! For an efficient use of these tables, first read `HowTo.pdf`.

T2.12B. Integrands of the form $\dfrac{x^n}{\sqrt{(a-x)(b-x)(c-x)(d-x)}}$, $n=0,1$;
and $\dfrac{1}{x\sqrt{(a-x)(b-x)(c-x)(d-x)}}$ and $\dfrac{1}{(p-x)\sqrt{(a-x)(b-x)(c-x)(d-x)}}$ on the intervals (y,b) and (b,y).

Notation used: $\kappa = \arcsin\sqrt{\dfrac{(a-c)(b-y)}{(b-c)(a-y)}}$, $\lambda = \arcsin\sqrt{\dfrac{(a-c)(y-b)}{(a-b)(y-c)}}$,

$$q = \sqrt{\frac{(b-c)(a-d)}{(a-c)(b-d)}}, \quad r = \sqrt{\frac{(a-b)(c-d)}{(a-c)(b-d)}}.$$

1. $$\int_y^b \frac{dx}{\sqrt{(a-x)(b-x)(x-c)(x-d)}} = \frac{2}{\sqrt{(a-c)(b-d)}} F(\kappa,\, q), \quad a>b>y\ge c>d.$$

2. $$\int_b^y \frac{dx}{\sqrt{(a-x)(x-b)(x-c)(x-d)}} = \frac{2}{\sqrt{(a-c)(b-d)}} F(\lambda,\, r), \quad a\ge y>b>c>d.$$

3. $$\int_y^b \frac{x\,dx}{\sqrt{(a-x)(b-x)(x-c)(x-d)}} = \frac{2}{\sqrt{(a-c)(b-d)}} \left\{(b-a)\Pi\left(\kappa,\, \frac{b-c}{a-c},\, q\right) + aF(\kappa,\, q)\right\},$$
$$a>b>y\ge c>d.$$

4. $$\int_b^y \frac{x\,dx}{\sqrt{(a-x)(x-b)(x-c)(x-d)}} = \frac{2}{\sqrt{(a-c)(b-d)}} \left\{(b-c)\Pi\left(\lambda,\, \frac{a-b}{a-c},\, r\right) + cF(\lambda,\, r)\right\},$$
$$a\ge y>b>c>d.$$

5. $$\int_y^b \frac{dx}{x\sqrt{(a-x)(b-x)(x-c)(x-d)}}$$
$$= \frac{2}{ab\sqrt{(a-c)(b-d)}} \left\{(a-b)\Pi\left(\kappa,\, \frac{a(b-c)}{b(a-c)},\, q\right) + bF(\kappa,\, q)\right\}, \quad a>b>y\ge c>d.$$

6. $$\int_b^y \frac{dx}{x\sqrt{(a-x)(x-b)(x-c)(x-d)}}$$
$$= \frac{2}{bc\sqrt{(a-c)(b-d)}} \left\{(c-b)\Pi\left(\lambda,\, \frac{c(a-b)}{b(a-c)},\, r\right) + bF\left(\lambda,\, r\right)\right\}, \quad a \geq y > b > c > d.$$

7. $$\int_y^b \frac{dx}{(p-x)\sqrt{(a-x)(b-x)(x-c)(x-d)}} = \frac{2}{(p-a)(p-b)\sqrt{(a-c)(b-d)}}$$
$$\times \left[(b-a)\Pi\left(\kappa,\, \frac{(b-c)(p-a)}{(a-c)(p-b)},\, q\right) + (p-b)\,F\left(\kappa,\, q\right)\right], \quad a > b > y \geq c > d,\ p \neq b.$$

8. $$\int_b^y \frac{dx}{(x-p)\sqrt{(a-x)(x-b)(x-c)(x-d)}} = \frac{2}{(b-p)(p-c)\sqrt{(a-c)(b-d)}}$$
$$\times \left[(b-c)\Pi\left(\lambda,\, \frac{(a-b)(p-c)}{(a-c)(p-b)},\, r\right) + (p-b)\,F\left(\lambda,\, r\right)\right], \quad a \geq y > b > c > d,\ p \neq b.$$

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