! For an efficient use of these tables, first read `HowTo.pdf`.

T2.76A. Integrands involving arctan, arccot and rational functions on the interval $(0,1)$.

1. $$\int_0^1 \frac{\arctan x}{x}\,dx = \int_1^\infty \frac{\operatorname{arccot} x}{x}\,dx = \mathbf{G}.$$

2. $$\int_0^1 \frac{\operatorname{arccot} x}{x(1+x)}\,dx = -\frac{\pi}{8}\ln 2 + \mathbf{G}.$$

3. $$\int_0^1 \arctan qx \frac{dx}{(1+px)^2} = \frac{1}{2}\frac{q}{p^2+q^2}\ln\frac{(1+p)^2}{1+q^2} + \frac{q^2-p}{(1+p)(p^2+q^2)}\arctan q, \quad p>-1.$$

4. $$\int_0^1 \operatorname{arccot} qx \frac{dx}{(1+px)^2} = \frac{1}{2}\frac{q}{p^2+q^2}\ln\frac{1+q^2}{(1+p)^2} + \frac{p}{p^2+q^2}\arctan q + \frac{1}{1+p}\operatorname{arccot} q, \quad p>-1.$$

5. $$\int_0^1 \frac{\arctan x}{x(1+x^2)}\,dx = \frac{\pi}{8}\ln 2 + \frac{1}{2}\mathbf{G}.$$

6. $$\int_0^1 \frac{\arctan x}{x\sqrt{1-x^2}}\,dx = \frac{\pi}{2}\ln(1+\sqrt{2}).$$

7. $$\int_0^1 \frac{x\arctan x\,dx}{\sqrt{(1+x^2)(1+k'^2x^2)}} = \frac{1}{k^2}\left[F\left(\frac{\pi}{4}, k\right) - \frac{\pi}{2\sqrt{2\,(1+k'^2)}}\right].$$

8. $$\int_0^1 x^p \arctan x\,dx = \frac{1}{2(p+1)}\left[\frac{\pi}{2} - \beta\left(\frac{p}{2}+1\right)\right], \quad p>-2.$$

9. $$\int_0^1 x^p \operatorname{arccot} x\,dx = \frac{1}{2(p+1)}\left[\frac{\pi}{2} + \beta\left(\frac{p}{2}+1\right)\right], \quad p>-1.$$

10. $$\int_0^1 \left(\frac{\pi}{4} - \arctan x\right)\frac{dx}{1-x} = -\frac{\pi}{8}\ln 2 + \mathbf{G}.$$

11. $$\int_0^1 \left(\frac{\pi}{4} - \arctan x\right) \frac{1+x}{1-x} \frac{dx}{1+x^2} = \frac{\pi}{8} \ln\, 2 + \frac{1}{2}\mathbf{G}.$$

12. $$\int_0^1 \left(x\, \operatorname{arccot} x - \frac{1}{x}\, \arctan x\right) \frac{dx}{1-x^2} = -\frac{\pi}{4} \ln\, 2.$$

13. $$\int_0^1 \frac{\arctan px}{1+p^2x}\, dx = \frac{1}{2p^2} \arctan p \ln(1+p^2).$$

14. $$\int_0^1 \frac{\operatorname{arccot} px}{1+p^2x}\, dx = \frac{1}{p^2}\, \left\{\frac{\pi}{4} + \frac{1}{2}\, \operatorname{arccot} p\right\} \ln(1+p^2), \quad p > 0.$$

15. $$\int_0^1 \frac{\arctan qx}{x\sqrt{1-x^2}}\, dx = \frac{\pi}{2} \ln\left(q + \sqrt{1+q^2}\right).$$

16. $$\int_0^1 \arctan(\sqrt{1-x^2}) \frac{dx}{1-x^2\cos^2\lambda} = \frac{\pi}{2\cos\lambda} \ln\left[\cos\left(\frac{\pi-4\lambda}{8}\right) \csc\left(\frac{\pi+4\lambda}{8}\right)\right].$$

17. $$\int_0^1 \arctan\left(p\sqrt{1-x^2}\right) \frac{dx}{1-x^2} = \frac{1}{2}\pi \ln\left(p + \sqrt{1+p^2}\right), \quad p > 0.$$

18. $$\int_0^1 \arctan\left(\tan\lambda\sqrt{1-k^2x^2}\right) \sqrt{\frac{1-x^2}{1-k^2x^2}}\, dx = \frac{\pi}{2k^2}\, [E(\lambda,\, k) - k'^2 F(\gamma,\, k)] \\ -\frac{\pi}{2k^2} \cot\gamma \left(1 - \sqrt{1-k^2\, \sin^2\gamma}\right).$$

19. $$\int_0^1 \arctan\left(\tan\lambda\sqrt{1-k^2x^2}\right) \sqrt{\frac{1-k^2x^2}{1-x^2}}\, dx = \frac{\pi}{2} E(\lambda,\, k) - \frac{\pi}{2} \cot\lambda \left(1 - \sqrt{1-k^2\sin^2\lambda}\right).$$

20. $$\int_0^1 \frac{\arctan\left(\tan\lambda\sqrt{1-k^2x^2}\right)}{\sqrt{(1-x^2)(1-k^2x^2)}}\, dx = \frac{\pi}{2} F(\lambda,\, k).$$