! For an efficient use of these tables, first read `HowTo.pdf`.

T3.03A. Integrands of the form $\dfrac{1}{\sqrt{(a-x)(b-x)(c-x)^n}}$, $\dfrac{1}{\sqrt{(a-x)(b-x)^n(c-x)}}$, and $\dfrac{1}{\sqrt{(a-x)^n(b-x)(c-x)}}$ for $n = 1, 3, 5$, on the interval (y, ∞) and $(-\infty, y)$.

Notation used: $\alpha = \arcsin\sqrt{\dfrac{a-c}{a-y}}$, $\beta = \arcsin\sqrt{\dfrac{c-y}{b-y}}$,

$$\nu = \arcsin\sqrt{\frac{a-c}{y-c}}, \quad p = \sqrt{\frac{a-b}{a-c}}, \quad q = \sqrt{\frac{b-c}{a-c}}.$$

1. $$\int_{-\infty}^{y} \frac{dx}{\sqrt{(a-x)(b-x)(c-x)}} = \frac{2}{\sqrt{a-c}}\, F(\alpha,\, p), \quad a > b > c \geq y.$$

2. $$\int_{y}^{\infty} \frac{dx}{\sqrt{(x-a)(x-b)(x-c)}} = \frac{2}{\sqrt{a-c}}\, F(\nu,\, q), \quad y \geq a > b > c.$$

3. $$\int_{-\infty}^{y} \frac{dx}{\sqrt{(a-x)^3(b-x)(c-x)}} = \frac{2}{(a-b)\sqrt{a-c}}\, [F(\alpha,\, p) - E(\alpha,\, p)], \quad a > b > c \geq y.$$

4. $$\int_{y}^{c} \frac{dx}{\sqrt{(a-x)^3(b-x)(c-x)}} = \frac{2}{(a-b)\sqrt{a-c}}\, [F(\beta,\, p) - E(\beta,\, p)] + \frac{2}{a-c}\sqrt{\frac{c-y}{(a-y)(b-y)}}, \quad a > b > c > y.$$

5. $$\int_{-\infty}^{y} \frac{dx}{\sqrt{(a-x)(b-x)^3(c-x)}} = \frac{2\sqrt{a-c}}{(a-b)(b-c)}\, E(\alpha,\, p) - \frac{2}{(a-b)\sqrt{a-c}} F(\alpha,\, p) - \frac{2}{b-c}\sqrt{\frac{c-y}{(a-y)(b-y)}}, \quad a > b > c \geq y.$$

6. $$\int_{y}^{\infty} \frac{dx}{\sqrt{(x-a)(x-b)^3(x-c)}} = \frac{2\sqrt{a-c}}{(a-b)(b-c)}\, E(\nu,\, q) - \frac{2}{(b-c)\sqrt{a-c}} F(\nu,\, q) - \frac{2}{a-b}\sqrt{\frac{y-a}{(y-b)(y-c)}}, \quad y \geq a > b > c.$$

7. $$\int_{-\infty}^{y} \frac{dx}{\sqrt{(a-x)(b-x)(c-x)^3}} = \frac{2}{(c-b)\sqrt{a-c}}\, E(\alpha,\, p) + \frac{2}{b-c}\sqrt{\frac{b-y}{(a-y)(c-y)}}, \qquad a > b > c > y.$$

8. $$\int_{y}^{\infty} \frac{dx}{\sqrt{(x-a)(x-b)(x-c)^3}} = \frac{2}{(b-c)\sqrt{a-c}}\,[F(\nu,\, q) - E(\nu,\, q)], \quad y \geq a > b > c.$$

9. $$\int_{-\infty}^{y} \frac{dx}{\sqrt{(a-x)^5(b-x)(c-x)}}$$
$$= \frac{2}{3(a-b)^2\sqrt{(a-c)^3}}[(3a-b-2c)\,F(\alpha,\, p) - 2(2a-b-c)E(\alpha,\, p)]$$
$$+ \frac{2}{3(a-c)(a-b)}\sqrt{\frac{(c-y)(b-y)}{(a-y)^3}}, \quad a > b > c \geq y.$$

10. $$\int_{y}^{\infty} \frac{dx}{\sqrt{(x-a)^5(x-b)(x-c)}} = \frac{2}{3(a-b)^2\sqrt{(a-c)^3}}[2(2a-b-c)E(\nu,\, q) - (a-b)\,F(\nu,\, q)]$$
$$+ \frac{2[4a^2-2ab-3ac+bc+y(b+2c-3a)]}{3(a-b)^2(a-c)}\sqrt{\frac{y-b}{(y-a)^3(y-c)}}, \quad y > a > b > c.$$

11. $$\int_{-\infty}^{y} \frac{dx}{\sqrt{(a-x)(b-x)^5(c-x)}} = \frac{2}{3(a-b)^2(b-c)^2\sqrt{a-c}}$$
$$\times[2(a-c)(a+c-2b)E(\alpha,\, p) + (b-c)(3b-a-2c)\,F(\alpha,\, p)]$$
$$- \frac{2[3ab-ac+2bc-4b^2-y(2a-3b+c)]}{3(a-b)(b-c)^2}\sqrt{\frac{c-y}{(a-y)(b-y)^3}}, \quad a > b > c \geq y.$$

12. $$\int_{y}^{\infty} \frac{dx}{\sqrt{(x-a)(x-b)^5(x-c)}}$$
$$= \frac{2}{3(a-b)^2(b-c)^2\sqrt{a-c}}\,[(a-b)(2a+c-3b)F(\nu,\, q) + 2(a-c)(2b-c-a)E(\nu,\, q)]$$
$$- \frac{2[3bc+2ab-ac-4b^2+y(3b-a-2c)]}{3(a-b)^2(b-c)}\sqrt{\frac{y-a}{(y-b)^3(y-c)}}, \quad y \geq a > b > c.$$

13. $$\int_{-\infty}^{y} \frac{dx}{\sqrt{(a-x)(b-x)(c-x)^5}} = \frac{2}{3(b-c)^2\sqrt{(a-c)^3}}[2(a+b-2c)E(\alpha,\, p) - (b-c)F(\alpha,\, p)]$$
$$+ \frac{2[ab-3ac-2bc+4c^2+y(2a+b-3c)]}{3(a-c)(b-c)^2}\sqrt{\frac{b-y}{(a-y)(c-y)^3}}, \quad a > b > c > y.$$

14. $$\int_y^\infty \frac{dx}{\sqrt{(x-a)(x-b)(x-c)^5}} = \frac{2}{3(b-c)^2\sqrt{(a-c)^3}}[(2a+b-3c)F(\nu,\, q)-2(a+b-2c)E(\nu,\, q)] + \frac{2}{3(a-c)(b-c)}\sqrt{\frac{(y-a)(y-b)}{(y-c)^3}}, \quad y \geq a > b > c.$$