! For an efficient use of these tables, first read `HowTo.pdf`.

T2.45D. Integrands involving powers of linear trigonometric functions on the interval $(0, 2\pi)$.

1. $$\int_0^{2\pi} (a\,\sin x + b\,\cos x)^{2n+1}\,dx = 0.$$

2. $$\int_0^{2\pi} (a\,\sin x + b\,\cos x)^{2n}\,dx = \frac{(2n-1)!!}{(2n)!!}\,2\pi(a^2+b^2)^n.$$

3. $$\int_0^{2\pi} \left[\beta + \sqrt{\beta^2-1}\,\cos(a-x)\right]^{\nu} (\gamma + \sqrt{\gamma^2-1}\,\cos x)^{\nu-1}\,dx$$
$$= 2\pi P_\nu\left[\beta\gamma - \sqrt{\beta^2-1}\sqrt{\gamma^2-1}\,\cos a\right], \quad \Re\{\beta\} > 0,\ \Re\{\gamma\} > 0.$$

4. $$\int_0^{2\pi} [\cos t + i\,\sin t\,\cos(a-x)]^{\nu}\,\cos mx\,dx = \frac{i^{3m}2\pi\Gamma(\nu+1)}{\Gamma(\nu+m+1)}\,\cos ma\,P_\nu^{(m)}(\cos t), \quad 0 < t < \frac{\pi}{2}.$$

5. $$\int_0^{2\pi} [\cos t + i + \sin t\,\cos(a-x)]^{\nu}\,\sin mx\,dx = \frac{i^{3m}2\pi\Gamma(\nu+1)}{\Gamma(\nu+m+1)}\,\sin ma\,P_\nu^{(m)}(\cos t), \quad 0 < t < \frac{\pi}{2}.$$

6. $$\int_0^{2\pi} \frac{dx}{(a+b\,\cos x)^{n+1}} = \begin{cases} \dfrac{2\pi}{(a^2-b^2)^{(n+1)/2}}\,P_n\left(\dfrac{a}{\sqrt{a^2-b^2}}\right), \\ \text{or} \\ \dfrac{2\pi}{2^n(a+b)^n\sqrt{a^2-b^2}}\displaystyle\sum_{k=0}^{n}\frac{(2n-2k-1)!!(2k-1)!!}{(n-k)!k!}\,\left(\dfrac{a+b}{a-b}\right)^k, \end{cases} \quad a > |b|.$$
