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! For an efficient use of these tables, first read `HowTo.pdf`.

T3.20C. Integrands involving exponential functions on the interval $(-\infty, \infty)$.

1. $$\int_{-\infty}^{\infty} \frac{e^{-px}}{1+e^{-qx}}\,dx = \frac{\pi}{|q|}\,\csc\frac{p\pi}{q}, \quad q > p > 0, \text{ or } 0 > p > q.$$

2. $$\int_{-\infty}^{\infty} \frac{e^{-\mu x}\,dx}{b-e^{-x}} = \pi b^{\mu-1}\,\cot(\mu\pi), \quad b > 0,\ 0 < \Re\{\mu\} < 1.$$

3. $$\int_{-\infty}^{\infty} \frac{e^{-\mu x}\,dx}{b+e^{-x}} = \pi b^{\mu-1}\csc(\mu\pi), \quad |\arg b| < \pi,\ 0 < \Re\{\mu\} < 1.$$

4. $$\int_{-\infty}^{\infty} \frac{e^{-\mu x}\,dx}{1-e^{-x}} = \pi\cot\pi\mu, \quad 0 < \Re\{\mu\} < 1.$$

5. $$\int_{-\infty}^{\infty} \frac{e^{-\mu x}\,dx}{(1+e^{-x})^{\nu}} = B(\mu,\,\nu-\mu), \quad 0 < \Re\{\mu\} < \Re\{\nu\}.$$

6. $$\int_{-\infty}^{\infty} \frac{e^{-\mu x}\,dx}{(e^{\beta/\gamma}+e^{-x/\gamma})^{\nu}} = \gamma\exp\left[\beta\left(\mu-\frac{\nu}{\gamma}\right)\right]\mathrm{B}(\gamma\mu,\,\nu-\gamma\mu),$$
$$\Re\{\nu/\gamma\} > \Re\{\mu\} > 0,\ |\Im\{\beta\}| < \pi\,\Re\{\gamma\}.$$

7. $$\int_{-\infty}^{\infty} \frac{e^{-\mu x}\,dx}{(e^{\beta}+e^{-x})^{\nu}(e^{\gamma}+e^{-x})^{\rho}} = \exp[\gamma(\mu-\rho)-\beta\nu]\mathrm{B}(\mu,\,\nu+\rho-\mu)\,{}_2F_1(\nu,\,\mu;\,\nu+\rho;\,1-e^{\nu-\beta}),$$
$$|\Im\{\beta\}| < \pi,\ |\Im\{\gamma\}| < \pi,\ 0 < \Re\{\mu\} < \Re\{\nu+\rho)\}.$$

8. $$\int_{-\infty}^{\infty} \frac{e^{-\mu x}\,dx}{(\beta+e^{-x})(\gamma+e^{-x})} = \frac{\pi(\beta^{\mu-1}-\gamma^{\mu-1})}{\gamma-\beta}\csc(\mu\pi),$$
$$|\arg\beta| < \pi,\ |\arg\gamma| < \pi,\ \beta\neq\gamma,\ 0 < \Re\{\mu\} < 2.$$

9. $$\int_{-\infty}^{\infty} \frac{(1+e^{-x})^{\nu}-1}{(1+e^{-x})^{\mu}}\,dx = \psi(\mu)-\psi(\mu-\nu), \quad \Re\{\mu\} > \Re\{\nu\} > 0.$$

10. $$\int_{-\infty}^{\infty} \left\{ \frac{1}{1+e^{-x}} - \frac{1}{(1+e^{-x})^{\mu}} \right\} dx = \gamma_e + \psi(\mu), \quad \Re\{\mu\} > 0.$$

11. $$\int_{-\infty}^{\infty} \left\{ \frac{1}{(1+e^{-x})^{\nu}} - \frac{1}{(1+e^{-x})^{\mu}} \right\} dx = \psi(\mu) - \psi(\nu), \quad \Re\{\mu\} > 0,\ \Re\{\nu\} > 0.$$

12. $$\int_{-\infty}^{\infty} e^{-p^2x^2 \pm qx}\, dx = \exp\left(\frac{q^2}{4p^2}\right) \frac{\sqrt{\pi}}{|p|}.$$

13. $$\int_{-\infty}^{\infty} \exp\left[-\left(x - \frac{b}{x} \right)^{2n} \right] dx = \frac{1}{n}\, \Gamma\left(\frac{1}{2n}\right), \quad b \geq 0.$$

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