! For an efficient use of these tables, first read `HowTo.pdf`.

T2.46B. Integrands involving square roots of expressions containing trigonometric functions on the intervals $(0, \pi/2)$ and $(y, \pi/2)$.

1. $$\int_0^{\pi/2} \sin^\alpha x \cos^\beta x \sqrt{1-k^2 \sin^2 x}\, dx = \frac{1}{2} \mathrm{B}\left(\frac{\alpha+1}{2}, \frac{\beta+1}{2}\right) F\left(\frac{\alpha+1}{2}, -\frac{1}{2}; \frac{\alpha+\beta+2}{2}; k^2\right),$$ $$\alpha > -1,\ \beta > -1,\ |k| < 1.$$

2. $$\int_0^{\pi/2} \frac{\sin^\alpha x \cos^\beta x}{\sqrt{1-k^2 \sin^2 x}}\, dx = \frac{1}{2} \mathrm{B}\left(\frac{\alpha+1}{2}, \frac{\beta+1}{2}\right) F\left(\frac{\alpha+1}{2}, \frac{1}{2}; \frac{\alpha+\beta+2}{2}; k^2\right),$$ $$\alpha > -1,\ \beta > -1,\ |k| < 1.$$

3. $$\int_0^{\pi/2} \frac{dx}{\sqrt{1-(p^2/2)(1-\cos 2x)}} = \mathbf{K}(p), \quad 0 < p < 1.$$

4. $$\int_0^{\pi/2} \frac{\sin x\, dx}{\sqrt{1+p^2 \sin^2 x}} = \frac{1}{p} \arctan p.$$

5. $$\int_0^{\pi/2} \tan^2 x \sqrt{1-p^2 \sin^2 x}\, dx = \infty.$$

6. $$\int_0^{\pi/2} \frac{dx}{\sqrt{p^2 \cos^2 x + q^2 \sin^2 x}} = \frac{1}{p} \mathbf{K}\left(\frac{\sqrt{p^2-q^2}}{p}\right), \quad 0 < q < p.$$

7. $$\int_0^{\pi/2} \frac{\sin^2 x\, dx}{\sqrt{1+\sin^2 x}} = \sqrt{2}\mathbf{E}\left(\frac{\sqrt{2}}{2}\right) - \frac{1}{\sqrt{2}} \mathbf{K}\left(\frac{\sqrt{2}}{2}\right).$$

8. $$\int_0^{\pi/2} \frac{\cos^2 x\, dx}{\sqrt{1+\sin^2 x}} = \sqrt{2}\left[\mathbf{K}\left(\frac{\sqrt{2}}{2}\right) - \mathbf{E}\left(\frac{\sqrt{2}}{2}\right)\right].$$

9. $$\int_0^{\pi/2} \frac{\cos^2 x}{1-\cos^2\beta\,\cos^2 x}\cdot\frac{dx}{\sqrt{1-k^2\sin^2 x}}$$
$$= \frac{1}{\sin\beta\,\cos\beta\sqrt{1-k'^2\sin^2\beta}}\left\{\frac{\pi}{2} - \mathbf{K}E(\beta,\,k') - \mathbf{E}F(\beta,\,k') + \mathbf{K}F(\beta,\,k')\right\},$$
$$\text{where } k' = \sqrt{1-k^2}.$$

10. $$\int_0^{\pi/2} \frac{\sin^2 x}{1-(1-k'^2\sin^2\beta)\sin^2 x}\cdot\frac{dx}{\sqrt{1-k^2\sin^2 x}}$$
$$= \frac{1}{k'^2\sin\beta\,\cos\beta\sqrt{1-k'^2\sin^2\beta}}\left\{\frac{\pi}{2} - \mathbf{K}E(\beta,\,k') - \mathbf{E}F(\beta,\,k') + \mathbf{K}F(\beta,\,k')\right\},$$
$$\text{where } k' = \sqrt{1-k^2}.$$

11. $$\int_0^{\pi/2} \frac{\sin^2 x}{1-k^2\sin^2\beta\,\sin^2 x}\cdot\frac{dx}{\sqrt{1-k^2\sin^2 x}} = \frac{\mathbf{K}E(\beta,\,k) - \mathbf{E}F(\beta,\,k)}{k^2\sin\beta\,\cos\beta\sqrt{1-k^2\sin^2\beta}}.$$

12. $$\int_y^{\pi/2} \frac{dx}{\sqrt{\sin x - \sin y}} = \sqrt{2}\mathbf{K}\left(\sin\frac{\pi-2y}{4}\right).$$