! For an efficient use of these tables, first read `HowTo.pdf`.

T3.55A. Integrands involving exponential and logarithm functions on the interval $(0, \infty)$.

1. $\displaystyle\int_0^\infty e^{-\mu x} \ln x \, dx = -\frac{1}{\mu} (\ln \mu + \gamma_e), \quad \Re\{\mu\} > 0.$

2. $\displaystyle\int_0^\infty \frac{\ln x \, dx}{e^x + e^{-x} - 1} = \frac{2\pi}{\sqrt{3}} \left[\frac{5}{6} \ln 2\pi - \ln \Gamma\left(\frac{1}{6}\right) \right].$

3. $\displaystyle\int_0^\infty \frac{\ln x \, dx}{e^x + e^{-x} + 1} = \frac{\pi}{\sqrt{3}} \ln \left[\frac{\Gamma(2/3)}{\Gamma(1/3)} \sqrt{2\pi} \right].$

4. $\displaystyle\int_0^\infty e^{-\mu x^2} \ln x \, dx = -\frac{1}{4} (\ln 4\mu + \gamma_e) \sqrt{\frac{\pi}{\mu}}, \quad \Re\{\mu\} > 0.$

5. $\displaystyle\int_0^\infty \frac{\ln x \, dx}{e^{x^2} + 1 + e^{-x^2}} = \frac{1}{2} \sqrt{\frac{\pi}{3}} \sum_{k=1}^\infty (-1)^k \frac{\gamma_e + \ln 4k}{\sqrt{k}} \sin \frac{k\pi}{3}.$

6. $\displaystyle\int_0^\infty e^{-\mu x} (\ln x)^2 \, dx = \frac{1}{\mu} \left[\frac{\pi^2}{6} + (\ln \mu + \gamma_e)^2 \right], \quad \Re\{\mu\} > 0.$

7. $\displaystyle\int_0^\infty e^{-x^2} (\ln x)^2 \, dx = \frac{\sqrt{\pi}}{8} \left[(\gamma_e + 2 \ln 2)^2 + \frac{\pi^2}{2} \right].$

8. $\displaystyle\int_0^\infty e^{-\mu x} (\ln x)^3 \, dx = -\frac{1}{\mu} \left[(\ln \mu + \gamma_e)^3 + \frac{\pi^2}{2} (\ln \mu + \gamma_e) + 2\zeta(3) \right].$

9. $\displaystyle\text{p.v.} \int_0^\infty \frac{e^{-x}}{\ln x} \, dx \approx -0.154479567.$

10. $\displaystyle\int_0^\infty \frac{e^{-\mu x} \, dx}{\pi^2 + (\ln x)^2} = \nu'(\mu) - e^\mu, \quad \Re\{\mu\} > 0.$

11. $\int_0^\infty e^{-\mu x} \ln(\beta + x) dx = \frac{1}{\mu} \left[\ln \beta - e^{\mu\beta}\mathrm{Ei}(-\beta\mu)\right], \quad |\arg\beta| < \pi,\ \Re\{\mu\} > 0.$

12. $\int_0^\infty e^{-\mu x} \ln(1 + \beta x)\, dx = -\frac{1}{\mu} e^{\mu/\beta}\, \mathrm{Ei}\left(-\frac{\mu}{\beta}\right), \quad |\arg\beta| < \pi,\ \Re\{\mu\} > 0.$

13. $\int_0^\infty e^{-\mu x} \ln|a - x|\, dx = \frac{1}{\mu} \left[\ln a - e^{-a\mu}\mathrm{Ei}(a\mu)\right], \quad a > 0,\ \Re\{\mu\} > 0.$

14. $\int_0^\infty e^{-\mu x} \ln\left|\frac{\beta}{\beta - x}\right| dx = \frac{1}{\mu} \left[e^{-\beta\mu}\mathrm{Ei}(\beta\mu)\right], \quad Re\{\mu\} > 0.$

15. $\int_0^\infty e^{-\mu x} \ln(\beta^2 + x^2)\, dx = \frac{2}{\mu} \left[\ln \beta - \mathrm{Ci}(\beta\mu)\cos(\beta\mu) - \mathrm{Si}\,(\beta\mu)\sin(\beta\mu)\right],$
$\Re\{\beta\} > 0,\ \Re\{\mu\} > 0.$

16. $\int_0^\infty e^{-\mu x} \ln(x^2 - \beta^2)^2\, dx = \frac{2}{\mu}[\ln \beta^2 - e^{\beta\mu}\mathrm{Ei}(-\beta\mu) - e^{\beta\mu}\mathrm{Ei}(\beta\mu)], \quad \Im\{\beta\} > 0,\ \Re\{\mu\} > 0.$

17. $\int_0^\infty e^{-\mu x} \ln\left|\frac{x+1}{x-1}\right| dx = \frac{1}{\mu} \left[e^{-\mu}(\ln 2\mu + \gamma) - e^{\mu}\mathrm{Ei}(-2\mu)\right], \quad \Re\{\mu\} > 0.$

18. $\int_0^\infty e^{-\mu x} \ln \frac{\sqrt{x + i\,a} + \sqrt{x - i\,a}}{\sqrt{2a}}\, dx = \frac{\pi}{4\mu} \left[\mathbf{H}_0(a\mu) - Y_0(a\mu)\right], \quad a > 0,\ \Re\{\mu\} > 0.$

19. $\int_0^\infty e^{-2nx} \ln(\sinh x)\, dx = \frac{1}{2n} \left[\frac{1}{n} + \ln 2 - 2\beta(2n+1)\right].$

20. $\int_0^\infty e^{-\mu x} \ln(\cosh x)\, dx = \frac{1}{\mu} \left[\beta\left(\frac{\mu}{2}\right) - \frac{1}{\mu}\right], \quad \Re\{\mu\} > 0.$

21. $\int_0^\infty e^{-\mu x}[\ln(\sinh x) - \ln x]\, dx = \frac{1}{\mu} \left[\ln \frac{\mu}{2} - \frac{1}{2\mu} - \psi\left(\frac{\mu}{2}\right)\right], \quad \Re\{\mu\} > 0.$