! For an efficient use of these tables, first read `HowTo.pdf`.

T3.21A. Integrands involving exponentials of exponential functions on the interval $(0, \infty)$.

1. $$\int_0^\infty \exp(-ae^{nx})\,dx = -\frac{1}{n}\,\mathrm{Ei}(-a), \quad n \ge 1,\ \Re\{a\} \ge 0,\ a \ne 0.$$

2. $$\int_0^\infty \left[\frac{a\exp(-c\,e^{ax})}{1-e^{-ax}} - \frac{b\exp(-c\,e^{bx})}{1-e^{-bx}}\right] dx = e^{-c}\,\ln\frac{b}{a}, \quad a>0,\ b>0,\ c>0.$$

3. $$\int_0^\infty \exp(-\beta e^{-x} - \mu x)\,dx = \beta^{-\mu}\gamma(\mu,\,\beta), \quad \Re\{\mu\} > 0.$$

4. $$\int_0^\infty \exp(-\beta e^{x} - \mu x)\,dx = \beta^{\mu}\Gamma(-\mu,\,\beta), \quad \Re\{\beta\} > 0.$$

5. $$\int_0^\infty (1-e^{-x})^{\nu-1}\,\exp\left(\beta e^{-x} - \mu x\right)dx = \mathrm{B}(\mu,\,\nu)\,\beta^{-(\mu+\nu)/2}\,e^{\beta/2}M_{(\nu-\mu)/2,\,(\nu+\mu-1)/2}(\beta),$$
$$\Re\{\mu\} > 0,\ \Re\{\nu\} > 0.$$

6. $$\int_0^\infty (1-e^{-x})^{\nu-1}\,\exp\left(-\beta e^{x} - \mu x\right)dx = \Gamma(\nu)\beta^{(\mu-1)/2}\,e^{-\beta/2}\,W_{(1-\mu-2\nu)/2,\,-\mu/2}(\beta),$$
$$\Re\{\beta >\}0,\ \Re\{\nu\} > 0.$$

7. $$\int_0^\infty (1-e^{-x})^{\nu-1}(1-\lambda e^{-x})^{-\varrho}\exp(\beta e^{-x} - \mu x)\,dx = \mathrm{B}(\mu,\,\nu)\Phi_1(\mu,\,\varrho,\,\nu,\,\lambda,\,\beta),$$
$$\Re\{\mu\} > 0,\ \Re\{\nu\} > 0,\ |\arg(1-\lambda)| < \pi.$$

8. $$\int_0^\infty (e^x-1)^{\nu-1}\exp\left[-\frac{\beta}{e^x-1} - \mu x\right]dx = \Gamma(\mu-\nu+1)\,e^{\beta/2}\,\beta^{(\nu-1)/2}\,W_{(\nu-2\mu-1)/2,\,-\nu/2}(\beta),$$
$$\Re\{\beta\} > 0,\ \Re\{\mu\} > \Re\{\nu\} - 1.$$
