! For an efficient use of these tables, first read `HowTo.pdf`.

T2.49A. Integrands involving trigonometric functions of arguments containing trigonometric and hyperbolic functions on the interval $(0, \pi/2)$.

1. $$\int_0^{\pi/2} \sin(z\,\sin x)\,\sin 2x\,dx = \frac{2}{z^2}(\sin z - z\,\cos z).$$

2. $$\int_0^{\pi/2} \cos(z\,\sin x)\,\cos^{2n}\,x\,dx = \frac{\pi}{2}\,\frac{(2n-1)!!}{z^n} J_n(z), \quad \Re\{n\} > -\tfrac{1}{2}.$$

3. $$\int_0^{\pi/2} \sin(z\,\cos x)\,\sin 2x\,dx = \frac{2}{z^2}(\sin z - z\,\cos z).$$

4. $$\int_0^{\pi/2} \sin(z\,\cos x)\,\cos ax\,dx = \begin{cases} \cos\dfrac{a\pi}{2}\,s_{0,a}(z), \\ \quad\text{or} \\ \dfrac{\pi}{4}\,\csc\dfrac{a\pi}{2}\left[\mathbf{J}_\nu(z) - \mathbf{J}_-\nu(z)\right], \\ \quad\text{or} \\ -\dfrac{\pi}{4}\,\sec\,\dfrac{a\pi}{4}\left[\mathbf{E}_\nu(z) + \mathbf{E}_{-\nu}(z)\right], \\ \quad\text{or} \\ \cos\dfrac{a\pi}{2}\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k-1}z^{2k-1}}{(1^2-a^2)(3^2-a^2)\ldots[(2k-1)^2-a^2]}, \quad a > 0. \end{cases}$$

5. $$\int_0^{\pi/2} \sin(z\,\cos x)\,\cos[(2n+1)x]\,dx = (-1)^n\frac{\pi}{2}J_{2n+1}(z).$$

6. $$\int_0^{\pi/2} \sin(a\,\cos x)\,\tan\,x\,dx = \mathrm{Si}(a) + \frac{\pi}{2}, \quad a > 0.$$

7. $$\int_0^{\pi/2} \sin(z\,\cos x)\,\sin^{2\nu}\,x\,dx = \frac{\sqrt{\pi}}{2}\left(\frac{2}{z}\right)^{\nu}\Gamma\left(\nu + \frac{1}{2}\right)\mathbf{H}_\nu(z), \quad \Re\{\nu\} > -\tfrac{1}{2}.$$

8. $$\int_0^{\pi/2} \cos(z\,\cos x)\,\cos ax\,dx = \begin{cases} -a\,\sin\dfrac{a\pi}{2}\,s_{-1,\,a}(z), \\ \qquad \text{or} \\ \dfrac{\pi}{4}\,\sec\,\dfrac{a\pi}{2}\,[\mathbf{J}_\nu(z) + \mathbf{J}_{-\nu}(z)]\,, \\ \qquad \text{or} \\ \dfrac{\pi}{4}\,\csc\,\dfrac{a\pi}{2}\,[\mathbf{E}_\nu(z) - \mathbf{E}_{-\nu}(z)]\,, \\ \qquad \text{or} \\ -a\,\sin\dfrac{a\pi}{2}\left\{-\dfrac{1}{a^2} + \displaystyle\sum_{k=1}^{\infty}\dfrac{(-1)^{k-1}z^{2k}}{a^2(2^2-a^2)(4^2-a^2)\dots[(2k)^2-a^2]}\right\}, \quad a > 0. \end{cases}$$

9. $$\int_0^{\pi/2} \cos(z\,\cos x)\,\cos 2nx\,dx = (-1)^n\,\frac{\pi}{2}J_{2n}(z).$$

10. $$\int_0^{\pi/2} \cos(z\,\cos x)\,\sin^{2\nu}\,x\,dx = \frac{\sqrt{\pi}}{2}\left(\frac{2}{z}\right)^{\nu}\Gamma\left(\nu+\frac{1}{2}\right)J_\nu(z), \quad Re\{\nu\} > -\tfrac{1}{2}.$$

11. $$\int_0^{\pi/2} \sin(a\,\tan x)\,dx = \frac{1}{2}\left[e^{-a}\,\overline{\mathrm{Ei}(a)} - e^{a}\,\mathrm{Ei}(-a)\right], \quad a > 0.$$

12. $$\int_0^{\pi/2} \cos(a\,\tan x)\,dx = \frac{\pi}{2}e^{-a}, \quad a \geq 0.$$

13. $$\int_0^{\pi/2} \sin(a\,\tan x)\,\sin 2x\,dx = \frac{a\pi}{2}e^{-a}, \quad a \geq 0.$$

14. $$\int_0^{\pi/2} \cos(a\,\tan x)\,\sin^2\,x\,dx = \frac{1-a}{4}\pi e^{-a}, \quad a \geq 0.$$

15. $$\int_0^{\pi/2} \cos(a\,\tan x)\,\cos^2\,x\,dx = \frac{1+a}{4}\pi e^{-a}, \quad a \geq 0.$$

16. $$\int_0^{\pi/2} \sin(a\,\tan x)\,\tan x\,dx = \frac{\pi}{2}e^{-a}, \quad a > 0.$$

17. $$\int_0^{\pi/2} \cos(a\,\tan x)\,\tan x\,dx = -\frac{1}{2}\left[e^{-a}\,\overline{\mathrm{Ei}(a)} + e^{a}\,\mathrm{Ei}(-a)\right], \quad a > 0.$$

18. $$\int_0^{\pi/2} \sin(a\,\tan x)\,\sin^2\,x\,\tan x\,dx = \frac{2-a}{4}\pi e^{-a}, \quad a > 0.$$

19. $$\int_0^{\pi/2} \sin^2(a\,\tan x)\,dx = \frac{\pi}{4}\left(1 - e^{-2a}\right), \quad a \geq 0.$$

20. $$\int_0^{\pi/2} \cos^2(a\,\tan x)\,dx = \frac{\pi}{4}\left(1 + e^{-2a}\right), \quad a \geq 0.$$

21. $$\int_0^{\pi/2} \sin^2(a\,\tan x)\,\cot^2 x\,dx = \frac{\pi}{4}\left(e^{-2a} + 2a - 1\right), \quad a \geq 0.$$

22. $$\int_0^{\pi/2} \left[1 - \sec^2 x\,\cos(\tan x)\right]\frac{dx}{\tan x} = \gamma_e.$$

23. $$\int_0^{\pi/2} \sin(a\,\cot x)\,\sin 2x\,dx = \frac{a\pi}{2}\,e^{-a}, \quad a \geq 0.$$

24. $$\int_0^{\pi/2} \sin(a\,\csc x)\,\sin(a\,\cot x)\,\frac{dx}{\cos x} = \int_0^{\pi/2} \sin(a\,\sec x)\,\sin(a\,\tan x)\,\frac{dx}{\sin x} = \frac{\pi}{2}\,\sin a, \quad a \geq 0.$$

25. $$\int_0^{\pi/2} \sin\left(\frac{\pi}{2}p - a\,\tan x\right)\tan^{p-1} x\,dx = \int_0^{\pi/2} \cos\left(\frac{\pi}{2}p - a\,\tan x\right)\tan^p x\,dx = \frac{\pi}{2}\,e^{-a}, \quad p^2 < 1,\ p \neq 0,\ a \geq 0.$$

26. $$\int_0^{\pi/2} \sin(a\,\tan x - \nu x)\,\sin^{\nu-2} x\,dx = 0, \quad \Re\{\nu\} > 0,\ a > 0.$$

27. $$\int_0^{\pi/2} \sin(n\,\tan x + \nu x)\,\frac{\cos^{\nu-1} x}{\sin x}\,dx = \frac{\pi}{2}, \quad \Re\{\nu\} > 0.$$

28. $$\int_0^{\pi/2} \cos(a\,\tan x - \nu x)\,\cos^{\nu-2} x\,dx = \frac{\pi e^{-a} a^{\nu-1}}{\Gamma(\nu)}, \quad \Re\{\nu\} > 1;\ a > 0.$$

29. $$\int_0^{\pi/2} \cos(a\,\tan x + \nu x)\,\cos^{\nu} x\,dx = 2^{-\nu-1}\pi e^{-a}, \quad \Re\{\nu\} > -1,\ a \geq 0.$$

30. $$\int_0^{\pi/2} \cos(a\,\tan x - \gamma x)\,\cos^{\nu} x\,dx = \frac{\pi a^{\nu/2}}{2^{\nu/2+1}} \cdot \frac{W_{\gamma/2,\,-(\nu+1)/2}(2a)}{\Gamma\left(1 + \frac{\gamma+\nu}{2}\right)}, \quad a > 0,\ \Re\{\nu\} > -1,\ \frac{\gamma+\nu}{2} \neq -1, -2, \ldots.$$

31. $$\int_0^{\pi/2} \frac{\sin nx - \sin(nx - a\tan x)}{\sin x} \cos^{n-1} x \, dx = \begin{cases} \pi/2, & n = 0,\ a > 0, \\ \pi\left(1 - e^{-a}\right), & n = 1,\ a \geq 0. \end{cases}$$