! For an efficient use of these tables, first read `HowTo.pdf`.

T3.32A. Integrands involving exponentials, hyperbolic functions and powers of $(a+b\,x)$ on the interval $(0,\infty)$.

1. $$\int_0^\infty x^{\mu-1}e^{-\beta x}\,\sinh\gamma x\,dx=\frac{1}{2}\Gamma(\mu)[(\beta-\gamma)^{-\mu}-(\beta+\gamma)^{-\mu}],\quad \Re\{\beta\}>-1,\ \Re\{\beta\}>|\Re\{\gamma\}|.$$

2. $$\int_0^\infty x^{\mu-1}e^{-\beta x}\,\cosh\gamma x\,dx=\frac{1}{2}\Gamma(\mu)[(\beta-\gamma)^{-\mu}+(\beta+\gamma)^{-\mu}],\quad \Re\{\mu\}>0,\ \Re\{\beta\}>|\Re\{\gamma\}|.$$

3. $$\int_0^\infty x^{\mu-1}e^{-\beta x}\,\coth\,x\,dx=\Gamma(\mu)\left[2^{1-\mu}\zeta\left(\mu,\,\frac{\beta}{2}\right)-\beta^{-\mu}\right],\quad \Re\{\mu\}>1,\ \Re\{\beta\}>0.$$

4. $$\int_0^\infty x^n e^{-(p+mq)x}\,\sinh^m\,qx\,dx=2^{-m}n!\sum_{k=0}^{m}\binom{m}{k}\frac{(-1)^k}{(p+2kq)^{n+1}},\quad p>0,\ q>0,\ m<p+qm.$$

5. $$\int_0^\infty \frac{e^{-\beta x}}{x}\,\sinh\gamma x\,dx=\frac{1}{2}\,\ln\,\frac{\beta+\gamma}{\beta-\gamma},\quad \Re\{\beta\}>|\Re\{\gamma\}|.$$

6. $$\int_0^\infty xe^{-x}\,\coth\,x\,dx=\frac{\pi^2}{4}-1.$$

7. $$\int_0^\infty e^{-\beta x}\,\tanh x\frac{dx}{x}=\ln\,\frac{\beta}{4}+2\,\ln\,\frac{\Gamma\left(\frac{\beta}{4}\right)}{\Gamma\left(\frac{\beta}{4}+\frac{1}{2}\right)},\quad \Re\{\beta\}>0.$$

8. $$\int_0^\infty xe^{-x}\,\coth\,(x/2)\,dx=\frac{\pi^2}{3}-1.$$

9. $$\int_0^\infty \frac{x^{\mu-1}e^{-\beta x}}{\sinh x}\,dx=2^{1-\mu}\Gamma(\mu)\zeta\left[\mu,\,\frac{1}{2}(\beta+1)\right],\quad \Re\{\mu\}>1,\ \Re\{\beta\}>-1.$$

10. $\displaystyle\int_0^\infty \frac{x^{2m-1}e^{-ax}}{\sinh ax}\,dx = \frac{1}{2m}|B_{2m}|\left(\frac{\pi}{a}\right)^{2m}, \quad a>0,\ m=1,2,\ldots.$

11. $\displaystyle\int_0^\infty \frac{x^{\mu-1}e^{-x}}{\cosh x}\,dx = \begin{cases} 2^{1-\mu}(1-2^{1-\mu})\Gamma(\mu)\zeta(\mu), & \mu\neq 1, \\ \ln 2, & \mu=1, \end{cases} \qquad \Re\{\mu\}>0.$

12. $\displaystyle\int_0^\infty \frac{x^{2m-1}e^{-ax}}{\cosh ax}\,dx = \frac{1-2^{1-2m}}{2m}|B_{2m}|\left(\frac{\pi}{a}\right)^{2m}, \quad a>0,\ m=1,2,\ldots.$

13. $\displaystyle\int_0^\infty \frac{x^2e^{-2nx}}{\sinh x}\,dx = 4\sum_{k=n}^{\infty}\frac{1}{(2k+1)^3}, \quad n=0,1,2,\ldots.$

14. $\displaystyle\int_0^\infty \frac{x^3e^{-2nx}}{\sinh x}\,dx = \frac{\pi^4}{8} - 12\sum_{k=1}^{n}\frac{1}{(2k+1)^4}, \quad n=0,1,\ldots.$

15. $\displaystyle\int_0^\infty \frac{\sinh^2\ ax}{\sinh x}\,\frac{e^{-x}\,dx}{x} = \frac{1}{2}\,\ln\left(a\pi\ \csc a\pi\right), \quad a<1.$

16. $\displaystyle\int_0^\infty \frac{\sinh^2\ \frac{\pi}{2}}{\cosh x}\cdot\frac{e^{-x}\,dx}{x} = \frac{1}{2}\,\ln\,\frac{4}{\pi}.$

17. $\displaystyle\int_0^\infty e^{-\beta x}(1-\text{sech}\ s)\frac{dx}{x} = 2\,\ln\,\frac{\Gamma\left(\frac{\beta+3}{4}\right)}{\Gamma\left(\frac{\beta+1}{4}\right)} - \ln\,\frac{\beta}{4}, \quad \Re\{\beta\}>0.$

18. $\displaystyle\int_0^\infty e^{-\beta x}\left(\frac{1}{x}-\text{csch}\ x\right)\,dx = \psi\left(\frac{\beta+1}{2}\right) - \ln\,\frac{\beta}{2}, \quad \Re\{\beta\}>0.$

19. $\displaystyle\int_0^\infty \left[\frac{\sinh\left(\frac{1}{2}-\beta\right)x}{\sinh\frac{x}{2}} - (1-2\beta)\,e^{-x}\right]\frac{dx}{x} = 2\,\ln\,\Gamma(\beta) - \ln\,\pi + \ln\left(\sin\pi\beta\right), \quad 0<\Re\{\beta\}<1.$

20. $\displaystyle\int_0^\infty e^{-\beta x}\left(\frac{1}{x}-\coth\ x\right)\,dx = \psi\left(\frac{\beta}{2}\right) - \ln\,\frac{\beta}{2} + \frac{1}{\beta}, \quad \Re\{\beta\}>0.$

21. $\displaystyle\int_0^\infty \left\{-\frac{\sinh qx}{\sinh\frac{x}{2}} + 2qe^{-x}\right\}\frac{dx}{x} = 2\,\ln\,\Gamma\left(q+\frac{1}{2}\right) + \ln\,\cos\pi q - \ln\,\pi, \quad q^2<\tfrac{1}{2}.$

22. $\displaystyle\int_0^\infty x^{\mu-1}e^{-\beta x}(\coth\ x-1)\,dx = 2^{1-\mu}\Gamma(\mu)\,\zeta\left(\mu,\frac{\beta}{2}+1\right), \quad \Re\{\beta\}>0;\ \Re\{\mu\}>1.$

23. $$\int_0^\infty \frac{\sinh^2\, ax}{1-e^{px}}\,\frac{dx}{x} = \frac{1}{4}\,\ln\left(\frac{p}{2a\pi}\,\sin\frac{2a\pi}{p}\right), \quad 0 < 2|a| < p.$$

24. $$\int_0^\infty \frac{\sinh^2\, ax}{e^{x}+1}\,\frac{dx}{x} = -\frac{1}{4}\,\ln\left(a\pi\,\cot a\pi\right), \quad a < \tfrac{1}{2}.$$

25. $$\int_{-\infty}^\infty x\frac{1-e^{px}}{\sinh x}\,dx = -\frac{\pi^2}{2}\,\tan^2\frac{p\pi}{2}, \quad p < 1.$$

26. $$\int_0^\infty \frac{1-e^{-px}}{\sinh x}\cdot\frac{1-e^{-(p+1)x}}{x}\,dx = 2p\,\ln 2, \quad p > -1.$$

27. $$\int_0^\infty \frac{e^{-px}-e^{-qx}}{\cosh x - \cos\frac{m}{n}\pi}\cdot\frac{dx}{x}$$
$$= \begin{cases} 2\,\csc\dfrac{m}{n}\pi\displaystyle\sum_{k=1}^{n-1}(-1)^{k-1}\,\sin\left(\frac{km}{n}\pi\right)\,\ln\frac{\Gamma\left(\frac{n+q+k}{2n}\right)\Gamma\left(\frac{p+k}{2n}\right)}{\Gamma\left(\frac{n+p+k}{2n}\right)\Gamma\left(\frac{q+k}{2n}\right)}, & m+n\ \text{odd}, \\ 2\,\csc\dfrac{m}{n}\pi\displaystyle\sum_{k=1}^{\frac{n-1}{2}}(-1)^{k-1}\,\sin\left(\frac{km}{n}\pi\right)\,\ln\frac{\Gamma\left(\frac{n+q-k}{n}\right)\Gamma\left(\frac{p+k}{n}\right)}{\Gamma\left(\frac{n+p-k}{n}\right)\Gamma\left(\frac{q+k}{n}\right)}, & m+n\ \text{even}, \end{cases} \quad p > -1,\ q > -1.$$

28. $$\int_0^\infty \frac{(1-e^{-x})^2}{\cosh x + \cos\frac{m}{n}\pi}\cdot\frac{dx}{x}$$
$$= \begin{cases} 2\,\csc\dfrac{m}{n}\pi\displaystyle\sum_{k=1}^{n-1}(-1)^{k-1}\,\sin\left(\frac{km}{n}\pi\right)\,\ln\frac{\left[\Gamma\left(\frac{n+k+1}{2n}\right)\right]^2\Gamma\left(\frac{k+2}{2n}\right)\Gamma\left(\frac{k}{2n}\right)}{\left[\Gamma\left(\frac{k+1}{2n}\right)\right]^2\Gamma\left(\frac{n+k}{2n}\right)\Gamma\left(\frac{n+k+2}{2n}\right)}, & m+n\ \text{odd}, \\ 2\,\csc\dfrac{m}{n}\pi\displaystyle\sum_{k=1}^{\frac{n-1}{2}}(-1)^{k-1}\,\sin\left(\frac{km}{n}\pi\right)\,\ln\frac{\left[\Gamma\left(\frac{n-k+1}{n}\right)\right]^2\Gamma\left(\frac{k+2}{n}\right)\Gamma\left(\frac{k}{n}\right)}{\left[\Gamma\left(\frac{k+1}{n}\right)\right]^2\Gamma\left(\frac{n-k}{n}\right)\Gamma\left(\frac{n-k+2}{n}\right)}, & m+n\ \text{even}. \end{cases}$$

29. $$\int_0^\infty \left[e^{-x}\,\tan\frac{m\pi}{2n} - \frac{e^{-px}\,\sin\frac{m}{n}\pi}{\cosh x + \cos\frac{m}{n}\pi}\right]\cdot\frac{dx}{x}$$
$$= \begin{cases} \tan\left(\dfrac{m\pi}{2n}\right)\,\ln(2n) + 2\displaystyle\sum_{k=1}^{n-1}(-1)^{k-1}\,\sin\left(\frac{km}{n}\pi\right)\,\ln\frac{\Gamma\left(\frac{p+n+k}{2n}\right)}{\Gamma\left(\frac{p+k}{2n}\right)}, & m+n\ \text{odd}, \\ \tan\left(\dfrac{m\pi}{2n}\right)\ln(n) + 2\displaystyle\sum_{k=1}^{(n-1)/2}(-1)^{k-1}\,\sin\left(\frac{km}{n}\pi\right)\,\ln\frac{\Gamma\left(\frac{p+n-k}{n}\right)}{\Gamma\left(\frac{p+k}{n}\right)}, & m+n\ \text{even}. \end{cases}$$

30. $$\int_0^\infty \frac{1+e^{-x}}{\cosh x+\cos a}\cdot\frac{dx}{x^{1-p}} = 2\,\sec\,\left(\frac{a}{2}\right)\Gamma(p)\sum_{k=1}^{\infty}(-1)^{k-1}\frac{\cos\left(k-1/2\right)a}{k^p},\quad p>0.$$

31. $$\int_0^\infty \frac{x^q e^{-x/2}\,\cosh\frac{x}{2}}{\cosh x+\cos\lambda}\,dx = \frac{\Gamma(q+1)}{\cos(\lambda/2)}\sum_{k=1}^{\infty}(-1)^{k-1}\frac{\cos\left(k-1/2\right)\lambda}{k^{q+1}},\quad q>-1.$$

32. $$\int_0^\infty x\frac{e^{-x}-\cos a}{\cosh x-\cos a}\,dx = |a|\pi-\frac{a^2}{2}-\frac{\pi^2}{3}.$$

33. $$\int_0^\infty x^{2m+1}\frac{e^{-x}-\cos a\pi}{\cosh x-\cos a\pi}\,dx = 2\,(2m+1)!\sum_{k=1}^{\infty}\frac{\cos ka\pi}{k^{2m+2}}.$$

34. $$\int_0^\infty x\frac{1-e^{-nx}}{\sinh^2\,\frac{x}{2}}\,dx = \frac{2n\pi^2}{3}-4\sum_{k=1}^{n-1}\frac{n-k}{k^2}.$$

35. $$\int_0^\infty x\frac{1-(-1)^n\,e^{-nx}}{\cosh^2\,\frac{x}{2}}\,dx = \frac{n\pi^2}{3}+4\sum_{k=1}^{n-1}(-1)^k\frac{n-k}{k^2}.$$

36. $$\int_0^\infty x^2\frac{1-e^{-nx}}{\sinh^2\,\frac{x}{2}}\,dx = 8n\,\zeta(3)-8\sum_{k=1}^{n-1}\frac{n-k}{k^3}.$$

37. $$\int_0^\infty x^2e^x\frac{1-e^{-2nx}}{\sinh^2\,x}\,dx = 8n\sum_{k=1}^{\infty}\frac{1}{(2k-1)^3}-8\sum_{k=1}^{n-1}\frac{n-k}{(2k-1)^3}.$$

38. $$\int_0^\infty x^2\frac{1+(-1)^ne^{-nx}}{\cosh^2\,\frac{x}{2}}\,dx = 6n\,\zeta(3)-8\sum_{k=1}^{n-1}\frac{n-k}{k^3}.$$

39. $$\int_0^\infty x^3\frac{1-e^{-nx}}{\sinh^2\,\frac{x}{2}}\,dx = \frac{4}{15}n\pi^4-24\sum_{k=1}^{n-1}\frac{n-k}{k^4}.$$

40. $$\int_0^\infty x^3\frac{1+(-1)^ne^{-nx}}{\cosh^2\,\frac{x}{2}}\,dx = \frac{7}{30}n\pi^4+24\sum_{k=1}^{n-1}(-1)^k\frac{n-k}{k^4}.$$

41. $$\int_0^\infty e^{-x}\left[a-\frac{1}{2}+\frac{(1-e^{-x})(1-ax)-xe^{-x}}{4\,\sinh^2\,\frac{x}{2}}e^{(2-a)x}\right]\frac{dx}{x} = a-\frac{1}{2}+\ln\,\Gamma(a)-\frac{1}{2}\,\ln\left(2\pi\right),\quad a>0.$$

42. $\displaystyle\int_0^\infty \frac{e^{-2x}\,\tanh\frac{x}{2}}{x\,\cosh x}\,dx = 2\,\ln\,\frac{\pi}{2\sqrt{2}}.$

43. $\displaystyle\int_0^\infty x^{2\mu-1}e^{-\beta x^2}\,\sinh\gamma x\,dx = \frac{1}{2}\Gamma(2\mu)(2\beta)^{-\mu}\,\exp\left(\frac{\gamma^2}{8\beta}\right)\left[D_{-2\mu}\left(-\frac{\gamma}{\sqrt{2\beta}}\right) - D_{-2\mu}\left(\frac{\gamma}{\sqrt{2\beta}}\right)\right]$

$\Re\{\mu\} > -\frac{1}{2},\ \Re\{\beta\} > 0.$

44. $\displaystyle\int_0^\infty x^{2\mu-1}e^{-\beta x^2}\,\cosh\gamma x\,dx = \frac{1}{2}\Gamma(2\mu)(2\beta)^{-\mu}\,\exp\left(\frac{\gamma^2}{8\beta}\right)\left[D_{-2\mu}\left(-\frac{\gamma}{\sqrt{2\beta}}\right) + D_{-2\mu}\left(\frac{\gamma}{\sqrt{2\beta}}\right)\right],$

$\Re\{\mu\} > 0,\ \Re\{\beta\} > 0.$

45. $\displaystyle\int_0^\infty xe^{-\beta x^2}\,\sinh\gamma x\,dx = \frac{\gamma}{4\beta}\sqrt{\frac{\pi}{\beta}}\,\exp\left(\frac{\gamma^2}{4\beta}\right), \quad \Re\{\beta\} > 0.$

46. $\displaystyle\int_0^\infty xe^{-\beta x^2}\,\cosh\gamma x\,dx = \frac{\gamma}{4\beta}\sqrt{\frac{\pi}{\beta}}\,\exp\left(\frac{\gamma^2}{4\beta}\right)\operatorname{erf}\left(\frac{\gamma}{2\sqrt{\beta}}\right) + \frac{1}{2\beta}, \quad \Re\{\beta\} > 0.$

47. $\displaystyle\int_0^\infty x^2e^{-\beta\,x^2}\,\sinh\gamma x\,dx = \frac{\sqrt{\pi}(2\beta+\gamma^2)}{8\beta^2\sqrt{\beta}}\,\exp\left(\frac{\gamma^2}{4\beta}\right)\operatorname{erf}\left(\frac{\gamma}{2\sqrt{\beta}}\right) + \frac{\gamma}{4\beta^2}, \quad \Re\{\beta\} > 0.$

48. $\displaystyle\int_0^\infty x^2e^{-\beta\,x^2}\,\cosh\gamma x\,dx = \frac{\sqrt{\pi}(2\beta+\gamma^2)}{8\beta^2\sqrt{\beta}}\,\exp\left(\frac{\gamma^2}{4\beta}\right), \quad \Re\{\beta\} > 0.$