! For an efficient use of these tables, first read `HowTo.pdf`.

T1.09. Integrand involving $1 \pm x^n$.

1. $\displaystyle\int \frac{dx}{1+x} = \ln(1+x).$

2. $\displaystyle\int \frac{dx}{1+x^2} = \arctan x = -\operatorname{arccot}(1/x).$

3. $\displaystyle\int \frac{dx}{1+x^3} = \frac{1}{3}\ln\frac{1+x}{\sqrt{1-x+x^2}} + \frac{1}{\sqrt{3}}\arctan\frac{x\sqrt{3}}{2-x}.$

4. $\displaystyle\int \frac{dx}{1+x^4} = \frac{1}{4\sqrt{2}}\ln\frac{1+x\sqrt{2}+x^2}{1-x\sqrt{2}+x^2} + \frac{1}{2\sqrt{2}}\arctan\frac{x\sqrt{2}}{1-x^2}$

$$\text{or} \quad = \frac{1}{4\sqrt{2}}\ln\frac{1+x\sqrt{2}+x^2}{1-x\sqrt{2}+x^2} + \frac{1}{2\sqrt{2}}\left[\arctan\frac{x}{\sqrt{2}-x} + \arctan\frac{x}{\sqrt{2}+x}\right].$$

5. $\displaystyle\int \frac{dx}{1+x^n} = -\frac{2}{n}\sum_{k=0}^{\lfloor n/2\rfloor-1} P_k \cos\left(\frac{2k+1}{n}\right)\pi + \frac{2}{n}\sum_{k=0}^{\lfloor n/2\rfloor-1} Q_k \sin\left(\frac{2k+1}{n}\right)\pi, \qquad n>0 \text{ even},$

$$= \frac{1}{n}\ln(1+x) - \frac{2}{n}\sum_{k=0}^{\lfloor n-3\rfloor/2} P_k \cos\left(\frac{2k+1}{n}\right)\pi + \frac{2}{n}\sum_{k=0}^{\lfloor n-3\rfloor/2} Q_k \sin\left(\frac{2k+1}{n}\right)\pi, \quad n>0 \text{ odd}.$$

where

$$P_k = \frac{1}{2}\ln\left(x^2 - 2x\cos\frac{2k+1}{n}\pi + 1\right) \text{ and } Q_k = \arctan\frac{x\sin\frac{2k+1}{n}\pi}{1-x\cos\frac{2k+1}{n}\pi} = \arctan\frac{x-\cos\frac{2k+1}{n}\pi}{\sin\frac{2k+1}{n}\pi}.$$

6. $\displaystyle\int \frac{dx}{1-x} = -\ln(1-x).$

7. $\displaystyle\int \frac{dx}{1-x^2} = \frac{1}{2}\ln\frac{1+x}{1-x} = \operatorname{arctanh} x, \qquad -1<x<1.$

8. $\displaystyle\int \frac{dx}{x^2-1} = \frac{1}{2}\ln\frac{x-1}{x+1} = -\operatorname{arccoth} x, \qquad x>1,\ x<-1.$

9. $\displaystyle\int \frac{dx}{1-x^3} = \frac{1}{3}\ln\frac{\sqrt{1+x+x^2}}{1-x} + \frac{1}{\sqrt{3}}\arctan\frac{x\sqrt{3}}{2+x}.$

10. $\displaystyle\int \frac{dx}{1-x^4} = \frac{1}{4}\ln\frac{1+x}{1-x} + \frac{1}{2}\arctan x = \frac{1}{2}(\operatorname{arctanh} x + \arctan x).$

11. $$\int \frac{dx}{1-x^n} = \frac{1}{n}\ln\frac{1+x}{1-x} - \frac{2}{n}\sum_{k=1}^{\frac{n}{2}-1} P_k \cos\frac{2k}{n}\pi + \frac{2}{n}\sum_{k=1}^{\frac{n}{2}-1} Q_k \sin\frac{2k}{n}\pi, \qquad \text{for } n>0 \text{ even,}$$

where $P_k = \frac{1}{2}\ln\left(x^2 - 2x\cos\frac{2k}{n}\pi + 1\right), \quad Q_k = \arctan\frac{x-\cos\frac{2k}{n}\pi}{\sin\frac{2k}{n}\pi}.$

12. $$\int \frac{dx}{1-x^n} = -\frac{1}{n}\ln(1-x) + \frac{2}{n}\sum_{k=0}^{\frac{n-3}{2}} P_k \cos\frac{2k+1}{n}\pi + \frac{2}{n}\sum_{k=0}^{\frac{n-3}{2}} Q_k \sin\frac{2k+1}{n}\pi \qquad \text{for } n>0 \text{ odd,}$$

where $P_k = \frac{1}{2}\ln\left(x^2 + 2x\cos\frac{2k+1}{n}\pi + 1\right), \quad Q_k = \arctan\frac{x+\cos\frac{2k+1}{n}\pi}{\sin\frac{2k+1}{n}\pi}.$

13. $$\int \frac{x\,dx}{1+x} = x - \ln(1+x).$$

14. $$\int \frac{x\,dx}{1+x^2} = \frac{1}{2}\ln(1+x^2).$$

15. $$\int \frac{x\,dx}{1+x^3} = -\frac{1}{6}\ln\frac{(1+x)^2}{1-x+x^2} + \frac{1}{\sqrt{3}}\arctan\frac{2x-1}{\sqrt{3}}.$$

16. $$\int \frac{x\,dx}{1+x^4} = \frac{1}{2}\arctan x^2.$$

17. $$\int \frac{x\,dx}{1-x} = -\ln(1-x) - x.$$

18. $$\int \frac{x\,dx}{1-x^2} = -\frac{1}{2}\ln(1-x^2).$$

19. $$\int \frac{x\,dx}{1-x^3} = -\frac{1}{6}\ln\frac{(1-x)^2}{1+x+x^2} - \frac{1}{\sqrt{3}}\arctan\frac{2x+1}{\sqrt{3}}.$$

20. $$\int \frac{x\,dx}{1-x^4} = \frac{1}{4}\ln\frac{1+x^2}{1-x^2}.$$

21. $$\int \frac{x^{m-1}\,dx}{1+x^{2n}} = -\frac{1}{2n}\sum_{k=1}^{n}\cos\frac{m\pi(2k-1)}{2n}\ln\left\{1-2x\cos\frac{2k-1}{2n}\pi + x^2\right\}$$
$$+\frac{1}{n}\sum_{k=1}^{n}\sin\frac{m\pi(2k-1)}{2n}\arctan\frac{x-\cos\frac{2k-1}{2n}\pi}{\sin\frac{2k-1}{2n}\pi} \qquad \text{for } m<2n;\ m,n\in \mathrm{N}.$$

22. $$\int \frac{x^{m-1}\,dx}{1+x^{2n+1}} = (-1)^{m+1}\frac{\ln(1+x)}{2n+1} - \frac{1}{2n+1}\sum_{k=1}^{n}\cos\frac{m\pi(2k-1)}{2n+1}\ln\left\{1-2x\cos\frac{2k-1}{2n+1}\pi + x^2\right\}$$
$$+\frac{2}{2n+1}\sum_{k=1}^{n}\sin\frac{m\pi(2k-1)}{2n+1}\arctan\frac{x-\cos\frac{2k-1}{2n+1}\pi}{\sin\frac{2k-1}{2n+1}\pi} \qquad \text{for } m\le 2n;\ m,n\in \mathrm{N}.$$

23. $$\int \frac{x^{m-1}\,dx}{1-x^{2n}} = \frac{1}{2n}\{(-1)^{m+1}\ \ln(1+x) - \ln(1-x)\} - \frac{1}{2n}\sum_{k=1}^{n-1}\cos\frac{km\pi}{n}\ln\left(1-2x\,\cos\frac{k\pi}{n}+x^2\right)$$
$$+\frac{1}{n}\sum_{k=1}^{n-1}\sin\frac{km\pi}{n}\arctan\frac{x-\cos\frac{k\pi}{n}}{\sin\frac{k\pi}{n}} \quad \text{for } m<2n;\ m,n\in \mathrm{N}.$$

24. $$\int \frac{x^{m-1}\,dx}{1-x^{2n+1}} = -\frac{1}{2n+1}\ln(1-x)$$
$$+(-1)^{m+1}\frac{1}{2n+1}\sum_{k=1}^{n}\cos\frac{m\pi(2k-1)}{2n+1}\ln\left(1+2x\,\cos\frac{2k-1}{2n+1}\pi+x^2\right)$$
$$+(-1)^{m+1}\frac{2}{2n+1}\sum_{k=1}^{n}\sin\frac{m\pi(2k-1)}{2n+1}\arctan\frac{x+\cos\frac{2k-1}{2n+1}\pi}{\sin\frac{2k-1}{2n+1}\pi} \qquad \text{for } m\le 2n;\ m,n\in \mathrm{N}.$$

25. $$\int \frac{x^m\,dx}{1-x^{2n}} = \frac{1}{2}\int\frac{x^m\,dx}{1-x^n}+\frac{1}{2}\int\frac{x^m\,dx}{1+x^n}.$$

26. $$\int \frac{x^m\,dx}{(1+x^2)^n} = -\frac{1}{2n-m-1}\cdot\frac{x^{m-1}}{(1+x^2)^{n-1}}+\frac{m-1}{2n-m-1}\int\frac{x^{m-2}\,dx}{(1+x^2)^n}.$$

27. $$\int \frac{x^m}{1+x^2}\,dx = \frac{x^{m-1}}{m-1}-\int\frac{x^{m-2}}{1+x^2}\,dx.$$

28. $$\int \frac{x^m\,dx}{(1-x^2)^n} = \frac{1}{2n-m-1}\frac{x^{m-1}}{(1-x^2)^{n-1}}-\frac{m-1}{2n-m-1}\int\frac{x^{m-2}\,dx}{(1-x^2)^n}$$
$$=\frac{1}{2n-2}\frac{x^{m-1}}{(1-x^2)^{n-1}}-\frac{m-1}{2n-2}\int\frac{x^{m-2}\,dx}{(1-x^2)^{n-1}}.$$

29. $$\int \frac{x^m\,dx}{1-x^2} = -\frac{x^{m-1}}{m-1}+\int\frac{x^{m-2}\,dx}{1-x^2}.$$

30. $$\int \frac{dx}{x^m(1+x^2)^n} = -\frac{1}{m-1}\frac{1}{x^{m-1}(1+x^2)^{n-1}}-\frac{2n+m-3}{m-1}\int\frac{dx}{x^{m-2}(1+x^2)^n}.$$

31. $$\int \frac{dx}{x(1+x^2)^n} = \frac{1}{2n-2}\frac{1}{(1+x^2)^{n-1}}+\int\frac{dx}{x(1+x^2)^{n-1}}.$$

32. $$\int \frac{dx}{x(1+x^2)} = \ln\frac{x}{\sqrt{1+x^2}}.$$

33. $$\int \frac{dx}{x^m(1+x^2)} = -\frac{1}{(m-1)x^{m-1}}-\int\frac{dx}{x^{m-2}(1+x^2)}.$$

34. $$\int \frac{dx}{(1+x^2)^n} = \frac{1}{2n-2}\frac{x}{(1+x^2)^{n-1}}+\frac{2n-3}{2n-2}\int\frac{dx}{(1+x^2)^{n-1}}.$$

35. $$\int \frac{dx}{(1+x^2)^n} = \frac{x}{2n-1}\sum_{k=1}^{n-1}\frac{(2n-1)(2n-3)(2n-5)\dots(2n-2k+1)}{2^k(n-1)(n-2)\dots(n-k)(1+x^2)^{n-k}}+\frac{(2n-3)!!}{2^{n-1}(n-1)!}\arctan x.$$

36. $\displaystyle\int \frac{dx}{x^m(1-x^2)^n} = -\frac{1}{(m-1)x^{m-1}(1-x^2)^{n-1}} + \frac{2n+m-3}{m-1}\int \frac{dx}{x^{m-2}(1-x^2)^n}.$

37. $\displaystyle\int \frac{dx}{x(1-x^2)^n} = \frac{1}{2(n-1)(1-x^2)^{n-1}} + \int \frac{dx}{x(1-x^2)^{n-1}}$

38. $\displaystyle\int \frac{dx}{x(1-x^2)} = \ln \frac{x}{\sqrt{1-x^2}}.$

39. $\displaystyle\int \frac{dx}{(1-x^2)^n} = \frac{1}{2n-2}\frac{x}{(1-x^2)^{n-1}} + \frac{2n-3}{2n-2}\int \frac{dx}{(1-x^2)^{n-1}}.$

40. $\displaystyle\int \frac{dx}{(1-x^2)^n} = \frac{x}{2n-1}\sum_{k=1}^{n-1}\frac{(2n-1)(2n-3)(2n-5)\dots(2n-2k+1)}{2^k(n-1)(n-2)\dots(n-k)(1-x^2)^{n-k}} + \frac{(2n-3)!!}{2^n\cdot(n-1)!}\ln\frac{1+x}{1-x}.$