! For an efficient use of these tables, first read `HowTo.pdf`.

T3.50A. Integrands involving logarithm functions and rational functions on the interval $(0, \infty)$.

1. $\displaystyle\int_0^\infty \ln x \frac{dx}{(a^2+b^2x^2)^n} = \frac{\Gamma(n-\frac{1}{2})\sqrt{\pi}}{4(n-1)!\, a^{2n-1}b}\left[2\ln\frac{a}{2b} - \psi\left(n-\frac{1}{2}\right) - \gamma_e\right], \quad a>0,\, b>0.$

2. $\displaystyle\int_0^\infty \frac{\ln x\, dx}{a^2+b^2x^2} = \frac{\pi}{2ab}\ln\frac{a}{b}, \quad ab>0.$

3. $\displaystyle\int_0^\infty \frac{\ln px}{q^2+x^2}\, dx = \frac{\pi}{2q}\ln pq, \quad p>0,\, q>0.$

4. $\displaystyle\int_0^\infty \frac{\ln x\, dx}{a^2-b^2x^2} = -\frac{\pi^2}{4ab}, \quad ab>0.$

5. $\displaystyle\int_0^\infty \frac{\ln x\, dx}{(x+\beta)(x+\gamma)} = \frac{(\ln\beta)^2-(\ln\gamma)^2}{2(\beta-\gamma)}, \quad |\arg\beta|<\pi,\ |\arg\gamma|<\pi.$

6. $\displaystyle\int_0^\infty \frac{\ln x}{x+a}\frac{dx}{x-1} = \frac{\pi^2+(\ln a)^2}{2(a+1)}, \quad a>0.$

7. $\displaystyle\int_0^\infty \frac{\ln x\, dx}{x^2+2xa\cos t+a^2} = \frac{t\ln a}{a\sin t}, \quad a>0,\ 0<t<\pi.$

8. $\displaystyle\int_0^\infty \frac{1+x^2}{(1-x^2)^2}\ln x\, dx = 0.$

9. $\displaystyle\int_0^\infty \frac{1-x^2}{(1+x^2)^2}\ln x\, dx = -\frac{\pi}{2}.$

10. $\displaystyle\int_0^\infty \frac{\ln x\, dx}{(a^2+b^2x^2)(1+x^2)} = \frac{b\pi}{2a(b^2-a^2)}\ln\frac{a}{b}, \quad ab>0.$

11. $$\int_0^\infty \frac{\ln\, x}{x^2+a^2} \cdot \frac{dx}{1+b^2x^2} = \frac{\pi}{2(1-a^2b^2)} \left(\frac{1}{a} \ln\, a + b \ln\, b\right), \quad a>0,\, b>0.$$

12. $$\int_0^\infty \frac{x^2 \ln\, x\, dx}{(a^2+b^2x^2)(1+x^2)} = \frac{a\pi}{2b(b^2-a^2)} \ln\, \frac{b}{a}, \quad ab>0.$$

13. $$\int_0^\infty \ln\, x \frac{(1-x)\, x^{n-2}}{1-x^{2n}}\, dx = -\frac{\pi^2}{4n^2}\, \tan^2 \frac{\pi}{2n}, \quad n>1.$$

14. $$\int_0^\infty \ln\, x \frac{(1-x^2)\, x^{m-1}}{1-x^{2n}}\, dx = -\frac{\pi^2 \sin \frac{m+1}{n}\pi \sin \frac{\pi}{n}}{4n^2 \sin^2 \frac{m\pi}{2n}\, \sin^2\left(\frac{m+2}{2n}\pi\right)}.$$

15. $$\int_0^\infty \ln\, x \frac{(1-x^2)\, x^{n-2}}{1-x^{2n}}\, dx = -\frac{\pi^2}{4n^2}\, \tan^2 \frac{\pi}{n}, \quad n>2.$$
