

**T1.17.** Integrand involving  $\sqrt{x^2 \pm a^2}$ ,  $\sqrt{a^2 - x^2}$ ,  $\sqrt{a + bx + cx^2}$  and integral powers of  $x$ .

$$1. \int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[ x \sqrt{x^2 \pm a^2} \pm a^2 \ln \left( x + \sqrt{x^2 \pm a^2} \right) \right].$$

$$2. \int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left( x + \sqrt{x^2 \pm a^2} \right).$$

$$3. \int \frac{dx}{x \sqrt{x^2 - a^2}} = \frac{1}{|a|} \operatorname{arcsec} \frac{x}{a}.$$

$$4. \int \frac{dx}{x \sqrt{x^2 + a^2}} = -\frac{1}{|a|} \ln \left( \frac{a + \sqrt{x^2 + a^2}}{x} \right).$$

$$5. \int \frac{\sqrt{x^2 + a^2}}{x} dx = \sqrt{x^2 + a^2} - a \ln \left( \frac{a + \sqrt{x^2 + a^2}}{x} \right).$$

$$6. \int \frac{\sqrt{x^2 - a^2}}{x} dx = \sqrt{x^2 - a^2} - |a| \operatorname{arcsec} \frac{x}{a}.$$

$$7. \int \frac{x}{\sqrt{x^2 \pm a^2}} dx = \sqrt{x^2 \pm a^2}.$$

$$8. \int x \sqrt{x^2 \pm a^2} dx = \frac{1}{3} \sqrt{(x^2 \pm a^2)^3}.$$

$$9. \int \sqrt{(x^2 \pm a^2)^3} dx = \frac{1}{4} \left[ x \sqrt{(x^2 \pm a^2)^3} \pm \frac{3a^2 x}{2} \sqrt{x^2 \pm a^2} + \frac{3a^4}{2} \ln \left( x + \sqrt{x^2 \pm a^2} \right) \right].$$

$$10. \int \frac{dx}{\sqrt{(x^2 + a^2)^3}} = \frac{\pm x}{a^2 \sqrt{x^2 \pm a^2}}.$$

$$11. \int \frac{x}{\sqrt{(x^2 + a^2)^3}} dx = \frac{-1}{\sqrt{x^2 \pm a^2}}.$$

$$12. \int x \sqrt{(x^2 + a^2)^3} dx = \frac{1}{5} \sqrt{(x^2 \pm a^2)^5}.$$

$$13. \int x^2 \sqrt{(x^2 \pm a^2)^3} dx = \frac{x}{4} \sqrt{(x^2 \pm a^2)^3} + \frac{a^2}{8} x \sqrt{x^2 \pm a^2} - \frac{a^4}{8} \ln \left( x + \sqrt{x^2 \pm a^2} \right).$$

$$14. \int x^3 \sqrt{x^2 + a^2} dx = \frac{1}{15} (3x^2 - 2a^2) \sqrt{(x^2 + a^2)^3}.$$

15.  $\int x^3 \sqrt{x^2 - a^2} dx = \frac{1}{5} \sqrt{(x^2 + a^2)^5} + \frac{a^2}{3} \sqrt{(x^2 - a^2)^3}.$
16.  $\int \frac{x^2}{\sqrt{x^2 \pm a^2}} dx = \frac{x}{2} \sqrt{x^2 \pm a^2} \mp \frac{a^2}{2} \ln \left( x + \sqrt{x^2 \pm a^2} \right).$
17.  $\int \frac{x^3}{\sqrt{x^2 \pm a^2}} dx = \frac{1}{3} \sqrt{(x^2 \pm a^2)^3} \mp a^2 \sqrt{x^2 \pm a^2}.$
18.  $\int \frac{dx}{x^2 \sqrt{x^2 \pm a^2}} = \mp \frac{\sqrt{x^2 \pm a^2}}{a^2 x}.$
19.  $\int \frac{dx}{x^3 \sqrt{x^2 + a^2}} = -\frac{\sqrt{x^2 + a^2}}{2a^2 x^2} + \frac{1}{2a^3} \ln \left( \frac{a + \sqrt{x^2 + a^2}}{x} \right).$
20.  $\int \frac{dx}{x^3 \sqrt{x^2 - a^2}} = \frac{\sqrt{x^2 - a^2}}{2a^2 x^2} + \frac{1}{2|a|^3} \operatorname{arcsec} \frac{x}{a}.$
21.  $\int x^2 \sqrt{(x^2 \pm a^2)^3} dx = \frac{x}{6} \sqrt{(x^2 \pm a^2)^5} + \frac{a^2 x}{24} \sqrt{(x^2 \pm a^2)^3} - \frac{a^4 x}{16} \sqrt{x^2 \pm a^2}$   
 $\mp \frac{a^6}{16} \ln \left( x + \sqrt{x^2 \pm a^2} \right).$
22.  $\int x^3 \sqrt{(x^2 \pm a^2)^3} dx = \frac{1}{7} \sqrt{(x^2 \pm a^2)^7} \mp \frac{a^2}{5} \sqrt{(x^2 \pm a^2)^5}.$
23.  $\int \frac{\sqrt{x^2 \pm a^2}}{x^2} dx = -\frac{\sqrt{x^2 \pm a^2}}{x} + \ln \left( x + \sqrt{x^2 \pm a^2} \right).$
24.  $\int \frac{\sqrt{x^2 + a^2}}{x^3} dx = -\frac{\sqrt{x^2 + a^2}}{2x^2} - \frac{1}{2a} \ln \left( \frac{x + \sqrt{x^2 + a^2}}{x} \right).$
25.  $\int \frac{\sqrt{x^2 - a^2}}{x^3} dx = -\frac{\sqrt{x^2 - a^2}}{2x^2} + \frac{1}{2a} \operatorname{arcsec} \frac{x}{a}.$
26.  $\int \frac{\sqrt{x^2 \pm a^2}}{x^4} dx = \mp \frac{\sqrt{(x^2 \pm a^2)^3}}{3a^2 x^3}.$
27.  $\int \frac{x^2}{\sqrt{(x^2 \pm a^2)^3}} dx = -\frac{x}{\sqrt{x^2 \pm a^2}} + \ln \left( x + \sqrt{x^2 \pm a^2} \right).$
28.  $\int \frac{x^3}{\sqrt{(x^2 \pm a^2)^3}} dx = \sqrt{x^2 \pm a^2} \pm \frac{a^2}{\sqrt{x^2 \pm a^2}}.$
29.  $\int \frac{dx}{x \sqrt{(x^2 + a^2)^3}} = \frac{1}{a^2 \sqrt{x^2 + a^2}} - \frac{1}{a^3} \ln \left( \frac{a + \sqrt{x^2 + a^2}}{x} \right).$

30.  $\int \frac{dx}{x \sqrt{(x^2 - a^2)^3}} = -\frac{1}{a^2 \sqrt{x^2 - a^2}} - \frac{1}{|a|^3} \operatorname{arcsec} \frac{x}{a}.$
31.  $\int \frac{dx}{x^2 \sqrt{(x^2 \pm a^2)^3}} = -\frac{1}{a^4} \left[ \frac{\sqrt{x^2 \pm a^2}}{x} + \frac{x}{\sqrt{x^2 \pm a^2}} \right].$
32.  $\int \frac{dx}{x^2 \sqrt{(x^2 + a^2)^3}} = -\frac{a^2 + 3}{2a^4 \sqrt{x^2 + a^2}} + \frac{3}{2a^5} \ln \left( \frac{a + \sqrt{x^2 + a^2}}{x} \right).$
33.  $\int \frac{dx}{x^2 \sqrt{(x^2 - a^2)^3}} = \frac{1}{2a^2 x^2 \sqrt{x^2 - a^2}} - \frac{3}{2a^4 \sqrt{x^2 - a^2}} - \frac{3}{2|a|^5} \operatorname{arcsec} \frac{x}{a}.$
34.  $\int \frac{x^m dx}{\sqrt{x^2 \pm a^2}} = \frac{1}{m} x^{m-1} \sqrt{x^2 \pm a^2} \mp \frac{m-1}{m} a^2 \int \frac{x^{m-2} dx}{\sqrt{x^2 \pm a^2}}.$
35.  $\int \frac{x^{2m} dx}{\sqrt{x^2 \pm a^2}} = \frac{(2m)!}{2^{2m} (m!)^2} \left[ \sqrt{x^2 \pm a^2} \sum_{k=1}^m \frac{k! (k-1)!}{(2k)!} (\mp a^2)^{m-k} (2x)^{2k-1} \right. \\ \left. + (\mp a^2)^m \ln \left( x + \sqrt{x^2 \pm a^2} \right) \right].$
36.  $\int \frac{x^{2m+1} dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2} \sum_{k=1}^m \frac{(2k)! (m!)^2}{(2m+1)! (k!)^2} (\mp 4a^2)^{m-k} x^{2k}.$
37.  $\int \frac{dx}{x^m \sqrt{x^2 \pm a^2}} = \mp \frac{\sqrt{x^2 \pm a^2}}{(m-1) a^2 x^{m-1}} \mp \frac{(m-2)}{(m-1) a^2} \int \frac{dx}{x^{m-2} \sqrt{x^2 \pm a^2}}.$
38.  $\int \frac{dx}{x^{2m} \sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2} \sum_{k=0}^{m-1} \frac{(m-1)! m! (2k)! 2^{2m-2k-1}}{(k!)^2 (2m)! (\mp a^2)^{m-k} x^{2k+1}}.$
39.  $\int \frac{dx}{x^{2m+1} \sqrt{x^2 + a^2}} = \frac{(2m)!}{(m!)^2} \left[ \frac{\sqrt{x^2 + a^2}}{a^2} \sum_{k=0}^m (-1)^{m-k+1} \frac{k! (k-1)!}{2(2k)! (4a^2)^{m-k} x^{2k}} \right. \\ \left. + \frac{(-1)^{m+1}}{2^{2m} a^{2m+1}} \ln \left( \frac{a + \sqrt{x^2 + a^2}}{x} \right) \right].$
40.  $\int \frac{dx}{x^{2m+1} \sqrt{x^2 - a^2}} = \frac{(2m)!}{(m!)^2} \left[ \frac{\sqrt{x^2 - a^2}}{a^2} \sum_{k=0}^m \frac{k! (k-1)!}{2(2k)! (4a^2)^{m-k} x^{2k}} \right. \\ \left. + \frac{1}{2^{2m} |a|^{2m+1}} \operatorname{arcsec} \frac{x}{a} \right].$
41.  $\int \frac{dx}{(x-a) \sqrt{x^2 - a^2}} = -\frac{\sqrt{x^2 - a^2}}{a(x-a)}.$

$$42. \int \frac{dx}{(x+a)\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{a(x+a)}.$$

$$43. \int \sqrt{a^2-x^2} dx = \frac{1}{2} \left[ x\sqrt{a^2-x^2} + a^2 \arcsin \frac{x}{|a|} \right].$$

$$44. \int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{|a|} = -\arccos \frac{x}{|a|}.$$

$$45. \int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a} \ln \left( \frac{a+\sqrt{a^2-x^2}}{x} \right).$$

$$46. \int \frac{\sqrt{a^2-x^2}}{x} dx = \sqrt{a^2-x^2} - a \ln \left( \frac{a+\sqrt{a^2-x^2}}{x} \right).$$

$$47. \int \frac{x}{\sqrt{a^2-x^2}} dx = -\sqrt{a^2-x^2}.$$

$$48. \int x\sqrt{a^2-x^2} dx = -\frac{1}{3}\sqrt{(a^2-x^2)^3}.$$

$$49. \int \sqrt{(a^2-x^2)^3} dx = -\frac{1}{4} \left[ x\sqrt{(a^2-x^2)^3} + \frac{3a^2x}{2}\sqrt{a^2-x^2} + \frac{3a^4}{2} \arcsin \frac{x}{|a|} \right].$$

$$50. \int \frac{dx}{\sqrt{(a^2-x^2)^3}} = \frac{x}{a^2\sqrt{a^2-x^2}}.$$

$$51. \int \frac{x}{\sqrt{(a^2-x^2)^3}} = \frac{1}{\sqrt{a^2-x^2}}.$$

$$52. \int x\sqrt{(a^2-x^2)^3} dx = -\frac{1}{5}\sqrt{(a^2-x^2)^5}.$$

$$53. \int x^2\sqrt{a^2-x^2} dx = -\frac{x}{4}\sqrt{(a^2-x^2)^3} + \frac{a^2}{8} \left[ x\sqrt{a^2-x^2} + a^2 \arcsin \frac{x}{|a|} \right].$$

$$54. \int x^3\sqrt{a^2-x^2} dx = \left( -\frac{1}{5}x^2 - \frac{2}{15}a^2 \right) \sqrt{a^2-x^2}.$$

$$55. \int x^2\sqrt{(a^2-x^2)^3} dx = -\frac{x}{6}\sqrt{(a^2-x^2)^5} + \frac{a^2x}{24}\sqrt{(a^2-x^2)^3} + \frac{a^4x}{16}\sqrt{a^2-x^2} + \frac{a^6}{16} \arcsin \frac{x}{|a|}.$$

$$56. \int x^3\sqrt{(a^2-x^2)^3} dx = -\frac{1}{7}\sqrt{(a^2-x^2)^7} - \frac{a^2}{5}\sqrt{(a^2-x^2)^5}.$$

$$57. \int \frac{x^2}{\sqrt{a^2-x^2}} dx = -\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2} \arcsin \frac{x}{|a|}.$$

$$58. \int \frac{dx}{x^2\sqrt{a^2-x^2}} = \frac{\sqrt{a^2-x^2}}{a^2x}.$$

59.  $\int \frac{\sqrt{a^2 - x^2}}{x^2} dx = -\frac{\sqrt{a^2 - x^2}}{x} - \arcsin \frac{x}{|a|}.$
60.  $\int \frac{\sqrt{a^2 - x^2}}{x^3} dx = -\frac{\sqrt{a^2 - x^2}}{2x^2} + \frac{1}{2a} \ln \left( \frac{a + \sqrt{a^2 - x^2}}{x} \right).$
61.  $\int \frac{\sqrt{a^2 - x^2}}{x^4} dx = -\frac{\sqrt{(a^2 - x^2)^3}}{3a^2 x^3}.$
62.  $\int \frac{x^2}{\sqrt{(a^2 - x^2)^3}} dx = \frac{x}{\sqrt{a^2 - x^2}} - \arcsin \frac{x}{|a|}.$
63.  $\int \frac{x^3}{\sqrt{a^2 - x^2}} dx = -\frac{2}{3} \sqrt{a^2 - x^2} - x^2 \sqrt{a^2 - x^2}.$
64.  $\int \frac{x^3}{\sqrt{(a^2 - x^2)^3}} dx = 2 \sqrt{a^2 - x^2} + \frac{x^2}{\sqrt{a^2 - x^2}}$   
 $= \frac{a^2}{\sqrt{a^2 - x^2}} + \sqrt{a^2 - x^2}.$
65.  $\int \frac{dx}{x^3 \sqrt{a^2 - x^2}} = -\frac{\sqrt{a^2 - x^2}}{2a^2 x^2} - \frac{1}{2a^3} \ln \left( \frac{a + \sqrt{a^2 - x^2}}{x} \right).$
66.  $\int \frac{dx}{x \sqrt{(a^2 - x^2)^3}} = \frac{1}{a^2 \sqrt{a^2 - x^2}} - \frac{1}{a^3} \ln \left( \frac{a + \sqrt{a^2 - x^2}}{x} \right).$
67.  $\int \frac{dx}{x^2 \sqrt{(a^2 - x^2)^3}} = \frac{1}{a^4} \left[ -\frac{\sqrt{a^2 - x^2}}{x} + \frac{x}{\sqrt{a^2 - x^2}} \right].$
68.  $\int \frac{dx}{x^3 \sqrt{(a^2 - x^2)^3}} = \frac{3 - a^2}{2a^4 \sqrt{a^2 - x^2}} - \frac{3}{2a^5} \ln \left( \frac{a + \sqrt{a^2 - x^2}}{x} \right).$
69.  $\int \frac{x^m}{\sqrt{a^2 - x^2}} dx = -\frac{x^{m-1} \sqrt{a^2 - x^2}}{m} + \frac{(m-1)a^2}{m} \int \frac{x^{m-2}}{\sqrt{a^2 - x^2}} dx.$
70.  $\int \frac{x^{2m}}{\sqrt{a^2 - x^2}} dx = \frac{(2m)!}{(m!)^2} \left[ -\sqrt{a^2 - x^2} \sum_{k=1}^m \frac{k! (k-1)!}{2^{2m-2k+1} (2k)!} a^{2m-2k} x^{2k-1} + \frac{a^{2m}}{2^{2m}} \arcsin \frac{x}{|a|} \right].$
71.  $\int \frac{x^{2m+1}}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2} \sum_{k=1}^m \frac{(2k)! (m!)^2}{(2m+1)! (k!)^2} (4a)^{m-k} x^{2k}.$
72.  $\int \frac{dx}{x^m \sqrt{a^2 - x^2}} = -\frac{\sqrt{a^2 - x^2}}{(m-1)a^2 x^{m-1}} + \frac{(m-2)}{(m-1)a^2} \int \frac{dx}{x^{m-2} \sqrt{a^2 - x^2}}.$

$$73. \int \frac{dx}{x^{2m} \sqrt{a^2 - x^2}} = -\sqrt{a^2 - x^2} \sum_{k=0}^{m-1} \frac{(m-1)! m! (2k)! 2^{2m-2k-1}}{(k!)^2 (2m)! a^{2m-2k} x^{2k+1}}.$$

$$74. \int \frac{dx}{x^{2m+1} \sqrt{a^2 - x^2}} = \frac{(2m)!}{(m!)^2} \left[ -\frac{\sqrt{a^2 - x^2}}{a^2} \sum_{k=1}^m \frac{k! (k-1)!}{2(2k)! (4a^2)^{m-k} x^{2k}} + \frac{1}{2^{2m} a^{2m+1}} \ln \left( \frac{a + \sqrt{a^2 - x^2}}{x} \right) \right].$$

$$75. \int \frac{dx}{(b^2 - x^2) \sqrt{a^2 - x^2}} = \begin{cases} \frac{1}{2b\sqrt{a^2 - b^2}} \ln \left( \frac{(b\sqrt{a^2 - x^2} + x\sqrt{a^2 - b^2})^2}{b^2 - x^2} \right), & a^2 > b^2, \\ \frac{1}{b\sqrt{b^2 - a^2}} \arctan \frac{x\sqrt{b^2 - a^2}}{b\sqrt{a^2 - x^2}}, & b^2 > a^2. \end{cases}$$

$$76. \int \frac{dx}{(b^2 + x^2) \sqrt{a^2 - x^2}} = \frac{1}{b\sqrt{a^2 + b^2}} \arctan \frac{x\sqrt{a^2 + b^2}}{b\sqrt{a^2 - x^2}}.$$

$$77. \int \frac{\sqrt{a^2 - x^2}}{b^2 + x^2} dx = \frac{\sqrt{a^2 + b^2}}{|b|} \arcsin \frac{x\sqrt{a^2 + b^2}}{|a|\sqrt{x^2 + b^2}} - \arcsin \frac{x}{|a|}, \quad b^2 > a^2.$$

For the following formulas we use the notation:  $X = a + bx + cx^2$ ,  $\Delta = 4ac - b^2$ .

$$78. \int \sqrt{X} dx = \frac{(2cx + b)\sqrt{X}}{4c} + \frac{\Delta}{8c} \int \frac{dx}{\sqrt{X}}.$$

$$79. \int x\sqrt{X} dx = \frac{\sqrt{X^3}}{3c} - \frac{(2cx + b)b}{8c^2} \sqrt{X} - \frac{b\Delta}{16c^2} \int \frac{dx}{\sqrt{X}}.$$

$$80. \int x^2 \sqrt{X} dx = \left( \frac{x}{4c} - \frac{5b}{24c^2} \right) \sqrt{X^3} + \left( \frac{5b^2}{16c^2} - \frac{a}{4c} \right) \frac{(2cx + b)\sqrt{X}}{4c} + \left( \frac{5b^2}{16c^2} - \frac{a}{4c} \right) \frac{\Delta}{8c} \int \frac{dx}{\sqrt{X}}.$$

$$81. \int x^3 \sqrt{X} dx = \left( \frac{x^2}{5c} - \frac{7bx}{40c^2} + \frac{7b^2}{48c^3} - \frac{2a}{15c^2} \right) \sqrt{X^3} - \left( \frac{7b^3}{32c^3} - \frac{3ab}{8c^2} \right) \frac{(2cx + b)\sqrt{X}}{4c} - \left( \frac{7b^3}{32c^3} - \frac{3ab}{8c^2} \right) \frac{\Delta}{8c} \int \frac{dx}{\sqrt{X}}.$$

$$82. \int \sqrt{X^{2n+1}} dx = \begin{cases} \frac{2cx+b}{4(n+1)c} \sqrt{X^{2n+1}} + \frac{2n+1}{8(n+1)} \frac{\Delta}{c} \int \sqrt{X^{2n-1}} dx, \\ \text{or} \\ \frac{(2cx+b)\sqrt{X}}{4(n+1)c} \left\{ X^n + \sum_{k=0}^{n-1} \frac{(2n+1)(2n-1)\dots(2n-2k+1)}{8^{k+1}n(n-1)\dots(n-k)} \left(\frac{\Delta}{c}\right)^{k+1} X^{n-k-1} \right\} \\ + \frac{(2n+1)!!}{8^{n+1}(n+1)!} \left(\frac{\Delta}{c}\right)^{n+1} \int \frac{dx}{\sqrt{X}} \end{cases}.$$

$$83. \int x^m \sqrt{X^{2n+1}} dx = \frac{x^{m-1} \sqrt{X^{2n+3}}}{(m+2n+2)c} - \frac{(2m+2n+1)b}{2(m+2n+2)c} \int x^{m-1} \sqrt{X^{2n+1}} dx \\ - \frac{(m-1)a}{(m+2n+2)c} \int x^{m-2} \sqrt{X^{2n+1}} dx.$$

$$84. \int \sqrt{X^3} dx = \left(\frac{X}{8c} + \frac{3\Delta}{64c^2}\right) (2cx+b) \sqrt{X} + \frac{3\Delta^2}{128c^2} \int \frac{dx}{\sqrt{X}}.$$

$$85. \int x \sqrt{X^3} dx = \frac{\sqrt{X^5}}{5c} - (2cx+b) \left(\frac{b}{16c^2} \sqrt{X^3} + \frac{3\Delta b}{128c^3} \sqrt{X}\right) - \frac{3\Delta^2 b}{256c^3} \int \frac{dx}{\sqrt{X}}.$$

$$86. \int x^2 \sqrt{X^3} dx = \left(\frac{x}{6c} - \frac{7b}{60c^2}\right) \sqrt{X^5} + \left(\frac{7b^2}{24c^2} - \frac{a}{6c}\right) \left(2x + \frac{b}{c}\right) \left(\frac{\sqrt{X^3}}{8} + \frac{3\Delta}{64c} \sqrt{X}\right) \\ + \left(\frac{7b^2}{4c} - a\right) \frac{\Delta^2}{256c^3} \int \frac{dx}{\sqrt{X}}.$$

$$87. \int x^3 \sqrt{X^3} dx = \left(\frac{x^2}{7c} - \frac{3bx}{28c^2} + \frac{3b^2}{40c^3} - \frac{2a}{35c^2}\right) \sqrt{X^5} \\ - \left(\frac{3b^3}{16c^3} - \frac{ab}{4c^2}\right) \left(2x + \frac{b}{c}\right) \left(\frac{\sqrt{X^3}}{8} + \frac{3\Delta}{64c} \sqrt{X}\right) - \left(\frac{3b^2}{4c} - a\right) \frac{3\Delta^2 b}{512c^4} \int \frac{dx}{\sqrt{X}}.$$

$$88. \int \frac{x^m dx}{\sqrt{X^{2n+1}}} = \frac{x^{m-1}}{(m-2n)c\sqrt{X^{2n-1}}} - \frac{(2m-2n-1)b}{2(m-2n)c} \int \frac{x^{m-1} dx}{\sqrt{X^{2n+1}}} - \frac{(m-1)a}{(m-2n)c} \int \frac{x^{m-2} dx}{\sqrt{X^{2n+1}}}.$$

$$89. \int \frac{x^{2n} dx}{\sqrt{X^{2n+1}}} = -\frac{x^{2n-1}}{(2n-1)c\sqrt{X^{2n-1}}} - \frac{b}{2c} \int \frac{x^{2n-1} dx}{\sqrt{X^{2n+1}}} + \frac{1}{c} \int \frac{x^{2n-2} dx}{\sqrt{X^{2n-1}}}.$$

$$90. \int \frac{dx}{\sqrt{X}} = \begin{cases} \frac{1}{\sqrt{c}} \ln(2\sqrt{cX} + 2cx + b), & c > 0, \\ \frac{1}{\sqrt{c}} \operatorname{arcsinh} \frac{2cx+b}{\sqrt{\Delta}}, & c > 0, \Delta > 0, \\ \frac{-1}{\sqrt{-c}} \arcsin \frac{2cx+b}{\sqrt{-\Delta}}, & c < 0, \Delta < 0, \\ \frac{1}{\sqrt{c}} \ln(2cx+b), & c > 0, \Delta = 0. \end{cases}$$

91.  $\int \frac{dx}{\sqrt{X^{2n+1}}} = \begin{cases} \frac{2(2cx+b)}{(2n-1)\Delta\sqrt{X^{2n-1}}} + \frac{8(n-1)c}{(2n-1)\Delta} \int \frac{dx}{\sqrt{X^{2n-1}}}, \\ \text{or} \\ \frac{2(2cx+b)}{(2n-1)\Delta\sqrt{X^{2n-1}}} \left\{ 1 + \sum_{k=1}^{n-1} \frac{8^k(n-1)(n-2)\dots(n-k)}{(2n-3)(2n-5)\dots(2n-2k-1)} \frac{c^k}{\Delta^k} X^k \right\}, \quad n \geq 1 \end{cases}$ .
92.  $\int \frac{x dx}{\sqrt{X}} = \frac{\sqrt{X}}{c} - \frac{b}{2c} \int \frac{dx}{\sqrt{X}}.$
93.  $\int \frac{x^2 dx}{\sqrt{X}} = \left( \frac{x}{2c} - \frac{3b}{4c^2} \right) \sqrt{X} + \left( \frac{3b^2}{8c^2} - \frac{a}{2c} \right) \int \frac{dx}{\sqrt{X}}.$
94.  $\int \frac{x^3 dx}{\sqrt{X}} = \left( \frac{x^2}{3c} - \frac{5bx}{12c^2} + \frac{5b^2}{8c^3} - \frac{2a}{3c^2} \right) \sqrt{X} - \left( \frac{5b^3}{16c^3} - \frac{3ab}{4c^2} \right) \int \frac{dx}{\sqrt{X}}.$
95.  $\int \frac{dx}{\sqrt{X^3}} = \frac{2(2cx+b)}{\Delta\sqrt{X}}.$
96.  $\int \frac{x dx}{\sqrt{X^3}} = -\frac{2(2a+bx)}{\Delta\sqrt{X}}.$
97.  $\int \frac{x^2 dx}{\sqrt{X^3}} = -\frac{(\Delta-b^2)x-2ab}{c\Delta\sqrt{X}} + \frac{1}{c} \int \frac{dx}{\sqrt{X}}.$
98.  $\int \frac{x^3 dx}{\sqrt{X^3}} = \frac{c\Delta x^2 + b(10ac-3b^2)x + a(8ac-3b^2)}{c^2\Delta\sqrt{X}} - \frac{3b}{2c^2} \int \frac{dx}{\sqrt{X}}.$
99.  $\int \frac{\sqrt{X^{2n+1}}}{x^m} dx = -\frac{\sqrt{X^{2n+3}}}{(m-1)ax^{m-1}} + \frac{(2n-2m+5)b}{2(m-1)a} \int \frac{\sqrt{X^{2n+1}}}{x^{m-1}} dx$   
 $+ \frac{(2n-m+4)c}{(m-1)a} \int \frac{\sqrt{X^{2n+1}}}{x^{m-2}} dx.$
100.  $\int \frac{\sqrt{X^{2n+1}}}{x} dx = \frac{\sqrt{X^{2n+1}}}{2n+1} + \frac{b}{2} \int \frac{\sqrt{X^{2n-1}}}{x} dx + a \int \frac{\sqrt{X^{2n-1}}}{x} dx.$
101.  $\int \frac{\sqrt{(bx+cx^2)^{2n+1}}}{x^m} dx = \frac{2\sqrt{(bx+cx^2)^{2n+3}}}{(2n-2m+3)bx^m} + \frac{2(m-2n-3)c}{(2n-2m+3)b} \int \frac{\sqrt{(bx+cx^2)^{2n+1}}}{x^{m-1}} dx.$
102.  $\int \frac{\sqrt{X} dx}{x} = \sqrt{X} + a \int \frac{dx}{x\sqrt{X}} + \frac{b}{2} \int \frac{dx}{\sqrt{X}}.$
103.  $\int \frac{\sqrt{X} dx}{x^2} = -\frac{\sqrt{X}}{x} + \frac{b}{2} \int \frac{dx}{x\sqrt{X}} + c \int \frac{dx}{\sqrt{X}}.$
104.  $\int \frac{\sqrt{bx+cx^2}}{x^2} dx = -\frac{2\sqrt{bx+cx^2}}{x} + c \int \frac{dx}{\sqrt{bx+cx^2}}.$
105.  $\int \frac{\sqrt{X} dx}{x^3} = -\left( \frac{1}{2x^2} + \frac{b}{4ax} \right) \sqrt{X} - \left( \frac{b^2}{8a} - \frac{c}{2} \right) \int \frac{dx}{x\sqrt{X}}.$



$$106. \int \frac{\sqrt{bx+cx^2}}{x^3} dx = -\frac{2\sqrt{(bx+cx^2)^3}}{3bx^3}.$$

$$107. \int \frac{\sqrt{X^3}}{x} dx = \frac{\sqrt{X^3}}{3} + \frac{2bcx+b^2+8ac}{8c}\sqrt{X} + a^2 \int \frac{dx}{x\sqrt{X}} + \frac{b(12ac-b^2)}{16c} \int \frac{dx}{\sqrt{X}}.$$

$$108. \int \frac{\sqrt{X^3}}{x^2} dx = -\frac{\sqrt{X^5}}{ax} + \frac{cx+b}{a}\sqrt{X^3} + \frac{3}{4}(2cx+3b)\sqrt{X} + \frac{3}{2}ab \int \frac{dx}{x\sqrt{X}} + \frac{3(4ac+b^2)}{8} \int \frac{dx}{\sqrt{X}}.$$

$$109. \int \frac{\sqrt{(bx+cx^2)^3}}{x^2} = \frac{\sqrt{(bx+cx^2)^3}}{2x} + \frac{3b}{4}\sqrt{bx+cx^2} + \frac{3b^2}{8} \int \frac{dx}{\sqrt{bx+cx^2}}.$$

$$110. \int \frac{\sqrt{X^3}}{x^3} dx = -\left(\frac{1}{2ax^2} + \frac{b}{4a^2x}\right)\sqrt{X^5} + \frac{bcx+2ac+b^2}{4a^2}\sqrt{X^3} + \frac{3(bcx+2ac+b^2)}{4a}\sqrt{X} \\ + \frac{3}{8}(4ac+b^2) \int \frac{dx}{x\sqrt{X}} + \frac{3}{2}bc \int \frac{dx}{\sqrt{X}}.$$

$$111. \int \frac{\sqrt{(bx+cx^2)^3}}{x^3} dx = \left(c - \frac{2b}{x}\right)\sqrt{bx+cx^2} + \frac{3bc}{2} \int \frac{dx}{\sqrt{bx+cx^2}}.$$

$$112. \int \frac{dx}{x^m\sqrt{X^{2n+1}}} = -\frac{1}{(m-1)ax^{m-1}\sqrt{X^{2n-1}}} \\ - \frac{(2n+2m-3)b}{2(m-1)a} \int \frac{dx}{x^{m-1}\sqrt{X^{2n+1}}} - \frac{(2n+m-2)c}{(m-1)a} \int \frac{dx}{x^{m-2}\sqrt{X^{2n+1}}}.$$

$$113. \int \frac{dx}{x\sqrt{X^{2n+1}}} = \frac{1}{(2n-1)a\sqrt{X^{2n-1}}} - \frac{b}{2a} \int \frac{dx}{\sqrt{X^{2n+1}}} + \frac{1}{a} \int \frac{dx}{x\sqrt{X^{2n-1}}}.$$

$$114. \int \frac{dx}{x^m\sqrt{(bx+cx^2)^{2n+1}}} = -\frac{2}{(2n+2m-1)bx^m\sqrt{(bx+cx^2)^{2n-1}}} \\ - \frac{(4n+2m-2)c}{(2n+2m-1)b} \int \frac{dx}{x^{m-1}\sqrt{(bx+cx^2)^{2n+1}}}.$$

$$115. \int \frac{dx}{x\sqrt{X}} = \begin{cases} -\frac{1}{\sqrt{a}} \ln \frac{2a+bx+2\sqrt{aX}}{x}, & a > 0, \\ \frac{1}{\sqrt{-a}} \arcsin \frac{2a+bx}{x\sqrt{b^2-4ac}}, & a < 0, \Delta < 0, \\ \frac{1}{\sqrt{-a}} \arctan \frac{2a+bx}{2\sqrt{-a}\sqrt{X}}, & a < 0, \\ -\frac{1}{\sqrt{a}} \operatorname{arcsinh} \frac{2a+bx}{x\sqrt{\Delta}}, & a > 0, \Delta > 0, \\ -\frac{1}{\sqrt{a}} \operatorname{arctanh} \frac{2a+bx}{2\sqrt{a}\sqrt{X}}, & a > 0, \\ \frac{1}{\sqrt{a}} \ln \frac{x}{2a+bx}, & a > 0, \Delta = 0, \\ -\frac{2\sqrt{bx+cx^2}}{bx}, & a = 0, b \neq 0, \\ -\frac{1}{\sqrt{a}} \operatorname{arccosh} \left( \frac{2a+bx}{x\sqrt{-\Delta}} \right), & a > 0, \Delta < 0. \end{cases}$$

$$116. \int \frac{dx}{x^2\sqrt{X}} = -\frac{\sqrt{X}}{ax} - \frac{b}{2a} \int \frac{dx}{x\sqrt{X}}.$$

$$117. \int \frac{dx}{x^3\sqrt{X}} = \left( -\frac{1}{2ax^2} + \frac{3b}{4a^2x} \right) \sqrt{X} + \left( \frac{3b^2}{8a^2} - \frac{c}{2a} \right) \int \frac{dx}{x\sqrt{X}}.$$

$$118. \int \frac{dx}{x^2\sqrt{bx+cx^2}} = \frac{2}{3} \left( -\frac{1}{bx^2} + \frac{2c}{b^2x} \right) \sqrt{bx+cx^2}.$$

$$119. \int \frac{dx}{x^3\sqrt{bx+cx^2}} = \frac{2}{5} \left( -\frac{1}{bx^3} + \frac{4c}{3b^2x^2} - \frac{8c^2}{3b^3x} \right) \sqrt{bx+cx^2}.$$

$$120. \int \frac{dx}{x\sqrt{X^3}} = -\frac{2(bc x - 2ac + b^2)}{a\Delta\sqrt{X}} + \frac{1}{a} \int \frac{dx}{x\sqrt{X}}.$$

$$121. \int \frac{dx}{x^2\sqrt{X^3}} = -\frac{A}{\sqrt{X}} - \frac{3b}{2a^2} \int \frac{dx}{x\sqrt{X}} - \frac{3b}{2a^2} \int \frac{dx}{x\sqrt{X}} \\ \text{where } A = \left( -\frac{1}{ax} - \frac{b(10ac - 3b^2)}{a^2\Delta} - \frac{c(8ac - 3b^2)x}{a^2\Delta} \right).$$

$$122. \int \frac{dx}{x^3\sqrt{X^3}} = \left( -\frac{1}{ax^2} + \frac{5b}{2a^2x} - \frac{15b^4 - 62acb^2 + 24a^2c^2}{2a^3\Delta} - \frac{bc(15b^2 - 52ac)x}{2a^3\Delta} \right) \frac{1}{2\sqrt{X}} \\ + \frac{15b^2 - 12ac}{8a^3} \int \frac{dx}{x\sqrt{X}}.$$

$$123. \int \frac{dx}{x\sqrt{(bx+cx^2)^3}} = \frac{2}{3} \left( -\frac{1}{bx} + \frac{4c}{b^2} + \frac{8c^2x}{b^3} \right) \frac{1}{\sqrt{bx+cx^2}}.$$

$$124. \int \frac{dx}{x^2 \sqrt{(bx + cx^2)^3}} = \frac{2}{5} \left( -\frac{1}{bx^2} + \frac{2c}{b^2x} - \frac{8c^2}{b^3} - \frac{16c^3x}{b^4} \right) \frac{1}{\sqrt{bx + cx^2}}.$$

$$125. \int \frac{dx}{x^3 \sqrt{(bx + cx^2)^3}} = \frac{2}{7} \left( -\frac{1}{bx^3} + \frac{8c}{5b^2x^2} - \frac{16c^2}{5b^3x} + \frac{64c^3}{5b^4} + \frac{128c^4x}{5b^5} \right) \frac{1}{\sqrt{bx + cx^2}}.$$


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