! For an efficient use of these tables, first read `HowTo.pdf`.

T2.58C. Integrands involving product of trigonometric and exponential functions of trigonometric functions on the interval $(0, 2\pi)$.

1. $$\int_0^{2\pi} e^{p\,\cos x}\,\cos(p\,\sin x - mx)\,dx = 2\int_0^{\pi} e^{p\,\cos x}\,\cos(p\,\sin x - mx)\,dx = \frac{2\pi p^m}{m!}.$$

2. $$\int_0^{2\pi} e^{p\,\sin x}\,\sin(p\,\cos x + mx)\,dx = \frac{2\pi p^m}{m!}\,\sin\frac{m\pi}{2}, \quad p > 0.$$

3. $$\int_0^{2\pi} e^{p\,\sin x}\,\cos(p\,\cos x + mx)\,dx = \frac{2\pi p^m}{m!}\,\cos\frac{m\pi}{2}, \quad p > 0.$$

4. $$\int_0^{2\pi} e^{\cos x}\,\sin(mx - \sin x)\,dx = 0.$$

5. $$\int_0^{2\pi} \exp(p\,\cos x + q\,\sin x)\,\sin(a\,\cos x + b\,\sin x - mx)\,dx$$
$$= i\,\pi[(b-p)^2+(a+q)^2]^{-m/2}\left\{(A+iB)^{m/2}\,I_m(\sqrt{C-iD}) - (A-iB)^{m/2}\,I_m(\sqrt{C+iD})\right\},$$
where $(b-p)^2+(a+q)^2>0,\ m=0,1,2,\dots,\quad A=p^2-q^2+a^2-b^2,\quad B=2(pq+ab),$
$C=p^2+q^2-a^2-b^2,\quad D=2(ap+bq).$

6. $$\int_0^{2\pi} \exp(p\,\cos x + q\,\sin x)\,\cos(a\,\cos x + b\,\sin x - mx)\,dx$$
$$= \pi[(b-p)^2+(a+q)^2]^{-m/2}\left\{(A+iB)^{m/2}\,I_m(\sqrt{C-iD}) + (A-iB)^{m/2}\,I_m(\sqrt{C+iD})\right\},$$
where $(b-p)^2+(a+q)^2>0,\ m=0,1,2,\dots,\quad A=p^2-q^2+a^2-b^2,\quad B=2(pq+ab),$
$C=p^2+q^2-a^2-b^2,\quad D=2(ap+bq).$

7. $$\int_0^{2\pi} \exp(p\,\cos x + q\,\sin x)\,\sin(q\,\cos x - p\,\sin x + mx)\,dx = \frac{2\pi}{m!}(p^2+q^2)\frac{m}{2}\,\sin\left(m\,\arctan\frac{q}{p}\right).$$

8. $\displaystyle\int_0^{2\pi} \exp(p\,\cos x + q\,\sin x)\,\cos(q\,\cos x - p\,\sin x + mx)\,dx = \frac{2\pi}{m!}(p^2+q^2)\frac{m}{2}\,\cos\left(m\,\arctan\frac{q}{p}\right).$
