}^{n}(-1)^{k}\binom{2n}{n+k}k^{m-1/2},$$

choose $+$ sign when $m \equiv 0 \pmod 4$ or $m \equiv 3 \pmod 4$;

choose $-$ sign when $m \equiv 2 \pmod 4$ or $m \equiv 1 \pmod 4$.

30. $$\int_0^\infty \left[\cos(ax^2\sqrt{n}) + \sin(ax^2\sqrt{n})\right]\left(\frac{\sin x^2}{x^2}\right)^n dx$$

$$= \frac{\sqrt{\pi}}{(2n-1)!!\sqrt{2}}\sum_{k=0}^{n}(-1)^k\binom{n}{k}(n-2k+a\sqrt{n})^{n-1/2}, \quad a > \sqrt{n} > 0.$$

31. $$\int_0^\infty x^2\cos(ax^4)\sin(2bx^2)\,dx = -\frac{\pi}{8}\sqrt{\frac{b^3}{a^3}}\left[\sin\left(\frac{b^2}{2a}-\frac{\pi}{8}\right)J_{-1/4}\left(\frac{b^2}{2a}\right)+\cos\left(\frac{b^2}{2a}-\frac{\pi}{8}\right)J_{3/4}\left(\frac{b^2}{2a}\right)\right],$$

$a > 0,\ b > 0.$

32. $$\int_0^\infty x^2\cos(ax^4)\cos(2bx^2)\,dx = -\frac{\pi}{8}\sqrt{\frac{b^3}{a^3}}\left[\sin\left(\frac{b^2}{2a}+\frac{\pi}{8}\right)J_{-3/4}\left(\frac{b^2}{2a}\right)+\cos\left(\frac{b^2}{2a}+\frac{\pi}{8}\right)J_{-1/4}\left(\frac{b^2}{2a}\right)\right],$$

$a > 0,\ b > 0.$

33. $$\int_0^\infty \sin\frac{b}{x}\,\sin ax\frac{dx}{x} = \frac{\pi}{2}Y_0(2\sqrt{ab}) + K_0(2\sqrt{ab}), \quad a > 0,\ b > 0.$$

34. $$\int_0^\infty \cos\frac{b}{x}\,\cos ax\frac{dx}{x} = -\frac{\pi}{2}Y_0(2\sqrt{ab}) + K_0(2\sqrt{ab}), \quad a > 0,\ b > 0.$$

35. $$\int_0^\infty x^{\mu-1}\sin\frac{b^2}{x}\,\sin(a^2x)\,dx = \frac{\pi}{4}\left(\frac{b}{a}\right)^\mu \csc\frac{\mu\pi}{2}[J_\mu(2ab) - J_{-\mu}(2ab) + I_{-\mu}(2ab) - I_\mu(2ab)],$$

$a > 0,\ b > 0,\ |\Re\{\mu\}| < 1.$

36. $$\int_0^\infty x^{\mu-1}\sin\frac{b^2}{x}\,\cos(a^2x)\,dx = \frac{\pi}{4}\left(\frac{b}{a}\right)^\mu \sec\frac{\mu\pi}{2}[J_\mu(2ab) + J_{-\mu}(2ab) + I_\mu(2ab) - I_{-\mu}(2ab)],$$

$a > 0,\ b > 0,\ |\Re\{\mu\}| < 1.$

37. $$\int_0^\infty x^{\mu-1}\,\cos\frac{b^2}{x}\,\cos(a^2x)\,dx = \frac{\pi}{4}\left(\frac{b}{a}\right)^\mu \csc\frac{\mu\pi}{2}\left[J_{-\mu}(2ab) - J_\mu(2ab) + I_{-\mu}(2ab) - I_\mu(2ab)\right], \quad a>0,\ b>0,\ |\Re\{\mu\}|<1.$$

38. $$\int_0^\infty \sin\left(a^2x + \frac{b^2}{x}\right)\frac{dx}{x} = \pi J_0(2ab), \quad a>0,\ b>0.$$

39. $$\int_0^\infty \cos\left(a^2x + \frac{b^2}{x}\right)\frac{dx}{x} = -\pi Y_0(2ab), \quad a>0,\ b>0.$$

40. $$\int_0^\infty \sin\left(a^2x - \frac{b^2}{x}\right)\frac{dx}{x} = 0, \quad a>0,\ b>0.$$

41. $$\int_0^\infty \cos\left(a^2x - \frac{b^2}{x}\right)\frac{dx}{x} = 2K_0(2ab), \quad a>0,\ b>0.$$

42. $$\int_0^\infty \sin\left(ax - \frac{b}{x}\right)\frac{x\,dx}{\beta^2+x^2} = \frac{\pi}{2}\exp\left(-\alpha\beta - \frac{b}{\beta}\right), \quad a>0,\ b>0,\ \Re\{\beta\}>0.$$

43. $$\int_0^\infty \cos\left(ax - \frac{b}{x}\right)\frac{dx}{\beta^2+x^2} = \frac{\pi}{2\beta}\exp\left(-a\beta - \frac{b}{\beta}\right), \quad a>0,\ b>0,\ \Re\{\beta\}>0.$$

44. $$\int_0^\infty x^{\mu-1}\sin\left[a\left(x + \frac{b^2}{x}\right)\right]dx = \pi b^\mu\left[J_\mu(2ab)\cos\frac{\mu\pi}{2} - Y_\mu(2ab)\sin\frac{\mu\pi}{2}\right], \quad a>0,\ b>0,\ \Re\{\mu\}<1.$$

45. $$\int_0^\infty x^{\mu-1}\cos\left[a\left(x + \frac{b^2}{x}\right)\right]dx = -\pi b^\mu\left[J_\mu(2ab)\sin\frac{\mu\pi}{2} + Y_\mu(2ab)\cos\frac{\mu\pi}{2}\right], \quad a>0,\ b>0,\ |\Re\{\mu\}|<1.$$

46. $$\int_0^\infty x^{\mu-1}\sin\left[a\left(x - \frac{b^2}{x}\right)\right]dx = 2b^\mu K_\mu(2ab)\sin\frac{\mu\pi}{2}, \quad a>0,\ b>0,\ |\Re\{\mu\}|<1.$$

47. $$\int_0^\infty x^{\mu-1}\cos\left[a\left(x - \frac{b^2}{x}\right)\right]dx = 2b^\mu K_\mu(2ab)\cos\frac{\mu\pi}{2}, \quad a>0,\ b>0,\ |\Re\{\mu\}|<1.$$

48. $$\int_0^\infty \sin\frac{a^2}{x^2}\,\cos b^2x^2\,\frac{dx}{x^2} = \frac{\sqrt{\pi}}{4\sqrt{2}a}\left[\sin(2ab) + \cos(2ab) + e^{-2ab}\right], \quad a>0,\ b>0.$$

49. $$\int_0^\infty \cos\frac{a^2}{x^2}\,\cos b^2x^2\,\frac{dx}{x^2} = \frac{\sqrt{\pi}}{4\sqrt{2}a}\left[\cos(2ab)-\sin(2ab)+e^{-2ab}\right], \quad a>0,\ b>0.$$

50. $$\int_0^\infty \sin\left(a^2x^2+\frac{b^2}{x^2}\right)\frac{dx}{x^2} = \frac{\sqrt{\pi}}{2b}\,\sin\left(2ab+\frac{\pi}{4}\right), \quad a>0,\ b>0.$$

51. $$\int_0^\infty \cos\left(a^2x^2+\frac{b^2}{x^2}\right)\frac{dx}{x^2} = \frac{\sqrt{\pi}}{2b}\,\cos\left(2ab+\frac{\pi}{4}\right), \quad a>0,\ b>0.$$

52. $$\int_0^\infty \sin\left(a^2x^2-\frac{b^2}{x^2}\right)\frac{dx}{x^2} = -\frac{\sqrt{\pi}}{2\sqrt{2}b}\,e^{-2ab}, \quad a\ge 0,\ b>0.$$

53. $$\int_0^\infty \cos\left(a^2x^2-\frac{b^2}{x^2}\right)\frac{dx}{x^2} = \frac{\sqrt{\pi}}{2\sqrt{2}b}e^{-2ab}, \quad a\ge 0,\ b>0.$$

54. $$\int_0^\infty \sin\left(ax-\frac{b}{x}\right)^2\frac{dx}{x^2} = \frac{\sqrt{2\pi}}{4b}, \quad a>0,\ b>0.$$

55. $$\int_0^\infty \cos\left(ax-\frac{b}{x}\right)^2\frac{dx}{x^2} = \frac{\sqrt{2\pi}}{4b}, \quad a>0,\ b>0.$$

56. $$\int_0^\infty \frac{\sin(p\sqrt{a^2+x^2})}{(a^2+x^2)^{3/2}}\,\cos bx\,dx = \frac{\pi p}{2a}\,e^{-ab}, \quad 0<p<b,\ a>0.$$

57. $$\int_0^\infty \frac{\sin(p\sqrt{x^2+a^2})}{\sqrt{x^2+a^2}}\,\cos bx\,dx = \begin{cases} \frac{\pi}{2}\,J_0(a\sqrt{p^2-b^2}), & 0<b<p, \\ 0, & b>p>0, \end{cases} \qquad a>0.$$

58. $$\int_0^\infty \frac{\cos(p\sqrt{x^2+a^2})}{\sqrt{x^2+a^2}}\,\cos bx\,dx = \begin{cases} -\frac{\pi}{2}Y_0(a\sqrt{p^2-b^2}), & 0<b<p, \\ K_0(a\sqrt{b^2-p^2}), & b>p>0, \end{cases} \qquad a>0.$$

59. $$\int_0^\infty \frac{\cos(p\sqrt{x^2+a^2})}{x^2+c^2}\,\cos bx\,dx = \frac{\pi}{2c}e^{-bc}\,\cos(p\sqrt{a^2-c^2}), \quad c>0,\ b>p.$$

60. $$\int_0^\infty \frac{\sin(p\sqrt{x^2+a^2})}{(x^2+c^2)\sqrt{x^2+a^2}}\,\cos bx\,dx = \begin{cases} \frac{\pi}{2c}\frac{e^{-bc}\,\sin(p\sqrt{a^2-c^2})}{\sqrt{a^2-c^2}}, & c\ne a, \\ \frac{\pi}{2}\,e^{-ba}\frac{p}{a}, & c=a, \end{cases} \qquad b>p,\ c>0.$$

61. $\displaystyle\int_0^\infty \frac{\cos(p\sqrt{x^2+a^2})}{x^2+a^2}\cos bx\,dx = \frac{\pi}{2a}e^{-ab}, \quad b>p>0;\ a>0.$

62. $\displaystyle\int_0^\infty \frac{x\,\cos(p\sqrt{x^2+a^2})}{x^2+a^2}\sin bx\,dx = \frac{\pi}{2}e^{-ab}, \quad a>0,\ b>p>0.$

63. $\displaystyle\int_0^\infty \frac{\sin(p\sqrt{x^4+a^4})}{\sqrt{x^4+a^4}}\cos bx^2\,dx = \frac{1}{2}\sqrt{b\left(\frac{\pi}{2}\right)^3}\,J_{-1/4}\left[\frac{a^2}{2}(p-\sqrt{p^2-b^2})\right]J_{1/4}\left[\frac{a^2}{2}(p+\sqrt{p^2-b^2})\right],$
$p>b>0.$

64. $\displaystyle\int_0^\infty \frac{\cos(p\sqrt{x^4+a^4})}{\sqrt{x^4+a^4}}\cos bx^2\,dx = -\frac{1}{2}\sqrt{b\left(\frac{\pi}{2}\right)^3}\,J_{-1/4}\left[\frac{a^2}{2}(p-\sqrt{p^2-b^2})\right]Y_{1/4}\left[\frac{a^2}{2}(p+\sqrt{p^2-b^2})\right],$
$a>0,\ p>b>0.$

65. $\displaystyle\int_0^\infty \sin ax^p\frac{dx}{x} = \frac{\pi}{2p}, \quad a>0,\ p>0.$

66. $\displaystyle\int_0^\infty \sin(a\tan x)\frac{dx}{x} = \frac{\pi}{2}(1-e^{-a}), \quad a>0.$

67. $\displaystyle\int_0^\infty \sin(a\tan x)\,\cos x\frac{dx}{x} = \frac{\pi}{2}(1-e^{-a}), \quad a>0.$

68. $\displaystyle\int_0^\infty \cos(a\tan x)\,\sin x\frac{dx}{x} = \frac{\pi}{2}e^{-a}, \quad a>0.$

69. $\displaystyle\int_0^\infty \sin(a\tan x)\,\sin 2x\frac{dx}{x} = \frac{1+a}{2}\pi e^{-a}, \quad a>0.$

70. $\displaystyle\int_0^\infty \cos(a\tan x)\,\sin^3 x\frac{dx}{x} = \frac{1-a}{4}\pi e^{-a}, \quad a>0.$

71. $\displaystyle\int_0^\infty \sin(a\tan x)\,\tan\frac{x}{2}\,\cos^2 x\frac{dx}{x} = \frac{1+a}{4}\pi e^{-a}, \quad a>0.$

72. $\displaystyle\int_0^\infty \cos(a\tan x)\,\tan x\frac{dx}{x} = \frac{\pi}{2}e^{-a}, \quad a>0.$

73. $\displaystyle\int_0^\infty \cos(a\tan x)\,\sin^2 x\,\tan x\frac{dx}{x} = \frac{1-a}{16}\pi\,e^{-a}, \quad a>0.$

74. $$\int_0^\infty \sin(a\,\tan x)\,\tan^2 x\frac{dx}{x} = \frac{\pi}{2}\,e^{-a}, \quad a > 0.$$

75. $$\int_0^\infty \cos(a\,\tan 2x)\,\tan x\frac{dx}{x} = \frac{\pi}{2}\,e^{-a}, \quad a > 0.$$

76. $$\int_0^\infty \sin(a\,\tan 2x)\,\cos^2\,2x\,\tan x\frac{dx}{x} = \frac{1+a}{4}\pi\,e^{-a}, \quad a > 0.$$

77. $$\int_0^\infty \sin(a\,\tan 2x)\,\tan x\,\tan 2x\frac{dx}{x} = \frac{\pi}{2}{}^{-a}, \quad a > 0.$$

78. $$\int_0^\infty \sin(a\,\tan 2x)\,\tan x\,\cot 2x\frac{dx}{x} = \frac{\pi}{2}(1-e^{-a}), \quad a > 0.$$

79. $$\int_0^\infty \sin(a\,\tan^2 x)\frac{x\,dx}{b^2+x^2} = \frac{\pi}{2}\left[\exp(-a\,\tanh b) - e^{-a}\right], \quad a > 0,\, b > 0.$$

80. $$\int_0^\infty \cos(a\,\tan^2 x)\,\cos x\frac{dx}{b^2+x^2} = \frac{\pi}{2b}\left[\cosh b\,\exp(-a\,\tanh b) - e^{-a}\,\sinh b\right], \quad a > 0,\, b > 0.$$

81. $$\int_0^\infty \cos(a\,\tan^2 x)\,\csc 2x\frac{x\,dx}{b^2+x^2} = \frac{\pi}{2\,\sinh 2b}\,\exp(-a\,\tanh b), \quad a > 0,\, b > 0.$$

82. $$\int_0^\infty \cos(a\,\tan^2 x)\,\tan x\frac{x\,dx}{b^2+x^2} = \frac{\pi}{2\,\cosh b}\left[e^{-a}\,\cosh b - \exp(-a\,\tanh b)\,\sinh b\right], \quad a > 0,\, b > 0.$$

83. $$\int_0^\infty \cos(a\,\tan^2 x)\,\cot x\frac{x\,dx}{b^2+x^2} = \frac{\pi}{2}\left[\coth\,b\,\exp(-a\,\tanh b) - e^{-a}\right], \quad a > 0,\, b > 0.$$

84. $$\int_0^\infty \cos(a\,\tan^2 x)\,\cot 2x\frac{x\,dx}{b^2+x^2} = \frac{\pi}{2}\left[\coth\,2b\,\exp(-a\,\tanh b) - e^{-a}\right], \quad a > 0,\, b > 0.$$