! For an efficient use of these tables, first read `HowTo.pdf`.

T2.02C. Powers of x and binomials of the form $(a+b\,x)$ on the interval (a,b).

1. $$\int_a^b (x-a)^{\mu-1}(b-x)^{\nu-1}\,dx = (b-a)^{\mu+\nu-1}\mathrm{B}(\mu,\,\nu), \quad b>a,\ \Re\{\mu\}>0,\ \Re\{\nu\}>0.$$

2. $$\int_a^b (x-a)^{\mu-1}(b-x)^{\nu-1}(x-c)^{-\mu-\nu}\,dx = (b-a)^{\mu+\nu-1}(b-c)^{-\mu}(a-c)^{-\nu}\mathrm{B}(\mu,\,\nu),$$ $$\Re\{\mu\}>0,\ \Re\{\nu\}>0,\ c<a<b.$$

3. $$\int_a^b \frac{(x-a)^{\nu}(b-x)^{-\nu}}{x-c}\,dx = \begin{cases} \pi\csc(\nu\pi)\left[1-\left(\dfrac{a-c}{b-c}\right)^{\nu}\right], & \text{for } c<a, \\ \pi\csc(\nu\pi)\left[1-\cos(\nu\pi)\left(\dfrac{c-a}{b-c}\right)^{\nu}\right], & \text{for } a<c<b, \\ \pi\csc(\nu\pi)\left[1-\left(\dfrac{c-a}{c-b}\right)^{\nu}\right], & \text{for } c>b, \quad |\Re\{\nu\}|<1. \end{cases}$$

4. $$\int_a^b \frac{(x-a)^{\nu-1}(b-x)^{-\nu}}{x-c}\,dx = \begin{cases} \dfrac{\pi\csc(\nu\pi)}{b-c}\left|\dfrac{a-c}{b-c}\right|^{\nu-1}, & \text{for } c<a \text{ or } c>b, \\ -\dfrac{\pi(c-a)^{\nu-1}}{(b-c)^{\nu}}\cot(\nu\pi), & \text{for } a<c<b, \qquad 0<\Re\{\nu\}<1. \end{cases}$$

5. $$\int_a^b \frac{(x-a)^{\nu-1}(b-x)^{\mu-1}}{x-c}\,dx = \begin{cases} \dfrac{(b-a)^{\mu+\nu-1}}{b-c}\mathrm{B}(\mu,\nu)\,{}_2F_1\left(1,\,\mu;\,\mu+\nu;\,\dfrac{b-a}{b-c}\right), & \text{for } c<a \text{ or } c>b, \\ \pi(c-a)^{\nu-1}(b-c)^{\mu-1}\cot\mu\pi-(b-a)^{\mu+\nu-2}\mathrm{B}(\mu-1,\,\nu) & \\ \qquad\times\,{}_2F_1\left(2-\mu-\nu,\,1;\,2-\mu;\,\frac{b-c}{b-a}\right), & \text{for } a<c<b, \end{cases}$$ $$\Re\{\mu\}>0,\ \Re\{\nu\}>0,\ \mu+\nu\neq 1,\ \mu\neq 1,2,\ldots.$$

6. $$\int_a^b \frac{(b-x)^{\mu-1}(x-a)^{\nu-1}}{|x-u|^{\mu+\nu}}\,dx = \frac{(b-a)^{\mu+\nu-1}}{|a-u|^{\mu}|b-u|^{\nu}}\frac{\Gamma(\mu)\Gamma(\nu)}{\Gamma(\mu+\nu)},$$

$$\Re\{\mu\} > 0,\ \Re\{\nu\} > 0,\ 0 < u < a < b \text{ or } 0 < a < b < u.$$