

! For an efficient use of these tables, first read [HowTo.pdf](#).

**T2.04A.** Integrands of the form  $\frac{1}{\sqrt{(a-x)(b-x)(c-x)^n}}$ ,  $\frac{1}{\sqrt{(a-x)(b-x)^n(c-x)}}$ ,  
and  $\frac{1}{\sqrt{(a-x)^n(b-x)(c-x)}}$  for  $n = 0, 1, 3, 5$ , on the intervals  $(y, a)$  and  $(a, y)$ .

Notation used:  $\lambda = \arcsin \sqrt{\frac{a-y}{a-b}}$ ,  $\mu = \arcsin \sqrt{\frac{y-a}{y-b}}$ ,  $p = \sqrt{\frac{a-b}{a-c}}$ ,  $q = \sqrt{\frac{b-c}{a-c}}$ .

$$1. \int_a^y \frac{dx}{\sqrt{(x-a)(y-x)}} = \pi, \quad 0 \leq a < y \leq 1.$$

$$2. \int_y^a \frac{dx}{\sqrt{(a-x)(y-x)}} = i\pi, \quad 0 \leq y < a \leq 1.$$

$$3. \int_y^a \frac{dx}{\sqrt{(a-x)(x-b)(x-c)}} = \frac{2}{\sqrt{a-c}} F(\lambda, p), \quad a > y \geq b > c.$$

$$4. \int_a^y \frac{dx}{\sqrt{(x-a)(x-b)(x-c)}} = \frac{2}{\sqrt{a-c}} F(\mu, q), \quad y > a > b > c.$$

$$5. \int_y^a \frac{x dx}{\sqrt{(a-x)(x-b)(x-c)}} = \frac{2c}{\sqrt{a-c}} F(\lambda, p) + 2\sqrt{a-c} E(\lambda, p), \quad a > y \geq b > c.$$

$$6. \int_a^y \frac{x dx}{\sqrt{(x-a)(x-b)(x-c)}} = \frac{2}{b\sqrt{a-c}} [a(a-b)\Pi(\mu, 1, q) + b^2 F(\mu, q)], \quad y > a > b > c.$$

$$7. \int_y^a \frac{dx}{\sqrt{(a-x)(x-b)^3(x-c)}} = \frac{2}{(a-b)\sqrt{a-c}} F(\lambda, p) - \frac{2\sqrt{a-c}}{(a-b)(b-c)} E(\lambda, p) \\ + \frac{2}{(a-b)(b-c)} \sqrt{\frac{(a-y)(y-c)}{y-b}}, \quad a > y > b > c.$$

$$8. \int_a^y \frac{dx}{\sqrt{(x-a)(x-b)^3(x-c)}} = \frac{2\sqrt{a-c}}{(a-b)(b-c)} E(\mu, q) - \frac{2}{(b-c)\sqrt{a-c}} F(\mu, q),$$

$$y > a > b > c.$$

$$9. \int_y^a \frac{dx}{\sqrt{(a-x)(x-b)(x-c)^3}} = \frac{2}{(b-c)\sqrt{a-c}} E(\lambda, p) - \frac{2}{(b-c)(a-c)} \sqrt{\frac{(a-y)(y-b)}{y-c}},$$

$$a > y \geq b > c.$$

$$10. \int_a^y \frac{dx}{\sqrt{(x-a)(x-b)(x-c)^3}} = \frac{2}{(b-c)\sqrt{a-c}} [F(\mu, q) - E(\mu, q)] + \frac{2}{a-c} \sqrt{\frac{y-a}{(y-b)(y-c)}},$$

$$y > a > b > c.$$

$$11. \int_y^a \frac{dx}{\sqrt{(a-x)(x-b)^5(x-c)}} = \frac{2}{3(a-b)^2(b-c)^2\sqrt{a-c}} [(b-c)(3b-2c-a)F(\lambda, p) + 2(a-c)(a+c-2b)E(\lambda, p)]$$

$$+ \frac{2[3ab+3bc-ac-5b^2+2y(2b-a-c)]}{3(a-b)^2(b-c)^2} \sqrt{\frac{(a-y)(y-c)}{(y-b)^3}}, \quad a > y > b > c.$$

$$12. \int_a^y \frac{dx}{\sqrt{(x-a)(x-b)^5(x-c)}} = \frac{2}{3(a-b)^2(b-c)^2\sqrt{a-c}} [(b-c)(3b-2c-a)F(\lambda, p) + 2(a-c)(a+c-2b)E(\lambda, p)]$$

$$- \frac{2[3ab+3bc-ac-5b^2+2y(2b-a-c)]}{3(a-b)^2(b-c)} \sqrt{\frac{(a-y)(y-c)}{(y-b)^3}}, \quad a > y > b > c.$$

$$13. \int_y^a \frac{dx}{\sqrt{(a-x)(x-b)(x-c)^5}} = \frac{2}{3(b-c)^2\sqrt{(a-c)^3}} [2(a+b-2c)E(\lambda, p) - (b-c)F(\lambda, p)]$$

$$- \frac{2[ab-3ac-3bc+5c^2+2y(a+b-2c)]}{3(b-c)^2(a-c)^2} \sqrt{\frac{(a-y)(y-b)}{(y-c)^3}}, \quad a > y \geq b > c.$$

$$14. \int_a^y \frac{dx}{\sqrt{(x-a)(x-b)(x-c)^5}} = \frac{2}{3(b-c)^2 \sqrt{(a-c)^3}} [(2a+b-3c)F(\mu, q) - 2(a+b-2c)E(\mu, q)] \\ + \frac{2[4c^2 - ab - 2ac - bc + y(3a+2b-5c)]}{3(b-c)(a-c)^2} \sqrt{\frac{y-a}{(y-b)(y-c)^3}}, \quad y > a > b > c.$$


---