! For an efficient use of these tables, first read `HowTo.pdf`.

T2.68A. Integrands involving rational functions of $\ln x$ and powers of $(a + b\,x)$ on the interval $(0, 1)$.

1. $\displaystyle\int_0^1 \left[\frac{1}{\ln x} + \frac{1}{1-x}\right] dx = \gamma_e.$

2. $\displaystyle\int_0^1 \frac{x^{p-1}\,dx}{q \pm \ln x} = \pm e^{\mp pq}\,\mathrm{Ei}(\pm pq), \quad p > 0,\, q > 0.$

3. $\displaystyle\int_0^1 \left[\frac{1}{\ln x} + \frac{x^{\mu-1}}{1-x}\right] dx = -\psi(\mu), \quad \Re\{\mu > 0.$

4. $\displaystyle\int_0^1 \left[\frac{x^{p-1}}{\ln x} + \frac{x^{q-1}}{1-x}\right] dx = \ln p - \psi(q), \quad p > 0,\, q > 0.$

5. $\displaystyle\int_0^1 \left[\frac{1}{1-x^2} + \frac{1}{2x\ln x}\right] \frac{dx}{\ln x} = \frac{\ln 2}{2}.$

6. $\displaystyle\int_0^1 \left[q - \frac{1}{2} + \frac{(1-x)(1+q\ln x) + x\ln x}{(1-x)^2}\,x^{q-1}\right] \frac{dx}{\ln x} = \frac{1}{2} - q - \ln\Gamma(q) + \frac{\ln 2\pi}{2}, \quad q > 0.$

7. $\displaystyle\int_0^1 \frac{\ln x}{4\pi^2 + (\ln x)^2} \cdot \frac{dx}{1-x} = \frac{1}{4} - \frac{1}{2}\,\gamma_e.$

8. $\displaystyle\int_0^1 \frac{1}{a^2 + (\ln x)^2} \cdot \frac{dx}{1+x^2} = \frac{1}{2a}\,\beta\left(\frac{2a+\pi}{4\pi}\right), \quad a > -\frac{\pi}{2}.$

9. $\displaystyle\int_0^1 \frac{1}{\pi^2 + (\ln x)^2}\,\frac{dx}{1+x^2} = \frac{4-\pi}{4\pi}.$

10. $\displaystyle\int_0^1 \frac{\ln\, x}{\pi^2+(\ln\, x)^2}\cdot\frac{dx}{1-x^2}=\frac{1}{2}\left(\frac{1}{2}-\ln\, 2\right).$

11. $\displaystyle\int_0^1 \frac{\ln\, x}{a^2+(\ln\, x)^2}\cdot\frac{x\,dx}{1-x^2}=\frac{1}{2}\left[\frac{\pi}{2a}+\ln\frac{\pi}{a}+\psi\left(\frac{a}{\pi}\right)\right],\quad a>0.$

12. $\displaystyle\int_0^1 \frac{\ln\, x}{\pi^2+(\ln\, x)^2}\cdot\frac{x\,dx}{1-x^2}=\frac{1}{2}\left(\frac{1}{2}-\gamma_e\right).$

13. $\displaystyle\int_0^1 \frac{1}{\pi^2+4(\ln\, x)^2}\cdot\frac{dx}{1+x^2}=\frac{\ln\, 2}{4\pi}.$

14. $\displaystyle\int_0^1 \frac{\ln\, x}{\pi^2+4(\ln\, x)^2}\cdot\frac{dx}{1-x^2}=\frac{2-\pi}{16}.$

15. $\displaystyle\int_0^1 \frac{1}{\pi^2+16(\ln\, x)^2}\cdot\frac{dx}{1+x^2}=\frac{1}{8\pi\sqrt{2}}\left[\pi+2\ln\left(\sqrt{2}-1\right)\right].$

16. $\displaystyle\int_0^1 \frac{\ln\, x}{\pi^2+16(\ln\, x)^2}\cdot\frac{dx}{1-x^2}=-\frac{\pi}{32\sqrt{2}}+\frac{1}{16}+\frac{1}{16\sqrt{2}}\ln\left(\sqrt{2}-1\right).$

17. $\displaystyle\int_0^1 \frac{\ln\, x}{[a^2+(\ln\, x)^2]^2}\,\frac{dx}{1-x}=-\frac{\pi^2}{a^4}\sum_{k=1}^{\infty}|B_{2k}|\left(\frac{2\pi}{a}\right)^{2k-2}.$

18. $\displaystyle\int_0^1 \frac{\ln\, x}{[a^2+(\ln\, x)^2]^2}\,\frac{x\,dx}{1-x^2}=-\frac{\pi^2}{4a^4}\sum_{k=1}^{\infty}|B_{2k}|\left(\frac{\pi}{a}\right)^{2k-2}.$

19. $\displaystyle\int_0^1 \frac{x^p-x^{-p}}{x^2-1}\,\frac{dx}{q^2+(\ln\, x)^2}=\frac{2\pi}{q}\sum_{k=1}^{\infty}(-1)^{k-1}\frac{\sin kp\pi}{2q+k\pi},\quad p^2<1.$

20. $\displaystyle\int_0^1 \left(\frac{x-1}{\ln\, x}-x\right)\frac{dx}{\ln\, x}=\ln\, 2-1.$

21. $\displaystyle\int_0^1 \left(\frac{1}{\ln\, x}+\frac{1}{1-x}-\frac{1}{2}\right)\frac{dx}{\ln\, x}=\frac{\ln\, 2\pi}{2}-1.$

22. $\displaystyle\int_0^1 \left(\frac{1}{\ln\, x}+\frac{x}{1-x}+\frac{x}{2}\right)\frac{dx}{x\ln\, x}=\frac{\ln\, 2\pi}{2}.$

23. $$\int_0^1 \left[\frac{1}{(\ln\,x)^2}-\frac{x}{(1-x)^2}\right]dx=\gamma_e-\frac{1}{2}.$$

24. $$\int_0^1 \left(\frac{1}{1-x^2}+\frac{1}{2\ln\,x}-\frac{1}{2}\right)\frac{dx}{\ln\,x}=\frac{\ln\,2-1}{2}.$$

25. $$\int_0^1 \left(\frac{1}{\ln\,x}+\frac{1}{2}\cdot\frac{1+x}{1-x}-\ln\,x\right)\frac{dx}{\ln\,x}=\frac{\ln\,2\pi}{2}.$$

26. $$\int_0^1 \left[\frac{1}{1-\ln\,x}-x\right]\frac{dx}{x\ln\,x}=-\gamma_e.$$

27. $$\int_0^1 \left[\frac{x^q-1}{x(\ln\,x)^2}-\frac{q}{\ln\,x}\right]dx=q\ln\,q-q,\quad q>0.$$

28. $$\int_0^1 \left[x+\frac{1}{a\ln\,x-1}\right]\frac{dx}{x\ln\,x}=\ln\,\frac{a}{q}+\gamma_e,\quad a>0,\,q>0.$$

29. $$\int_0^1 \left[\frac{1}{\ln\,x}+\frac{1+x}{2(1-x)}\right]\frac{x^{p-1}}{\ln\,x}\,dx=-\ln\,\Gamma(p)+\left(p-\frac{1}{2}\right)\ln\,p-p+\frac{\ln\,2\pi}{2},\quad p>0.$$

30. $$\int_0^1 \left[p-1-\frac{1}{1-x}+\left(\frac{1}{2}-\frac{1}{\ln\,x}\right)x^{p-1}\right]\frac{dx}{\ln\,x}=\left(\frac{1}{2}-p\right)\ln\,p+p-\frac{\ln\,2\pi}{2},\quad p>0.$$

31. $$\int_0^1 \left[-\frac{1}{(\ln\,x)^2}+\frac{(p-2)x^p-(p-1)x^{p-1}}{(1-x)^2}\right]dx=-\psi(p)+p-\frac{3}{2},\quad p>0.$$

32. $$\int_0^1 \left[\left(p-\frac{1}{2}\right)x^3+\frac{1}{2}\left(1-\frac{1}{\ln\,x}\right)\left(x^{2p-1}-1\right)\right]\frac{dx}{\ln\,x}=\left(\frac{1}{2}-p\right)(\ln\,p-1),\quad p>0.$$

33. $$\int_0^1 \left[\left(q-\frac{1}{2}\right)\frac{x^{p-1}-x^{r-1}}{\ln\,x}+\frac{px^{pq-1}}{1-x^p}-\frac{rx^{rq-1}}{1-x^r}\right]\frac{dx}{\ln\,x}=(p-r)\left[\frac{1}{2}-q-\ln\,\Gamma(q)+\frac{\ln\,2\pi}{2}\right],\quad q>0.$$

34. $$\int_0^1 \left[\frac{x^q-1}{x(\ln\,x)^3}-\frac{q}{x(\ln\,x)^2}-\frac{q^2}{2\ln\,x}\right]dx=\frac{q^2}{2}\ln\,q-\frac{3}{4}q^2,\quad q>0.$$

35. $$\int_0^1 \left[\frac{x^q-1}{x(\ln\,x)^4}-\frac{q}{x(\ln\,x)^3}-\frac{q^2}{2x(\ln\,x)^2}-\frac{q^3}{6\ln\,x}\right]dx=\frac{q^3}{6}\ln\,q-\frac{11}{36}q^3,\quad q>0.$$

36. $\displaystyle\int_0^1 \frac{x^{p-1}\,dx}{(q+\ln x)^n} = \frac{p^{n-1}}{(n-1)!}\,e^{-pq}\,\mathrm{Ei}(pq) - \frac{1}{(n-1)!q^{n-1}}\sum_{k=1}^{n-1}(n-k-1)!\,(pq)^{k-1}, \quad p>0,\ q<0.$