! For an efficient use of these tables, first read `HowTo.pdf`.

T2.43B. Integrands involving powers of trigonometric functions and liner trigonometric functions on the interval $(0, \pi/2)$.

1. $$\int_0^{\pi/2} \sin^{\nu-2} x \, \sin \nu x \, dx = \frac{1}{1-\nu} \cos \frac{\nu\pi}{2}, \quad \Re\{\nu\} > 1.$$

2. $$\int_0^{\pi/2} \cos^{\nu-1} x \, \cos ax \, dx = \frac{\pi}{2^\nu \nu \, \mathrm{B}\left(\frac{\nu+a+1}{2}, \frac{\nu-a+1}{2}\right)}, \quad \Re\{\nu\} > 0.$$

3. $$\int_0^{\pi/2} \sin^{\nu-2} x \, \cos \nu x \, dx = \frac{1}{\nu-1} \sin \frac{\nu\pi}{2}, \quad \Re\{\nu\} > 1.$$

4. $$\int_0^{\pi/2} \cos^{\nu-2} x \, \sin \nu x \, dx = \frac{1}{\nu-1}, \quad \Re\{\nu\} > 1.$$

5. $$\int_0^{\pi/2} \cos^{n} x \, \sin nx \, dx = \frac{1}{2^{n+1}} \sum_{k=1}^{n} \frac{2^k}{k}.$$

6. $$\int_0^{\pi/2} \cos^{\nu-2} x \, \cos \nu x \, dx = 0, \quad \Re\{\nu\} > 1.$$

7. $$\int_0^{\pi/2} \cos^{n} x \, \cos nx \, dx = \frac{\pi}{2^{n+1}}, \quad \Re\{n\} > -1.$$

8. $$\int_0^{\pi/2} \cos^{p} x \, \sin[(p+2n)x] \, dx = (-1)^{n-1} \sum_{k=0}^{n-1} \frac{(-1)^k 2^k}{p+k+1} \binom{n-1}{k}, \quad n > 0.$$

8. $$\int_0^{\pi/2} \cos^{p+q-2} x \, \cos[(p-q)x] \, dx = \frac{\pi}{2^{p+q-1}(p+q-1)\, \mathrm{B}\,(p,\, q)}, \quad p+q > 1.$$

10. $$\int_0^{\pi/2} \cos^{p-1} x \, \sin ax \, \sin x \, dx = \frac{a\pi}{2^{p+1} p(p+1)\, \mathrm{B}\left(\frac{p+a}{2}+1,\, \frac{p-a}{2}+1\right)}.$$

11. $$\int_0^{\pi/2} \cos^n x \, \sin nx \, \sin 2mx \, dx = \int_0^{\frac{\pi}{2}} \cos^n x \, \cos nx \, \cos 2mx \, dx = \frac{\pi}{2^{n+2}} \binom{n}{m}.$$

12. $$\int_0^{\pi/2} \cos^{n-1} x \, \cos[(n+1)x] \, \cos 2mx \, dx = \frac{\pi}{2^{n+1}} \binom{n-1}{m-1}, \quad n > m-1.$$

13. $$\int_0^{\pi/2} \cos^{p+q} x \, \cos px \, \cos qx \, dx = \frac{\pi}{2^{p+q+2}} \left[1 + \frac{1}{(p+q+1)\, \mathrm{B}\,(p+1,\, q+1)}\right], \quad p+q > -1.$$

14. $$\int_0^{\pi/2} \cos^{p+q} x \, \sin px \, \sin qx \, dx = \frac{\pi}{2^{p+q+2}} \sum_{k=1}^{\infty} \binom{p}{k} \binom{q}{k} = \frac{\pi}{2^{p+q+2}} \left[\frac{\Gamma(p+q+1)}{\Gamma(p+1)\Gamma(q+1)} - 1\right],$$
$$p+q > -1.$$

15. $$\int_0^{\pi/2} \sin^{\mu-1} x \, \cos^{\nu-1} x \, \sin(\mu+\nu)x \, dx = \sin \frac{\mu\pi}{2} \, \mathrm{B}\,(\mu,\, \nu), \quad \Re\{\mu\} > 0,\ \Re\{\nu\} > 0.$$

16. $$\int_0^{\pi/2} \sin^{\mu-1} x \, \cos^{\nu-1} x \, \cos(\mu+\nu)x \, dx = \cos \frac{\mu\pi}{2} \, \mathrm{B}\,(\mu,\, \nu), \quad \Re\{\mu\} > 0,\ \Re\{\nu\} > 0.$$

17. $$\int_0^{\pi/2} \cos^{p+n-1} x \, \sin px \, \cos[(n+1)x] \, \sin x \, dx = \frac{\pi}{2^{p+n+1}} \frac{\Gamma(p+n)}{n!\Gamma(p)}, \quad p > -n.$$

18. $$\int_0^{\pi/2} \cos^{p+2n} x \, \sin px \, \tan x \, dx = \begin{cases} \dfrac{\pi}{2^{p+2n+1}\Gamma(p)} \displaystyle\sum_{k=0}^{\infty} \binom{n}{k} \frac{\Gamma(p+n-k)}{(n-k)!}, \\ \text{or} \\ \dfrac{p\pi}{2^{p+2n+1}} \dfrac{\Gamma(p+2n)}{\Gamma(n+1)\Gamma(p+n+1)}, \quad p > -2n. \end{cases}$$

19. $$\int_0^{\pi/2} \cos^{n-1} x \, \sin[(n+1)x] \, \cot x \, dx = \frac{\pi}{2}.$$

20. $$\int_0^{\pi/2} \tan^{\pm\mu} x \, \sin 2x \, dx = \frac{\mu\pi}{2} \csc \frac{\mu\pi}{2}, \quad 0 < \Re\{\mu\} < 2.$$

21. $$\int_0^{\pi/2} \tan^{\pm\mu} x \, \cos 2x \, dx = \mp \frac{\mu\pi}{2} \sec \frac{\mu\pi}{2}, \quad |\Re\{\mu\}| < 1.$$

22. $$\int_0^{\pi/2} \frac{\tan^{2\mu} x}{\cos x}\,dx = \int_0^{\pi/2} \frac{\cot^{2\mu} x}{\sin x}\,dx = \frac{\Gamma\left(\mu+\frac{1}{2}\right)\Gamma(-\mu)}{2\sqrt{\mu}}, \quad -\frac{1}{2} < \Re\{\mu\} < 1.$$

23. $$\int_0^{\pi/2} \tan^p x \sin^{q-2} x \sin qx\,dx = -\cos\frac{(p+q)\pi}{2}\,\mathrm{B}\,(p+q-1,\,1-p), \quad p+q>1>p.$$

24. $$\int_0^{\pi/2} \tan^p x \sin^{q-2} x \cos qx\,dx = \sin\frac{(p+q)\pi}{2}\,\mathrm{B}\,(p+q-1,\,1-p), \quad p+q>1>p.$$

25. $$\int_0^{\pi/2} \cot^p x \cos^{q-2} x \sin qx\,dx = \cos\frac{p\pi}{2}\,\mathrm{B}(p+q-1,\,1-p), \quad p+q>1>p.$$

26. $$\int_0^{\pi/2} \cot^p x \cos^{q-2} x \cos qx\,dx = \sin\frac{p\pi}{2}\,\mathrm{B}\,(p+q-1,\,1-p), \quad p+q>1>p.$$

27. $$\int_0^{\pi/2} \frac{\cos^{p-1} x \sin px}{\sin x}\,dx = \frac{\pi}{2}, \quad p>0.$$

28. $$\int_0^{\pi/2} \sin^{2n} x \sin(2m+1)x\,dx$$
$$= \begin{cases} \dfrac{(-1)^m 2^{n+1} n!(2n-1)!!}{(2n-2m-1)!!(2m+2n+1)!!}, & m \le n, \\ \dfrac{(-1)^n 2^{n+1} n!(2m-2n-1)!!(2n-1)!!}{(2m+2n+1)!!}, & m \ge n. \end{cases}$$
Note that $(2n-2m-1)!! = 1$ for $m=n$.

29. $$\int_0^{\pi/2} \sin^{2n+1} x \sin(2m+1)x\,dx$$
$$= \begin{cases} \dfrac{(-1)^m \pi}{2^{2n+2}}\dbinom{2n+1}{n-m}, & n \ge m, \\ 0, & n < m. \end{cases}$$

30. $$\int_0^{\pi/2} \cos^m x \sin nx\,dx = \sum_{k=0}^{r-1} \frac{m!}{(m-k)!}\,\frac{(m+n-2k-2)!!}{(m+n)!!} + s\,\frac{m!(n-m-2)!!}{(m+n)!!},$$
where $r = \begin{cases} m & \text{if } m \le n, \\ n & \text{if } m \ge n, \end{cases} \quad s = \begin{cases} 2 & \text{if } n-m = 4j+2 > 0, \\ 1 & \text{if } n-m = 2j+1 > 0, \\ 0 & \text{if } n-m = 4j \text{ or } n-m < 0, \end{cases}$

and j is a positive integer.

31. $$\int_0^{\pi/2} \cos^n x \cos mx \, dx = \begin{cases} \dfrac{s\, n!}{(m-n)(m-n+2)\dots(m+n)} & \text{if } n < m, \\ \dfrac{\pi}{2^{n+1}} \dbinom{n}{k} & \text{if } m \le n \text{ and } n-m = 2k, \\ \dfrac{n!}{(2k+1)!!\,(2m+2k+1)!!} & \text{if } m < n \text{ and } n-m = 2k+1, \end{cases}$$

$$\text{where } s = \begin{cases} 0 & \text{if } m-n = 2k, \\ 1 & \text{if } m-n = 4k+1, \\ 0 & \text{if } m-n = 2k, \\ -1 & \text{if } m-n = 4k-1. \end{cases}$$