! For an efficient use of these tables, first read `HowTo.pdf`.

T2.19C. Integrands of the form $\dfrac{1}{\sqrt{(a^2 \pm x^2)^5\,(b^2 \pm x^2)}}$ on the interval $(0, y)$.

Notation used: $\alpha = \arctan \dfrac{y}{b}$, $\gamma = \arcsin \dfrac{y}{b}\sqrt{\dfrac{a^2+b^2}{a^2+y^2}}$, $\eta = \arcsin \dfrac{y}{b}$,

$$q = \frac{\sqrt{a^2-b^2}}{a}, \quad r = \frac{b}{\sqrt{a^2+b^2}}, \quad t = \frac{b}{a}.$$

1. $$\int_0^y \frac{dx}{\sqrt{(x^2+a^2)^5(x^2+b^2)}} = \frac{1}{3a^3(a^2-b^2)^2}\{(3a^2-b^2)F(\alpha,\, q) - 2(2a^2-b^2)E(\alpha,\, q)\} + \frac{y[a^2(4a^2-3b^2)+y^2(3a^2-2b^2)]}{3a^4(a^2-b^2)\sqrt{(y^2+a^2)^3(y^2+b^2)}}, \quad a > b,\ y > 0.$$

2. $$\int_0^y \frac{dx}{\sqrt{(x^2+a^2)(x^2+b^2)^5}} = \frac{3b^2-a^2}{3ab^2(a^2-b^2)^2}F(\alpha,\, q) + \frac{a(2a^2-4b^2)}{3b^4(a^2-b^2)^2}E(\alpha,\, q) + \frac{y}{3b^2(a^2-b^2)}\sqrt{\frac{y^2+a^2}{(y^2+b^2)^3}}, \quad a > b,\ y > 0.$$

3. $$\int_0^y \frac{dx}{\sqrt{(a^2+x^2)^5(b^2-x^2)}} = \frac{1}{3a^4\sqrt{(a^2+b^2)^3}}\{2(b^2+2a^2)E(\gamma,\, r) - a^2F(\gamma,\, r)\} + \frac{y}{3a^2(a^2+b^2)}\sqrt{\frac{b^2-y^2}{(a^2+y^2)^3}}, \quad b \ge y > 0.$$

4. $$\int_0^y \frac{dx}{\sqrt{(a^2+x^2)(b^2-x^2)^5}} = \frac{1}{3b^4\sqrt{(a^2+b^2)^3}}\left\{(2a^2+3b^2)F(\gamma,\,r)-(2a^2+4b^2)E(\gamma,\,r)\right\}$$
$$+\frac{y[(3a^3+4b^2)b^2-(2a^2+3b^2)y^2]}{3b^4(a^2+b^2)\sqrt{(a^2+y^2)(b^2-y^2)^3}}, \quad b>y>0.$$

5. $$\int_0^y \frac{dx}{\sqrt{(a^2-x^2)(b^2-x^2)^5}} = \frac{2a^2-3b^2}{3ab^4(a^2-b^2)}F(\eta,\,t)+\frac{2a(2b^2-a^2)}{3b^4(a^2-b^2)^2}E(\eta,\,t)$$
$$+\frac{y[(3a^2-5b^2)b^2-2(a^2-2b^2)y^2]}{3b^4(a^2-b^2)^2(b^2-y^2)}\sqrt{\frac{a^2-y^2}{b^2-y^2}}, \quad a>b>a>0.$$

6. $$\int_0^y \frac{dx}{\sqrt{(a^2-x^2)^5(b^2-x^2)}} = \frac{1}{3a^3(a^2-b^2)^2}\left\{(4a^2-2b^2)E(\eta,\,t)-(a^2-b^2)F(\eta,\,t)\right.$$
$$\left.-\frac{y[(5a^2-3b^2)a^2-(4a^2-2b^2)y^2]}{a(a^2-y^2)}\sqrt{\frac{b^2-y^2}{a^2-y^2}}\right\}, \quad a>b\geq y>0.$$