38 Test Functions

Test Functions of D'Amore, Lacceti and Murli, §7.2.5(f):

1. $\bar{f}(s) = \dfrac{1}{s}; \quad f(t) = 1,$

2. $\bar{f}(s) = 2\left(\sqrt{s+1} - \sqrt{s}\right); \quad f(t) = \dfrac{1-e^{-t}}{\sqrt{\pi t^3}},$

3. $\bar{f}(s) = \dfrac{1}{\sqrt{s}}; \quad f(t) = \dfrac{1}{\sqrt{\pi t}},$

4. $\bar{f}(s) = \dfrac{s^2-1}{\left(s^2+1\right)^2}; \quad f(t) = t\,\cos t,$

5. $\bar{f}(s) = \dfrac{1}{(s+1)^2}; \quad f(t) = t\,e^{-t},$

6. $\bar{f}(s) = \dfrac{1}{s^2}; \quad f(t) = t,$

7. $\bar{f}(s) = \dfrac{1}{s^2+1}; \quad f(t) = \sin t,$

8. $\bar{f}(s) = \dfrac{1}{s+0.5}; \quad f(t) = e^{-0.5t},$

9. $\bar{f}(s) = \dfrac{1}{\sqrt{s^2+1}}; \quad f(t) = J_0(t),$

10. $\bar{f}(s) = \dfrac{e^{-1/s}}{\sqrt{s}}; \quad f(t) = \dfrac{\cos\left(2\sqrt{t}\,\right)}{\sqrt{\pi t}},$

11. $\bar{f}(s) = e^{-4\sqrt{s}}; \quad f(t) = \dfrac{2e^{-4/t}}{t\,\sqrt{\pi t}},$

12. $\bar{f}(s) = \arctan(1/s);\quad f(t) = \dfrac{\sin t}{t},$

13. $\bar{f}(s) = \dfrac{1}{(s+0.2)^2+1};\quad f(t) = e^{-0.2t}\,\sin t,$

14. $\bar{f}(s) = \dfrac{1}{s^3};\quad f(t) = \frac{1}{2}\,t^2,$

15. $\bar{f}(s) = \dfrac{e^{-2s}}{s};\quad f(t) = U(t-2),$

16. $\bar{f}(s) = \dfrac{1}{s\,(1+e^{-s})};\quad f(t) = \sum_{n=0}^{\infty}(-1)^n\,U(t-n)$ (square wave),

17. $\bar{f}(s) = \dfrac{1}{s^2+s+1};\quad f(t) = \dfrac{2}{\sqrt{3}}\,e^{-t/2}\,\sin\dfrac{t\sqrt{3}}{2},$

18. $\bar{f}(s) = \dfrac{3}{s^2-9};\quad f(t) = \sinh(3t),$

19. $\bar{f}(s) = \dfrac{120}{s^6};\quad f(t) = t^5,$

20. $\bar{f}(s) = \dfrac{s}{(s^2+1)^2};\quad f(t) = \dfrac{t}{2}\,\sin t,$

21. $\bar{f}(s) = \dfrac{1}{s+1} - \dfrac{1}{s+1000};\quad f(t) = e^{-t} - e^{-1000t},$

22. $\bar{f}(s) = \dfrac{s}{s^2+1};\quad f(t) = \cos t,$

23. $\bar{f}(s) = \dfrac{1}{(s-0.25)^2};\quad f(t) = te^{-t/4},$

24. $\bar{f}(s) = \dfrac{1}{s\sqrt{s}};\quad f(t) = 2\sqrt{\dfrac{t}{\pi}},$

25. $\bar{f}(s) = \dfrac{1}{\sqrt{s+1}};\quad f(t) = \dfrac{e^{-t}}{\sqrt{\pi t}},$

26. $\bar{f}(s) = \dfrac{s+2}{s\sqrt{s}};\quad f(t) = \dfrac{1+4t}{\sqrt{\pi t}},$

27. $\bar{f}(s) = \dfrac{1}{(s^2+1)^2};\quad f(t) = \frac{1}{2}\,(\sin t - t\,\cos t),$

28. $\bar{f}(s) = \dfrac{1}{s\,(s+1)^2};\quad f(t) = 1 - (1+t)\,e^{-t},$

29. $\bar{f}(s) = \dfrac{1}{s^3-8};\quad f(t) = \dfrac{e^{-t}}{12}\left[e^{3t} - \cos\left(\sqrt{3t}\right) - \sqrt{3}\,\sin\left(\sqrt{3t}\right)\right],$

30. $\bar{f}(s) = \log\dfrac{s^2+1}{s^2+4};\quad f(t) = \dfrac{2}{t}\,(\cos(2t) - \cos t),$

31. $\bar{f}(s) = \log(s+1)/s; \quad f(t) = \dfrac{1-e^{-t}}{t},$

32. $\bar{f}(s) = \dfrac{\log\, s}{s}; \quad f(t) = -\gamma - \log\, t,$

33. $\bar{f}(s) = \dfrac{1-e^{-s}}{s^2}; \quad f(t) = t\,U(t) - (t-1)\,U(t-1),$

34. $\bar{f}(s) = \dfrac{1}{s\,(1+e^{-s})}; \quad f(t) = \sum_{n=0}^{\infty}(-1)^n\,U(t-n),$

35. $\bar{f}(s) = \dfrac{1}{s(s+1)}\left[\dfrac{1}{2s} - \dfrac{e^{-2s}}{1-e^{-2s}}\right]; \quad f(t) = \tfrac{1}{2} + e^{-t}\left[\tfrac{1}{2} - \dfrac{e^2}{e^2-1} - \dfrac{1}{\pi}\sum_{n=1}^{\infty}\dfrac{\sin(n\pi t) - \arctan(n\pi)}{n\sqrt{n^2\pi^2+1}}\right],$

36. $\bar{f}(s) = \dfrac{1}{s}\, e^{-a\sqrt{s(1+s)/(1+cs)}}$, with $a = 0.5$ and $c = 0.4$;

$$f(t) = \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} e^{-am\sqrt{m/2}\,(\cos\theta - \sin\theta)}\,\sin\left(tu - am\sqrt{u/2}\right)\,(\cos\theta + \sin\theta)\,\frac{du}{u},$$

where $m = \left(\dfrac{1+u^2}{1+c^2u^2}\right)^{1/4}$ and $2\theta = \arctan(u) - \arctan(cu)$.

37. $\bar{f}(s) = \dfrac{e^{-2\Psi}}{s}$, where $\cosh\Psi = \sqrt{1 + s^2 + s^2/16}$;

$$f(t) = 1 - \frac{1}{\pi}\int_0^{u_1}\Big[\sin(ut+2k) - \sin(ut-2k)\Big]\,\frac{du}{u} + \frac{1}{\pi}\int_{u_2}^{4}\Big[\sin(ut+2k) - \sin(ut-2k)\Big]\,\frac{du}{u},$$

where $\cos k = \frac{1}{4}\sqrt{(u_1^2 - u^2)\,(u_2^2 - u^2)}$, and $u_{1,2} = 2\sqrt{2 \mp \sqrt{3}}$.

38. $\bar{f}(s) = \dfrac{s - \sqrt{s^2-c^2}}{\sqrt{s}\,\sqrt{s^2-c^2}\,\sqrt{s - N\sqrt{s^2-c^2}}}, \quad N < 1;$

$$f(t) = \frac{2}{\pi}\int_0^c \cosh(tu)\,\frac{u\sqrt{(R+u)/2} + \sqrt{c^2 - u^2}\,\sqrt{(R-u)/2}}{R\sqrt{c^2-u^2}\,\sqrt{u}}$$
$$+\frac{2}{\pi}\int_0^b \frac{u - \sqrt{c^2+u^2}}{\sqrt{u}\,\sqrt{c^2+u^2}\,\sqrt{N\sqrt{(c^2+u^2} - u}}\,\cos(tu)\,du,$$

where $R = \sqrt{(u^2 + N^2\sqrt{c^2-u^2}}$, $b = \sqrt{(1-N)/(1+N)}$, and $c = (1-N)/N$.

The integral 36, 37 and 36 are the same as the integral 3, 4 and 5 in §7.5.2.