T1.21. Integrand involving trigonometric functions and their powers.

1. $\displaystyle\int \sin^2 x\,dx = -\frac{1}{4}\sin 2x + \frac{1}{2}x = -\frac{1}{2}\sin x\,\cos x + \frac{1}{2}x.$

2. $\displaystyle\int \sin^3 x\,dx = \frac{1}{12}\cos 3x - \frac{3}{4}\cos x = \frac{1}{3}\cos^3 x - \cos x.$

3. $\displaystyle\int \sin^4 x\,dx = \frac{3x}{8} - \frac{\sin 2x}{4} + \frac{\sin 4x}{32} = -\frac{3}{8}\sin x\,\cos x - \frac{1}{4}\sin^3 x\,\cos x + \frac{3}{8}x.$

4. $\displaystyle\int \sin^5 x\,dx = -\frac{5}{8}\cos x + \frac{5}{48}\cos\,3x - \frac{1}{80}\cos\,5x = -\frac{1}{5}\sin^4 x\,\cos x + \frac{4}{15}\cos^3 x - \frac{4}{5}\cos x.$

5. $$\int \sin^6 x\,dx = \frac{5}{16}x - \frac{15}{64}\sin 2x + \frac{3}{64}\sin 4x - \frac{1}{192}\sin 6x$$
$$= -\frac{1}{6}\sin^5 x\,\cos x - \frac{5}{24}\sin^3 x\,\cos x - \frac{5}{16}\sin x\,\cos x + \frac{5}{16}x.$$

6. $$\int \sin^7 x\,dx = -\frac{35}{64}\cos x + \frac{7}{64}\cos 3x - \frac{7}{320}\cos 5x + \frac{1}{448}\cos 7x$$
$$= -\frac{1}{7}\sin^6 x\,\cos x - \frac{6}{35}\sin^4 x\,\cos x + \frac{8}{35}\cos^3 x - \frac{24}{35}\cos x.$$

7. $\displaystyle\int \cos^2 x\,dx = \frac{1}{4}\sin 2x + \frac{x}{2} = \frac{1}{2}\sin x\cos x + \frac{1}{2}x.$

8. $\displaystyle\int \cos^3 x\,dx = \frac{1}{12}\sin 3x + \frac{3}{4}\sin x = \sin x - \frac{1}{3}\sin^3 x.$

9. $\displaystyle\int \cos^4 x\,dx = \frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x = \frac{3}{8}x + \frac{3}{8}\sin x\,\cos x + \frac{1}{4}\sin x\,\cos^3 x.$

10. $\displaystyle\int \cos^5 x\,dx = \frac{5}{8}\sin x + \frac{5}{48}\sin 3x + \frac{1}{80}\sin 5x = \frac{4}{5}\sin x - \frac{4}{15}\sin^3 x + \frac{1}{5}\cos^4 x\,\sin x.$

11. $$\int \cos^6 x\,dx = \frac{5}{16}x + \frac{15}{64}\sin 2x + \frac{3}{64}\sin 4x + \frac{1}{192}\sin 6x$$
$$= \frac{5}{16}x + \frac{5}{16}\sin x\,\cos x + \frac{5}{24}\sin x\,\cos^3 x + \frac{1}{6}\sin x\,\cos^5\,x.$$

12. $$\int \cos^7 x\,dx = \frac{35}{64}\sin x + \frac{7}{64}\sin 3x + \frac{7}{320}\sin 5x + \frac{1}{448}\sin 7x$$
$$= \frac{24}{35}\sin x - \frac{8}{35}\sin^3 x + \frac{6}{35}\sin x\,\cos^4 x + \frac{1}{7}\sin x\,\cos^6 x.$$

13. $$\int \sin^{2n} x\,dx = \frac{1}{2^{2n}}\binom{2n}{n}x + \frac{(-1)^n}{2^{2n-1}}\sum_{k=0}^{n-1}(-1)^k\binom{2n}{k}\frac{\sin(2n-2k)x}{2n-2k}.$$

14. $$\int \sin^{2n+1} x\,dx = \frac{1}{2^{2n}}(-1)^{n+1}\sum_{k=0}^{n}(-1)^k\binom{2n+1}{k}\frac{\cos(2n+1-2k)x}{2n+1-2k}.$$

15. $$\int \cos^{2n} x\,dx = \frac{1}{2^{2n}}\binom{2n}{n}x + \frac{1}{2^{2n-1}}\sum_{k=0}^{n-1}\binom{2n}{k}\frac{\sin(2n-2k)x}{2n-2k}.$$

16. $$\int \cos^{2n+1} x\,dx = \frac{1}{2^{2n}}\sum_{k=0}^{n}\binom{2n+1}{k}\frac{\sin(2n-2k+1)x}{2n-2k+1}.$$

17. $$\begin{aligned}\int \sin^p x\,\cos^q x\,dx &= -\frac{\sin^{p-1} x\,\cos^{q+1} x}{q+1} + \frac{p-1}{q+1}\int \sin^{p-2} x\,\cos^{q+2} x\,dx\\ &= -\frac{\sin^{p-1} x\,\cos^{q+1} x}{p+q} + \frac{p-1}{p+q}\int \sin^{p-2} x\,\cos^{q} x\,dx\\ &= \frac{\sin^{p+1} x\,\cos^{q+1} x}{p+1} + \frac{p+q+2}{p+1}\int \sin^{p+2} x\,\cos^{q} x\,dx\\ &= \frac{\sin^{p+1} x\,\cos^{q-1} x}{p+1} + \frac{q-1}{p+1}\int \sin^{p+2} x\,\cos^{q-2} x\,dx\\ &= \frac{\sin^{p+1} x\,\cos^{q-1} x}{p+q} + \frac{q-1}{p+q}\int \sin^{p} x\,\cos^{q-2} x\,dx\\ &= -\frac{\sin^{p+1} x\,\cos^{q+1} x}{q+1} + \frac{p+q+2}{q+1}\int \sin^{p} x\,\cos^{q+2} x\,dx\\ &= \frac{\sin^{p-1} x\,\cos^{q-1} x}{p+q}\left\{\sin^2 x - \frac{q-1}{p+q-2}\right\}\\ &\quad + \frac{(p-1)(q-1)}{(p+q)(p+q-2)}\int \sin^{p-2} x\,\cos^{q-2} x\,dx.\end{aligned}$$

18. $$\begin{aligned}\int \cos^{2m} x\,dx = \frac{\sin x}{2m}&\left\{\cos^{2m-1} x + \sum_{k=1}^{m-1}\frac{(2m-1)(2m-3)\dots(2m-2k+1)}{2^k(m-1)(m-2)\dots(m-k)}\cos^{2m-2k-1} x\right\}\\ &+\frac{(2m-1)!!}{2^m m!}\,x.\end{aligned}$$

19. $$\int \cos^{2m+1} x\,dx = \frac{\sin x}{2m+1}\left\{\cos^{2m} x + \sum_{k=0}^{m-1}\frac{2^{k+1}m(m-1)\dots(m-k)}{(2m-1)(2m-3)\dots(2m-2k-1)}\cos^{2m-2k-2} x\right\}.$$

20. $\displaystyle\int \cos^p x\, \sin^{2n+1} x\, dx$

$$= -\frac{\cos^{p+1} x}{2n+p+1}\left\{\sin^{2n} x + \sum_{k=1}^{n} \frac{2^k n(n-1)\ldots(n-k+1)\sin^{2n-2k} x}{(2n+p-1)(2n+p-3)\ldots(2n+p-2k+1)}\right\}$$

$$p \neq -1, -3, \ldots, -(2n+1).$$

21. $\displaystyle\int \sin^p x\, \cos^{2n} x\, dx$

$$= \frac{\sin^{p+1} x}{2n+p}\left\{\cos^{2n-1} x + \sum_{k=1}^{n-1} \frac{(2n-1)(2n-3)\ldots(2n-2k+1)\cos^{2n-2k-1} x}{(2n+p-2)(2n+p-4)\ldots(2n+p-2k)}\right\}$$

$$+ \frac{(2n-1)!!}{(2n+p)(2n+p-2)\ldots(p+2)}\int \sin^p x\, dx, \qquad p \neq -2, -4, \ldots, -2n.$$

22. $\displaystyle\int \sin^{2m} x\, dx$

$$= -\frac{\cos x}{2m}\left\{\sin^{2m-1} x + \sum_{k=1}^{m-1} \frac{(2m-1)(2m-3)\ldots(2m-2k+1)}{2^k(m-1)(m-2)\ldots(m-k)}\sin^{2m-2k-1} x\right\} + \frac{(2m-1)!!}{2^m m!}x.$$

23. $\displaystyle\int \sin^{2m+1} x\, dx = -\frac{\cos x}{2m+1}\left\{\sin^{2m} x + \sum_{k=0}^{m-1} \frac{2^{k+1} m(m-1)\ldots(m-k)}{(2m-1)(2m-3)\ldots(2m-2k-1)}\sin^{2m-2k-2} x\right\}.$

24. $\displaystyle\int \sin x\, \cos^2 x\, dx = -\frac{1}{4}\left\{\frac{1}{3}\cos 3x + \cos x\right\} = -\frac{\cos^3 x}{3}.$

25. $\displaystyle\int \sin x\, \cos^3 x\, dx = -\frac{\cos^4 x}{4}.$

26. $\displaystyle\int \sin x\, \cos^4 x\, dx = -\frac{\cos^5 x}{5}.$

27. $\displaystyle\int \sin^2 x\, \cos x\, dx = -\frac{1}{4}\left\{\frac{1}{3}\sin 3x - \sin x\right\} = \frac{\sin^3 x}{3}.$

28. $\displaystyle\int \sin^2 x\, \cos^2 x\, dx = -\frac{1}{8}\left\{\frac{1}{4}\sin 4x - x\right\}.$

29. $\displaystyle\int \sin^2 x\, \cos^3 x\, dx = -\frac{1}{16}\left\{\frac{1}{5}\sin 5x + \frac{1}{3}\sin 3x - 2\,\sin x\right\}$

$$= \frac{\sin^3 x}{5}\left(\cos^2 x + \frac{2}{3}\right) = \frac{\sin^3 x}{5}\left(\frac{5}{3} - \sin^2 x\right).$$

30. $\displaystyle\int \sin^2 x\, \cos^4 x\, dx = \frac{x}{16} + \frac{1}{64}\sin 2x - \frac{1}{64}\sin 4x - \frac{1}{192}\sin 6x.$

31. $\displaystyle\int \sin^3 x\, \cos x\, dx = \frac{1}{8}\left(\frac{1}{4}\cos 4x - \cos 2x\right) = \frac{\sin^4 x}{4}.$

32. $\displaystyle\int \sin^3 x \, \cos^2 x \, dx = \frac{1}{16}\left(\frac{1}{5}\cos 5x - \frac{1}{3}\cos 3x - 2\,\cos x\right) = \frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x.$

33. $\displaystyle\int \sin^3 x \, \cos^3 x \, dx = \frac{1}{32}\left(\frac{1}{6}\cos\, 6x - \frac{3}{2}\cos 2x\right).$

34. $\displaystyle\int \sin^3 x \, \cos^4 x \, dx = \frac{1}{7}\cos^3 x\left(-\frac{2}{5} - \frac{3}{5}\sin^2 x + \sin^4 x\right).$

35. $\displaystyle\int \sin^4 x \, \cos x \, dx = \frac{\sin^5 x}{5}.$

36. $\displaystyle\int \sin^4 x \, \cos^2 x \, dx = \frac{1}{16}x - \frac{1}{64}\sin 2x - \frac{1}{64}\sin 4x + \frac{1}{192}\sin 6x.$

37. $\displaystyle\int \sin^4 x \, \cos^3 x \, dx = \frac{1}{7}\sin^3 x\left(\frac{2}{5} + \frac{3}{5}\cos^2 x - \cos^4 x\right).$

38. $\displaystyle\int \sin^4 x \, \cos^4 x \, dx = \frac{3}{128}x - \frac{1}{128}\sin 4x + \frac{1}{1024}\sin 8x.$

39. $\displaystyle\int \cos^p x \, \sin^{2n} x \, dx$

$$= -\frac{\cos^{p+1} x}{2n+p}\left\{\sin^{2n-1} x + \sum_{k=1}^{n-1}\frac{(2n-1)(2n-3)\dots(2n-2k+1)\sin^{2n-2k-1} x}{(2n+p-2)(2n+p-4)\dots(2n+p-2k)}\right\}$$
$$+\frac{(2n-1)!!}{(2n+p)(2n+p-2)\dots(p+2)}\int \cos^p x \, dx, \quad p \neq -2, -4, \dots, -2n.$$

40. $\displaystyle\int \sin^p x \cos^{2n+1} x \, dx = \frac{\sin^{p+1} x}{2n+p+1}\left\{\cos^{2n} x + \sum_{k=1}^{n}\frac{2^k n(n-1)\dots(n-k+1)\cos^{2n-2k} x}{(2n+p-1)(2n+p-3)\dots(2n+p-2k+1)}\right\},$

$p \neq -1, -3, \dots, -(2n+1).$

41. $\displaystyle\int \frac{dx}{\sin x \, \cos^{2m+1} x} = \sum_{k=1}^{m}\frac{1}{(2m-2k+2)\cos^{2m-2k+2} x} + \ln\,\tan\, x.$

42. $\displaystyle\int \frac{dx}{\sin x \, \cos^{2m} x} = \sum_{k=1}^{m}\frac{1}{(2m-2k+1)\cos^{2m-2k+1} x} + \ln\,\tan\,\frac{x}{2}.$

43. $\displaystyle\int \frac{\cos^m x}{\sin^2 x}\, dx = -\frac{\cos^{m-1} x}{\sin x} - (m-1)\int \cos^{m-2} x \, dx.$

44. $\displaystyle\int \frac{\sin^{2n+1} x}{\cos^m x}\, dx = \sum_{\substack{k=0\\ k\neq(m-1)/2}}^{n} (-1)^{k+1}\binom{n}{k}\frac{\cos^{2k-m+1} x}{2k-m+1} + s(-1)^{(m+1)/2}\binom{n}{\frac{m-1}{2}}\ln\,\cos x,$

where $s = 1$ for m odd and $m < 2n+1$; otherwise $s = 0$.

45. $$\int \frac{\cos^{2n+1} x}{\sin^m x}\, dx = \sum_{\substack{k=0\\ k\neq(m-1)/2}}^{n} (-1)^k \binom{n}{k} \frac{\sin^{2k-m+1} x}{2k-m+1} + s(-1)^{(m-1)/2}\binom{n}{\frac{m-1}{2}} \ln \sin x,$$

where $s = 1$ for m odd and $m < 2n+1$; otherwise $s = 0$.

46. $$\int \frac{dx}{\sin^{2m} x\, \cos^{2n} x} = \sum_{k=0}^{m+n-1} \binom{m+n-1}{k} \frac{\tan^{2k-2m+1} x}{2k-2m+1}.$$

47. $$\int \frac{dx}{\sin^{2m+1} x\, \cos^{2n+1} x} = \sum_{k=0}^{m+n} \binom{m+n}{k} \frac{\tan^{2k-2m} x}{2k-2m} + \binom{m+n}{m} \ln \tan x.$$

48. $$\int \frac{dx}{\sin x} = \ln \tan \frac{x}{2}.$$

49. $$\int \frac{dx}{\sin^2 x} = -\cot x.$$

50. $$\int \frac{dx}{\sin^3 x} = -\frac{1}{2}\frac{\cos x}{\sin^2 x} + \frac{1}{2} \ln \tan \frac{x}{2}.$$

51. $$\int \frac{dx}{\sin^4 x} = -\frac{\cos x}{3 \sin^3 x} - \frac{2}{3} \cot x = -\frac{1}{3}\cot^3 x - \cot x.$$

52. $$\int \frac{dx}{\sin^5 x} = -\frac{\cos x}{4 \sin^4 x} - \frac{3}{8}\frac{\cos x}{\sin^2 x} + \frac{3}{8} \ln \tan \frac{x}{2}.$$

53. $$\int \frac{dx}{\sin^6 x} = -\frac{\cos x}{5 \sin^5 x} - \frac{4}{15}\cot^3 x - \frac{4}{5}\cot x = -\frac{1}{5}\cot^5 x - \frac{2}{3}\cot^3 x - \cot x.$$

54. $$\int \frac{dx}{\sin^7 x} = -\frac{\cos x}{6 \sin^2 x}\left(\frac{1}{\sin^4 x} + \frac{5}{4 \sin^2 x} + \frac{15}{8}\right) + \frac{5}{16} \ln \tan \frac{x}{2}.$$

55. $$\int \frac{dx}{\sin^8 x} = -\left(\frac{1}{7}\cot^7 x + \frac{3}{5}\cot^5 x + \cot^3 x + \cot x\right).$$

56. $$\int \frac{dx}{\cos x} = \ln \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) = \ln \cot\left(\frac{\pi}{4} - \frac{x}{2}\right) = \ln \sqrt{\frac{1+\sin x}{1-\sin x}}.$$

57. $$\int \frac{dx}{\cos^2 x} = \tan x.$$

58. $$\int \frac{dx}{\cos^3 x} = \frac{1}{2}\frac{\sin x}{\cos^2 x} + \frac{1}{2} \ln \tan\left(\frac{\pi}{4} + \frac{x}{2}\right).$$

59. $$\int \frac{dx}{\cos^4 x} = \frac{\sin x}{3 \cos^3 x} + \frac{2}{3} \tan x = \frac{1}{3}\tan^3 x + \tan x.$$

60. $$\int \frac{dx}{\cos^5 x} = \frac{\sin x}{4 \cos^4 x} + \frac{3}{8}\frac{\sin x}{\cos^2 x} + \frac{3}{8} \ln \tan\left(\frac{x}{2} + \frac{\pi}{4}\right).$$

61. $$\int \frac{dx}{\cos^6 x} = \frac{\sin x}{5 \cos^5 x} + \frac{4}{15}\tan^3 x + \frac{4}{5}\tan x = \frac{1}{5}\tan^5 x + \frac{2}{3}\tan^3 x + \tan x.$$

62. $\int \frac{dx}{\cos^7 x} = \frac{\sin x}{6\ \cos^6 x} + \frac{5\ \sin x}{24\ \cos^4 x} + \frac{5\ \sin x}{16\ \cos^2 x} + \frac{5}{16} \ln\ \tan\left(\frac{x}{2} + \frac{\pi}{4}\right).$

63. $\int \frac{dx}{\cos^8 x} = \frac{1}{7}\ \tan^7 x + \frac{3}{5} \tan^5 x + \tan^3 x + \tan\ x.$

64. $\int \frac{\sin x}{\cos x}\, dx = -\ln\ \cos x.$

65. $\int \frac{\sin^2 x}{\cos x}\, dx = -\sin x + \ln\ \tan\left(\frac{\pi}{4} = \frac{x}{2}\right).$

66. $\int \frac{\sin^3 x}{\cos x}\, dx = -\frac{\sin^2 x}{2} - \ln\ \cos x = \frac{1}{2} \cos^2 x - \ln\ \cos x.$

67. $\int \frac{\sin^4 x}{\cos x}\, dx = -\frac{1}{3} \sin^3 x - \sin x + \ln\ \tan\left(\frac{x}{2} + \frac{\pi}{4}\right).$

68. $\int \frac{\sin^2 x\, dx}{\cos^2 x} = \frac{1}{\cos x}.$

69. $\int \frac{\sin^2 x\, dx}{\cos^2 x} = \tan\ x - x.$

70. $\int \frac{\sin^3 x\, dx}{\cos^2 x} = \cos x + \frac{1}{\cos x}.$

71. $\int \frac{\sin^4 x\, dx}{\cos^2 x} = \tan\ x + \frac{1}{2} \sin x\ \cos x - \frac{3}{2} x.$

72. $\int \frac{\sin x\, dx}{\cos^3 x} = \frac{1}{2\ \cos^2 x} = \frac{1}{2} \tan^2 x.$

73. $\int \frac{\sin^2 x\, dx}{\cos^3 x} = \frac{\sin x}{2\ \cos^2 x} - \frac{1}{2} \ln\ \tan\left(\frac{\pi}{4} + \frac{x}{2}\right).$

74. $\int \frac{\sin^3 x\, dx}{\cos^3 x} = \frac{1}{2} \frac{\sin x}{\cos^2 x} + \ln\ \cos\ x.$

75. $\int \frac{\sin^4 x\, dx}{\cos^3 x} = \frac{1}{2} \frac{\sin x}{\cos^2 x} + \sin x - \frac{3}{2} \ln\ \tan\left(\frac{x}{2} + \frac{\pi}{4}\right).$

76. $\int \frac{\sin x\, dx}{\cos^4 x} = \frac{1}{3\ \cos^3 x}.$

77. $\int \frac{\sin^2 x\, dx}{\cos^4 x} = \frac{1}{3} \tan^3 x.$

78. $\int \frac{\sin^3 x\, dx}{\cos^4 x} = -\frac{1}{\cos x} + \frac{1}{3\ \cos^3 x}.$

79. $\int \frac{\sin^4 x\, dx}{\cos^4 x} = \frac{1}{3} \tan^3 x - \tan\ x + x.$

80. $\displaystyle\int \frac{\cos x\, dx}{\sin x} = \ln\, \sin x.$

81. $\displaystyle\int \frac{\cos^2 x\, dx}{\sin x} = \cos x + \ln\, \tan \frac{x}{2}.$

82. $\displaystyle\int \frac{\cos^3 x\, dx}{\sin x} = \frac{\cos^2 x}{2} + \ln\, \sin x.$

83. $\displaystyle\int \frac{\cos^4 x\, dx}{\sin x} = \frac{1}{3} \cos^3 x + \cos x + \ln\, \tan \left(\frac{x}{2}\right).$

84. $\displaystyle\int \frac{\cos x}{\sin^2 x}\, dx = -\frac{1}{\sin x}.$

85. $\displaystyle\int \frac{\cos^2 x}{\sin^2 x}\, dx = -\cot\, x - x.$

86. $\displaystyle\int \frac{\cos^3 x}{\sin^2 x}\, dx = -\sin x - \frac{1}{\sin x}.$

87. $\displaystyle\int \frac{\cos^4 x}{\sin^2 x}\, dx = -\cot\, x - \frac{1}{2} \sin x\, \cos x - \frac{3}{2} x.$

88. $\displaystyle\int \frac{\cos x}{\sin^3 x}\, dx = -\frac{1}{2\, \sin^2 x}.$

89. $\displaystyle\int \frac{\cos^2 x}{\sin^3 x}\, dx = -\frac{\cos x}{2\, \sin^2 x} - \frac{1}{2} \ln\, \tan \frac{x}{2}.$

90. $\displaystyle\int \frac{\cos^3 x}{\sin^3 x}\, dx = -\frac{1}{2\, \sin^2 x} - \ln\, \sin x.$

91. $\displaystyle\int \frac{\cos^4 x}{\sin^3 x}\, dx = -\frac{1}{2} \frac{\cos x}{\sin^2 x} - \cos x - \frac{3}{2} \ln\, \tan \frac{x}{2}.$

92. $\displaystyle\int \frac{\cos x}{\sin^4 x}\, dx = -\frac{1}{3\, \sin^3 x}.$

93. $\displaystyle\int \frac{\cos^2 x}{\sin^4 x}\, dx = -\frac{1}{3} \cot^3 x.$

94. $\displaystyle\int \frac{\cos^3 x}{\sin^4 x}\, dx = \frac{1}{\sin x} - \frac{1}{3\, \sin^3 x}.$

95. $\displaystyle\int \frac{\cos^4 x}{\sin^4 x}\, dx = -\frac{1}{3} \cot^3 x + \cot\, x + x.$

96. $\displaystyle\int \frac{dx}{\sin\, x\, \cos\, x} = \ln\, \tan\, x.$

97. $\displaystyle\int \frac{dx}{\sin x\, \cos^2 x} = \frac{1}{\cos x} + \ln\, \tan \frac{x}{2}.$

98. $\displaystyle\int \frac{dx}{\sin x\,\cos^3 x} = \frac{1}{2\,\cos^2 x} + \ln\,\tan\,x.$

99. $\displaystyle\int \frac{dx}{\sin x\,\cos^4 x} = \frac{1}{\cos x} + \frac{1}{3\,\cos^3 x} + \ln\,\tan\,\frac{x}{2}.$

100. $\displaystyle\int \frac{dx}{\sin^2 x\,\cos x} = \ln\,\tan\left(\frac{\pi}{4} + \frac{x}{2}\right) - \csc\,x.$

101. $\displaystyle\int \frac{dx}{\sin^2 x\,\cos^2 x} = -2\,\cot\,2x.$

102. $\displaystyle\int \frac{dx}{\sin^2 x\,\cos^3 x} = \left(\frac{1}{2\,\cos^2 x} - \frac{3}{2}\right)\frac{1}{\sin x} + \frac{3}{2}\ln\left(\frac{\pi}{4} + \frac{x}{2}\right).$

103. $\displaystyle\int \frac{dx}{\sin^2 x\,\cos^4 x} = \frac{1}{3\,\sin x\,\cos^3 x} - \frac{8}{3}\cot\,2x.$

104. $\displaystyle\int \frac{dx}{\sin^3 x\,\cos x} = -\frac{1}{2\,\sin^2 x} + \ln\,\tan\,x.$

105. $\displaystyle\int \frac{dx}{\sin^3 x\,\cos^2 x} = -\frac{1}{\cos x}\left(\frac{1}{2\,\sin^2 x} - \frac{3}{2}\right) + \frac{3}{2}\ln\,\tan\,\frac{x}{2}.$

106. $\displaystyle\int \frac{dx}{\sin^3 x\,\cos^3 x} = -\frac{2\,\cos 2x}{\sin^2\,2x} + 2\,\ln\,\tan\,x.$

107. $\displaystyle\int \frac{dx}{\sin^3 x\,\cos^4 x} = \frac{2}{\cos x} + \frac{1}{3\,\cos^3 x} - \frac{\cos x}{2\,\sin^2 x} + \frac{5}{2}\ln\,\tan\,\frac{x}{2}.$

108. $\displaystyle\int \frac{dx}{\sin^4 x\,\cos x} = -\frac{1}{\sin x} - \frac{1}{3\,\sin^3\,x} + \ln\,\tan\left(\frac{x}{2} + \frac{\pi}{4}\right).$

109. $\displaystyle\int \frac{dx}{\sin^4 x\,\cos^2 x} = -\frac{1}{3\,\cos x\,\sin^3 x} - \frac{8}{3}\cot\,2x.$

110. $\displaystyle\int \frac{dx}{\sin^4 x\,\cos^3 x} = -\frac{2}{\sin x} - \frac{1}{3\,\sin^3\,x} + \frac{\sin x}{2\,\cos^2\,x} + \frac{5}{2}\,\ln\,\tan\left(\frac{x}{2} + \frac{\pi}{4}\right).$

111. $\displaystyle\int \frac{dx}{\sin^4 x\,\cos^4 x} = -8\,\cot\,2x - \frac{8}{3}\cot^3\,2x.$

112. $\displaystyle\int \frac{\sin^p x}{\cos^{2n} x}\,dx$

$$= \frac{\sin^{p+1} x}{2n-1}\left\{\sec^{2n-1} x + \sum_{k=1}^{n-1}\frac{(2n-p-2)(2n-p-4)\dots(2n-p-2k)}{(2n-3)(2n-5)\dots(2n-2k-1)}\sec^{2n-2k-1} x\right\}$$

$$+\frac{(2n-p-2)(2n-p-4)\dots(-p+2)(-p)}{(2n-1)!!}\int \sin^p x\,dx, \quad \text{for any real } p.$$

113. $\displaystyle\int \frac{dx}{\sin^{2m}\,x} = -\frac{\cos x}{2m-1}\left\{\csc^{2m-1} x + \sum_{k=1}^{m-1}\frac{2^k(m-1)(m-2)\dots(m-k)}{(2m-3)(2m-5)\dots(2m-2k-1)}\csc^{2m-2k-1} x\right\}$

114. $$\int \frac{dx}{\sin^{2m+1} x} = -\frac{\cos x}{2m} \left\{ \csc^{2m} x + \sum_{k=1}^{m-1} \frac{(2m-1)(2m-3)\dots(2m-2k+1)}{28^k (m-1)(m-2)\dots(m-k)} \csc^{2m-2k} x \right\} + \frac{(2m-1)!!}{2^m m!} \ln \tan \frac{x}{2}.$$

115. $$\int \frac{\sin^p x\,dx}{\cos^{2n+1} x} = \frac{\sin^{p+1}}{2n} \left\{ \sec^{2n} x + \sum_{k=1}^{n-1} \frac{(2n-p-1)(2n-p-3)\dots(2n-p-2k+1)}{2^k (n-1)(n-2)\dots(n-k)} \sec^{2n-2k} x \right\} + \frac{(2n-p-1)(2n-p-3)\dots(3-p)(1-p)}{2^n n!} \int \frac{\sin^p x}{\cos x}\,dx, \quad \text{for any real } p.$$

116. $$\int \frac{\sin^{2m+1} x\,dx}{\cos x} = -\sum_{k=1}^{m} \frac{\sin^{2k} x}{2k} - \ln \cos x.$$

117. $$\int \frac{\sin^{2m} x\,dx}{\cos x} = -\sum_{k=1}^{m} \frac{\sin^{2k-1} x}{2k-1} + \ln \tan\left(\frac{\pi}{4} + \frac{x}{2}\right).$$

118. $$\int \frac{dx}{\sin^{2m+1} x \,\cos x} = -\sum_{k=1}^{m} \frac{1}{(2m-2k+2)\sin^{2m-2k+2} x} + \ln \tan x.$$

119. $$\int \frac{dx}{\sin^{2m} x \,\cos x} = -\sum_{k=1}^{m} \frac{1}{(2m-2k+1)\sin^{2m-2k+1} x} + \ln \tan\left(\frac{\pi}{4} - \frac{x}{2}\right).$$

120. $$\int \frac{\sin^p x}{\cos^2 x}\,dx = \frac{\sin^{p-1} x}{\cos x} - (p-1) \int \sin^{p-2} x\,dx.$$

121. $$\int \frac{\cos^p x\,dx}{\sin^{2n} x} = \frac{\cos^{p+1} x}{2n-1} \left\{ \csc^{2n-1} x + \sum_{k=1}^{n-1} \frac{(2n-p-2)(2n-p-4)\dots(2n-p-2k)}{(2n-3)(2n-5)\dots(2n-2k-1)} \csc^{2n-2k-1} x \right\} + \frac{(2n-p-2)(2n-p-4)\dots(2-p)(-p)}{(2n-1)!!} \int \cos^p x\,dx, \quad \text{for any real } p.$$

122. $$\int \frac{dx}{\cos^{2m} x} = \frac{\sin x}{2m-1} \left\{ \sec^{2m-1} x + \sum_{k=1}^{m-1} \frac{2^k (m-1)(m-2)\dots(m-k)}{(2m-3)(2m-5)\dots(2m-2k-1)} \sec^{2m-2k-1} x \right\}.$$

123. $$\int \frac{dx}{\cos^{2m+1} x} = \frac{\sin x}{2m} \left\{ \sec^{2m} x + \sum_{k=1}^{m-1} \frac{(2m-1)(2m-3)\dots(2m-2k+1)}{2^k (m-1)(m-2)\dots(m-k)} \sec^{2m-2k} x \right\} + \frac{(2m-1)!!}{2^m m!} \ln \tan\left(\frac{\pi}{4} + \frac{x}{2}\right).$$

124. $$\int \frac{\cos^p x\,dx}{\sin^{2n+1} x} = -\frac{\cos^{p+1} x}{2n}\left\{\csc^{2n} x + \sum_{k=1}^{n-1} \frac{(2n-p-1)(2n-p-3)\dots(2n-p-2k+1)}{2^k(n-1)(n-2)\dots(n-k)}\csc^{2n-2k} x\right\}$$
$$+\frac{(2n-p-1)(2n-p-3)\dots(3-p)(1-p)}{2^n\cdot n!}\int \frac{\cos^p x}{\sin x}\,dx, \quad p \text{real.}$$

125. $$\int \frac{\cos^{2m+1} x\,dx}{\sin x} = \sum_{k=1}^{m} \frac{\cos^{2k} x}{2k} + \ln \sin x.$$

126. $$\int \frac{\cos^{2m} x\,dx}{\sin x} = \sum_{k=1}^{m} \frac{\cos^{2k-1} x}{2k-1} + \ln \tan \frac{x}{2}.$$

127. $$\int \tan^p x\,dx = \frac{\tan^{p-1} x}{p-1} - \int \tan^{p-2} x\,dx, \quad p \neq 1.$$

128. $$\int \tan^{2n+1} x\,dx = \sum_{k=1}^{n} (-1)^{n+k}\binom{n}{k}\frac{1}{2k\cos^{2k} x} - (-1)^n \ln \cos x$$
$$= \sum_{k=1}^{n} \frac{(-1)^{k-1}\tan^{2n-2k+2} x}{2n-2k+2} - (-1)^n \ln \cos x.$$

129. $$\int \tan^{2n} x\,dx = \sum_{k=1}^{n} (-1)^{k-1}\frac{\tan^{2n-2k+1} x}{2n-2k+1} + (-1)^n x.$$

130. $$\int \cot^p x\,dx = -\frac{\cot^{p-1} x}{p-1} - \int \cot^{p-2} x\,dx, \quad p \neq 1.$$

131. $$\int \cot^{2n+1} x\,dx = \sum_{k=1}^{n} (-1)^{n+k+1}\binom{n}{k}\frac{1}{2k\sin^{2k} x} + (-1)^n \ln \sin x$$
$$= \sum_{k=1}^{n} (-1)^k \frac{\cot^{2n-2k+2} x}{2n-2k+2} + (-1)^n \ln \sin x.$$

132. $$\int \cot^{2n} x\,dx = \sum_{k=1}^{n} (-1)^k \frac{\cot^{2n-2k+1} x}{2n-2k+1} + (-1)^n x.$$

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