! For an efficient use of these tables, first read `HowTo.pdf`.

T2.62A. Integrands involving logarithm functions on the interval $(0,1)$.

1. $$\int_0^1 \frac{dx}{a+\ln x} = e^{-a}\,\mathrm{Ei}\,(a), \quad a>0.$$

2. $$\int_0^1 \frac{dx}{a-\ln x} = -e^{a}\,\mathrm{Ei}\,(-a), \quad a>0.$$

3. $$\int_0^1 \frac{dx}{(a\ln x)^2} = -\frac{1}{a} + e^{-a}\,\mathrm{Ei}\,(a), \quad a>0.$$

4. $$\int_0^1 \frac{dx}{(a-\ln x)^2} = \frac{1}{a} + e^{a}\,\mathrm{Ei}\,(-a), \quad a>0.$$

5. $$\int_0^1 \frac{\ln x\,dx}{(a+\ln x)^2} = 1+(1-a)\,e^{-a}\,\mathrm{Ei}\,(a), \quad a>0.$$

6. $$\int_0^1 \frac{\ln x\,dx}{(a-\ln x)^2} = 1+(1+a)e^{a}\mathrm{Ei}\,(-a), \quad a>0.$$

7. $$\int_0^1 \frac{dx}{(a+\ln x)^n} = \frac{1}{(n-1)!}e^{-a}\mathrm{Ei}\,(a) - \frac{1}{(n-1)!}\sum_{k=1}^{n-1}(n-k-1)!\,a^{k-n}, \quad a>0,\ n>1 \text{ and odd.}$$

8. $$\int_0^1 \frac{dx}{(a-\ln x)^n} = \frac{(-1)^n}{(n-1)!}e^{a}\mathrm{Ei}\,(-a) + \frac{(-1)^{n-1}}{(n-1)!}\sum_{k=1}^{n-1}(n-k-1)!(-a)^{k-n}, \quad a>0,\ n \text{ odd.}$$

9. $$\int_0^1 \frac{dx}{a^2+(\ln x)^2} = \frac{1}{a}[\mathrm{Ci}\,(a)\sin a - \mathrm{Si}\,(a)\cos a], \quad a>0.$$

10. $$\int_0^1 \frac{dx}{a^2-(\ln x)^2} = \frac{1}{2a}[e^{-a}\,\mathrm{Ei}\,(a) - e^{a}\mathrm{Ei}\,(-a)], \quad a>0.$$

11. $\displaystyle\int_0^1 \frac{\ln\, x\, dx}{a^2+(\ln\, x)^2} = \mathrm{Ci}\,(a)\cos a + \mathrm{Si}\,\,(a)\sin a, \quad a>0.$

12. $\displaystyle\int_0^1 \frac{\ln\, x\, dx}{a^2-(\ln\, x)^2} = -\frac{1}{2}[e^{-a}\,\mathrm{Ei}\,\,(a)+e^{a}\mathrm{Ei}\,(-a)], \quad a>0.$

13. $\displaystyle\int_0^1 \frac{dx}{[a^2+(\ln\, x)^2]^2} = \frac{1}{2a^3}[\mathrm{Ci}\,(a)\sin a - \mathrm{Si}\,\,(a)\cos a] - \frac{1}{2a^2}[\mathrm{Ci}\,(a)\cos a + \mathrm{Si}\,\,(a)\sin a], \quad a>0.$

14. $\displaystyle\int_0^1 \frac{\ln\, x\, dx}{[a^2+(\ln\, x)^2]^2} = \frac{1}{2a}[\mathrm{Ci}\,(a)\sin a - \mathrm{Si}\,\,(a)\cos a] - \frac{1}{2a^2}, \quad a>0.$

15. $\displaystyle\int_0^1 \frac{dx}{[a^2-(\ln\, x)^2]^2} = \infty.$

16. $\displaystyle\int_0^1 \frac{\ln\, x\, dx}{[a^2-(\ln\, x)^2]^2} = \infty.$

17. $\displaystyle\int_0^1 \frac{dx}{a^4-(\ln\, x)^4} = -\frac{1}{4a^3}\left[e^{a}\mathrm{Ei}\,(-a) - e^{-a}\,\mathrm{Ei}\,\,(a) - 2\,\,\mathrm{Ci}\,(a)\sin a + 2\,\,\mathrm{Si}\,\,(a)\cos a\right], \quad a>0.$

18. $\displaystyle\int_0^1 \frac{\ln\, x\, dx}{a^4-(\ln\, x)^4} = -\frac{1}{4a^2}\left[e^{a}\mathrm{Ei}\,(-a) + e^{-a}\,\mathrm{Ei}\,\,(a) - 2\,\,\mathrm{Ci}\,(a)\cos a - 2\,\,\mathrm{Si}\,\,(a)\sin a\right], \quad a>0.$

19. $\displaystyle\int_0^1 \frac{(\ln\, x)^2\, dx}{a^4-(\ln\, x)^4} = -\frac{1}{4a}\left[e^{a}\mathrm{Ei}\,(-a) - e^{-a}\,\mathrm{Ei}\,\,(a) + 2\,\,\mathrm{Ci}\,(a)\sin a - 2\,\,\mathrm{Si}\,\,(a)\cos a\right], \quad a>0.$

20. $\displaystyle\int_0^1 \frac{(\ln\, x)^3\, dx}{a^4-(\ln\, x)^4} = -\frac{1}{4}\left[e^{a}\mathrm{Ei}\,(-a) + e^{-a}\,\mathrm{Ei}\,\,(a) + 2\,\,\mathrm{Ci}\,(a)\cos a + 2\,\,\mathrm{Si}\,\,(a)\sin a\right], \quad a>0.$

21. $\displaystyle\int_0^1 \left(\ln\,\frac{1}{x}\right)^{\mu-1}\, dx = \Gamma(\mu), \quad \Re\{\mu\}>0.$

22. $\displaystyle\int_0^1 \frac{dx}{\left(\ln\,\frac{1}{x}\right)^{\mu}} = \frac{\pi}{\Gamma(\mu)}\,\csc\mu\pi, \quad \Re\{\mu\}<1.$

23. $\displaystyle\int_0^1 \sqrt{\ln(1/x)}\,dx = \frac{\sqrt{\pi}}{2}.$

24. $\displaystyle\int_0^1 \frac{dx}{\sqrt{\ln(1/x)}} = \sqrt{\pi}.$