! For an efficient use of these tables, first read `HowTo.pdf`.

T2.44C. Integrands involving powers of trigonometric functions and rational trigonometric functions on the interval $(0, \pi)$.

1. $$\int_0^{\pi} \frac{\sin^m x}{p+q\,\cos x}\,dx = 2^{m-2}\frac{p}{q^2}\sum_{\nu=1}^{k}\left(\frac{p^2-q^2}{-4q^2}\right)^{\nu-1}\mathrm{B}\left(\frac{m+1-2\nu}{2},\,\frac{m+1-2\nu}{2}\right)+A\left(\frac{p^2-q^2}{-q^2}\right)^{k},$$
$$\text{where } A=\begin{cases}\dfrac{\pi p}{q^2}\left(1-\sqrt{1-\dfrac{q^2}{p^2}}\right), & m=2k+2,\\[2ex] \dfrac{1}{q}\,\ln\dfrac{p+q}{p-q}, & m=2k+1,\end{cases}\qquad k\ge 1,\ q\ne 0,\ p^2-q^2\ge 0.$$

2. $$\int_0^{\pi} \frac{\sin^m x}{1+\cos x}\,dx = 2^{m-1}\,\mathrm{B}\left(\frac{m-1}{2},\,\frac{m+1}{2}\right),\qquad m\ge 2.$$

3. $$\int_0^{\pi} \frac{\sin^m x}{1-\cos x}\,dx = 2^{m-1}\,\mathrm{B}\left(\frac{m-1}{2},\,\frac{m+1}{2}\right),\qquad m\ge 2.$$

4. $$\int_0^{\pi} \frac{\sin^2 x}{p+q\,\cos x}\,dx = \frac{p\pi}{q^2}\left(1-\sqrt{1-\frac{q^2}{p^2}}\right).$$

5. $$\int_0^{\pi} \frac{\sin^3 x}{p+q\,\cos x}\,dx = 2\frac{p}{q^2}+\frac{1}{q}\left(1-\frac{p^2}{q^2}\right)\ln\frac{p+q}{p-q}.$$

6. $$\int_0^{\pi} \frac{\cos^n x\,dx}{(a+b\,\cos x)^{n+1}} = \frac{\pi}{2^n(a+b)^n\sqrt{a^2-b^2}}\sum_{k=0}^{n}(-1)^k\frac{(2n-2k-1)!!(2k-1)!!}{(n-k)!k!}\left(\frac{a+b}{a-b}\right)^{k},$$
$$a^2>b^2.$$
