

! For an efficient use of these tables, first read [HowTo.pdf](#).

T1.07. Integrand involving $a + bx^4 \equiv X_4$ and $a^4 \pm x^4$.

Notation used: $B = \left(\frac{a}{b}\right)^{1/4}$, $B' = \left(-\frac{a}{b}\right)^{1/4}$.

1. $\int \frac{dx}{X_4} = \begin{cases} \frac{B}{4a\sqrt{2}} \left\{ \ln \frac{x^2 + Bx\sqrt{2} + B^2}{x^2 - Bx\sqrt{2} + B^2} + 2 \arctan \frac{Bx\sqrt{2}}{B^2 - x^2} \right\} & \text{for } ab > 0, \\ \frac{B'}{4a} \left\{ \ln \frac{x + B'}{x - B'} + 2 \arctan \frac{x}{B'} \right\} & \text{for } ab < 0. \end{cases}$
2. $\int \frac{x dx}{X_4} = \begin{cases} \frac{1}{2\sqrt{ab}} \arctan x^2 \sqrt{\frac{b}{a}} & \text{for } ab > 0, \\ \frac{1}{4\sqrt{-ab}} \ln \frac{a + x^2\sqrt{-ab}}{a - x^2\sqrt{-ab}} & \text{for } ab < 0. \end{cases}$
3. $\int \frac{x^2 dx}{X_4} = \begin{cases} \frac{1}{4bB\sqrt{2}} \left\{ \ln \frac{x^2 - Bx\sqrt{2} + B^2}{x^2 + Bx\sqrt{2} + B^2} + 2 \arctan \frac{Bx\sqrt{2}}{B^2 - x^2} \right\} & \text{for } ab > 0, \\ -\frac{1}{4bB'} \left\{ \ln \frac{x + B'}{x - B'} - 2 \arctan \frac{x}{B'} \right\} & \text{for } ab < 0. \end{cases}$
4. $\int \frac{x^3 dx}{X_4} = \frac{1}{4b} \ln X_4.$
5. $\int \frac{x^n dx}{X_4^m} = \begin{cases} \frac{x^{n+1}}{4a(m-1)X_4^{m-1}} + \frac{4m-n-5}{4a(m-1)} \int \frac{x^n dx}{X_4^{m-1}}, \\ \text{or} \\ \frac{x^{n-3}}{X_4^{m-1}(n+1-4m)b} - \frac{(n-3)a}{b(n+1-4m)} \int \frac{x^{n-4} dx}{X_4^m}. \end{cases}$
6. $\int \frac{dx}{X_4^2} = \frac{x}{4aX_4} + \frac{3}{4a} \int \frac{dx}{X_4}.$
7. $\int \frac{x dx}{X_4^2} = \frac{x^2}{4aX_4} + \frac{1}{2a} \int \frac{x dx}{X_4}.$
8. $\int \frac{x^2 dx}{X_4^2} = \frac{x^3}{4aX_4} + \frac{1}{4a} \int \frac{x^2 dx}{X_4}.$

$$9. \int \frac{x^3 dx}{X_4^2} = \frac{x^4}{4a X_4} = -\frac{1}{4b X_4}.$$

$$10. \int \frac{dx}{x X_4} = \frac{\ln x}{a} - \frac{\ln X_4}{4a} = \frac{1}{4a} \ln \frac{x^4}{X_4}.$$

$$11. \int \frac{dx}{x^2 X_4} = -\frac{1}{ax} - \frac{b}{a} \int \frac{x^2 dx}{X_4}.$$

$$12. \int \frac{dx}{x X_4^m} = \frac{1}{a} \int \frac{dx}{x X_4^{m-1}} - \frac{b}{a} \int \frac{dx}{x^{-3} X_4^m}.$$

$$13. \int \frac{dx}{x^n X_4^m} = -\frac{1}{(n-1)ax^{n-1} X_4^{m-1}} - \frac{b(4m+n-5)}{(n-1)a} \int \frac{dx}{x^{n-4} X_4^m}.$$

$$14. \int \frac{dx}{a^4 + x^4} = \frac{1}{2a^3\sqrt{2}} \left[\frac{1}{2} \ln \left(\frac{x^2 + ax\sqrt{2} + a^2}{x^2 - ax\sqrt{2} + a^2} \right) + \arctan \frac{ax\sqrt{2}}{a^2 - x^2} \right].$$

$$15. \int \frac{dx}{a^4 - x^4} = \frac{1}{2a^3} \left[\frac{1}{2} \ln \frac{a+x}{a-x} + \arctan \frac{x}{a} \right].$$

$$16. \int \frac{x dx}{a^4 + x^4} = \frac{1}{2a^2} \arctan \frac{x^2}{a^2}.$$

$$17. \int \frac{x dx}{a^4 - x^4} = \frac{1}{4a^2} \ln \frac{a^2 + x^2}{a^2 - x^2}.$$

$$18. \int \frac{x^2 dx}{a^4 + x^4} = \frac{1}{2a\sqrt{2}} \left[\frac{1}{2} \ln \frac{x^2 - ax\sqrt{2} + a^2}{x^2 + ax\sqrt{2} + a^2} + \arctan \frac{ax\sqrt{2}}{a^2 - x^2} \right].$$

$$19. \int \frac{x^2 dx}{a^4 - x^4} = \frac{1}{2a} \left[\frac{1}{2} \ln \frac{a+x}{a-x} - \arctan \frac{x}{a} \right].$$

$$20. \int \frac{x^3 dx}{a^4 \pm x^4} = \pm \frac{1}{4} \ln (a^4 \pm x^4).$$