! For an efficient use of these tables, first read `HowTo.pdf`.

T1.13. Integrand involving $a + bx + cx^2$ and $\alpha + \beta x$.

Notation used: $R = a + bx + cx^2$, $X = \alpha + \beta x$, $A = \alpha\beta^2 - ab\beta + c\,\alpha^2$, $B = b\beta - 2c\alpha$, $\Delta = 4ac - b^2$.

1. $\displaystyle\int \frac{dx}{XR} = \frac{\beta}{2A} \ln \frac{X^2}{R} - \frac{B}{2A} \int \frac{dx}{R}.$

2. $\displaystyle\int X^m R^n\, dx = \frac{\beta X^{m-1} R^{n+1}}{(m+2n+1)c} - \frac{(m+n)B}{(m+2n+1)c} \int X^{m-1} R^n\, dx - \frac{(m-1)A}{(m+2n+1)c} \int X^{m-2} R^n\, dx.$

3. $$\begin{aligned}\int \frac{R^n\, dx}{X^m} &= -\frac{1}{(m-2n-1)\beta} \frac{R^n}{X^{m-1}} - \frac{2nA}{(m-2n-1)\beta^2} \int \frac{R^{n-1}\, dx}{X^m} \\ &\qquad - \frac{nB}{(m-2n-1)\beta^2} \int \frac{R^{n-1}\, dx}{X^{m-1}} \\ &= \frac{-\beta}{(m-1)A} \frac{R^{n+1}}{X^{m-1}} - \frac{(m-n-2)B}{(m-1)A} \int \frac{R^n\, dx}{X^{m-1}} - \frac{(m-2n-3)c}{(m-1)A} \int \frac{R^n\, dx}{X^{m-2}} \\ &= -\frac{1}{(m-1)\beta} \frac{R^n}{X^{m-1}} + \frac{nB}{(m-1)\beta^2} \int \frac{R^{n-1}\, dx}{X^{m-1}} + \frac{2nc}{(m-1)\beta^2} \int \frac{R^{n-1}\, dx}{X^{m-2}}.\end{aligned}$$

4. $$\begin{aligned}\int \frac{X^m\, dx}{R^n} &= \frac{\beta}{(m-2n+1)c} \frac{X^{m-1}}{R^{n-1}} - \frac{(m-n)B}{(m-2n+1)c} \int \frac{X^{m-1}\, dx}{R^n} - \frac{(m-1)A}{(m-2n+1)c} \int \frac{X^{m-2}\, dx}{R^n} \\ &= \frac{b+2cx}{(n-1)\Delta} \frac{X^m}{R^{n-1}} - \frac{2(m-2n+3)c}{(n-1)\Delta} \int \frac{X^m\, dx}{R^{n-1}} - \frac{Bm}{(n-1)\Delta} \int \frac{X^{m-1}\, dx}{R^{n-1}}\end{aligned}$$

5. $$\int \frac{dx}{X^m R^n} = \begin{cases} -\dfrac{\beta}{(m-1)A} \dfrac{1}{X^{m-1}R^{n-1}} - \dfrac{(m+n-2)B}{(m-1)A} \displaystyle\int \frac{dx}{X^{m-1}R^n} - \dfrac{(m+2n-3)c}{(m-1)A} \displaystyle\int \frac{dx}{X^{m-2}R^n} & \\ = \dfrac{\beta}{2(n-1)A} \dfrac{1}{X^{m-1}R^{n-1}} - \dfrac{B}{2A} \displaystyle\int \frac{dx}{X^{m-1}R^n} + \dfrac{(m+2n-3)\,\beta^2}{2(n-1)A} \displaystyle\int \frac{dx}{X^m R^{n-1}}, & A \neq 0, \\ -\dfrac{\beta}{(m+n-1)B} \dfrac{1}{X^m R^{n-1}} - \dfrac{(m+2n-2)\,c}{(m+n-1)\,B} \displaystyle\int \frac{dx}{X^{m-1}R^n}, & A = 0. \end{cases}$$