! For an efficient use of these tables, first read `HowTo.pdf`.

T2.43C. Integrands involving powers of trigonometric functions and liner trigonometric functions on the interval $(0, \pi)$.

1. $\displaystyle\int_0^\pi \sin^{\nu-1} x \sin ax\, dx = \frac{\pi \sin(a\pi/2)}{2^{\nu-1}\nu\, \mathrm{B}\left(\frac{\nu+a+1}{2}, \frac{\nu-a+1}{2}\right)}, \quad \Re\{\nu\} > 0.$

2. $\displaystyle\int_0^\pi \sin^{\nu} x \sin \nu x\, dx = 2^{-\nu}\pi \sin \frac{\nu\pi}{2}, \quad \Re\{\nu\} > -1.$

3. $\displaystyle\int_0^\pi \sin^{n} x \sin 2mx\, dx = 0.$

4. $\displaystyle\int_0^\pi \sin^{2n} x \sin(2m+1)x\, dx = \begin{cases} \dfrac{(-1)^m 2^{n+1} n!(2n-1)!!}{(2n-2m-1)!!(2m+2n+1)!!}, & m \le n, \\ \dfrac{(-1)^n 2^{n+1} n!(2m-2n-1)!!(2n-1)!!}{(2m+2n+1)!!}, & m \ge n. \end{cases}$

Note that $(2n-2m-1)!! = 1$ for $m = n$.

5. $\displaystyle\int_0^\pi \sin^{2n+1} x \sin(2m+1)x\, dx = \begin{cases} \dfrac{(-1)^m \pi}{2^{2n+1}} \dbinom{2n+1}{n-m}, & n \ge m, \\ 0, & n < m. \end{cases}$

6. $\displaystyle\int_0^\pi \sin^{n} x \cos(2m+1)x\, dx = 0.$

7. $\displaystyle\int_0^\pi \sin^{\nu-1} x \cos ax\, dx = \frac{\pi \cos \frac{a\pi}{2}}{2^{\nu-1}\nu\, \mathrm{B}\left(\frac{\nu+a+1}{2}, \frac{\nu-a+1}{2}\right)}, \quad \Re\{\nu\} > 0.$

8. $\displaystyle\int_0^\pi \sin^{\nu} x \cos \nu x\, dx = \frac{\pi}{2^\nu} \cos \frac{\nu\pi}{2}, \quad \Re\{\nu\} > -1.$

9. $$\int_0^{\pi} \sin^{2n} x \cos 2mx\, dx = 2\int_0^{\pi/2} \sin^{2n} x \cos 2mx\, dx = \begin{cases} \dfrac{(-1)^m}{2^{2n}} \dbinom{2n}{n-m} \pi, & n \ge m, \\ 0, & n < m. \end{cases}$$

10. $$\int_0^{\pi} \sin^{2n+1} x \cos 2mx\, dx = \begin{cases} \dfrac{(-1)^m 2^{n+1} n!(2n+1)!!}{(2n-2m+1)!!(2m+2n+1)!!}, & n \ge m-1, \\ \dfrac{(-1)^{n+1} 2^{n+1} n!(2m-2n-3)!!(2n+1)!!}{(2m+2n+1)!!}, & n < m-1. \end{cases}$$

11. $$\int_0^{\pi} \cos^m x \sin nx\, dx = \left[1-(-1)^{m+n}\right]\left\{\sum_{k=0}^{r-1} \frac{m!}{(m-k)!} \frac{(m+n-2k-2)!!}{(m+n)!!} + s\frac{m!(n-m-2)!!}{(m+n)!!}\right\},$$

where $r = \begin{cases} m & \text{if } m \le n, \\ n & \text{if } m \ge n, \end{cases} \qquad s = \begin{cases} 2 & \text{if } n-m = 4j+2 > 0, \\ 1 & \text{if } n-m = 2j+1 > 0, \\ 0 & \text{if } n-m = 4j \text{ or } n-m < 0, \end{cases}$

and j is a positive integer.

12. $$\int_0^{\pi} \cos^n x \cos mx\, dx = \left[1+(-1)^{m+n}\right] \begin{cases} \dfrac{s\, n!}{(m-n)(m-n+2)\dots(m+n)} & \text{if } n < m, \\ \dfrac{\pi}{2^{n+1}}\dbinom{n}{k} & \text{if } m \le n \text{ and } n-m = 2k, \\ \dfrac{n!}{(2k+1)!!\,(2m+2k+1)!!} & \text{if } m < n \text{ and } n-m = 2k+1, \end{cases}$$

where $s = \begin{cases} 0 & \text{if } m-n = 2k, \\ 1 & \text{if } m-n = 4k+1, \\ 0 & \text{if } m-n = 2k, \\ -1 & \text{if } m-n = 4k-1. \end{cases}$

13. $$\int_0^{\pi} \cos^m x \cos ax\, dx = \frac{(-1)^m \sin a\pi}{2^m(m+a)}\, {}_2F_1\left(-m, -\frac{a+m}{2}; 1-\frac{a+m}{2}; -1\right), \quad a \ne 0, \pm 1, \pm 2, \dots.$$

14. $$\int_0^{\pi} \sin^{p-1} x \cos\left[a\left(\frac{\pi}{2}-x\right)\right] dx = 2^{p-1}\frac{\Gamma\left(\frac{p-a}{2}\right)\Gamma\left(\frac{p+a}{2}\right)}{\Gamma(p-a)\Gamma(p+a)}\Gamma(p), \quad p^2 < a^2.$$