! For an efficient use of these tables, first read `HowTo.pdf`.

T2.63E. Integrands involving logarithm functions of complicated arguments, like $\left(1+a^2/x^2\right)$, $(1\pm e^{-x})$, $\left(1+2e^{-x}\cos t+e^{-2x}\right)$ and others, on the intervals $(0,\pi)$, $(0,2\pi)$ and $(0,n\pi)$.

1. $\displaystyle\int_0^{\pi}\ln\left(a+b\cos x\right)dx=\pi\ln\frac{a+\sqrt{a^2-b^2}}{2},\quad a\geq|b|>0.$

2. $\displaystyle\int_0^{\pi}\ln\left(1\pm\sin x\right)dx=-\pi\ln 2\pm 4\mathbf{G}.$

3. $\displaystyle\int_0^{\pi}\ln\left(1+a\cos x\right)^2dx=2\pi\ln\frac{1+\sqrt{1-a^2}}{2},\quad a^2\leq 1.$

4. $\displaystyle\int_0^{\pi}\ln\left(a^2+b^2\tan^2 x\right)dx=2\pi\ln\left(a+b\right),\quad a>0,\,b>0.$

5. $\displaystyle\int_0^{2\pi}\ln\left(1+a\sin x+b\cos x\right)dx=2\pi\ln\frac{1+\sqrt{1-a^2-b^2}}{2},\quad a^2+b^2<1.$

6. $\displaystyle\int_0^{2\pi}\ln\left(1+a^2+b^2+2a\sin x+2b\cos x\right)dx=\begin{cases}0, & a^2+b^2\leq 1,\\ 2\pi\ln\left(a^2+b^2\right), & a^2+b^2\geq 1.\end{cases}$

7. $\displaystyle\int_0^{n\pi}\ln\left(b^2-2ab\cos x+a^2\right)dx=\begin{cases}n\pi\ln a^2, & a^2\geq b^2>0,\\ n\pi\ln b^2, & b^2>a^2>0.\end{cases}$

8. $\displaystyle\int_0^{n\pi}\ln\left(1-2a\cos x+a^2\right)dx=\begin{cases}0, & a^2\leq 1,\\ n\pi\ln a^2, & a^2\geq 1.\end{cases}$
