! For an efficient use of these tables, first read `HowTo.pdf`.

T3.41A. Integrands involving product of exponentials, trigonometric functions and powers of trigonometric functions on the interval $(0, \infty)$.

1. $$\int_0^\infty e^{-px} \sin(qx + \lambda)\, dx = \frac{1}{p^2 + q^2}(q \cos\lambda + p \sin\lambda), \quad p > 0.$$

2. $$\int_0^\infty e^{-px} \cos(qx + \lambda)\, dx = \frac{1}{p^2 + q^2}(p \cos\lambda - q \sin\lambda), \quad p > 0.$$

3. $$\int_0^\infty e^{-x \cos t} \cos(t - x \sin t)\, dx = 1.$$

4. $$\int_0^\infty \frac{e^{-\beta x} \sin ax}{\sin bx}\, dx = \Re\left\{ \frac{1}{2bi} \left[\psi\left(\frac{a+b}{2b} - i\,\frac{\beta}{2b} \right) - \psi\left(\frac{b-a}{2b} - i\,\frac{\beta}{2b} \right) \right] \right\}, \quad \Re\{\beta\} > 0,\ b \neq 0.$$

5. $$\int_0^\infty \frac{e^{-2px} \sin[(2n+1)x]}{\sin x}\, dx = \frac{1}{2p} + \sum_{k=1}^{n} \frac{p}{p^2 + k^2}, \quad p > 0.$$

6. $$\int_0^\infty \frac{e^{-px} \sin 2nx}{\sin x}\, dx = 2p \sum_{k=0}^{n-1} \frac{1}{p^2 + (2k+1)^2}, \quad p > 0.$$

7. $$\int_0^\infty e^{-px} \cos[(2n+1)x] \tan x\, dx = \frac{2n+1}{p^2 + (2n+1)^2} + (-1)^n \sum_{k=0}^{n-1} \frac{(-1)^k\, 2\,(2k+1)}{p^2 + (2k+1)^2}, \quad p > 0.$$

8. $$\int_0^\infty e^{-\beta x} \sin^{2m} x\, dx = \frac{(2m)!}{\beta(\beta^2 + 2^2)(\beta^2 + 4^2)\ldots[\beta^2 + (2m)^2]}, \quad \Re\{\beta\} > 0.$$

9. $$\int_0^\infty e^{-\beta x} \sin^{2m+1} x\, dx = \frac{(2m+1)!}{(\beta^2 + 1^2)(\beta^2 + 3^2)\ldots[\beta^2 + (2m+1)^2]}, \quad \Re\{\beta\} > 0.$$

10. $$\int_0^\infty e^{-p\,x}\,\cos^{2m} x\,dx = \frac{(2m)!}{p(p^2+2^2)\dots[p^2+(2m)^2]} \times\left\{1+\frac{p^2}{2!}+\frac{p^2(p^2+2^2)}{4!}+\dots+\frac{p^2(p^2+2^2)\dots[p^2+(2m-2)^2]}{(2m)!}\right\},\quad p>0.$$

11. $$\int_0^\infty e^{-px}\,\cos^{2m+1} x\,dx = \frac{(2m+1)!p}{(p^2+1^2)(p^2+3^2)\dots[p^2+(2m+1)^2]} \times\left\{1+\frac{p^2+1^2}{3!}+\frac{(p^2+1^2)(p^2+3^2)}{5!}+\dots+\frac{(p^2+1^2)(p^2+3^2)\dots[p^2+(2m-1)^2]}{(2m+1)!}\right\},\quad p>0.$$

12. $$\int_0^\infty e^{-\beta x}\,sin^{2n}x\,\sin ax\,dx = -\frac{1}{(-4)^{n+1}(2n+1)}\left\{\binom{\frac{a}{2}+i\frac{\beta}{2}+n}{2n+1}^{-1}+\binom{\frac{a}{2}-i\frac{\beta}{2}+n}{2n+1}^{-1}\right\},\quad \Re\{\beta\}>0,\ a>0.$$

13. $$\int_0^\infty e^{-\beta x}\,\sin^{2n-1} x\,\sin ax\,dx = \frac{-i}{(-4)^{n+1}n}\left\{\binom{\frac{a}{2}-i\frac{\beta}{2}+n-\frac{1}{2}}{2n}^{-1}-\binom{\frac{a}{2}+i\frac{\beta}{2}+n-\frac{1}{2}}{2n}^{-1}\right\},\quad \Re\{\beta\}>0,\ a>0,\ n>0.$$

13. $$\int_0^\infty e^{-\beta x}\,\sin^{2n} x\,\cos ax\,dx = \frac{i\,(-1)^n}{(2n+1)2^{2n+2}}\left\{\binom{\frac{a}{2}+i\frac{\beta}{2}+n}{2n+1}^{-1}-\binom{\frac{a}{2}-i\frac{\beta}{2}+n}{2n+1}^{-1}\right\},\quad \Re\{\beta\}>0,\ a\ge 0.$$

14. $$\int_0^\infty e^{-\beta x}\,\sin^{2n-1} x\,\cos ax\,dx = \frac{(-1)^n}{2^{2n+2}n}\left\{\binom{\frac{a}{2}-i\frac{\beta}{2}+n-\frac{1}{2}}{2n}^{-1}+\binom{\frac{a}{2}+i\frac{\beta}{2}+n-\frac{1}{2}}{2n}^{-1}\right\},\quad \Re\{\beta\}>0,\ a\ge 0,\ n>0.$$

15. $$\int_0^\infty e^{-\beta x}\,\sin^n x\left\{\begin{matrix}\sin ax\\ \cos ax\end{matrix}\right\}dx = \frac{2^{-n-2}}{a(n+1)}\,e^{(1\mp 1+2n)\,i\,\pi/4}\left\{\binom{\frac{b+na+i\,\beta}{2a}}{n+1}^{-1}\pm\binom{\frac{b+na-i\,\beta}{2a}}{n+1}^{-1}\right\},\quad \Re\{\beta\}>0,\ a>0,\ b>0.$$

16. $$\int_0^\infty e^{-ax}\,\cos^2 mx\,dx = \frac{a^2+2m^2}{a\,(a^2+4m^2)}.$$

17. $$\int_0^\infty e^{-ax}\,\cos mx\,\cos nx\,dx = \frac{a\left(a^2+m^2+n^2\right)}{\left(a^2+(m-n)^2\right)\left(a^2+(m+n)^2\right)}.$$

18. $$\int_0^\infty e^{-ax}\,\sin mx\,\cos nx\,dx = \frac{m\left(a^2+m^2-n^2\right)}{\left(a^2+(m-n)^2\right)\left(a^2+(m+n)^2\right)}.$$

19. $$\int_0^\infty e^{-ax}\,\sin^2 mx\,dx = \frac{2m}{a\left(a^2+4m^2\right)}, \quad a>0.$$

20. $$\int_0^\infty e^{-ax}\,\sin mx\,\sin nx\,dx = \frac{2amn}{\left(a^2+(m-n)^2\right)\left(a^2+(m+n)^2\right)}.$$

21. $$\int_{-\infty}^\infty e^{-q^2x^2}\,\cos[p(x+\lambda)]\,dx = \frac{\sqrt{\pi}}{q}\,e^{-p^2/\left(4q^2\right)}\,\cos p\lambda.$$

22. $$\begin{aligned}\int_0^\infty e^{-a\,x^2}\,\sin bx\,dx &= \frac{b}{2a}\,\exp\left(-\frac{b^2}{4a}\right)\,{}_1F_1\left(\frac{1}{2};\,\frac{3}{2};\,\frac{b^2}{4a}\right)\\ &= \frac{b}{2a}\,{}_1F_1\left(1;\,\frac{3}{2};\,-\frac{b^2}{4a}\right)\\ &= \frac{b}{2a}\sum_{k=1}^\infty\frac{1}{(2k-1)!!}\left(-\frac{b^2}{2a}\right)^{k-1}, \quad a>0.\end{aligned}$$

23. $$\int_0^\infty e^{-\beta\,x^2}\,\cos bx\,dx = \frac{1}{2}\sqrt{\frac{\pi}{\beta}}\,\exp\left(-\frac{b^2}{4\beta}\right), \quad \Re\{\beta\}>0.$$

24. $$\begin{aligned}\int_0^\infty e^{-\beta\,x^2-\nu x}\,\sin bx\,dx = -\frac{i}{4}\sqrt{\frac{\pi}{\beta}}\biggl\{&\exp\frac{(\gamma-ib)^2}{4\beta}\left[1-\operatorname{erf}\left(\frac{\gamma-ib}{2\sqrt{\beta}}\right)\right]\\ &-\exp\frac{(\gamma+ib)^2}{4\beta}\left[1-\operatorname{erf}\left(\frac{\gamma+ib}{2\sqrt{\beta}}\right)\right]\biggr\}, \quad \Re\{\beta\}>0.\end{aligned}$$

25. $$\begin{aligned}\int_0^\infty e^{-\beta x^2-\gamma x}\,\cos bx\,dx = \frac{1}{4}\sqrt{\frac{\pi}{\beta}}\biggl\{&\exp\frac{(\gamma-ib)^2}{4\beta}\left[1-\operatorname{erf}\left(\frac{\gamma-ib}{2\sqrt{\beta}}\right)\right]\\ &+\exp\frac{(\gamma+ib)^2}{4\beta}\left[1-\operatorname{erf}\left(\frac{\gamma+ib}{2\sqrt{\beta}}\right)\right]\biggr\}, \quad \Re\{\beta\}>0.\end{aligned}$$

26. $$\int_0^\infty e^{-\beta\,x^2}\,\sin ax\,\sin bx\,dx = \frac{1}{4}\sqrt{\frac{\pi}{\beta}}\left\{e^{-(a-b)^2/(4\beta)} - e^{-(a+b)^2/(4\beta)}\right\}, \quad \Re\{\beta\}>0.$$

27. $$\int_0^\infty e^{-\beta x^2} \cos ax \cos bx \, dx = \frac{1}{4}\sqrt{\frac{\pi}{\beta}} \left\{ e^{-(a-b)^2/(4\beta)} + e^{-(a+b)^2/(4\beta)} \right\}, \quad \Re\{\beta\} > 0.$$

28. $$\int_0^\infty e^{-p x^2} \sin^2 ax \, dx = \frac{1}{4}\sqrt{\frac{\pi}{p}} \left(1 - e^{-a^2/p}\right), \quad p > 0.$$

29. $$\int_0^\infty \frac{e^{-p^2x^2} \sin[(2n+1)x]}{\sin x} \, dx = \frac{\sqrt{\pi}}{p} \left[\frac{1}{2} + \sum_{k=1}^{n} \exp\left(-\frac{k^2}{p^2}\right)\right], \quad p > 0.$$

30. $$\int_0^\infty \frac{e^{-p^2x^2} \cos[(4n+1)x]}{\cos x} \, dx = \frac{\sqrt{\pi}}{p} \left[\frac{1}{2} + \sum_{k=1}^{2n} (-1)^{k-n} \exp\left(-\frac{k^2}{p^2}\right)\right], \quad p > 0.$$

31. $$\int_0^\infty \frac{e^{-p x^2} \, dx}{1 - 2a \cos x + a^2} = \begin{cases} \dfrac{\sqrt{\pi/p}}{1-a^2} \left\{\dfrac{1}{2} + \displaystyle\sum_{k=1}^{\infty} a^k \exp\left(-\dfrac{k^2}{4p}\right)\right\}, & a^2 < 1, \ p > 0, \\ \dfrac{\sqrt{\pi/p}}{a^2-1} \left\{\dfrac{1}{2} + \displaystyle\sum_{k=1}^{\infty} a^{-k} \exp\left(-\dfrac{k^2}{4p}\right)\right\}, & a^2 > 1, \ p > 0. \end{cases}$$

32. $$\int_0^\infty \frac{\sin ax}{e^{\beta x} + 1} \, dx = \frac{1}{2a} - \frac{\pi}{2\beta \sinh(a\pi/\beta)}, \quad a > 0, \ \Re\{\beta\} > 0.$$

33. $$\int_0^\infty \frac{\sin ax}{e^{\beta x} - 1} \, dx = \frac{\pi}{2\beta} \coth\left(\frac{\pi a}{\beta}\right) - \frac{1}{2a}, \quad a > 0, \ \Re\{\beta\} > 0.$$

34. $$\int_0^\infty \frac{\sin ax}{e^x - 1} e^{x/2} \, dx = \frac{1}{2}\pi \tanh(a\pi), \quad a > 0.$$

35. $$\int_0^\infty \frac{\sin ax}{1 - e^{-x}} e^{-nx} \, dx = \frac{\pi}{2} - \frac{1}{2a} + \frac{\pi}{e^{2\pi a} - 1} - \sum_{k=1}^{n-1} \frac{a}{a^2 + k^2}, \quad a > 0.$$

36. $$\int_0^\infty e^{-\lambda x} \sin x \, \frac{dx}{x} = \arctan(1/\lambda).$$

37. $$\int_0^\infty \left(1 + e^{-x}\right) \sin x \, \frac{dx}{x} = \frac{3\pi}{4}.$$

38. $$\int_0^\infty \frac{1 - \cos ax}{e^x - 1} \, \frac{dx}{x} = \frac{1}{2} \ln\left[\frac{\sinh(\pi a)}{\pi a}\right], \quad a > 0.$$

39. $\displaystyle\int_0^\infty \frac{x\,e^{-2x}}{(1+e^{-x})^4}\,dx = \frac{1}{6}\left[\ln 2 - \frac{1}{4}\right].$

40. $\displaystyle\int_0^\infty \frac{\sin ax}{e^{\beta x} - e^{\gamma x}}\,dx = \frac{1}{2i\,(\beta-\gamma)}\left[\psi\left(\frac{\beta + i\,a}{\beta-\gamma}\right) - \psi\left(\frac{\beta - i\,a}{\beta-\gamma}\right)\right], \quad \Re\{\beta\} > 0,\, \Re\{\gamma\} > 0.$

41. $\displaystyle\int_0^\infty \frac{\sin ax\,dx}{e^{\beta x}(e^{-x}-1)} = \frac{i}{2}\left[\psi(\beta + i\,a) - \psi(\beta - i\,a)\right], \quad \Re\{\beta\} > -1.$

42. $\displaystyle\int_0^\infty e^{-\beta x}\left(1 - e^{-\gamma x}\right)^{\nu-1}\sin ax\,dx = -\frac{i}{2\gamma}\left[\mathrm{B}\left(\nu,\, \frac{\beta - i\,a}{\gamma}\right) - \mathrm{B}\left(\nu,\, \frac{\beta + i\,a}{\gamma}\right)\right],$
$\Re\{\beta\} > 0,\ \Re\{\gamma\} > 0,\ \Re\{\nu\} > 0,\ a > 0.$

43. $\displaystyle\int_0^\infty e^{-\beta x}\left(1 - e^{-\gamma x}\right)^{\nu-1}\cos ax\,dx = \frac{1}{2\gamma}\left[\mathrm{B}\left(\nu,\, \frac{\beta - i\,a}{\gamma}\right) + \mathrm{B}\left(\nu,\, \frac{\beta + i\,a}{\gamma}\right)\right],$
$\Re\{\beta\} > 0,\ \Re\{\gamma\} > 0,\ \Re\{\nu\} > 0,\ a > 0.$

44. $\displaystyle\int_0^\infty e^{-\beta\sqrt{\gamma^2+x^2}}\cos bx\,dx = \frac{\beta\gamma}{\sqrt{\beta^2+b^2}}\,K_1(\gamma\sqrt{\beta^2+b^2}), \quad \Re\{\beta\} > 0,\ \Re\{\gamma\} > 0.$
