! For an efficient use of these tables, first read `HowTo.pdf`.

T3.57B. Integrands involving logarithm and hyperbolic functions on the interval $(1,\infty)$.

1. $$\int_1^\infty \ln x \,\frac{\sinh mx}{\sinh nx}\,dx = \begin{cases} \dfrac{\pi}{2n}\tan\dfrac{m\pi}{2n}\ln 2\pi + \dfrac{\pi}{n}\displaystyle\sum_{k=1}^{n-1}(-1)^{k-1}\sin\dfrac{km\pi}{n}\ln\dfrac{\Gamma\left(\frac{n+k}{2n}\right)}{\Gamma\left(\frac{k}{2n}\right)}, & m+n \text{ is odd},\\[2ex] \dfrac{\pi}{2n}\tan\dfrac{m\pi}{2n}\ln \pi + \dfrac{\pi}{n}\displaystyle\sum_{k=1}^{\frac{n-1}{2}}(-1)^{k-1}\sin\dfrac{km\pi}{n}\ln\dfrac{\Gamma\left(\frac{n-k}{n}\right)}{\Gamma\left(\frac{k}{n}\right)}, & m+n \text{ is even}.\end{cases}$$

2. $$\int_1^\infty \ln x \,\frac{\cosh mx}{\cosh nx}\,dx = \begin{cases} \dfrac{\pi}{2n}\dfrac{\ln 2\pi}{\cos\frac{m\pi}{2n}} + \dfrac{\pi}{n}\displaystyle\sum_{k=1}^{n}(-1)^{k-1}\cos\dfrac{(2k-1)m\pi}{2n}\ln\dfrac{\Gamma\left(\frac{2n+2k-1}{4n}\right)}{\Gamma\left(\frac{2k-1}{4n}\right)}, & m+n \text{ is odd},\\[2ex] \dfrac{\pi}{2n}\dfrac{\ln \pi}{\cos\frac{m\pi}{2n}} + \dfrac{\pi}{n}\displaystyle\sum_{k=1}^{\frac{n-1}{2}}(-1)^{k-1}\cos\dfrac{(2k-1)m\pi}{2n}\ln\dfrac{\Gamma\left(\frac{2n-2k+1}{2n}\right)}{\Gamma\left(\frac{2k-1}{2n}\right)}, & m+n \text{ is even},\end{cases}$$

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