! For an efficient use of these tables, first read `HowTo.pdf`.

T2.25B. Integrands of the form $\sqrt{\dfrac{x^2+a^2}{x^2\pm b^2}}$ and $\sqrt{\dfrac{b^2-x^2}{a^2\pm x^2}}$ on the intervals (y,b) and (b,y).

Notation used: $\delta = \arccos\dfrac{y}{b}$, $\varepsilon = \arccos\dfrac{b}{y}$,

$$\zeta = \arcsin\frac{a}{b}\sqrt{\frac{b^2-y^2}{a^2-y^2}}, \quad \kappa = \arcsin\frac{a}{y}\sqrt{\frac{y^2-b^2}{a^2-b^2}},$$

$$r = \frac{b}{\sqrt{a^2+b^2}}, \quad s = \frac{a}{\sqrt{a^2+b^2}} \quad t = \frac{b}{a}.$$

1. $\displaystyle\int_y^b \sqrt{\frac{a^2+x^2}{b^2-x^2}}\,dx = \sqrt{a^2+b^2}E(\delta,\, r), \quad b > y \geq 0.$

2. $\displaystyle\int_b^y \sqrt{\frac{a^2+x^2}{x^2-b^2}}\,dx = \sqrt{a^2+b^2}\left\{F(\varepsilon,\, s) - E(\varepsilon,\, s)\right\} + \frac{1}{y}\sqrt{(y^2+a^2)(y^2-b^2)}, \quad y > b > 0.$

3. $\displaystyle\int_y^b \sqrt{\frac{b^2-x^2}{a^2+x^2}}\,dx = \sqrt{a^2+b^2}\left\{F(\delta,\, r) - E(\delta,\, r)\right\}, \quad b > y \geq 0.$

4. $\displaystyle\int_b^y \sqrt{\frac{x^2-b^2}{a^2+x^2}}\,dx = \frac{1}{y}\sqrt{(a^2+y^2)(y^2-b^2)} - \sqrt{a^2+b^2}E(\varepsilon,\, s), \quad y > b > 0.$

5. $\displaystyle\int_y^b \sqrt{\frac{b^2-x^2}{a^2-x^2}}\,dx = aE(\zeta,\, t) - \frac{a^2-b^2}{a}F(\zeta,\, t) - y\sqrt{\frac{b^2-y^2}{a^2-y^2}}, \quad a > b > y \geq 0.$

6. $\displaystyle\int_b^y \sqrt{\frac{x^2-b^2}{a^2-x^2}}\,dx = aE(\kappa,\, q) - \frac{b^2}{a}F(\kappa,\, q) - \frac{1}{y}\sqrt{(a^2-y^2)(y^2-b^2)}, \quad a \geq y > b > 0.$

7. $$\int_y^b \sqrt{\frac{a^2-x^2}{b^2-x^2}}\,dx = a\left\{E(\zeta,\,t) - \frac{y}{a}\sqrt{\frac{b^2-y^2}{a^2-y^2}}\right\}, \quad a > b > y \geq 0.$$

8. $$\int_b^y \sqrt{\frac{a^2-x^2}{x^2-b^2}}\,dx = a\,\{F(\kappa,\,q) - E(\kappa,\,q)\} + \frac{1}{y}\sqrt{(a^2-y^2)(y^2-b^2)}, \quad a \geq y > b > 0.$$