! For an efficient use of these tables, first read `HowTo.pdf`.

T3.20A. Integrands involving exponential functions on the interval $(0, \infty)$.

1. $\displaystyle\int_0^\infty e^{-px}\,dx = \frac{1}{p}, \quad \Re\{p\} > 0.$

2. $\displaystyle\int_0^\infty \frac{dx}{1+e^{px}} = \frac{\ln 2}{p}.$

3. $\displaystyle\int_0^\infty \frac{e^{-\mu x}}{1+e^{-x}}\,dx = \beta(\mu), \quad \Re\{\mu\} > 0.$

4. $\displaystyle\int_0^\infty \frac{e^{-qx}\,dx}{1-ae^{-px}} = \sum_{k=0}^{\infty} \frac{a^k}{q+kp}, \quad 0 < a < 1.$

5. $\displaystyle\int_0^\infty \frac{1-e^{\nu x}}{e^x - 1}\,dx = \psi(\nu) + \gamma_e + \pi\,\cot(\pi\nu), \quad \Re\{\nu\} < 1.$

6. $\displaystyle\int_0^\infty \frac{e^{-x} - e^{-\nu x}}{1-e^{-x}}\,dx = \psi(\nu) + \gamma_e, \quad \Re\{\nu\} > 0.$

7. $\displaystyle\int_0^\infty \frac{e^{-\mu x} - e^{-\nu x}}{1-e^{-x}}\,dx = \psi(\nu) - \psi(\mu), \quad \Re\{\mu\} > 0,\ \Re\{\nu\} > 0.$

8. $\displaystyle\int_0^\infty \frac{e^{-px} - e^{-qx}}{1-e^{-(p+q)x}}\,dx = \frac{\pi}{p+q}\,\cot\frac{p\pi}{p+q}, \quad p > 0,\ q > 0.$

9. $\displaystyle\int_0^\infty \frac{e^{-px} + e^{-qx}}{1+e^{-(p+q)x}}\,dx = \frac{\pi}{p+q}\,\cot\frac{p\pi}{p+q}, \quad p > 0,\ q > 0.$

10. $\displaystyle\int_0^\infty \frac{e^{px} - e^{qx}}{e^{rx} - e^{sx}}\,dx = \frac{1}{r-s}\left[\psi\left(\frac{r-q}{r-s}\right) - \psi\left(\frac{r-p}{r-s}\right)\right], \quad r > s,\ r > p,\ r > q.$

11. $\displaystyle\int_0^\infty \frac{a^x - b^x}{c^x - d^x}\,dx = \frac{1}{\ln(c/d)}\left\{\psi\left(\frac{\ln(c/b)}{\ln(c/d)}\right) - \psi\left(\frac{\ln(c/a)}{\ln(c/d)}\right)\right\}, \quad c > a > 0,\ b > 0,\ d > 0.$

12. $\displaystyle\int_0^\infty \left(1 - e^{-x/\beta}\right)^{\nu-1} e^{-\mu x}\,dx = \beta\,\mathrm{B}(\beta\mu,\,\nu), \quad \Re\{\beta > 0,\ \Re\{\nu\} > 0,\ \Re\{\mu\} > 0.$

13. $\displaystyle\int_0^\infty (1 - e^{-x})^{-1}(1 - e^{-\alpha x})(1 - e^{-\beta x})e^{-px}\,dx = \psi(p+\alpha) + \psi(p+\beta) - \psi(p+\alpha+\beta) - \psi(p),$

$$\Re\{p\} > 0,\ \Re\{p\} > -\Re\{\alpha\},\ \Re\{p\} > -\Re\{\beta\},\ \Re\{p\} > -\Re\{\alpha+\beta\}.$$

14. $\displaystyle\int_0^\infty (1 - e^{-x})^{\nu-1}(1 - \beta e^{-x})^{-\rho} e^{-\mu x}\,dx = \mathrm{B}(\mu,\,\nu)\,{}_2F_1(\rho,\,\mu;\,\mu+\nu;\,\beta),$

$$\Re\{\mu\} > 0,\ \Re\{\nu\} > 0,\ |\arg(1-\beta)| < \pi.$$

15. $\displaystyle\int_0^\infty \frac{[\beta + \sqrt{1 - e^{-x}}]^{-\nu} + [\beta - \sqrt{1 - e^{-x}}]^{-\nu}}{\sqrt{1 - e^{-x}}} e^{-\mu x}\,dx$

$$= \frac{2^{\mu+1} e^{(\mu-\nu)\pi i}(\beta^2 - 1)^{(\mu-\nu)/2}\Gamma(\mu) Q_{\mu-1}^{(\nu-\mu)}(\beta)}{\Gamma(\nu)}, \quad \Re\{\mu\} > 0.$$

16. $\displaystyle\int_0^\infty e^{-q^2x^2}\,dx = \frac{\sqrt{\pi}}{2q}, \quad q > 0.$

17. $\displaystyle\int_0^\infty \exp\left(-\frac{x^2}{4\beta} - \gamma x\right)\,dx = \sqrt{\pi\beta}\exp(\beta\gamma^2)\left[1 - \operatorname{erf}(\gamma\sqrt{\beta})\right], \quad \Re\{\beta\} > 0.$

18. $\displaystyle\int_0^\infty e^{-i\lambda x^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{\lambda}}\,e^{-i\pi/4}, \quad \lambda > 0.$

19. $\displaystyle\int_0^\infty e^{-\beta^2x^4 - 2\gamma^2x^2}\,dx = 2^{-3/2}\,\frac{\gamma}{\beta}\,e^{\gamma^4/(2\beta^2)} K_{1/4}\left(\frac{\gamma^4}{2\beta^2}\right), \quad |\arg\beta| < \frac{\pi}{4},\ |\arg\gamma| < \frac{\pi}{4}.$

20. $\displaystyle\int_0^\infty \exp\left(-\frac{\beta}{4x} - \gamma x\right)\,dx = \sqrt{\frac{\beta}{\gamma}}\,K_1(\sqrt{\beta\gamma}), \quad \Re\{\beta\} \geq 0,\ \Re\{\gamma\} > 0.$

21. $\displaystyle\int_0^\infty e^{-ax^2 - b/x^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}\,e^{-2\sqrt{ab}}, \quad a > 0,\ b > 0.$

22. $\displaystyle\int_0^\infty \exp(-x^\mu)\,dx = \frac{1}{\mu}\,\Gamma\left(\frac{1}{\mu}\right), \quad \Re\{\mu\} > 0.$

23. $\displaystyle\int_0^\infty \alpha\, x^m \, \exp\left(-\beta\, x^n\right)\, dx = \frac{\alpha\Gamma(\gamma)}{n\,\beta^\gamma}, \quad \gamma = \frac{m+1}{n},\ a > 0,\ \beta > 0,\ m,\ n > 0, \gamma > 0.$