! For an efficient use of these tables, first read `HowTo.pdf`.

T3.39A. Integrands involving powers of trigonometric functions and powers of $(a + b\,x^n)$ for $n = 1, 2, 3, 4$, on the interval $(0, \infty)$.

1. $$\int_0^\infty \frac{\sin^p\, x}{x}\,dx = \frac{\sqrt{\pi}}{2}\,\frac{\Gamma\left(\frac{p}{2}\right)}{\Gamma\left(\frac{p+1}{2}\right)} = 2^{p-2}\,\mathrm{B}\left(\frac{p}{2}, \frac{p}{2}\right),$$

   where p is a fraction with odd numerator and denominator.

2. $$\int_0^\infty \frac{\sin^{2n+1}\, x}{x}\,dx = \frac{(2n-1)!!}{(2n)!!}\,\frac{\pi}{2}.$$

3. $$\int_0^\infty \frac{\sin^{2n}\, x}{x}\,dx = \infty.$$

4. $$\int_0^\infty \frac{\sin^2\, ax}{x^2}\,dx = \frac{a\pi}{2}, \quad a > 0.$$

5. $$\int_0^\infty \frac{\sin^{2m}\, ax}{x^2}\,dx = \frac{(2m-3)!!}{(2m-2)!!}\,\frac{a\pi}{2}, \quad a > 0.$$

6. $$\int_0^\infty \frac{\sin^{2m+1}\, ax}{x^3}\,dx = \frac{(2m-3)!!}{(2m)!!}(2m+1)\frac{a^2\pi}{4}, \quad a > 0.$$

7. $$\int_0^\infty \frac{\sin^p\, x}{x^m}\,dx = \begin{cases} \dfrac{p}{m-1}\displaystyle\int_0^\infty \frac{\sin^{p-1}\, x}{x^{m-1}}\cos x\,dx, & p > m-1 > 0, \\ \dfrac{p(p-1)}{(m-1)(m-2)}\displaystyle\int_0^\infty \frac{\sin^{p-2}\, x}{x^{m-2}}\,dx - \frac{p^2}{(m-1)(m-2)}\int_0^\infty \frac{\sin^p\, x}{x^{m-2}}\,dx, & p > m-1 > 1. \end{cases}$$

8. $$\int_0^\infty \frac{\sin^{2n}\, px}{\sqrt{x}}\,dx = \infty.$$

9. $\displaystyle\int_0^\infty \sin^{2n+1} px \frac{dx}{\sqrt{x}} = \frac{1}{2^{2n}}\sqrt{\frac{\pi}{2p}}\sum_{k=0}^{n}(-1)^k\binom{2n+1}{n+k+1}\frac{1}{\sqrt{2k+1}}.$

10. $\displaystyle\int_0^\infty x^{-\frac{1}{2}} \cos^{2n+1}(px)\,dx = \frac{1}{2^{2n}}\sqrt{\frac{\pi}{2p}}\sum_{k=0}^{n}\binom{2n+1}{n+k+1}\frac{1}{\sqrt{2k+1}}.$

11. $\displaystyle\int_0^\infty x^{\mu-1} \sin^2 ax\,dx = -\frac{\Gamma(\mu)\cos(\mu\pi/2)}{2^{\mu+1}a^{\mu}}, \quad a>0,\ -2<\Re\{\mu\}<0.$

12. $\displaystyle\int_0^\infty \frac{\sin^2 ax}{x^2+\beta^2}\,dx = \frac{\pi}{4\beta}(1-e^{-2a\beta}), \quad a>0,\ \Re\{\beta\}>0.$

13. $\displaystyle\int_0^\infty \frac{\cos^2 ax}{x^2+\beta^2}\,dx = \frac{\pi}{4\beta}(1+e^{-2a\beta}), \quad a>0,\ \Re\{\beta\}>0.$

14. $$\int_0^\infty \sin^{2m} x\frac{dx}{a^2+x^2} = \frac{(-1)^m}{2^{2m+1}}\frac{\pi}{a}\left\{2^{2m}\sinh^{2m} a - 2\sum_{k=0}^{m}(-1)^k\binom{2m}{k}\sinh\left[2(m-k)a\right]\right\},$$
$$a>0.$$

15. $$\int_0^\infty \sin^{2m+1} x\frac{dx}{a^2+x^2} = \frac{(-1)^{m-1}}{2^{2m+2}a}\left\{e^{(2m+1)a}\sum_{k=0}^{2m+1}(-1)^k\binom{2m+1}{k}e^{-2ka}\,\mathrm{Ei}\left[(2k-2m-1)a\right]\right.$$
$$\left.+e^{-(2m+1)a}\sum_{k=0}^{2m+1}(-1)^{k-1}\binom{2m+1}{k}e^{2ka}\,\mathrm{Ei}\left[(2m+1-2k)a\right]\right\}, \quad a>0.$$

16. $$\int_0^\infty \sin^{2m+1} x\frac{x\,dx}{a^2+x^2} = \frac{\pi}{2^{2m+1}}e^{-(2m+1)a}\sum_{k=0}^{m}(-1)^{m+k}\binom{2m+1}{k}e^{2ka},$$
$$|\arg a|<\pi/2,\ m=0,1,2,\ldots.$$

17. $\displaystyle\int_0^\infty \cos^{2m} x\frac{dx}{a^2+x^2} = \frac{\pi}{2^{2m+1}a}\binom{2m}{m} + \frac{\pi}{2^{2m}a}\sum_{k=1}^{m}\binom{2m}{m+k}e^{-2ka}, \quad a>0.$

18. $\displaystyle\int_0^\infty \cos^{2m+1} x\frac{dx}{a^2+x^2} = \frac{\pi}{2^{2m+1}a}\sum_{k=1}^{m}\binom{2m+1}{m+k+1}e^{-(2k+1)a}, \quad a>0.$

19. $$\int_0^\infty \cos^{2m+1} x \frac{x\,dx}{a^2+x^2} = -\frac{e^{-(2m+1)a}}{2^{2m+2}} \sum_{k=0}^{2m+1} \binom{2m+1}{k} e^{2ka}\,\mathrm{Ei}[(2m-2k+1)a] - \frac{e^{(2m+1)a}}{2^{2m+2}} \sum_{k=0}^{2m+1} \binom{2m+1}{k} e^{-2ka}\,\mathrm{Ei}[(2k-2m-1)a].$$

20. $$\int_0^\infty \frac{\cos^2 ax}{b^2-x^2}\,dx = \frac{\pi}{4b}\sin 2ab, \quad a>0,\ b>0.$$

21. $$\int_0^\infty \frac{\sin^2 ax\,\cos^2 bx}{\beta^2+x^2}\,dx = \begin{cases} \dfrac{\pi}{8\beta}\left[1-\dfrac{1}{2}e^{-2(a+b)\beta}+e^{-2b\beta}-\dfrac{1}{2}e^{2(b-a)\beta}-e^{-2a\beta}\right], & a>b, \\ \dfrac{\pi}{16\beta}\left[1-e^{-4a\beta}\right], & a=b, \\ \dfrac{\pi}{8\beta}\left[1-\dfrac{1}{2}e^{-2(a+b)\beta}+e^{-2b\beta}-\dfrac{1}{2}e^{2(a-b)\beta}-e^{-2a\beta}\right], & a<b, \end{cases}$$
$$a>0,\ b>0.$$

22. $$\int_0^\infty \frac{x\,\sin 2ax\,\cos^2 bx}{\beta^2+x^2}\,dx = \begin{cases} \dfrac{\pi}{8}\left[2e^{-2a\beta}+e^{-2(a+b)\beta}+e^{2(b-a)\beta}\right], & a>0, \\ \dfrac{\pi}{8}\left[e^{-4a\beta}+2e^{-2a\beta}\right], & a=b, \\ \dfrac{\pi}{8}\left[2e^{-2a\beta}+e^{-2(a+b)\beta}-e^{2(a-b)\beta}\right], & a<b. \end{cases}$$

23. $$\int_0^\infty \frac{\sin^2 ax\,dx}{(b^2+x^2)(c^2+x^2)} = \frac{\pi(b-c+ce^{-2ab}-be^{-2ac})}{4bc(b^2-c^2)}, \quad a>0,\ b>0,\ c>0.$$

24. $$\int_0^\infty \frac{\cos^2 ax\,dx}{(b^2+x^2)(c^2+x^2)} = \frac{\pi(b-c+be^{-2ac}-ce^{-2ab})}{4bc(b^2-c^2)}, \quad a>0,\ b>0,\ c>0.$$

25. $$\int_0^\infty \frac{\sin^2 ax\,dx}{(b^2-x^2)(c^2-x^2)} = \frac{\pi(c\,\sin 2ab-b\,\sin 2ac)}{4bc(b^2-c^2)}, \quad a>0,\ b>0,\ c>0,\ b\neq c.$$

26. $$\int_0^\infty \frac{\cos^2 ax\,dx}{(b^2-x^2)(c^2-x^2)} = \frac{\pi(b\,\sin 2ac-c\,\sin 2ab)}{4bc(b^2-c^2)}, \quad a>0,\ b>0,\ c>0,\ b\neq c.$$

27. $$\int_0^\infty \frac{\sin^2 ax\,dx}{x^2(b^2+x^2)} = \frac{\pi}{4b^2}\left[2a-\frac{1}{b}(1-e^{-2ab})\right], \quad a>0,\ b>0.$$

28. $$\int_0^\infty \frac{\sin^2 ax\,dx}{x^2(b^2-x^2)} = \frac{\pi}{4b^2}\left(2a-\frac{1}{b}\sin 2ab\right), \quad a>0,\ b>0.$$

29. $$\int_0^\infty \frac{\sin^3 ax}{x^\nu}\,dx = \frac{3-3^{\nu-1}}{4}a^{\nu-1}\cos\frac{\nu\pi}{2}\Gamma(1-\nu), \quad a>0,\ 0<\Re\{\nu\}<4.$$

30. $$\int_0^\infty \frac{\sin^3 ax}{x}\,dx = \frac{\pi}{4}\ \operatorname{sgn} a.$$

31. $$\int_0^\infty \frac{\sin^3 ax}{x^2}\,dx = \frac{3}{4}a\ \ln 3.$$

32. $$\int_0^\infty \frac{\sin^3 ax}{x^3}\,dx = \frac{3}{8}a^2\pi\ \operatorname{sgn} a.$$

33. $$\int_0^\infty \frac{\sin^4 ax}{x^2}\,dx = \frac{a\pi}{4}, \quad a>0.$$

34. $$\int_0^\infty \frac{\sin^4 ax}{x^3}\,dx = a^2\ \ln 2.$$

35. $$\int_0^\infty \frac{\sin^4 ax}{x^4}\,dx = \frac{a^3\pi}{3}, \quad a>0.$$

36. $$\int_0^\infty \frac{\sin^5 ax}{x^2}\,dx = \frac{5}{16}a(3\ \ln 3-\ln 5).$$

37. $$\int_0^\infty \frac{\sin^5 ax}{x^3}\,dx = \frac{5}{32}a^2\pi, \quad a>0.$$

38. $$\int_0^\infty \frac{\sin^5 ax}{x^4}\,dx = \frac{5}{96}a^3(25\ \ln 5-27\ \ln 3).$$

39. $$\int_0^\infty \frac{\sin^5 ax}{x^5}\,dx = \frac{115}{384}a^4\pi, \quad a>0.$$

40. $$\int_0^\infty \frac{\sin^6 ax}{x^2}\,dx = \frac{3}{16}a\pi, \quad a>0.$$

41. $$\int_0^\infty \frac{\sin^6 ax}{x^3}\,dx = \frac{3}{16}a^2(8\ \ln 2-3\ \ln 3).$$

42. $$\int_0^\infty \frac{\sin^6 ax}{x^5}\,dx = \frac{1}{16}a^4(27\,\ln 3 - 32\,\ln 2).$$

43. $$\int_0^\infty \frac{\sin^6 ax}{x^6}\,dx = \frac{11}{40}a^5\pi, \quad a > 0.$$

44. $$\int_0^\infty \frac{\sin px\,\sin qx}{x}\,dx = \ln\sqrt{\frac{p+q}{|p-q|}}, \quad p \neq q.$$

45. $$\int_0^\infty \sin qx\,\sin px\frac{dx}{x^2} = \begin{cases} \frac{1}{2}p\pi, & p \le q, \\ \frac{1}{2}q\pi, & p \ge q. \end{cases}$$

46. $$\int_0^\infty \frac{\sin^2 ax\,\sin bx}{x}\,dx = \begin{cases} \frac{\pi}{4}, & 0 < b < 2a, \\ \frac{\pi}{8}, & b = 2a, \\ 0, & b > 2a. \end{cases}$$

47. $$\int_0^\infty \frac{\sin^2 ax\,\cos bx}{x}\,dx = \frac{1}{4}\,\ln\frac{4a^2 - b^2}{b^2}.$$

48. $$\int_0^\infty \frac{\sin^2 ax\,\cos 2bx}{x^2}\,dx = \begin{cases} \frac{\pi}{2}(a-b), & b < a, \\ 0, & b \ge a. \end{cases}$$

49. $$\int_0^\infty \frac{\sin 2ax\,\cos^2 bx}{x}\,dx = \begin{cases} \frac{\pi}{2}, & a > b, \\ \frac{3}{8}\pi, & a = b, \\ \frac{\pi}{4}, & a < b. \end{cases}$$

50. $$\int_0^\infty \frac{\sin^2 ax\,\sin bx\,\sin cx}{x^2}\,dx = \frac{\pi}{16}\left(|b-2a-c| - |2a-b-c| + 2c\right), \quad a > 0,\, b > 0,\, c > 0.$$

51. $$\int_0^\infty \frac{\sin^2 ax\,\sin bx\,\sin cx}{x}\,dx = \frac{1}{4}\,\ln\frac{b+c}{b-c} + \frac{1}{8}\,\ln\frac{(2a-b+c)(2a+b-c)}{(2a+b+c)(2a-b-c)}, \quad a > 0,\, b > 0,\, c > 0,\, b \neq c.$$

52. $$\int_0^\infty \frac{\sin^2 ax\,\sin^2 bx}{x^2}\,dx = \begin{cases} \frac{\pi}{4}\,a, & 0 \le a \le b, \\ \frac{\pi}{4}\,b, & 0 \le b \le a. \end{cases}$$

$$53.\ \int_0^\infty \frac{\sin^2 ax\,\sin^2 bx}{x^4}\,dx = \begin{cases} \dfrac{1}{6}\,a^2\pi(3b-a), & 0 \le a \le b, \\ \dfrac{1}{6}\,b^2\pi(3a-b), & 0 \le b \le a. \end{cases}$$

$$54.\ \int_0^\infty \frac{\sin^2 ax\,\cos^2 bx}{x^2}\,dx = \begin{cases} \dfrac{2a-b}{4}\,\pi, & a \ge b > 0, \\ \dfrac{a\pi}{4}, & 0 < a \le b. \end{cases}$$

$$55.\ \int_0^\infty \frac{\sin^3 ax\,\sin 3bx}{x^4}\,dx = \begin{cases} \dfrac{a^3\pi}{2}, & b > a, \\ \dfrac{\pi}{16}\left[8a^3 - 9(a-b)^3\right], & a \le 3b \le 3a, \\ \dfrac{9b\pi}{8}(a^2-b^2), & 3b \le a. \end{cases}$$

$$56.\ \int_0^\infty \frac{\sin^3 ax\,\cos bx}{x}\,dx = \begin{cases} 0, & b > 3a, \\ -\dfrac{\pi}{16}, & b = 3a, \\ -\dfrac{\pi}{8}, & 3a > b > a, \\ \dfrac{\pi}{16}, & b = a, \\ \quad\dfrac{\pi}{4}, & a > b, \end{cases} \qquad a > 0,\ b > 0.$$

$$57.\ \int_0^\infty \frac{\sin^3 ax\,\cos 3bx}{x^2}\,dx = \frac{3}{8}\Big\{(a+b)\,\ln[3(a+b)] + (b-a)\,\ln[3(b-a)] - \frac{1}{3}(a+3b)\,\ln(a+3b)$$
$$-\frac{1}{3}(3b-a)\,\ln(3b-a)\Big\}, \quad a > 0,\ b > 0.$$

$$58.\ \int_0^\infty \frac{\sin^3 ax\,\cos bx}{x^3}\,dx = \begin{cases} \dfrac{\pi}{8}(3a^2-b^2), & b < a, \\ \dfrac{\pi b^2}{4}, & a = b, \\ \dfrac{\pi}{16}(3a-b)^2, & a < b < 3a, \\ 0, & 3a < b, \end{cases} \qquad a > 0,\ b > 0.$$

$$59.\ \int_0^\infty \frac{\sin^3 ax\,\sin bx}{x^4}\,dx = \begin{cases} \dfrac{b\pi}{24}(9a^2-b^2), & 0 < b \le a, \\ \dfrac{\pi}{48}\left[24a^3 - (3a-b)^3\right], & 0 < a \le b \le 3a, \\ \dfrac{\pi a^3}{2}, & 0 < 3a \le b. \end{cases}$$

60. $$\int_0^\infty \frac{\sin^3 ax\,\sin^2 bx}{x}\,dx = \begin{cases} \dfrac{\pi}{8}, & 2b > 3a, \\ \dfrac{5\pi}{32}, & 2b = 3a, \\ \dfrac{3\pi}{16}, & 3a > 2b > a, \\ \dfrac{3\pi}{32}, & 2b = a, \\ 0, & a > 2b, \end{cases} \qquad a > 0,\ b > 0.$$

61. $$\int_0^\infty \frac{\sin^2 ax\,\cos^3 bx}{x}\,dx = \begin{cases} \dfrac{1}{16}\ln\dfrac{(2a+b)^3(b-2a)^3(2a+3b)(3b-2a)}{9b^8}, & b > 2a > 0 \text{ or } 2a > 3b > 0, \\ \dfrac{1}{16}\ln\dfrac{(2a+b)^3(2a-b)^3(2a+3b)(3b-2a)}{9b^8}, & 3b > 2a > b. \end{cases}$$

62. $$\int_0^\infty \frac{\sin^2 ax\,\sin^2 bx\,\sin 2cx}{x}\,dx = \frac{\pi}{16}\big[1 + \operatorname{sgn}(c-a+b) + \operatorname{sgn}(c+a-b) - 2\operatorname{sgn}(c-a) \\ -2\operatorname{sgn}(c-b)\big], \quad a > 0,\ b > 0,\ c > 0.$$

63. $$\int_0^\infty \frac{\sin^2 ax \sin^2 bx \sin 2cx\,dx}{x^2} = \frac{a-b-c}{16}\ln 4(a-b-c)^2 - \frac{a+b+c}{16}\ln 4(a+b+c)^2 \\ + \frac{a+b-c}{16}\ln 4(a+b-c)^2 - \frac{a-b+c}{16}\ln 4(a-b+c)^2 + \frac{a+c}{8}\ln 4(a+c)^2 \\ - \frac{a-c}{8}\ln 4(a-c)^2 + \frac{b+c}{8}\ln 4(b+c)^2 - \frac{b-c}{8}\ln 4(b-c)^2 - \frac{1}{2}c\ln 2c, \quad a > 0,\ b > 0,\ c > 0.$$

64. $$\int_0^\infty \frac{\sin^2 ax\,\sin^3 bx}{x^3}\,dx = \begin{cases} \dfrac{3b^2\pi}{16}, & 2a > 3b, \\ \dfrac{a^2\pi}{12}, & 2a = 3b, \\ \dfrac{6b^2-(3b-2a)^2}{32}\pi, & 3b > 2a > b, \\ \dfrac{a^2\pi}{4}, & b > 2a, \end{cases} \qquad a > 0,\ b > 0.$$

65. $$\int_0^\infty \frac{x^n - \sin^n x}{x^{n+2}}\,dx = \frac{\pi}{2^n(n+1)!}\sum_{k=0}^{[(n-1)/2]}(-1)^k\binom{n}{k}(n-2k)^{n+1}.$$

66. $$\int_0^\infty (1-\cos^{2m-1}x)\frac{dx}{x^2} = \int_0^\infty (1-\cos^{2m}x)\frac{dx}{x^2} = \frac{m\pi}{2^{2m}}\binom{2m}{m}.$$

67. $\displaystyle\int_0^\infty \frac{\sin^{2n} ax - \sin^{2n} bx}{x}\,dx = \frac{(2n-1)!!}{(2n)!!}\ \ln\frac{b}{a}, \quad ab>0,\ n=1,2,\ldots.$

68. $\displaystyle\int_0^\infty \frac{\cos^{2n} ax - \cos^{2n} bx}{x}\,dx = \left[1-\frac{(2n-1)!!}{(2n)!!}\right]\ln\frac{b}{a}, \quad ab>0,\ n=0,1,\ldots.$

69. $\displaystyle\int_0^\infty \frac{\cos^{2m+1} ax - \cos^{2m+1} bx}{x}\,dx = \ln\frac{b}{a}, \quad ab>0,\ m=0,1,\ldots.$

70. $\displaystyle\int_0^\infty \frac{\cos^{m} ax\,\cos max - \cos^{m} bx\,\cos mbx}{x}\,dx = \left(1-\frac{1}{2^m}\right)\ln\frac{b}{a}, \quad ab>0,\ m=0,1,\ldots.$

71. $\displaystyle\int_0^{\pi/2} x\,\cos^{p-1} x\,\sin ax\,dx = \frac{\pi}{2^{p+1}}\Gamma(p)\frac{\psi\left(\frac{p+a+1}{2}\right)-\psi\left(\frac{p-a+1}{2}\right)}{\Gamma\left(\frac{p+a+1}{2}\right)\Gamma\left(\frac{p-a+1}{2}\right)},$

$p>0,\ -(p+1)<a<p+1.$

72. $\displaystyle\int_0^\infty \sin^{2m+1} x\,\sin 2mx\frac{dx}{a^2+x^2} = \frac{(-1)^m\pi}{2^{2m+1}a}\left[(1-e^{-2a})^{2m}-1\right]\sinh a, \quad a>0,\ m=0,1,\ldots.$

73. $\displaystyle\int_0^\infty \sin^{2m-1} x\,\sin[(2m-1)x]\frac{dx}{a^2+x^2} = \frac{(-1)^{m+1}\pi}{2^{2m}a}(1-e^{-2a})^{2m-1}, \quad a>0,\ m=1,\ 2,\ldots.$

74. $\displaystyle\int_0^\infty \sin^{2m-1} x\,\sin[(2m+1)x]\frac{dx}{a^2+x^2} = \frac{(-1)^{m-1}\pi}{2^{2m}a}e^{-2a}\left(1-e^{-2a}\right)^{2m-1},$

$a>0,\ m=1,\ 2,\ldots.$

75. $\displaystyle\int_0^\infty \sin^{2m+1} x\,\sin[3(2m+1)x]\frac{dx}{a^2+x^2} = \frac{(-1)^m\pi}{2a}\,e^{-3(2m+1)a}\,\sinh^{2m+1} a, \quad a>0.$

76. $\displaystyle\int_0^\infty \sin^{2m} x\,\sin[(2m-1)x]\frac{x\,dx}{a^2+x^2} = \frac{(-1)^m\pi}{2^{2m+1}}e^{a}\left[(1-e^{-2a})^{2m}-\left(1+e^{-2a}\right)\right],$

$a\ge 0,\ m=0,1,\ldots.$

77. $\displaystyle\int_0^\infty \sin^{2m} x\,\sin(2mx)\frac{x\,dx}{a^2+x^2} = \frac{(-1)^m\pi}{2^{2m+1}}\left[(1-e^{-2a})^{2m}-1\right], \quad a>0,\ m=0,1,\ldots.$

78. $\displaystyle\int_0^\infty \sin^{2m} x\,\sin[(2m+2)x]\frac{x\,dx}{a^2+x^2} = \frac{(-1)^m\pi}{2^{2m+1}}\,e^{-2a}(1-e^{-2a})^{2m}, \quad a>0,\ m=0,1,\ldots.$

79. $\displaystyle\int_0^\infty \sin^{2m} x \, \sin 4mx \frac{x\,dx}{a^2+x^2} = \frac{(-1)^m \pi}{2} e^{-4ma} \sinh^{2m} a, \quad a>0,\ m=1,2,\ldots.$

80. $\displaystyle\int_0^\infty \sin^{2m} x \, \cos x \frac{dx}{x^2} = \frac{(2m-3)!!}{(2m)!!}\,\frac{\pi}{2}, \quad m=1,2,\ldots.$

81. $\displaystyle\int_0^\infty \sin^{2m} x \, \cos[(2m-1)x] \frac{dx}{a^2+x^2} = \frac{(-1)^m \pi}{2^{2m} a}\left[(1-e^{-2a})^{2m-1}-1\right] \sinh a,$
$a>0,\ m=1,2,\ldots.$

82. $\displaystyle\int_0^\infty \sin^{2m} x \, \cos(2mx) \frac{dx}{a^2+x^2} = \frac{(-1)^m \pi}{2^{2m+1} a}\left(1-e^{-2a}\right)^{2m}, \quad a>0,\ m=0,1,\ldots.$

83. $\displaystyle\int_0^\infty \sin^{2m} x \, \cos[(2m+2)x] \frac{dx}{a^2+x^2} = \frac{(-1)^m \pi}{2^{2m+1} a} e^{-2a}\left(1-e^{-2a}\right)^{2m}, \quad a>0,\ m=0,1,\ldots.$

84. $\displaystyle\int_0^\infty \sin^{2m} x \, \cos 4mx \frac{dx}{a^2+x^2} = \frac{(-1)^m \pi}{2a} e^{-4ma} \sinh^{2m} a, \quad a>0,\ m=0,1,\ldots.$

85. $\displaystyle\int_0^\infty \sin^{2m+1} x \, \cos x \frac{dx}{x} = \frac{(2m-1)!!}{(2m+2)!!}\,\frac{\pi}{2}, \quad m=0,1,\ldots.$

86. $\displaystyle\int_0^\infty \sin^{2m+1} x \, \cos x \frac{dx}{x^3} = \frac{(2m-3)!!}{(2m)!!}\,\frac{\pi}{2}, \quad m=1,2,\ldots.$

87. $\displaystyle\int_0^\infty \sin^{2m-1} x \, \cos[(2m-1)x] \frac{x\,dx}{a^2+x^2} = \frac{(-1)^m \pi}{2^{2m}}\left[(1-e^{-2a})^{2m-1}-1\right],$
$a>0,\ m=1,2,\ldots.$

88. $\displaystyle\int_0^\infty \sin^{2m+1} x \, \cos 2mx \frac{x\,dx}{a^2+x^2} = \frac{(-1)^{m-1} \pi}{2^{2m+2}}\left\{e^a[(1-e^{-2a})^{2m+1}-1]-e^{-a}\right\},$
$m=0,;1,\ldots,\ a\geq 0.$

89. $\displaystyle\int_0^\infty \sin^{2m-1} x \, \cos[(2m+1)x] \frac{x\,dx}{a^2+x^2} = \frac{(-1)^m \pi}{2^{2m}} e^{-2a}\left(1-e^{-2a}\right)^{2m-1},$
$m=1,2,\ldots;\ a>0.$

90. $\displaystyle\int_0^\infty \sin^{2m+1} x \, \cos[2(2m+1)x] \frac{x\,dx}{a^2+x^2} = \frac{(-1)^{m-1} \pi}{2}\, e^{-2(2m+1)a} \sinh^{2m+1} a,$
$m=0,1,\ldots;\ a>0.$

91. $$\int_0^\infty \cos^m x \,\sin mx \frac{x\,dx}{a^2+x^2} = \frac{1}{2^{m+1}a}\sum_{k=1}^{m}\binom{m}{k}\left[e^{-2ka}\,\mathrm{Ei}(2ka) - e^{2ka}\,\mathrm{Ei}(-2ka)\right], \quad a>0.$$

92. $$\int_0^\infty \cos^n sx \,\sin nsx \frac{x\,dx}{a^2+x^2} = \frac{\pi}{2^{n+1}}[(1+e^{-2as})^n - 1], \quad s>0,\ \Re\{a\}>0,\ n\ge 0.$$

93. $$\int_0^\infty \cos^n sx \,\sin nsx \frac{x\,dx}{a^2-x^2} = \frac{\pi}{2}(2^{-n} - \cos^n as\,\cos nas), \quad n=0,1,\dots.$$

94. $$\int_0^\infty \cos^{m-1} x \,\sin[(m+1)x] \frac{x\,dx}{a^2+x^2} = \frac{\pi}{2^m}e^{-2a}(1+e^{-2a})^{m-1}, \quad a>0,\ m=1,2,\dots.$$

95. $$\int_0^\infty \cos^m x \,\sin[(m+1)x] \frac{x\,dx}{a^2+x^2} = \frac{\pi}{2^{m+1}}e^{-a}(1+e^{-2a})^m, \quad m=0,1,\dots,\ a>0.$$

96. $$\int_0^\infty \cos^m x \,\sin[(m-1)x] \frac{x\,dx}{a^2+x^2} = \frac{\pi}{2^m}\cosh a[(1+e^{-2a})^{m-1} - 1], \quad m=0,1,\dots;\ a\ge 0.$$

97. $$\int_0^\infty \cos^m x \,\sin(3mx) \frac{x\,dx}{a^2+x^2} = \frac{\pi}{2}\,e^{-3ma}\,\cosh^m a, \quad a>0,\ m=1,2,\dots.$$

98. $$\int_0^\infty \cos^n sx \,\cos nxs \frac{dx}{a^2+x^2} = \frac{\pi}{2^{n+1}a}(1+e^{-2as})^n, \quad n=0,1,\dots.$$

99. $$\int_0^\infty \cos^n sx \,\cos nsx \frac{dx}{a^2-x^2} = \frac{\pi}{2a}\cos^n as\,\sin nas, \quad n=0,1,\dots.$$

100. $$\int_0^\infty \cos^{m-1} x \,\cos[(m+1)x] \frac{dx}{a^2+x^2} = \frac{\pi}{2^m a}e^{-2a}(1+e^{-2a})^{m-1}, \quad m=1,2,\dots;\ a>0.$$

101. $$\int_0^\infty \cos^m x \,\cos[(m-1)x] \frac{dx}{a^2+x^2} = \frac{\pi}{2^{m+1}a}\,e^a\left[(1+e^{-2a})^m - (1-e^{-2a})\right],$$
$$m=0,1,\dots;\ a>0.$$

102. $$\int_0^\infty \cos^m x \,\cos[(m+1)x] \frac{dx}{a^2+x^2} = \frac{\pi}{2^{m+1}a}e^{-a}(1+e^{-2a})^m, \quad m=0,1,\dots,\ a>0.$$

103. $$\int_0^\infty \sin^p x\,\cos x\frac{dx}{x^q}$$
$$=\begin{cases} \dfrac{p}{q-1}\displaystyle\int_0^\infty \frac{\sin^{p-1}x}{x^{q-1}}dx-\frac{p+1}{q-1}\int_0^\infty\frac{\sin^{p+1}x}{x^{q-1}}\,dx, & p>q-1>0,\\ \dfrac{p(p-1)}{(q-1)(q-2)}\displaystyle\int_0^\infty \sin^{p-2}x\,\cos x\frac{dx}{x^{q-2}} & \\ \qquad -\dfrac{(p+1)^2}{(q-1)(q-2)}\displaystyle\int_0^\infty \sin^p x\,\cos x\frac{dx}{x^{q-2}}, & p>q-1>1. \end{cases}$$

104. $$\int_0^\infty \cos^{2m}x\,\cos 2nx\,\sin x\frac{dx}{x}=\int_0^\infty \cos^{2m-1}x\,\cos 2nx\,\sin x\frac{dx}{x}=\frac{\pi}{2^{2m+1}}\binom{2m}{m+n}.$$

105. $$\int_0^\infty \cos^p ax\,\sin bx\,\cos x\frac{dx}{x}=\frac{\pi}{2},\quad b>ap,\ p>-1.$$

106. $$\int_0^\infty \cos^p ax\,\sin pax\,\cos x\frac{dx}{x}=\frac{\pi}{2^{p+1}}(2^p-1),\quad p>-1.$$

107. $$\int_0^\infty \left(\prod_{k=1}^n \cos^{p_k} a_k x\right)\sin bx\,\sin x\,\frac{dx}{x^2}=\frac{\pi}{2},\quad b>\sum_{k=1}^n a_k p_k;\ a_k>0,\ p_k>0.$$

108. $$\int_0^\infty \sin^{2m+1}x\,\cos^{2n}x\frac{dx}{x}=\int_0^\infty \sin^{2m+1}x\,\cos^{2n-1}x\frac{dx}{x}=\frac{(2m-1)!!(2n-1)!!}{2^{m+n+1}(m+n)!!}\pi$$
$$=\frac{1}{2}\,\mathrm{B}\left(m+\tfrac{1}{2},\,n+\tfrac{1}{2}\right).$$

109. $$\int_0^\infty \sin^{2m+1}2x\,\cos^{2n-1}2x\,\cos^2 x\frac{dx}{x}=\frac{\pi}{2}\cdot\frac{(2m-1)!!(2n-1)!!}{(2m+2n)!!}.$$

110. $$\int_0^\infty \frac{\sin^{2m+1}x}{1-2a\,\cos x+a^2}\cdot\frac{dx}{x}=\frac{(-1)^m\pi(1+a)^{4m}}{2^{2m+2}a^{2m+1}}\left\{\left|\frac{1-a}{1+a}\right|^{2m-1}\right.$$
$$\left.-\sum_{k=0}^{2m}(-1)^k\binom{m-\frac{1}{2}}{k}\left(\frac{4a}{(1+a)^2}\right)^k\right\},\quad |a|\neq 1.$$

111. $$\int_0^\infty \frac{\sin^{2m+1} x \, \cos^n x}{(1 - 2a \, \cos x + a^2)^p} \cdot \frac{dx}{x}$$
$$= \frac{n!\pi}{2^{n+1}(2m+n+1)!(1+a)^{2p}} \sum_{k=0}^{n} \frac{(-1)^k (2m+2n-2k+1)!!(2m+2k-1)!!}{k!(n-k)!}$$
$$\times F\left(m+n-k+\frac{3}{2},\, p;\, 2m+n+2;\, \frac{4a}{(1+a)^2}\right), \quad a \neq \pm 1.$$

112. $$\int_0^\infty \frac{\cos^{2m} x \, \cos 2mx \, \sin x}{a^2 \, \cos^2 x + b^2 \, \sin^2 x} \cdot \frac{dx}{x} = \frac{\pi}{2} \, \frac{b^{2m-1}}{a(a+b)^{2m}}, \quad ab > 0.$$

113. $$\int_0^\infty \frac{\cos^{2m-1} x \, \cos 2mx \, \sin x}{a^2 \, \cos^2 x + b^2 \, \sin^2 x} \cdot \frac{dx}{x} = \frac{\pi}{2a} \, \frac{b^{2m-1}}{(a+b)^{2m}}, \quad ab > 0.$$

114. $$\int_0^\infty \left(\frac{\sin x}{x}\right)^n \frac{\sin mx}{x} \, dx = \frac{\pi}{2}, \quad m \geq n.$$

115. $$\int_0^\infty \left(\frac{\sin x}{x}\right)^n \cos mx \, dx = \begin{cases} \dfrac{n\pi}{2^n} \displaystyle\sum_{k=0}^{\lfloor (m+n)/2 \rfloor} \dfrac{(-1)^k (n+m-2k)^{n-1}}{k!(n-k)!}, & 0 \leq m < n, \\ 0, & m \geq n \geq 2, \\ \dfrac{\pi}{4}, & m = n = 1. \end{cases}$$

116. $$\int_0^\infty \left(\frac{\sin x}{x}\right)^{n-1} \sin nx \, \cos x \frac{dx}{x} = \frac{\pi}{2}, \quad n \geq 1.$$

117. $$\int_0^\infty \left(\frac{\sin x}{x}\right)^n \frac{\sin(anx)}{x} \, dx = \frac{\pi}{2} \left[1 - \frac{1}{2^{n-1} n!} \sum_{k=0}^{\lfloor n(1-a)/2 \rfloor} (-1)^k \binom{n}{k} (n + an - 2k)^n \right],$$
all real a, $n \geq 1$.

118. $$\int_0^\infty \left(\frac{\sin x}{x}\right)^n \cos bx \, dx = \frac{n\,\pi}{2} \, (2^{n-1} n!)^{-1} \sum_{k=0}^{\lfloor r \rfloor} (-1)^k \binom{n}{k} (n-b-2k)^{n-1},$$
$0 \leq b < n,\ n \geq 1$, and r is the largest integer contained in $\ln r$.

119. $$\int_0^\infty \left(\frac{\sin x}{x}\right)^n \cos anx \, dx = 0, \quad a \leq -1 \ \text{or} \ a \geq 1,\ n \geq 2.$$

120. $$\int_0^\infty \frac{\cos 2nx}{\cos x}\,\sin^{2n} x \frac{dx}{x^m} = 0, \quad n > \frac{m-1}{2},\ m > 0.$$

121. $$\int_0^\infty \frac{\cos 2nx}{\cos x}\,\sin^{2n+1} x \frac{dx}{x^m} = 0, \quad n > \frac{m-2}{2},\ m > 0.$$

122. $$\int_0^\infty \sin^{2n} x\,\tan x \frac{dx}{x} = \frac{\pi}{2} \cdot \frac{(2n-1)!!}{(2n)!!}.$$

123. $$\int_0^\infty \cos^s rx\,\tan qx \frac{dx}{x} = \frac{\pi}{2}, \quad s > -1.$$

124. $$\int_0^\infty \frac{\cos[(2n-1)x]}{\cos x} \cdot \left(\frac{\sin x}{x}\right)^{2n} dx = (-1)^{n-1} \frac{2^{2n}-1}{(2n)!} \cdot 2^{2n-1}\pi |B_{2n}|.$$

125. $$\int_0^\infty \tan^m px \frac{dx}{q^2+x^2} = \frac{\pi}{2q} \sec \frac{m\pi}{2}\,\tanh^m pq, \quad m^2 < 1.$$