C4282

! For an efficient use of these tables, first read `HowTo.pdf`.

T2.66A. Integrands involving logarithm functions and powers of $(a+b\,x)$ on the interval $(0,1)$.

1. $$\int_0^1 \frac{x^{\mu-1}\ln x}{x+1}\,dx = \beta'(\mu), \quad \Re\{\{\mu\} > 0.$$

2. $$\int_0^1 \frac{x^{\mu-1}\ln x}{1-x}\,dx = -\psi'(\mu) = -\zeta(2,\,\mu), \quad \Re\{\{\mu\} > 0.$$

3. $$\int_0^1 \ln x\frac{x^{2n}\,dx}{1+x} = -\frac{\pi^2}{12} + \sum_{k=1}^{2n}\frac{(-1)^{k-1}}{k^2}.$$

4. $$\int_0^1 \ln x\frac{x^{2n-1}\,dx}{1+x} = \frac{\pi^2}{12} + \sum_{k=1}^{2n-1}\frac{(-1)^{k}}{k^2}.$$

5. $$\int_0^1 x^{\mu-1}(1-x^r)^{\nu-1}\ln x\,dx = \frac{1}{r^2}\mathrm{B}\left(\frac{\mu}{r},\,\nu\right)\left[\psi\left(\frac{\mu}{r}\right) - \psi\left(\frac{\mu}{r}+\nu\right)\right],$$
$$\Re\{\{\mu\} > 0,\ \Re\{\nu\} > 0,\ r > 0.$$

6. $$\int_0^1 \frac{x^{p-1}}{(1-x)^{p+1}}\ln x\,dx = -\frac{\pi}{p}\csc p\pi, \quad 0 < p < 1.$$

7. $$\int_0^1 \ln x\left(\frac{x}{a^2+x^2}\right)^p\frac{dx}{x} = \frac{\ln a}{2a^p}\,\mathrm{B}\left(\frac{p}{2},\,\frac{p}{2}\right), \quad a > 0,\, p > 0.$$

8. $$\int_0^1 \frac{x^{p-1}\ln x}{1-x^q}\,dx = -\frac{1}{q^2}\,\psi'\left(\frac{p}{q}\right), \quad p > 0,\, q > 0.$$

9. $$\int_0^1 \frac{x^{p-1}\ln x}{1+x^q}\,dx = \frac{1}{q^2}\,\beta'\left(\frac{p}{q}\right), \quad p > 0,\, q > 0.$$

C4282

10. $\displaystyle\int_0^1 \frac{x^{q-1}\ln x}{1-x^{2q}}\,dx = -\frac{\pi^2}{8q^2}, \quad q>0.$

11. $\displaystyle\int_0^1 \ln x \frac{(1-x^2)x^{p-2}}{1+x^{2p}}\,dx = -\left(\frac{\pi}{2p}\right)^2 \frac{\sin\frac{\pi}{2p}}{\cos^2\frac{\pi}{2p}}, \quad p>1.$

12. $\displaystyle\int_0^1 \ln x \frac{(1+x^2)x^{p-2}}{1-x^{2p}}\,dx = -\left(\frac{\pi}{2p}\right)^2 \sec^2\frac{\pi}{2p}, \quad p>1.$

13. $\displaystyle\int_0^1 \ln\frac{1}{x}\frac{x^{\mu-1}\,dx}{(1-x^n)^{(n-m)/n}} = \frac{1}{n^2}\,\mathrm{B}\left(\frac{\mu}{n},\frac{m}{n}\right)\left[\psi\left(\frac{\mu+m}{n}\right)-\psi\left(\frac{\mu}{n}\right)\right], \quad \Re\{\{\mu\}>0.$

C4282
C4282
C4282
C4282
C4282
C4282