! For an efficient use of these tables, first read `HowTo.pdf`.

T2.14B. Integrands of the form $\sqrt{(a^2 \pm x^2)\,(x^2 \pm b^2)}$ and $\sqrt{(a^2 \pm x^2)\,(x^2 \pm b^2)}$ on the intervals (y, b) and (b, y).

Notation used: $\delta = \arccos\dfrac{y}{b} \quad \varepsilon = \arccos\dfrac{b}{y},$

$$\zeta = \arcsin\frac{a}{b}\sqrt{\frac{b^2-y^2}{a^2-y^2}}, \quad \kappa = \arcsin\frac{a}{y}\sqrt{\frac{y^2-b^2}{a^2-b^2}},$$

$$q = \sqrt{\frac{a^2-b^2}{a}}, \quad r = \frac{b}{\sqrt{a^2+b^2}}, \quad s = \frac{a}{\sqrt{a^2+b^2}}, \quad t = \frac{b}{a}.$$

1. $$\int_y^b \sqrt{(a^2+x^2)(b^2-x^2)}\,dx = \frac{1}{3}\sqrt{a^2+b^2}\{a^2F(\delta,\,r) + (a^2-b^2)E(\delta,\,r)\} + \frac{y}{3}\sqrt{(a^2+y^2)(b^2-y^2)}, \quad b > y \geq 0.$$

2. $$\int_b^y \sqrt{(a^2+x^2)(x^2-b^2)}\,dx = \frac{1}{3}\sqrt{a^2+b^2}\{(b^2-a^2)E(\varepsilon,\,s) - b^2F(\varepsilon,\,s)\} + \frac{y^2+a^2-b^2}{3y}\sqrt{(a^2+y^2)(y^2-b^2)}, \quad y > b > 0.$$

3. $$\int_y^b \sqrt{(a^2-x^2)(b^2-x^2)}\,dx = \frac{a}{3}\{(a^2+b^2)E(\zeta,\,t) - (a^2-b^2)F(\zeta,\,t)\} + \frac{y}{3}(y^2-2a^2-b^2)\sqrt{\frac{b^2-y^2}{a^2-y^2}}, \quad a > b > y \geq 0.$$

4. $$\int_b^y \sqrt{(a^2 - x^2)(x^2 - b^2)}\, dx = \frac{a}{3}\{(a^2 + b^2)E(\kappa,\, q) - 2b^2 F(\kappa,\, q)\} + \frac{y^2 - a^2 - b^2}{3y}\sqrt{(a^2 - y^2)(y^2 - b^2)}, \quad a \geq y > b > 0.$$