

! For an efficient use of these tables, first read [HowTo.pdf](#).

T2.24B. Integrands of the form $\sqrt{\frac{d-x}{(a-x)(b-x)(c-x)^3}}$, $\sqrt{\frac{c-x}{(a-x)(b-x)(d-x)^3}}$, $\sqrt{\frac{b-x}{(a-x)(c-x)(d-x)^3}}$, and $\sqrt{\frac{a-x}{(b-x)(c-x)(d-x)^3}}$, and similar expressions with cubes of one of the factors in the denominator, on the intervals (y, b) and (b, y) .

Notation used: $\kappa = \arcsin \sqrt{\frac{(a-c)(b-y)}{(b-c)(a-y)}}$, $\lambda = \arcsin \sqrt{\frac{(a-c)(y-b)}{(a-b)(y-c)}}$,

$$q = \sqrt{\frac{(b-c)(a-d)}{(a-c)(b-d)}}, \quad r = \sqrt{\frac{(a-b)(c-d)}{(a-c)(b-d)}}.$$

$$1. \int_y^b \sqrt{\frac{x-c}{(a-x)(b-x)(x-d)^3}} dx = \frac{2}{a-d} \sqrt{\frac{a-c}{b-d}} [F(\kappa, q) - E(\kappa, q)] + \frac{2}{b-d} \sqrt{\frac{(b-y)(y-c)}{(a-y)(y-d)}},$$

$$a > b > y \geq c > d.$$

$$2. \int_b^y \sqrt{\frac{x-c}{(a-x)(x-b)(x-d)^3}} dx = \frac{2}{a-d} \left[\sqrt{\frac{a-c}{b-d}} E(\lambda, r) - \frac{c-d}{b-d} \sqrt{\frac{(a-y)(y-b)}{(y-c)(y-d)}} \right],$$

$$a \geq y > b > c > d.$$

$$3. \int_y^b \sqrt{\frac{b-x}{(a-x)(x-c)(x-d)^3}} dx = \frac{2}{(a-d)(c-d)\sqrt{(a-c)(b-d)}} \\ \times [(a-c)(b-d)E(\kappa, q) - (a-b)(c-d)F(\kappa, q)] - \frac{2}{c-d} \sqrt{\frac{(b-y)(y-c)}{(a-y)(y-d)}},$$

$$a > b > y \geq c > d.$$

$$\begin{aligned}
4. \int_b^y \sqrt{\frac{x-b}{(a-x)(x-c)(x-d)^3}} dx &= \frac{2}{(a-d)(c-d)\sqrt{(a-c)(b-d)}} \\
&\times [(a-c)(b-d)E(\lambda, r) - (a-d)(b-c)F(\lambda, r)] - \frac{2}{a-d} \sqrt{\frac{(a-y)(y-b)}{(y-c)(y-d)}}, \\
&a \geq y > b > c > d.
\end{aligned}$$

$$\begin{aligned}
5. \int_y^b \sqrt{\frac{a-x}{(b-x)(x-c)(x-d)^3}} dx &= \frac{2}{c-d} \sqrt{\frac{a-c}{b-d}} E(\kappa, q) - \frac{2(a-d)}{(b-d)(c-d)} \sqrt{\frac{(b-y)(y-c)}{(a-y)(y-d)}}, \\
&a > b > y \geq c > d.
\end{aligned}$$

$$\begin{aligned}
6. \int_b^y \sqrt{\frac{a-x}{(x-b)(x-c)(x-d)^3}} dx &= \frac{2}{c-d} \sqrt{\frac{a-c}{b-d}} [F(\lambda, r) - E(\lambda, r)] + \frac{2}{b-d} \sqrt{\frac{(a-y)(y-b)}{(y-c)(y-d)}}, \\
&a \geq y > b > c > d.
\end{aligned}$$

$$\begin{aligned}
7. \int_y^b \sqrt{\frac{x-d}{(a-x)(b-x)(x-c)^3}} dx &= \frac{2}{b-c} \sqrt{\frac{b-d}{a-c}} [F(\kappa, q) - E(\kappa, q)] + \frac{2}{b-c} \sqrt{\frac{(b-y)(y-d)}{(a-y)(y-c)}}, \\
&a > b > y > c > d.
\end{aligned}$$

$$8. \int_b^y \sqrt{\frac{x-d}{(a-x)(x-b)(x-c)^3}} dx = \frac{2}{b-c} \sqrt{\frac{b-d}{a-c}} E(\lambda, r), \quad a \geq y > b > c > d.$$

$$\begin{aligned}
9. \int_y^b \sqrt{\frac{b-x}{(a-x)(x-c)^3(x-d)}} dx &= \frac{2}{d-c} \sqrt{\frac{b-d}{a-c}} E(\kappa, q) + \frac{2}{c-d} \sqrt{\frac{(b-y)(y-d)}{(a-y)(y-c)}}, \\
&a > b > y > c > d.
\end{aligned}$$

$$10. \int_b^y \sqrt{\frac{x-b}{(a-x)(x-c)^3(x-d)}} dx = \frac{2}{c-d} \sqrt{\frac{b-d}{a-c}} [F(\lambda, r) - E(\lambda, r)]. \quad a \geq y > b > c > d.$$

$$\begin{aligned}
11. \int_y^b \sqrt{\frac{a-x}{(b-x)(x-c)^3(x-d)}} dx &= \frac{2(a-b)}{(b-c)\sqrt{(a-c)(b-c)}} F(\kappa, q) - 2 \sqrt{\frac{(a-c)(b-d)}{(b-c)(c-d)}} E(\kappa, q) \\
&+ \frac{2(a-c)}{(b-c)(c-d)} \sqrt{\frac{(b-y)(y-d)}{(a-y)(y-c)}}, \quad a > b > y > c > d.
\end{aligned}$$

12.
$$\int_b^y \sqrt{\frac{a-x}{(x-b)(x-c)^3(x-d)}} dx = \frac{2\sqrt{(a-c)(b-d)}}{(b-c)(c-d)} E(\lambda, r) - \frac{2(a-d)}{(c-d)\sqrt{(a-c)(b-d)}} F(\lambda, r), \quad a \geq y > b > c > d.$$
13.
$$\int_y^b \sqrt{\frac{x-d}{(a-x)^3(b-x)(x-c)}} dx = \frac{2}{a-b} \sqrt{\frac{b-d}{a-c}} E(\kappa, q), \quad a > by \geq c > d.$$
14.
$$\int_b^y \sqrt{\frac{x-d}{(a-x)^3(x-b)(x-c)}} dx = \frac{2}{a-b} \sqrt{\frac{b-d}{a-c}} [F(\lambda, r) - E(\lambda, r)] + \frac{2}{a-b} \sqrt{\frac{(y-b)(y-d)}{(a-y)(y-c)}}, \quad a > y > b > c > d.$$
15.
$$\int_y^b \sqrt{\frac{x-c}{(a-x)^3(b-x)(x-d)}} dx = \frac{2\sqrt{(a-c)(b-d)}}{(a-b)(a-d)} E(\kappa, q) - \frac{2(c-d)}{(a-d)\sqrt{(a-c)(b-d)}} F(\kappa, q), \quad a > b > y \geq c > d.$$
16.
$$\int_b^y \sqrt{\frac{x-c}{(a-x)^3(x-b)(x-d)}} dx = \frac{2(b-c)}{(a-b)\sqrt{(a-c)(b-d)}} F(\lambda, r) - \frac{2\sqrt{(a-c)(b-d)}}{(a-b)(a-d)} E(\lambda, r) + \frac{2(a-c)}{(a-b)(a-d)} \sqrt{\frac{(y-b)(y-d)}{(a-y)(y-c)}}, \quad a > y > b > c > d.$$
17.
$$\int_y^b \sqrt{\frac{b-x}{(a-x)^3(x-c)(x-d)}} dx = \frac{2}{a-d} \sqrt{\frac{b-d}{a-c}} [F(\kappa, q) - E(\kappa, q)], \quad a > b > y \geq c > d.$$
18.
$$\int_b^y \sqrt{\frac{x-b}{(a-x)^3(x-c)(x-d)}} dx = \frac{-2}{a-d} \sqrt{\frac{b-d}{a-c}} E(\lambda, r) + \frac{2}{a-d} \sqrt{\frac{(y-b)(y-d)}{(a-y)(y-c)}}, \quad a \geq y > b > c > d.$$
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