

## Cotesian Numbers

For the Newton-Cotes quadrature, defined by  $\int_{-1}^1 f(x) dx = \sum_{i=0}^n C_i f(x_i)$ , the Cotesian numbers  $C_i$  are rational numbers, which can be expressed in the form  $C_i = N_i/D_n$ , where the values of  $N_i$  and  $D_n$  are given below for  $n = 1(1)20$ .

$n = 1 :$	$N_0 = N_1 = 1, D_1 = 2.$
$n = 2 :$	$N_0 = N_2 = 1, N_1 = 4, D_2 = 6.$
$n = 3 :$	$N_0 = N_3 = 1, N_1 = N_2 = 3, D_3 = 8.$
$n = 4 :$	$N_0 = N_4 = 7, N_1 = N_3 = 32, N_2 = 12, D_4 = 90.$
$n = 5 :$	$N_0 = N_5 = 19, N_1 = N_4 = 75, N_2 = N_3 = 50, D_5 = 288.$
$n = 6 :$	$N_0 = N_6 = 41, N_1 = N_5 = 216, N_2 = N_4 = 27, N_3 = 272,$ $D_6 = 840.$
$n = 7 :$	$N_0 = N_7 = 751, N_1 = N_6 = 3577, N_2 = N_5 = 1323,$ $N_3 = N_4 = 2989, D_7 = 17280.$
$n = 8 :$	$N_0 = N_8 = 989, N_1 = N_7 = 5888, N_2 = N_6 = -928,$ $N_3 = N_5 = 10496, N_4 = -4540, D_8 = 28350.$
$n = 9 :$	$N_0 = N_9 = 2857, N_1 = N_8 = 15741, N_2 = N_7 = 1080,$ $N_3 = N_6 = 19344, N_4 = N_5 = 5778, D_9 = 89600.$
$n = 10 :$	$N_0 = N_{10} = 16067, N_1 = N_9 = 106300, N_2 = N_8 = -48525,$ $N_3 = N_7 = 272400, N_4 = N_6 = -260550, N_5 = 427368,$ $D_{10} = 598752.$
$n = 11 :$	$N_0 = N_{11} = 2171465, N_1 = N_{10} = 13486539,$ $N_2 = N_9 = -3237113, N_3 = N_8 = 25226685,$ $N_4 = N_7 = -9595542, N_5 = N_6 = 15493566,$ $D_{11} = 87091200.$
$n = 12 :$	$N_0 = N_{12} = 1364651, N_1 = N_{11} = 9903168,$ $N_2 = N_{10} = -7587864, N_3 = N_9 = 35725120,$ $N_4 = N_8 = -51491295, N_5 = N_7 = 87516288,$ $N_6 = -87797136, D_{12} = 63063000.$

$$\begin{aligned}
n = 13 : N_0 = N_{13} &= 61376\,98213, \quad N_1 = N_{12} = 4\,21942\,38652, \\
N_2 = N_{11} &= -2\,33615\,40993, \quad N_3 = N_{10} = 11\,67782\,74403, \\
N_4 = N_9 &= -11\,32197\,77650, \\
N_5 = N_8 &= 15\,44245\,90209, \quad N_6 = N_7 = -3\,20679\,788334, \\
D_{13} &= 30\,17710\,08000.
\end{aligned}$$


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$$\begin{aligned}
n = 14 : N_0 = N_{14} &= 902\,41897, \quad N_1 = N_{13} = 7109\,86864, \\
N_2 = N_{12} &= -7707\,20657, \quad N_3 = N_{11} = 35014\,42784, \\
N_4 = N_{10} &= -66250\,93363, \quad N_5 = N_9 = 1\,26301\,21616 \\
N_6 = N_8 &= -1\,68022\,70373, \quad N_7 = 1\,95344\,38464, \\
D_{14} &= 50038\,56000.
\end{aligned}$$


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$$\begin{aligned}
n = 15 : N_0 = N_{15} &= 1059\,30069, \quad N_1 = N_{14} = 7966\,61595, \\
N_2 = N_{13} &= -6988\,08195, \quad N_3 = N_{12} = 31433\,32755, \\
N_4 = N_{11} &= -46885\,22055, \\
N_5 = N_{10} &= 73856\,54007, \quad N_6 = N_9 = -60009\,98415, \\
N_7 = N_8 &= 30564\,22815, \\
D_{15} &= 61993\,45152.
\end{aligned}$$


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$$\begin{aligned}
n = 16 : N_0 = N_{16} &= 1\,50436\,11773, \quad N_1 = N_{15} = 12\,76266\,06592, \\
N_2 = N_{14} &= -17\,97311\,34720, \quad N_3 = N_{13} = 83\,22118\,55360, \\
N_4 = N_{12} &= -192\,94986\,07520, \\
N_5 = N_{11} &= 417\,75888\,93696, \quad N_6 = N_{10} = -680\,65344\,07936, \\
N_7 = N_9 &= 936\,88750\,18240, \quad N_8 = -1023\,42389\,72220 \\
D_{16} &= 97\,69246\,98750.
\end{aligned}$$


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$$\begin{aligned}
n = 17 : N_0 = N_{17} &= 5529\,47208\,74657, \quad N_1 = N_{16} = 45018\,55154\,46285, \\
N_2 = N_{15} &= -54202\,34370\,08852, \quad N_3 = N_{14} = 2\,42863\,65257\,64260, \\
N_4 = N_{13} &= -4\,76891\,68001\,23440, \\
N_5 = N_{12} &= 8\,85541\,66486\,84984, \quad N_6 = N_{11} = -10\,90537\,18597\,96660, \\
N_7 = N_{10} &= 10\,06961\,57501\,32836, \quad N_8 = N_9 = -3\,75978\,59740\,54070 \\
D_{17} &= 3\,76610\,21798\,40000.
\end{aligned}$$


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$$\begin{aligned}
n = 18 : N_0 = N_{18} &= 20\,37323\,52169, \quad N_1 = N_{17} = 184\,87302\,21900, \\
N_2 = N_{16} &= -321\,27443\,74395, \quad N_3 = N_{15} = 1552\,98303\,12096, \\
N_4 = N_{14} &= -4236\,86306\,85840, \\
N_5 = N_{13} &= 10368\,05634\,65808, \quad N_6 = N_{12} = -19864\,84298\,67720, \\
N_7 = N_{11} &= 31903\,57844\,79840, \quad N_8 = N_{10} = -41912\,79511\,14198 \\
N_9 &= 46132\,73443\,40680, \quad D_{18} = 1520\,91139\,20000.
\end{aligned}$$


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$$\begin{aligned}
n = 19 : N_0 = N_{19} &= 69\,02876\,31556\,44023, \quad N_1 = N_{18} = 603\,65208\,22708\,08125, \\
N_2 = N_{17} &= -926\,84051\,57002\,22955, \quad N_3 = N_{16} = 4301\,58153\,84505\,00095, \\
N_4 = N_{15} &= -10343\,69223\,42431\,92788, \\
N_5 = N_{14} &= 22336\,42032\,84799\,61316, \quad N_6 = N_{13} = -35331\,88842\,11147\,81580, \\
N_7 = N_{12} &= 43920\,76837\,05651\,35580, \quad N_8 = N_{11} = -37088\,37026\,13798\,51390 \\
N_9 = N_{10} &= 15148\,33730\,59217\,59574, \quad D_{19} = 5377\,99391\,28115\,20000.
\end{aligned}$$


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$n = 20 : N_0 = N_{20} = 1947\,01402\,41329$ ,  $N_1 = N_{19} = 18792\,60903\,80000$ ,  
 $N_2 = N_{18} = -38935\,81941\,77509$ ,  $N_3 = N_{17} = 1\,98596\,91593\,40000$ ,  
 $N_4 = N_{16} = -6\,20894\,88358\,89375$ ,  
 $N_5 = N_{15} = 17\,01938\,77765\,17504$ ,  $N_6 = N_{14} = -37\,38973\,46712\,90000$ ,  
 $N_7 = N_{13} = 68\,86928\,75743\,20000$ ,  $N_8 = N_{12} = -105\,49901\,48137\,01250$   
 $N_9 = N_{11} = 136\,42452\,17984\,40000$ ,  $N_{10} = -148\,19252\,66072\,80936$ ,  
 $D_{20} = 1\,64648\,54410\,80480$ .

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