! For an efficient use of these tables, first read `HowTo.pdf`.

T2.65A. Integrands involving logarithm functions and algebraic functions on the intervals $(0,1)$ and $(0,1/\sqrt{2})$.

1. $\displaystyle\int_0^1 \frac{x^{2n}\ln x}{\sqrt{1-x^2}}\,dx = \frac{(2n-1)!!}{(2n)!!}\,\frac{\pi}{2}\left(\sum_{k=1}^{2n}\frac{(-1)^{k-1}}{k} - \ln 2\right).$

2. $\displaystyle\int_0^1 \frac{x^{2n+1}\ln x}{\sqrt{1-x^2}}\,dx = \frac{(2n)!!}{(2n+1)!!}\left(\ln 2 + \sum_{k=1}^{2n+1}\frac{(-1)^{k}}{k}\right).$

3. $\displaystyle\int_0^1 x^{2n}\sqrt{1-x^2}\ln x\,dx = \frac{(2n-1)!!}{(2n+2)!!}\,\frac{\pi}{2}\left(\sum_{k=1}^{2n}\frac{(-1)^{k-1}}{k} - \frac{1}{2n+2} - \ln 2\right).$

4. $\displaystyle\int_0^1 x^{2n+1}\sqrt{1-x^2}\,\ln x\,dx = \frac{(2n)!!}{(2n+3)!!}\left(\ln 2 + \sum_{k=1}^{2n+1}\frac{(-1)^{k}}{k} - \frac{1}{2n+3}\right).$

5. $\displaystyle\int_0^1 \ln x\,\sqrt{(1-x^2)^{2n-1}}\,dx = -\frac{(2n-1)!!}{4\,(2n)!!}\,\pi\,[\psi(n+1) + \ln 4 + \gamma_e].$

6. $\displaystyle\int_0^1 \frac{\ln x\,dx}{\sqrt{1-x^2}} = -\frac{\pi}{2}\ln 2.$

7. $\displaystyle\int_0^1 \sqrt{1-x^2}\ln x\,dx = -\frac{\pi}{8} - \frac{\pi}{4}\ln 2.$

8. $\displaystyle\int_0^1 x\sqrt{1-x^2}\ln x\,dx = \frac{1}{3}\ln 2 - \frac{4}{9}.$

9. $\displaystyle\int_0^1 \frac{\ln x\,dx}{\sqrt{x(1-x^2)}} = -\frac{\sqrt{2\pi}}{8}\left[\Gamma\left(\frac{1}{4}\right)\right]^2.$

10. $\displaystyle\int_0^1 \frac{x\ln x}{\sqrt{1-x^4}}\,dx = -\frac{\pi}{8}\ln 2.$

11. $\displaystyle\int_0^1 \frac{\ln x\,dx}{x^{1/3}(1-x^2)^{2/3}} = -\frac{1}{8}\left[\Gamma\left(\frac{1}{3}\right)\right]^3.$

12. $\displaystyle\int_0^1 \frac{\ln x\,dx}{\left(1-x^3\right)^{1/3}} = -\frac{\pi}{3\sqrt{3}}\left(\ln 3+\frac{\pi}{3\sqrt{3}}\right).$

13. $\displaystyle\int_0^1 \frac{x\ln x\,dx}{\left(1-x^3\right)^{2/3}} = \frac{\pi}{3\sqrt{3}}\left(\frac{\pi}{3\sqrt{3}}-\ln 3\right).$

14. $\displaystyle\int_0^1 \frac{x^{4n+1}\ln x}{\sqrt{1-x^4}}\,dx = \frac{(2n-1)!!}{(2n)!!}\,\frac{\pi}{8}\left(\sum_{k=1}^{2n}\frac{(-1)^{k-1}}{k}-\ln 2\right).$

15. $\displaystyle\int_0^1 \frac{x^{4n+3}\ln x}{\sqrt{1-x^4}}\,dx = \frac{(2n)!!}{4\,(2n+1)!!}\left(\ln 2+\sum_{k=1}^{2n+1}\frac{(-1)^k}{k}\right).$

16. $\displaystyle\int_0^1 (1-x^2)^{n-1/2}\ln x\,dx = -\frac{(2n-1)!!}{(2n)!!}\,\frac{\pi}{4}\left[2\ln 2+\sum_{k=1}^{n}\frac{1}{k}\right].$

17. $\displaystyle\int_0^1 \frac{\ln x}{\sqrt[n]{1-x^{2n}}}\,dx = -\frac{\pi\mathrm{B}\left(\frac{1}{2n},\frac{1}{2n}\right)}{8n^2\sin\frac{\pi}{2n}}, \quad n>1.$

18. $\displaystyle\int_0^1 \frac{\ln x\,dx}{x^{(n-1)/n}\,\left(1-x^2\right)^{1/n}} = -\frac{\pi\mathrm{B}\left(\frac{1}{2n},\frac{1}{2n}\right)}{8\sin\frac{\pi}{2n}}.$

19. $\displaystyle\int_0^{1/\sqrt{2}} \frac{\ln x\,dx}{\sqrt{1-x^2}} = -\frac{\pi}{4}\ln 2-\frac{1}{2}\mathbf{G}.$