! For an efficient use of these tables, first read `HowTo.pdf`.

T2.08B. Integrands of the form $\dfrac{1}{\sqrt{1-x^3}}$, $\dfrac{1}{\sqrt{x^3-1}}$, $\dfrac{x}{\sqrt{1-x^3}}$ and $\dfrac{x}{\sqrt{x^3-1}}$ on the intervals $(y,1)$ and $(1,y)$.

Notation used: $\beta = \arccos \dfrac{\sqrt{3}-1+y}{\sqrt{3}+1-y}$, $\gamma = \arccos \dfrac{\sqrt{3}+1-y}{\sqrt{3}-1+y}$.

1. $$\int_y^1 \frac{dx}{\sqrt{1-x^3}} = \frac{1}{(3)^{1/4}}\, F\left(\beta,\, \sin\frac{5\pi}{12}\right).$$

2. $$\int_1^y \frac{dx}{\sqrt{x^3-1}} = \frac{1}{(3)^{1/4}}\, F\left(\gamma,\, \sin\frac{\pi}{12}\right).$$

3. $$\int_y^1 \sqrt{1-x^3}\, dx = \frac{1}{5}\left\{(27)^{1/4}\, F\left(\beta,\, \sin\frac{5\pi}{12} - 2y\sqrt{1-y^3}\right)\right\}.$$

4. $$\int_y^1 \frac{x\, dx}{\sqrt{1-x^3}} = (3^{-1/4} - 3^{1/4})\, F\left(\beta,\, \sin\frac{5\pi}{12}\right) + 2\,(3)^{1/4}\, E\left(\beta,\, \sin\frac{5\pi}{12}\right) - \frac{2\sqrt{1-y^3}}{\sqrt{3}+1-y}.$$

5. $$\int_y^1 \frac{x^m\, dx}{\sqrt{1-x^3}} = \frac{2y^{m-2}\sqrt{1-y^3}}{2m-1} + \frac{2(m-2)}{2m-1}\int_y^1 \frac{x^{m-3}\, dx}{\sqrt{1-x^3}}.$$

6. $$\int_1^y \frac{x\, dx}{\sqrt{x^3-1}} = (3^{-1/4} + 3^{1/4})\, F\left(\gamma,\, \sin\frac{\pi}{12}\right) - 2\,(3)^{1/4}\, E\left(\gamma,\, \sin\frac{\pi}{12}\right) + \frac{2\sqrt{y^3-1}}{\sqrt{3}-1+y}.$$

7. $$\int_y^1 \frac{(1-x)\, dx}{(1+\sqrt{3}-x)^2\sqrt{1-x^3}} = \frac{2-\sqrt{3}}{(27)^{1/4}}\left[F\left(\beta,\, \sin\frac{5\pi}{12}\right) - E\left(\beta,\, \sin\frac{5\pi}{12}\right)\right].$$

8. $$\int_1^y \frac{(x-1)\, dx}{(1+\sqrt{3}-x)^2\sqrt{x^3-1}} = \frac{2(\sqrt{3}-2)}{\sqrt{3}}\,\frac{\sqrt{y^3-1}}{y^2-2y-2} - \frac{2-\sqrt{3}}{(27)^{1/4}}\, E\left(\gamma,\, \sin\frac{\pi}{12}\right).$$

9. $$\int_1^y \frac{(x-1)\,dx}{(1-\sqrt{3}-x)^2\sqrt{x^3-1}} = \frac{2+\sqrt{3}}{(27)^{1/4}}\left[F\left(\gamma,\,\sin\frac{\pi}{12}\right) - E\left(\gamma,\,\sin\frac{\pi}{12}\right)\right].$$

10. $$\int_y^1 \frac{(x^2+x+1)\,dx}{(x-1+\sqrt{3})^2\sqrt{1-x^3}} = \frac{1}{(3)^{1/4}}\,E\left(\beta,\,\sin\frac{5\pi}{12}\right).$$

11. $$\int_1^y \frac{(x^2+x+1)\,dx}{(\sqrt{3}+x-1)^2\sqrt{x^3-1}} = \frac{1}{(3)^{1/4}}\,E\left(\gamma,\,\sin\frac{\pi}{12}\right).$$

12. $$\int_1^y \frac{(x-1)\,dx}{(x^2+x+1)\sqrt{x^3-1}} = \frac{4}{(27)^{1/4}}\,E\left(\gamma,\,\sin\frac{\pi}{12}\right) - \frac{2+\sqrt{3}}{(27)^{1/4}}\,F\left(\gamma,\,\sin\frac{\pi}{12}\right) - \frac{2-\sqrt{3}}{\sqrt{3}}\,\frac{2(y-1)(\sqrt{3}+1-y)}{(\sqrt{3}-1+y)\sqrt{y^3-1}}.$$

13. $$\int_y^1 \frac{(1+\sqrt{3}-x)^2\,dx}{[(1+\sqrt{3}-x)^2-4\sqrt{3}p^2(1-x)]\sqrt{1-x^3}} = \frac{1}{(3)^{1/4}}\,\Pi\left(\beta,\,p^2,\,\sin\frac{5\pi}{12}\right).$$

14. $$\int_1^y \frac{(1-\sqrt{3}-x)^2\,dx}{[(1-\sqrt{3}-x)^2-4\sqrt{3}p^2(x-1)]\sqrt{x^3-1}} = \frac{1}{(3)^{1/4}}\,\Pi\left(\gamma,\,p^2,\,\sin\frac{\pi}{12}\right).$$