! For an efficient use of these tables, first read `HowTo.pdf`.

T3.40B. Integrands involving product of trigonometric functions of linear and quadratic arguments and sum of powers and square roots of $(a + b\,x^n)$ on the interval (y, ∞).

1. $$\int_y^\infty \frac{\sin(a^2x^2)}{\sqrt{x^4-y^4}}\,dx = -\frac{a}{4}\sqrt{\frac{\pi^3}{2}}\,J_{1/4}\left(\frac{a^2y^2}{2}\right)Y_{1/4}\left(\frac{a^2y^2}{2}\right), \quad a>0.$$

2. $$\int_y^\infty \frac{\cos(a^2x^2)}{\sqrt{x^4-y^4}}\,dx = -\frac{a}{4}\sqrt{\frac{\pi^3}{2}}\,J_{-1/4}\left(\frac{a^2y^2}{2}\right)Y_{-1/4}\left(\frac{a^2y^2}{2}\right).$$

3. $$\int_y^\infty \frac{(x^2+\sqrt{x^4-y^4})^\nu + (x^2-\sqrt{x^4-y^4})^\nu}{\sqrt{x^4-y^4}}\sin(a^2x^2)\,dx$$
$$= -\frac{a}{4}\sqrt{\frac{\pi^3}{a}}\,y^{2\nu}\left[J_{1/4+\nu/2}\left(\frac{a^2y^2}{2}\right)Y_{1/4-\nu/2}\left(\frac{a^2y^2}{2}\right)+J_{1/4-\nu/2}\left(\frac{a^2y^2}{2}\right)Y_{1/4+\nu/2}\left(\frac{a^2y^2}{2}\right)\right],$$
$$\Re\{\nu\}<\tfrac{3}{2}.$$

4. $$\int_y^\infty \frac{(x^2+\sqrt{x^4-y^4})^\nu + (x^2-\sqrt{x^4-y^4})^\nu}{\sqrt{x^4-y^4}}\cos(a^2x^2)\,dx$$
$$= -\frac{a}{4}\sqrt{\frac{\pi^3}{a}}\,y^{2\nu}\left[J_{-1/4+\nu/2}\left(\frac{a^2y^2}{2}\right)Y_{-1/4-\nu/2}\left(\frac{a^2y^2}{2}\right)+J_{-1/4-\nu/2}\left(\frac{a^2y^2}{2}\right)Y_{-1/4+\nu/2}\left(\frac{a^2y^2}{2}\right)\right],$$
$$\Re\{\nu\}<\tfrac{3}{2}.$$

5. $$\int_y^\infty \frac{(x-y)^{\mu-1}}{x^{2\mu}}\sin\frac{a}{x}\,dx = \sqrt{\frac{\pi}{y}}a^{1/2-\mu}\Gamma(\mu)\sin\frac{a}{2y}J_{\mu-1/2}\left(\frac{a}{2y}\right), \quad a>0,\ y>0,\ \Re\{\mu\}>0.$$

6. $$\int_0^y \frac{(y^2-x^2)^{\mu-1}}{x^{2\mu}}\cos\frac{a}{x}\,dx = -\frac{\sqrt{\pi}}{2}\left(\frac{2}{a}\right)^{\mu-1/2}\Gamma(\mu)y^{\mu-3/2}Y_{1/2-\mu}\left(\frac{a}{y}\right),$$
$$a>0,\ y>0,\ 0<\Re\{\mu\}<1.$$

7. $\displaystyle\int_y^\infty \frac{(x-y)^{\mu-1}}{x^{2\mu}} \cos\frac{a}{x}\,dx = \sqrt{\frac{\pi}{y}}a^{1/2-\mu}\Gamma(\mu)\,\cos\frac{a}{2y}J_{\mu-1/2}\left(\frac{a}{2y}\right), \quad a>0,\ y>0,\ \Re\{\mu\}>0.$

8. $\displaystyle\int_y^\infty \frac{x\,\sin(p\sqrt{x^2-y^2})}{x^2+a^2}\cos bx\,dx = \frac{\pi}{2}\,\exp(-p\sqrt{a^2+y^2})\,\cosh ab, \quad 0<b<p.$

9. $\displaystyle\int_y^\infty \frac{x\,\sin(p\sqrt{x^2-y^2})}{a^2+x^2-y^2}\cos bx\,dx = \frac{\pi}{2}\,e^{-ap}\,\cos(b\sqrt{y^2-a^2}), \quad 0<b<p,\ a>0.$

10. $\displaystyle\int_y^\infty \frac{\cos(p\sqrt{x^2-y^2})}{\sqrt{x^2-y^2}}\cos bx\,dx = \begin{cases} K_0\left(y\sqrt{p^2-b^2}\right), & 0<b<|p|,\\ -\dfrac{\pi}{2}Y_0\left(y\sqrt{b^2-p^2}\right), & b>|p|. \end{cases}$

11. $\displaystyle\int_y^\infty \frac{\sin(p\sqrt{x^2-y^2})}{\sqrt[4]{(x^2-y^2)^3}}\cos bx\,dx = -\sqrt{\frac{\pi^3 p}{8}}\,J_{1/4}\left[\frac{y}{2}(b-\sqrt{b^2-p^2})\right]Y_{1/4}\left[\frac{y}{2}(b+\sqrt{b^2-p^2})\right],$

$b>p>0.$

12. $\displaystyle\int_y^\infty \frac{\cos(p\sqrt{x^2-y^2})}{\sqrt[4]{(x^2-y^2)^3}}\cos bx\,dx = -\sqrt{\frac{\pi^3 p}{8}}\,J_{-1/4}\left[\frac{y}{2}(b-\sqrt{b^2-p^2})\right]Y_{1/4}\left[\frac{y}{2}(b+\sqrt{b^2-p^2})\right],$

$b>p>0.$