! For an efficient use of these tables, first read `HowTo.pdf`.

T3.35C. Integrands involving product and division of trigonometric functions by powers of $(a+b\,x)$ on the interval (y,∞).

1. $$\int_y^\infty x^{\mu-1}\,\sin x\,dx = \frac{i}{2}\left[e^{-i\,\mu\pi/2}\Gamma(\mu,\,iy) - e^{i\,\mu\pi/2}\Gamma(\mu,\,-iy)\right],\quad \Re\{\mu\} > -1.$$

2. $$\int_y^\infty x^{\mu-1}\,\cos x\,dx = \frac{1}{2}\left[e^{-i\,\mu\pi/2}\Gamma(\mu,\,iy) + e^{i\,\mu\pi/2}\Gamma(\mu,\,-i\,y)\right],\quad \Re\{\mu\} < 1.$$

3. $$\int_y^\infty (x-y)^{\mu-1}\,\sin(ax)\,dx = \frac{\Gamma(\mu)}{a^\mu}\,\sin\left(ay + \frac{\mu\pi}{2}\right),\quad a>0,\ 0<\Re\{\mu\}<1.$$

4. $$\int_y^\infty (x-y)^{\mu-1}\,\cos(ax)\,dx = \frac{\Gamma(\mu)}{a^\mu}\,\cos\left(ay + \frac{\mu\pi}{2}\right),\quad a>0,\ 0<\Re\{\mu\}<1.$$

5. $$\int_y^\infty x^{\mu-1}(x-y)^{\mu-1}\,\sin(ax)\,dx$$
$$= \frac{\sqrt{\pi}}{2}\,\left(\frac{y}{a}\right)^{\mu-1/2}\Gamma(\mu)\left[\cos\frac{ay}{2}\,J_{1/2-\mu}\left(\frac{ay}{2}\right) - \sin\frac{ay}{2}\,Y_{1/2-\mu}\left(\frac{ay}{2}\right)\right],$$
$$a>0,\ 0<\Re\{\mu\}<\tfrac{1}{2}.$$

6. $$\int_y^\infty x^{\mu-1}(x-y)^{\mu-1}\,\cos(ax)\,dx = -\frac{\sqrt{\pi}}{2}\left(\frac{y}{a}\right)^{\mu-1/2}\Gamma(\mu)\left[\sin\frac{ay}{2}\,J_{1/2-\mu}\left(\frac{ay}{2}\right) - \cos\frac{ay}{2}\,Y_{1/2-\mu}\left(\frac{ay}{2}\right)\right],$$
$$a>0,\ 0<\Re\{\mu\}<\tfrac{1}{2}.$$

7. $$\int_y^\infty (x^2-y^2)^{\nu-1/2}\,\sin(ax)\,dx = \frac{\sqrt{\pi}}{2}\left(\frac{2y}{a}\right)^\nu\Gamma\left(\nu+\frac{1}{2}\right)J_{-\nu}(ay),\quad a>0,\ y>0,\ |\Re\{\nu\}|<\tfrac{1}{2}.$$

8. $$\int_y^\infty x(x^2-y^2)^{\nu-1/2}\,\sin(ax)\,dx = \frac{\sqrt{\pi}}{2} y \left(\frac{2y}{a}\right)^\nu \Gamma\left(\nu+\frac{1}{2}\right) Y_{-\nu-1}(ay),$$
$$a>0,\ y>0,\ -\frac{1}{2}<\Re\{\nu\}<0.$$

9. $$\int_y^\infty (x^2-y^2)^{\nu-1/2}\,\cos(ax)\,dx = -\frac{\sqrt{\pi}}{2} \left(\frac{2y}{a}\right)^\nu \Gamma\left(\nu+\frac{1}{2}\right) Y_{-\nu}(ay),$$
$$a>0,\ y>0,\ |\Re\{\nu\}|<\tfrac{1}{2}.$$

10. $$\int_y^\infty x(x^2-y^2)^{\nu-1/2}\,\cos(ax)\,dx = \frac{\sqrt{\pi}y}{2} \left(\frac{2y}{a}\right)^\nu \Gamma\left(\nu+\frac{1}{2}\right) J_{-\nu-1}(ay),$$
$$a>0,\ y>0,\ 0<\Re\{\nu\}<\tfrac{1}{2}.$$

11. $$\int_y^\infty \frac{(x+\sqrt{x^2-y^2})^\nu+(x-\sqrt{x^2-y^2})^\nu}{\sqrt{x^2-y^2}}\,\sin(ax)\,dx = \pi y^\nu\left[J_\nu(ay)\,\cos\frac{\nu\pi}{2} - Y_\nu(ay)\,\sin\frac{\nu\pi}{2}\right],$$
$$a>0,\ y>0,\ |\Re\{\nu\}|<1.$$

12. $$\int_y^\infty \frac{(x+\sqrt{x^2-y^2})^\nu+(x-\sqrt{x^2-y^2})^\nu}{\sqrt{x^2-y^2}}\,\cos(ax)\,dx = -\pi y^\nu\left[Y_\nu(ay)\,\cos\frac{\nu\pi}{2} + J_\nu(ay)\,\sin\frac{\nu\pi}{2}\right],$$
$$a>0,\ y>0,\ |\Re\{\nu\}|<1.$$

13. $$\int_y^\infty \frac{(x+\sqrt{x^2-y^2})^\nu+(x-\sqrt{x^2-y^2})^\nu}{\sqrt{x(x^2-y^2)}}\,\sin(ax)\,dx$$
$$= -\sqrt{a}\,\left(\frac{\pi}{2}\right)^{3/2} y^\nu\left[J_{1/4+\nu/2}\left(\frac{ay}{2}\right)Y_{1/4-\nu/2}\left(\frac{ay}{2}\right)+J_{1/4-\nu/2}\left(\frac{ay}{2}\right)Y_{1/4+\nu/2}\left(\frac{ay}{2}\right)\right],$$
$$a>0,\ y>0,\ |\Re\{\nu\}|<\tfrac{3}{2}.$$

14. $$\int_y^\infty \frac{(x+\sqrt{x^2-y^2})^\nu+(x-\sqrt{x^2-y^2})^\nu}{\sqrt{x(x^2-y^2)}}\,\cos(ax)\,dx$$
$$= -\sqrt{a}\,\left(\frac{\pi}{2}\right)^{3/2} y^\nu\left[J_{-1/4+\nu/2}\left(\frac{ay}{2}\right)Y_{-1/4-\nu/2}\left(\frac{ay}{2}\right)+J_{-1/4-\nu/2}\left(\frac{ay}{2}\right)Y_{-1/4+\nu/2}\left(\frac{ay}{2}\right)\right],$$
$$a>0,\ y>0,\ |\Re\{\,\nu\}|<\tfrac{3}{2}.$$

15. $$\int_y^\infty (x^2-2bx)^{\nu-1/2}\,\sin(ax)\,dx = \frac{\sqrt{\pi}}{2}\left(\frac{2b}{a}\right)^\nu\Gamma\left(\nu+\frac{1}{2}\right)\left[J_{-\nu}\left(\frac{ay}{2}\right)\,\cos\left(\frac{ay}{2}\right)-Y_{-\nu}\left(\frac{ay}{2}\right)\,\sin\left(\frac{ay}{2}\right)\right],$$
$$a>0,\ y>0,\ |\Re\{\nu\}|<\tfrac{1}{2}.$$

16. $$\int_y^{\infty} (x^2 - 2bx)^{\nu - 1/2} \cos(ax)\, dx = -\frac{\sqrt{\pi}}{2} \left(\frac{2b}{a}\right)^{\nu} \Gamma\left(\nu + \frac{1}{2}\right) \left[J_{-\nu}\left(\frac{ay}{2}\right) \sin\left(\frac{ay}{2}\right) + Y_{-\nu}\left(\frac{ay}{2}\right) \cos\left(\frac{ay}{2}\right) \right],$$

$$a > 0,\ y > 0,\ |\Re\{\nu\}| < \tfrac{1}{2}.$$