! For an efficient use of these tables, first read `HowTo.pdf`.

T2.35A. Integrands involving exponentials and arbitrary powers of $(a + b\,x)$ on the intervals $(0, y)$ and $(0, 2y)$.

1. $$\int_0^y x^{\nu-1} e^{-\mu x}\,dx = \mu^{-\nu}\gamma(\nu,\,\mu y), \quad \Re\{\nu\} > 0.$$

2. $$\int_0^y x^{p-1} e^{-x}\,dx = \begin{cases} \displaystyle\sum_{k=0}^{\infty} (-1)^k \frac{y^{p+k}}{k!(p+k)}, \\ \text{or} \\ \displaystyle = e^{-y} \sum_{k=0}^{\infty} \frac{y^{p+k}}{p(p+1)\dots(p+k)}. \end{cases}$$

3. $$\int_0^y (y-x)^{\nu} e^{-\mu x}\,dx = (-\mu)^{-\nu-1} e^{-y\mu}\gamma(\nu+1,\,-y\mu), \quad \Re\{\nu\} > -1,\ y > 0.$$

4. $$\int_0^y (a+x)^{\mu-1} e^{-x}\,dx = e^a[\gamma(\mu,\,a+y) - \gamma(\mu,\,a)], \quad \Re\{\mu\} > 0.$$

5. $$\int_0^y x^{\nu-1}(y-x)^{\mu-1} e^{\beta x}\,dx = \mathrm{B}(\mu,\,\nu)\,y^{\mu+\nu-1}\,{}_1F_1(\nu;\,\mu+\nu;\,\beta y), \quad \Re\{\mu\} > 0,\ \Re\{\nu\} > 0.$$

6. $$\int_0^y x^{\mu-1}(y-x)^{\mu-1} e^{\beta x}\,dx = \sqrt{\pi}\left(\frac{y}{\beta}\right)^{\mu-1/2} \exp\left(\frac{\beta y}{2}\right)\Gamma(\mu) I_{\mu-1/2}\left(\frac{\beta y}{2}\right), \quad \Re\{\mu\} > 0.$$

7. $$\int_0^y (y^2-x^2)^{\nu-1} e^{\mu x}\,dx = \frac{\sqrt{\pi}}{2}\left(\frac{2y}{\mu}\right)^{\nu-1/2} \Gamma(\nu)\,\left[I_{\nu-1/2}(y\mu) + \mathbf{L}_{\nu-1/2}(y\mu)\right],$$
$$y > 0,\ \Re\{\nu\} > 0.$$

8. $\displaystyle\int_0^y x^{2\nu-1}(y^2-x^2)^{\rho-1}e^{\mu x}\,dx = \frac{1}{2}\mathrm{B}(\nu,\,\rho)y^{2\nu+2\rho-2}\,{}_1F_2\left(\nu;\,\frac{1}{2},\,\nu+\rho;\,\frac{\mu^2y^2}{4}\right)$

$$+\frac{\mu}{2}\mathrm{B}\left(\nu+\frac{1}{2},\,\rho\right)y^{2\nu+2\rho-1}\,{}_1F_2\left(\nu+\frac{1}{2};\,\frac{3}{2},\,\nu+\rho+\frac{1}{2};\,\frac{\mu^2y^2}{4}\right),\quad \Re\{\rho\}>0,\ \Re\{\nu\}>0.$$

9. $\displaystyle\int_0^y x(y^2-x^2)^{\nu-1}e^{\mu x}\,dx = \frac{y^{2\nu}}{2\nu}+\frac{\sqrt{\pi}}{2}\left(\frac{\mu}{2}\right)^{1/2-\nu}y^{\nu+1/2}\Gamma(\nu)[I_{\nu+1/2}(\mu y)+\mathbf{L}_{\nu+1/2}(\mu y)],$

$$\Re\{\nu\}>0.$$

10. $\displaystyle\int_0^{2y}(2yx-x^2)^{\nu-1}e^{-\mu x}\,dx = \sqrt{\pi}\left(\frac{2y}{\mu}\right)^{\nu-1/2}e^{-y\mu}\Gamma(\nu)\,I_{\nu-1/2}(y\mu),\quad y>0,\ \Re\{\nu\}>0.$