! For an efficient use of these tables, first read `HowTo.pdf`.

T2.03A. Powers of x, binomials of the form $(a+b\,x)$ and polynomials in x on the interval $(0,1)$.

1. $\displaystyle\int_0^1 \frac{x^{\mu-1}\,dx}{1+x^p} = \frac{1}{p}\,\beta\left(\frac{\mu}{p}\right), \quad \Re\{\mu\}>0,\ p>0.$

2. $\displaystyle\int_0^1 \frac{x^{p-1}+x^{q-p-1}}{1+x^q}\,dx = \frac{\pi}{q}\,\csc\frac{p\pi}{q}, \quad q>p>0.$

3. $\displaystyle\int_0^1 \frac{x^{p-1}-x^{q-p-1}}{1-x^q}\,dx = \frac{\pi}{q}\,\cot\frac{p\pi}{q}, \quad q>p>0.$

4. $\displaystyle\int_0^1 \frac{x^{\nu-1}-x^{\mu-1}}{1-x^\nu}\,dx = \frac{1}{\nu}\left[\gamma_e+\psi\left(\frac{\mu}{\nu}\right)\right], \quad \Re\{\mu\}>\Re\{\nu\}>0.$

5. $\displaystyle\int_0^1 \frac{x^{\alpha-1}(1-x)^{n-1}}{1-\xi x^b}\,dx = (n-1)!\sum_{k=0}^{\infty}\frac{\xi^k}{(\alpha+kb)\,(\alpha+kb+1)\ldots(\alpha+kb+k-1)},$
$b>0,\ |\xi|<1.$

6. $\displaystyle\int_0^1 \frac{x^{2n+1}\,dx}{\sqrt{1-x^2}} = \frac{(2n)!!}{(2n+1)!!}.$

7. $\displaystyle\int_0^1 \frac{x^{2n}\,dx}{\sqrt{1-x^2}} = \frac{(2n-1)!!}{(2n)!!}\,\frac{\pi}{2}.$

8. $\displaystyle\int_0^1 \frac{x^\mu\,dx}{1+x^2} = \frac{1}{2}\,\beta\left(\frac{\mu+1}{2}\right), \quad \Re\{\mu\}>-1.$

9. $\displaystyle\int_0^1 (1-x^2)^{\mu-1}\,dx = 2^{2\mu-2}\mathrm{B}(\mu,\,\mu) = \frac{1}{2}\mathrm{B}\left(\tfrac{1}{2},\,\mu\right), \quad \Re\{\mu\}>0.$

10. $\displaystyle\int_0^1 (1-\sqrt{x}\,)^{p-1}\,dx = \frac{2}{p(p+1)}, \quad p > 0.$

11. $\displaystyle\int_0^1 (1-x^{\mu})^{-1/\nu}\,dx = \frac{1}{\mu}\mathrm{B}\left(\frac{1}{\mu},\, 1-\frac{1}{\nu}\right), \quad \Re\{\mu\} > 0,\ |\nu| > 1.$

12. $\displaystyle\int_0^1 x^{\mu-1}(1-x^{\lambda})^{\nu-1}\,dx = \frac{1}{\lambda}\,\mathrm{B}\left(\frac{\mu}{\lambda},\, \nu\right), \quad \Re\{\mu\} > 0,\ \Re\{\nu\} > 0,\ \lambda > 0.$

13. $\displaystyle\int_0^1 \frac{x^{\mu}\,dx}{(1+x^2)^2} = -\frac{1}{4} + \frac{\mu-1}{4}\beta\left(\frac{\mu-1}{2}\right), \quad \Re\{\mu\} > 1.$

14. $\displaystyle\int_0^1 x^{q+p-1}(1-x^{q})^{-p/q}\,dx = \frac{p\pi}{q^2}\,\csc\frac{p\pi}{q}, \quad q > p.$

15. $\displaystyle\int_0^1 x^{q/p-1}(1-x^{q})^{-1/p}\,dx = \frac{\pi}{q}\csc\frac{\pi}{p}, \quad p > 1,\ q > 0.$

16. $\displaystyle\int_0^1 x^{p-1}(1-x^{q})^{-p/q}\,dx = \frac{\pi}{q}\,\csc\frac{p\pi}{q}, \quad q > p > 0.$

17. $\displaystyle\int_0^1 \frac{x^{p-1}+x^{q-1}}{(1-x^2)^{(p+q)/2}}\,dx = \frac{1}{2}\,\cos\left(\frac{q-p}{4}\pi\right)\sec\left(\frac{q+p}{4}\pi\right)\mathrm{B}\left(\frac{p}{2},\, \frac{q}{2}\right), \quad p > 0,\ q > 0,\ p+q < 2.$

19. $\displaystyle\int_0^1 \frac{x^{p-1}-x^{q-1}}{(1-x^2)^{(p+q)/2}}\,dx = \frac{1}{2}\,\sin\left(\frac{q-p}{4}\pi\right)\csc\left(\frac{q+p}{4}\pi\right)\mathrm{B}\left(\frac{p}{2},\, \frac{q}{2}\right), \quad p > 0,\ q > 0,\ p+q < 2.$

19. $\displaystyle\int_0^1 x^{p-1}(1-x)^{n-1}(1+bx^{m})^{j}\,dx = (n-1)!\sum_{k=0}^{\infty}\binom{j}{k}\frac{b^k\,\Gamma(p+km)}{\Gamma(p+n+km)},$

$|b| < 1$ unless $j = 0, 1, 2, \ldots;\, p,\ n,\ p+jm > 0.$

20. $\displaystyle\text{p.v.}\int_0^1 \frac{(1-x\,\cos t)x^{\mu-1}\,dx}{1-2x\,\cos t+x^2} = \sum_{k=0}^{\infty}\frac{\cos kt}{\mu+k}, \quad \Re\{\mu\} > 0,\ t \neq 2n\pi.$

21. $\displaystyle\int_0^1 \frac{(x^{\nu}+x^{-\nu})\,dx}{1+2x\,\cos t+x^2} = \frac{\pi\,\sin\nu t}{\sin t\,\sin\nu\pi}, \quad \nu^2 < 1,\ t \neq (2n+1)\pi.$

22. $\displaystyle\int_0^1 \frac{(x^{1+p}+x^{1-p})\,dx}{(1+2x\,\cos t+x^2)^2} = \frac{\pi(p\,\sin t\,\cos pt-\cos t\,\sin pt)}{2\sin^3 t\,\sin p\pi}, \quad p^2 < 1,\ t \neq (2n+1)\pi.$

23. $$\int_0^1 \frac{x^{\mu-1}}{1+2ax\,\cos t+a^2x^2}\cdot\frac{dx}{(1-x)^\mu}=\frac{2\pi\csc t\csc\mu\pi}{\mu\,(1+2a\,\cos t+a^2)}\,\sin\left(t-\mu\,\arctan\frac{a\,\sin t}{1+a\,\cos t}\right),\quad a>0,\ 0<\Re\{\mu\}<1.$$

24. $$\int_0^1 \frac{1-x^{\mu-1}}{1-x}\,dx=\begin{cases}\psi(\mu)+\gamma_e, & \Re\{\mu\}>0,\\ \psi(1-\mu)+\gamma_e-\pi\,\cot(\mu\pi), & \Re\{\mu\}>0.\end{cases}$$

25. $$\int_0^1 \frac{x^{3n}\,dx}{\sqrt[3]{1-x^3}}=\frac{2\pi}{3\sqrt{3}}\frac{\Gamma\left(n+\frac{1}{3}\right)}{\Gamma\left(\frac{1}{3}\right)\Gamma(n+1)}.$$

26. $$\int_0^1 \frac{x^{3n-1}\,dx}{\sqrt[3]{1-x^3}}=\frac{(n-1)!\Gamma\left(\frac{2}{3}\right)}{3\Gamma\left(n+\frac{2}{3}\right)}.$$

27. $$\int_0^1 \frac{x^{3n-2}\,dx}{\sqrt[3]{1-x^3}}=\frac{\Gamma(n-1/3)\,\Gamma(2/3)}{3\Gamma(n+1/3)}.$$

28. $$\int_0^1 \left(\frac{1}{1-x}-\frac{px^{p-1}}{1-x^p}\right)\,dx=\ln p.$$

29. $$\int_0^1 \frac{1-x^\mu}{1-x}x^{\nu-1}\,dx=\psi(\mu+\nu)-\psi(\nu),\quad \Re\{\nu\}>0,\ \Re\{\mu\}>0.$$

30. $$\int_0^1 \left[\frac{n}{1-x}-\frac{x^{\mu-1}}{1-\sqrt[n]{x}}\right]\,dx=n\gamma_e+\sum_{k=1}^{n}\psi\left(\mu+\frac{n-k}{n}\right),\quad \Re\{\mu\}>0.$$

31. $$\int_0^1 \frac{x^p-x^{-p}}{1-x^2}x\,dx=\frac{\pi}{2}\,\cot\frac{p\pi}{2}-\frac{1}{p},\quad p^2<1.$$

32. $$\int_0^1 \frac{x^p-x^{-p}}{1+x^2}x\,dx=\frac{1}{p}-\frac{\pi}{2}\csc\frac{p\pi}{2},\quad p^2<1.$$

33. $$\int_0^1 \frac{x^\mu-x^\nu}{1-x^2}\,dx=\frac{1}{2}\psi\left(\frac{\nu+1}{2}\right)-\frac{1}{2}\psi\left(\frac{\mu+1}{2}\right),\quad \Re\{\mu\}>-1,\ \Re\{\nu\}>-1.$$

34. $$\int_0^1 \frac{x^{n-1}+x^{n-\frac{1}{2}}-2x^{2n-1}}{1-x}\,dx=2\,\ln 2.$$

35. $$\int_0^1 \frac{x^{n-1}+x^{n-\frac{2}{3}}+x^{n-\frac{1}{3}}-3x^{3n-1}}{1-x}\,dx=3\,\ln 3.$$

36. $$\int_0^1 \frac{\sin t - a^n x^n \ \sin[(n+1)t] + a^{n+1}x^{n+1} \ \sin nt}{1 - 2ax \ \cos t + a^2x^2}(1-x)^{p-1}\,dx = \Gamma(p)\sum_{k=1}^{n}\frac{(k-1)!a^{k-1} \ \sin kt}{\Gamma(p+k)},$$
$$p > 0.$$

37. $$\int_0^1 \frac{\cos t - ax - a^n x^n \ \cos[(n+1)t] + a^{n+1}x^{n+1} \ \cos nt}{1 - 2ax \ \cos t + a^2x^2(1-x)^{p-1}}\,dx = \Gamma(p)\sum_{k=1}^{n}\frac{(k-1)!\,a^{k-1} \ \cos kt}{\Gamma(p+k)},$$
$$p > 0.$$

38. $$\int_0^1 x\frac{\sin t - x^n \ \sin[(n+1)t] + x^{n+1} \ \sin nt}{1 - 2x \ \cos t + x^2}\,dx = \sum_{k=1}^{n}\frac{\sin kt}{k+1}.$$

39. $$\int_0^1 \frac{1 - x \ \cos t - x^{n+1} \ \cos[(n+1)t] + x^{n+2} \ \cos nt}{1 - 2x \ \cos t + x^2}\,dx = \sum_{k=0}^{n}\frac{\cos kt}{k+1}.$$

40. $$\int_0^1 \frac{1-x^n}{(1+x)^{n+1}}\frac{dx}{1-x} = \frac{1}{2^{n+1}}\sum_{k=1}^{n}\frac{2^k}{k}.$$

41. $$\int_0^1 \left\{\frac{x^{n-1}}{1-x^{1/p}} - \frac{px^{np-1}}{1-x}\right\}dx = p \ \ln p, \quad p > 0.$$

42. $$\int_0^1 \left\{\frac{nx^{n-1}}{1-x^n} - \frac{x^{mn-1}}{1-x}\right\}dx = \gamma_e + \frac{1}{n}\sum_{k=1}^{n}\psi\left(m + \frac{n-k}{n}\right).$$

43. $$\int_0^1 \left(\frac{x^{p-1}}{1-x} - \frac{qx^{pq-1}}{1-x^q}\right)dx = \ln q, \quad q > 0.$$

44. $$\int_0^1 \frac{x^{\mu+1/2}(1-x)^{\mu-1/2}}{(c+2bx-ax^2)^{\mu+1}}\,dx = \frac{\sqrt{\pi}}{\left[a + (\sqrt{c+2b-a} + \sqrt{c})^2\right]^{\mu+1/2}\sqrt{c+2b-a}}\frac{\Gamma(\mu+1/2)}{\Gamma(\mu+1)},$$
$$a + (\sqrt{c+2b-a} + \sqrt{c})^2 > 0, \ c + 2b - a > 0, \ \Re\{\mu\} > -\tfrac{1}{2}.$$