

! For an efficient use of these tables, first read [HowTo.pdf](#).

**T2.04C.** Integrands of the form  $\frac{1}{\sqrt{(a-x)(b-x)(c-x)^n}}$ ,  $\frac{1}{\sqrt{(a-x)(b-x)^n(c-x)}}$ ,  
and  $\frac{1}{\sqrt{(a-x)^n(b-x)(c-x)}}$  for  $n = 0, 1, 3, 5$ , on the intervals  $(y, c)$  and  $(c, y)$ .

Notation used:  $\beta = \arcsin \sqrt{\frac{c-y}{b-y}}$ ,  $\gamma = \arcsin \sqrt{\frac{y-c}{b-c}}$ ,  $p = \sqrt{\frac{a-b}{a-c}}$ ,  $q = \sqrt{\frac{b-c}{a-c}}$ .

$$1. \int_c^y \frac{dx}{\sqrt{(x-c)(y-x)}} = \pi, \quad 0 \leq c < y \leq 1.$$

$$2. \int_y^c \frac{dx}{\sqrt{(c-x)(y-x)}} = i\pi, \quad 0 \leq y < c \leq 1.$$

$$3. \int_y^c \frac{dx}{\sqrt{(a-x)(b-x)(c-x)}} = \frac{2}{\sqrt{a-c}} F(\beta, p), \quad a > b > c > y.$$

$$4. \int_c^y \frac{dx}{\sqrt{(a-x)(b-x)(x-c)}} = \frac{2}{\sqrt{a-c}} F(\gamma, q), \quad a > b \geq y > c.$$

$$5. \int_y^c \frac{x dx}{\sqrt{(a-x)(b-x)(c-x)}} = \frac{2}{\sqrt{a-c}} [c F(\beta, p) + (a-c) E(\beta, p)] - 2 \sqrt{\frac{(a-y)(c-y)}{b-y}},$$

$a > b > c > y.$

$$6. \int_c^y \frac{x dx}{\sqrt{(a-x)(b-x)(x-c)}} = \frac{2a}{\sqrt{a-c}} F(\gamma, q) - 2 \sqrt{a-c} E(\gamma, q), \quad a > b \geq y > c.$$

$$7. \int_c^y \frac{dx}{\sqrt{(a-x)^3(b-x)(x-c)}} = \frac{2}{(a-b)\sqrt{a-c}} E(\gamma, q) - \frac{2}{(a-b)(a-c)} \sqrt{\frac{(b-y)(y-c)}{a-y}},$$

$a > b \geq y > c.$

$$8. \int_y^\infty \frac{dx}{\sqrt{(x-a)^3(x-b)(x-c)}} = \frac{2}{(b-a)\sqrt{a-c}} E(\nu, q) + \frac{2}{a-b} \sqrt{\frac{y-b}{(y-a)(y-c)}},$$

$$y > a > b > c.$$

$$9. \int_y^c \frac{dx}{\sqrt{(a-x)(b-x)^3(c-x)}} = \frac{2\sqrt{a-c}}{(a-b)(b-c)} E(\beta, p) - \frac{2}{(a-b)\sqrt{a-c}} F(\beta, p),$$

$$a > b > c > y.$$

$$10. \int_c^y \frac{dx}{\sqrt{(a-x)(b-x)^3(x-c)}} = \frac{2}{(b-c)\sqrt{a-c}} F(\gamma, q) - \frac{2\sqrt{a-c}}{(a-b)(b-c)} E(\gamma, q)$$

$$+ \frac{2}{(a-b)(b-c)} \sqrt{\frac{(a-y)(y-c)}{b-y}}, \quad a > b > y > c.$$

$$11. \int_y^c \frac{dx}{\sqrt{(a-x)^5(b-x)(c-x)}} = \frac{2}{3(a-b)^2\sqrt{(a-c)^3}} [(3a-b-2c)F(\beta, p) - 2(2a-b-c)E(\beta, p)]$$

$$+ \frac{2[4a^2 - 3ab - 2ac + bc - y(3a - 2b - c)]}{3(a-b)(a-c)^2} \sqrt{\frac{c-y}{(a-y)^3(b-y)}}, \quad a > b > c > y.$$

$$12. \int_c^y \frac{dx}{\sqrt{(a-x)^5(b-x)(x-c)}} = \frac{2}{3(a-b)^3\sqrt{(a-c)^3}} [2(2a-b-c)E(\gamma, q) - (a-b)F(\gamma, q)]$$

$$- \frac{2[5a^2 - 3ab - 3ac + bc - 2y(2a-b-c)]}{3(a-b)^2(a-c)^2} \sqrt{\frac{(b-y)(y-c)}{(a-y)^3}}, \quad a > b \geq y > c.$$

$$13. \int_y^c \frac{dx}{\sqrt{(a-x)(b-x)^5(c-x)}} = \frac{2}{3(a-b)^2(b-c)^2\sqrt{a-c}} [(b-c)(3b-a-2c)F(\beta, p) + 2(a-c)(a-2b+c)E(\beta, p)]$$

$$+ \frac{2}{3(a-b)(b-c)} \sqrt{\frac{(a-y)(c-y)}{(b-y)^3}}, \quad a > b > c > y.$$

$$\begin{aligned}
 14. \int_c^y \frac{dx}{\sqrt{(a-x)(b-x)^5(x-c)}} \\
 = \frac{2}{3(a-b)^2(b-c)^2\sqrt{a-c}} [(a-b)(2a-3b+c)F(\gamma, q) + 2(a-c)(2b-a-c)E(\gamma, q)] \\
 + \frac{2[3ab+3bc-ac-5b^2-2y(a-2b+c)]}{3(a-b)^2(b-c)^2} \sqrt{\frac{(a-y)(y-c)}{(b-y)^3}}, \quad a > b > y > c.
 \end{aligned}$$


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