! For an efficient use of these tables, first read `HowTo.pdf`.

T2.72A. Integrands involving logarithms and exponentials on the interval $(0, 1)$.

1. $\displaystyle\int_0^1 \ln x \sin ax\, dx = -\frac{1}{a}\left[\ln a - \mathrm{Ci}(a) + \gamma_e\right], \quad a > 0.$

2. $\displaystyle\int_0^1 \ln x \cos ax\, dx = -\frac{1}{a}\left[\mathrm{Si}(a) + \frac{\pi}{2}\right], \quad a > 0.$

3. $\displaystyle\int_0^1 \ln(\sin \pi x) \sin 2n\pi x\, dx = 0.$

4. $\displaystyle\int_0^1 \ln(\sin \pi x) \sin(2n+1)\pi x\, dx = 2\int_0^{1/2} \ln(\sin \pi x) \sin(2n+1)\pi x\, dx$
$$= \frac{2}{(2n+1)\pi}\left[\ln 2 - \frac{1}{2n+1} - 2\sum_{k=1}^{n} \frac{1}{2k-1}\right].$$

5. $\displaystyle\int_0^1 \ln(\sin \pi x) \cos 2n\pi x\, dx = 2\int_0^{1/2} \ln(\sin \pi x) \cos 2n\pi x\, dx = \begin{cases} -\ln 2, & n = 0, \\ -\dfrac{1}{2n}, & n > 0. \end{cases}$

6. $\displaystyle\int_0^1 \ln(\sin \pi x) \cos(2n+1)\pi x\, dx = 0.$
