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! For an efficient use of these tables, first read `HowTo.pdf`.

T3.49A. Integrands involving logarithm functions of complicated arguments, like $\left(1+a^2/x^2\right)$, $(1 \pm e^{-x})$, $\left(1+2e^{-x}\cos t + e^{-2x}\right)$ and others, on the interval $(0, \infty)$.

1. $\displaystyle\int_0^\infty \ln \frac{a^2+x^2}{b^2+x^2}\,dx = (a-b)\pi, \quad a>0,\ b>0.$

2. $\displaystyle\int_0^\infty \ln x \ln \frac{a^2+x^2}{b^2+x^2}\,dx = \pi(b-a) + \pi \ln \frac{a^a}{b^b}, \quad a>0,\ b>0.$

3. $\displaystyle\int_0^\infty \ln x \ln \left(1+\frac{b^2}{x^2}\right) dx = \pi b(\ln b - 1), \quad b>0.$

4. $\displaystyle\int_0^\infty \ln(1+a^2x^2) \ln \left(1+\frac{b^2}{x^2}\right) dx = 2\pi \left[\frac{1+ab}{a}\ln(1+ab) - b\right], \quad a>0,\ b>0.$

5. $\displaystyle\int_0^\infty \ln(a^2+x^2) \ln \left(1+\frac{b^2}{x^2}\right) dx = 2\pi[(a+b)\ln(a+b) - a\ln a - b], \quad a>0,\ b>0.$

6. $\displaystyle\int_0^\infty \ln \left(1+\frac{a^2}{x^2}\right) \ln \left(1+\frac{b^2}{x^2}\right) dx = 2\pi[(a+b)\ln(a+b) - a\ln a - b\ln b], \quad a>0,\ b>0.$

7. $\displaystyle\int_0^\infty \ln \left(a^2+\frac{1}{x^2}\right) \ln \left(1+\frac{b^2}{x^2}\right) dx = 2\pi \left[\frac{1+ab}{a}\ln(1+ab) - b\ln b\right], \quad a>0,\ b>0.$

8. $\displaystyle\int_0^\infty \ln(1+e^{-x})\,dx = \frac{\pi^2}{12}.$

9. $\displaystyle\int_0^\infty \ln(1-e^{-x})\,dx = -\frac{\pi^2}{6}.$

10. $\displaystyle\int_0^\infty \ln\left(1+2e^{-x}\cos t+e^{-2x}\right)dx=\frac{\pi^2}{6}-\frac{t^2}{2},\quad |t|<\pi.$

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