! For an efficient use of these tables, first read `HowTo.pdf`.

T3.58A. Integrands involving logarithms and exponentials on the interval $(0, \infty)$.

1. $$\int_0^\infty \ln\left|\frac{x+a}{x-a}\right| \sin bx \, dx = \frac{\pi}{b} \sin ab, \quad a < 0,\ b > 0.$$

2. $$\int_0^\infty \ln\left|\frac{x+a}{x-a}\right| \cos bx \, dx = \frac{2}{b}[\cos ab \ \mathrm{Si}(ab) - \sin ab \ \mathrm{Ci}(ab)], \quad a > 0,\ b > 0.$$

3. $$\int_0^\infty \ln \frac{a^2+x^2}{b^2+x^2} \cos cx \, dx = \frac{\pi}{c}(e^{-bc} - e^{-ac}), \quad a > 0,\ b > 0,\ c > 0.$$

4. $$\int_0^\infty \ln \frac{x^2+x+a^2}{x^2-x+a^2} \sin bx \, dx = \frac{2\pi}{b} \ \exp\left(-b\sqrt{a^2 - \tfrac{1}{4}}\right) \sin \frac{b}{2}, \quad b > 0.$$

5. $$\int_0^\infty \ln \frac{(x+\beta)^2+\gamma^2}{(x-\beta)^2+\gamma^2} \sin bx \, dx = \frac{2\pi}{b} e^{-\gamma b} \sin \beta b, \quad \Re\{\gamma\} > 0,\ |\Im\{\beta\}| \le \Re\{\gamma\},\ b > 0.$$

6. $$\int_0^\infty \ln(1+e^{-\beta x}) \cos bx \, dx = \frac{\beta}{2b^2} - \frac{\pi}{2b \ \sinh\left(\frac{\pi b}{\beta}\right)}, \quad \Re\{\beta\} > 0,\ b > 0.$$

7. $$\int_0^\infty \ln(1-e^{-\beta x}) \cos bx \, dx = \frac{\beta}{2b^2} - \frac{\pi}{2b} \ \coth\left(\frac{\pi b}{\beta}\right), \quad \Re\{\beta\} > 0,\ b > 0.$$

8. $$\int_0^\infty \ln x \sin ax^2 \, dx = -\frac{1}{4}\sqrt{\frac{\pi}{2a}}\left(\ln 4a + \gamma_e - \frac{\pi}{2}\right), \quad a > 0.$$

9. $$\int_0^\infty \ln x \cos ax^2 \, dx = -\frac{1}{4}\sqrt{\frac{\pi}{2a}}\left(\ln 4a + \gamma_e - \frac{\pi}{2}\right), \quad a > 0.$$
