T1.23. Integrand involving rational functions of sines and cosines.

1. $$\int \frac{dx}{a+b\sin x} = \begin{cases} \dfrac{2}{\sqrt{a^2-b^2}} \arctan \dfrac{a\tan\frac{x}{2}+b}{\sqrt{a^2-b^2}}, & a^2 > b^2, \\ = \dfrac{1}{\sqrt{b^2-a^2}} \ln \dfrac{a\tan\frac{x}{2}+b-\sqrt{b^2-a^2}}{a\tan\frac{x}{2}+b+\sqrt{b^2-a^2}}, & a^2 < b^2. \end{cases}$$

2. $$\int \frac{A+B\sin x}{a+b\sin x}\,dx = \frac{B}{b}x + \frac{Ab-aB}{b}\int \frac{dx}{a+b\sin x}.$$

3. $$\int \frac{A+B\sin x}{(a+b\sin x)^n}\,dx = \frac{1}{(n-1)(a^2-b^2)}\left[\frac{(Ab-aB)\cos x}{(a+b\sin x)^{n-1}} + \int \frac{(Aa-Bb)(n-1)+(aB-bA)(n-2)\sin x}{(a+b\sin x)^{n-1}}\,dx\right].$$

4. $$\int \frac{dx}{(a+b\sin x)^n} = \frac{1}{(n-1)(a^2-b^2)}\left\{\frac{b\cos x}{(a+b\sin x)^{n-1}} + \int \frac{(n-1)a-(n-2)b\sin x}{(a+b\sin x)^{n-1}}\,dx\right\}.$$

5. $$\int \frac{A+B\cos x}{a+b\sin x}\,dx = \frac{B}{b}\ln(a+b\sin x) + A\int \frac{dx}{a+b\sin x}.$$

6. $$\int \frac{A+B\cos x}{(a+b\sin x)^n}\,dx = -\frac{B}{(n-1)b(a+b\sin x)^{n-1}} + A\int \frac{dx}{(a+b\sin x)^n}.$$

7. $$\int \frac{dx}{a+b\cos x} = \begin{cases} \dfrac{2}{\sqrt{a^2-b^2}} \arctan \dfrac{\sqrt{a^2-b^2}\tan\frac{x}{2}}{a+b}, & a^2 > b^2, \\ = \dfrac{1}{\sqrt{b^2-a^2}} \ln \dfrac{\sqrt{b^2-a^2}\tan\frac{x}{2}+a+b}{\sqrt{b^2-a^2}\tan\frac{x}{2}-a-b}, & a^2 < b^2. \end{cases}$$

8. $$\int \frac{A+B\sin x}{a+b\cos x}\,dx = -\frac{B}{b}\ln(a+b\cos x) + A\int \frac{dx}{a+b\cos x}.$$

9. $$\int \frac{A+B\sin x}{(a+b\cos x)^n}\,dx = \frac{B}{(n-1)b(a+b\cos x)^{n-1}} + A\int \frac{dx}{(a+b\cos x)^n}.$$

10. $$\int \frac{dx}{(a+b\cos x)^n} = -\frac{1}{(n-1)(a^2-b^2)}\left\{\frac{b\sin x}{(a+b\cos x)^{n-1}} - \int \frac{(n-1)a-(n-2)b\cos x}{(a+b\cos x)^{n-1}}\,dx\right\}.$$

11. $$\int \frac{A+B\cos x}{a+b\cos x}\,dx=\frac{B}{b}x+\frac{Ab-aB}{b}\int\frac{dx}{a+b\cos x}.$$

12. $$\int \frac{A+B\cos x}{(a+b\cos x)^n}\,dx=\frac{1}{(n-1)(a^2-b^2)}\left[\frac{(aB-Ab)\sin x}{(a+b\cos x)^{n-1}}\right.$$
$$\left.+\int\frac{(Aa-bB)(n-1)+(n-2)(aB-bA)\cos x}{(a+b\cos x)^{n-1}}\,dx\right].$$

13. $$\int \frac{A+B\sin x}{1\pm\sin x}\,dx=\pm Bx+(A\mp B)\tan\left(\frac{\pi}{4}\mp\frac{x}{2}\right).$$

14. $$\int \frac{A+B\sin x}{(1\pm\sin x)^n}\,dx=-\frac{1}{2^{n-1}}\left\{2B\sum_{k=0}^{n-2}\binom{n-2}{k}\frac{\tan^{2k+1}\left(\frac{\pi}{4}\mp\frac{x}{2}\right)}{2k+1}\right.$$
$$\left.\pm(A\mp B)\sum_{k=0}^{n-1}\binom{n-1}{k}\frac{\tan^{2k+1}\left(\frac{\pi}{4}\mp\frac{x}{2}\right)}{2k+1}\right\}.$$

15. $$\int \frac{A+B\cos x}{1\pm\cos x}\,dx=\pm Bx\pm(A\mp B)\tan\left[\frac{\pi}{4}\mp\left(\frac{\pi}{4}-\frac{x}{2}\right)\right].$$

16. $$\int \frac{A+B\cos x}{(1\pm\cos x)^n}\,dx=\frac{1}{2^{n-1}}\left\{2B\sum_{k=0}^{n-2}\binom{n-2}{k}\frac{\tan^{2k+1}\left[\frac{\pi}{4}\mp\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]}{2k+1}\right.$$
$$\left.\pm(A\mp B)\sum_{k=0}^{n-1}\binom{n-1}{k}\frac{\tan^{2k+1}\left[\frac{\pi}{4}\mp\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]}{2k+1}\right\}.$$

17. $$\int \frac{(1-a^2)\,dx}{1-2a\cos x+a^2}=2\arctan\left(\frac{1+a}{1-a}\tan\frac{x}{2}\right),\quad 0<a<1,\ |x|<\pi.$$

18. $$\int \frac{(1-a\cos x)\,dx}{1-2a\cos x+a^2}=\frac{x}{2}+\arctan\left(\frac{1+a}{1-a}\tan\frac{x}{2}\right),\quad 0<a<1,\ |x|<\pi.$$

19. $$\int \frac{dx}{a\cos x+b\sin x}=\frac{\ln\tan\left[\frac{1}{2}\left(x+\arctan\frac{a}{b}\right)\right]}{\sqrt{a^2+b^2}}.$$

20. $$\int \frac{dx}{(a\cos x+b\sin x)^2}=-\frac{\cot\left(x+\arctan\frac{a}{b}\right)}{a^2+b^2}=+\frac{1}{a^2+b^2}\cdot\frac{a\sin x-b\cos x}{a\cos x+b\sin x}.$$

21. $$\int \frac{dx}{(a\cos x+b\sin x)^n}=\frac{1}{\sqrt{(a^2+b^2)^n}}\int\frac{dx}{\sin^n\left(x+\arctan\frac{a}{b}\right)}.$$

22. $$\int \frac{\sin x\,dx}{a\sin x+b\cos x}=\frac{ax-b\ln\sin\left(x+\arctan\frac{b}{a}\right)}{a^2+b^2}.$$

23. $$\int \frac{\cos x\,dx}{a\cos x+b\sin x}=\frac{ax+b\ln\sin\left(x+\arctan\frac{a}{b}\right)}{a^2+b^2}.$$

24. $$\int \frac{A+B\cos x+C\sin x}{a+b\cos x+c\sin x}\,dx=\frac{Bc-Cb}{b^2+c^2}\ln(a+b\cos x+c\sin x)+\frac{Bb+Cc}{b^2+c^2}x$$
$$+\left(A-\frac{Bb+Cc}{B^2+c^2}a\right)\int\frac{dx}{a+b\cos x+c\sin x}.$$

25. $$\int \frac{A + B\,\cos x + C\,\sin x}{(a + b\,\cos x + c\,\sin x)^n}\,dx$$

$$= \begin{cases} \dfrac{(Bc - Cb) + (Ac - Ca)\,\cos x - (Ab - Ba)\,\sin x}{(n-1)(a^2 - b^2 - c^2)(a + b\,\cos x + c\,\sin x)^{n-1}} + \dfrac{1}{(n-1)(a^2 - b^2 - c^2)} \\ \times \displaystyle\int \frac{(n-1)(Aa - Bb - Cc) - (n-2)[(Ab - Ba)\,\cos x - (Ac - Ca)\,\sin x]}{(a + b\,\cos x + c\,\sin x)^{n-1}}\,dx, & n \neq 1,\ a^2 \neq b^2 + c^2, \\ \dfrac{Cb - Bc + Ca\,\cos x - Ba\,\sin x}{(n-1)a(a + b\,\cos x + c\,\sin x)^n} + \left(\dfrac{A}{a} + \dfrac{n(Bb + Cc)}{(n-1)a^2}\right)(-c\,\cos x + b\,\sin x) \\ \times \dfrac{(n-1)!}{(2n-1)!!}\displaystyle\sum_{k=0}^{n-1}\frac{(2n-2k-3)!!}{(n-k-1)!a^k}\,\frac{1}{(a + b\,\cos x + c\,\sin x)^{n-k}}, & n \neq 1,\ a^2 = b^2 + c^2. \end{cases}$$

26. $$\int \frac{dx}{a + b\,\cos x + c\,\sin x} = \begin{cases} \dfrac{2}{\sqrt{a^2 - b^2 - c^2}}\ \arctan \dfrac{(a-b)\,\tan\frac{x}{2} + c}{\sqrt{a^2 - b^2 - c^2}}, & a^2 > b^2 + c^2, \\ \dfrac{1}{\sqrt{b^2 + c^2 - a^2}}\ \ln \dfrac{(a-b)\,\tan\frac{x}{2} + c - \sqrt{b^2 + c^2 - a^2}}{(a-b)\,\tan\frac{x}{2} + c + \sqrt{b^2 + c^2 - a^2}}, & a^2 < b^2 + c^2, \\ \dfrac{1}{c}\ \ln\left(a + c \cdot \tan\dfrac{x}{2}\right), & a = b, \\ \dfrac{-2}{c + (a-b)\,\tan\frac{x}{2}}, & a^2 = b^2 + c^2. \end{cases}$$

27. $$\int \frac{dx}{(a + b\,\cos x + c\,\sin x)^n} = \int \frac{d(x - \alpha)}{[a + r\,\cos(x - \alpha)]^n}, \quad \text{where} b = r\,\cos\alpha,\ c = r\,\sin\alpha.$$

28. $$\int \frac{dx}{[a(1 + \cos x) + c\,\sin x]^2} = \frac{1}{c^3}\left[\frac{c(a\,\sin x - c\,\cos x)}{a(1 + \cos x) + c\,\sin x} - a\,\ln\left(a + c\,\tan\frac{x}{2}\right)\right].$$

29. $$\int \frac{A + B\,\cos x + C\,\sin x}{(a_1 + b_1\,\cos x + c_1\,\sin x)(a_2 + b_2\,\cos x + c_2\,\sin x)}\,dx$$

$$= A_0\,\ln\frac{a_1 + b_1\,\cos x + c_1\,\sin x}{a_2 + b_2\,\cos x + c_2\,\sin x} + A_1\int\frac{dx}{a_1 + b_1\,\cos x + c_1\,\sin x} + A_2\int\frac{dx}{a_2 + b_2\,\cos x + c_2\,\sin x},$$

where $$A_0 = \frac{\begin{vmatrix} A & B & C \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\Delta}, \quad A_1 = \frac{\begin{vmatrix} \begin{vmatrix} B & C \\ b_1 & c_1 \end{vmatrix} & \begin{vmatrix} A & C \\ a_1 & c_1 \end{vmatrix} & \begin{vmatrix} B & A \\ b_1 & a_1 \end{vmatrix} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\Delta},$$

$$A_2 = \frac{\begin{vmatrix} \begin{vmatrix} C & B \\ c_2 & b_2 \end{vmatrix} & \begin{vmatrix} C & A \\ c_2 & a_2 \end{vmatrix} & \begin{vmatrix} A & B \\ a_2 & b_2 \end{vmatrix} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\Delta}, \quad \Delta = \left[\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}^2 + \begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}^2 \neq \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}^2\right].$$

30. $$\int \frac{A\cos^2 x + 2B\sin x\cos x + C\sin^2 x}{a\cos^2 x + 2b\sin x\cos x + c\sin^2 x}\,dx$$
$$= \frac{1}{4b^2+(a-c)^2}\{[4Bb+(A-C)(a-c)]x + [(A-C)b - B(a-c)]$$
$$\times \ln(a\cos^2 x + 2b\sin x\cos x + c\sin^2 x)$$
$$+[2(A+C)b^2 - 2Bb(a+c) + (aC-Ac)(a-c)]f(x)\},$$
where $$f(x) = \begin{cases} \dfrac{1}{2\sqrt{b^2-ac}}\ln\dfrac{c\tan x + b - \sqrt{b^2-ac}}{c\tan x + b + \sqrt{b^2-ac}}, & b^2 > ac, \\ \dfrac{1}{\sqrt{ac-b^2}}\arctan\dfrac{c\tan x + b}{\sqrt{ac-b^2}}, & b^2 < ac, \\ -\dfrac{1}{c\tan x + b}, & b^2 = ac. \end{cases}$$

31. $$\int \frac{(A+B\sin x)\,dx}{(\sin x)(a+b\sin x)} = \frac{A}{a}\ln\tan\frac{x}{2} + \frac{Ba-Ab}{a}\int\frac{dx}{a+b\sin x}.$$

32. $$\int \frac{(A+B\sin x)\,dx}{(\sin x)(a+b\cos x)} = \frac{A}{a^2-b^2}\left\{a\ln\tan\frac{x}{2} + b\ln\frac{a+b\cos x}{\sin x}\right\} + B\int\frac{dx}{a+b\cos x}.$$

33. $$\int \frac{(A+B\sin x)\,dx}{(\sin x)(a+b\cos x)} = \frac{A}{2}\left\{\ln\tan\frac{x}{2} + \frac{1}{1+\cos x}\right\} + B\tan\frac{x}{2}.$$

34. $$\int \frac{(A+B\sin x)\,dx}{(\sin x)(1-\cos x)} = \frac{A}{2}\left\{\ln\tan\frac{x}{2} - \frac{1}{1-\cos x}\right\} - B\cot\frac{x}{2}.$$

35. $$\int \frac{(A+B\sin x)\,dx}{(\cos x)(a+b\sin x)} = \frac{1}{a^2-b^2}\left\{(Aa-Bb)\ln\tan\left(\frac{\pi}{4}+\frac{x}{2}\right) - (Ab-aB)\ln\frac{a+b\sin x}{\cos x}\right\}.$$

36. $$\int \frac{(A+B\sin x)\,dx}{(\cos x)(1\pm\sin x)} = \frac{A\pm B}{2}\ln\tan\left(\frac{\pi}{4}+\frac{x}{2}\right) \mp \frac{A\mp B}{2(1\pm\sin x)}.$$

37. $$\int \frac{(A+B\sin x)\,dx}{(\cos x)(a+b\cos x)} = \frac{A}{a}\ln\tan\left(\frac{\pi}{4}+\frac{x}{2}\right) + \frac{B}{a}\ln\frac{a+b\cos x}{\cos x} - \frac{Ab}{a}\int\frac{dx}{a+b\cos x}.$$

38. $$\int \frac{(A+B\cos x)\,dx}{(\sin x)(a+b\sin x)} = \frac{A}{a}\ln\tan\frac{x}{2} - \frac{B}{a}\ln\frac{a+b\sin x}{\sin x} - \frac{Ab}{a}\int\frac{dx}{a+b\sin x}.$$

39. $$\int \frac{(A+B\cos x)\,dx}{(\sin x)(a+b\cos x)} = \frac{1}{a^2-b^2}\left\{(Aa-Bb)\ln\tan\frac{x}{2} + (Ab-Ba)\ln\frac{a+b\cos x}{\sin x}\right\}.$$

40. $$\int \frac{(A+B\cos x)\,dx}{(\sin x)(1\pm\cos x)} = \pm\frac{A\mp B}{2(1\pm\cos x)} + \frac{A\pm B}{2}\ln\tan\frac{x}{2}.$$

41. $$\int \frac{(A+B\cos x)\,dx}{(\cos x)(a+b\sin x)} = \frac{A}{a^2-b^2}\left\{a\ln\tan\left(\frac{\pi}{4}+\frac{x}{2}\right) - b\ln\frac{a+b\sin x}{\cos x}\right\} + B\int\frac{dx}{a+b\sin x}.$$

42. $$\int \frac{(A+B\sin x)\,dx}{(\cos x)(1\pm\sin x)} = \frac{A\pm B}{2}\ln\tan\left(\frac{\pi}{4}+\frac{x}{2}\right) \mp \frac{A\mp B}{2(1\pm\sin x)}.$$

43. $$\int \frac{(A+B\cos x)\,dx}{(\cos x)(a+b\cos x)} = \frac{A}{a}\ln\tan\left(\frac{\pi}{4}+\frac{x}{2}\right) + \frac{Ba-Ab}{a}\int\frac{dx}{a+b\cos x}.$$

44. $$\int \frac{dx}{a+b\,\sin^2\,x} = \begin{cases} \dfrac{\text{sgn}\ a}{\sqrt{a(a+b)}}\ \arctan\left(\sqrt{\dfrac{a+b}{a}}\ \tan x\right), & \dfrac{b}{a} > -1, \\ \dfrac{\text{sgn}\ a}{\sqrt{-a(a+b)}}\ \text{arctanh}\left(\sqrt{-\dfrac{a+b}{a}}\ \tan x\right), & \dfrac{b}{a} < -1,\ \sin^2\,x < -\dfrac{a}{b}, \\ \dfrac{\text{sgn}\ a}{\sqrt{-a(a+b)}}\ \text{arccoth}\left(\sqrt{-\dfrac{a+b}{a}}\ \tan x\right), & \dfrac{b}{a} < -1,\ \sin^2 x > -\dfrac{a}{b}. \end{cases}$$

45. $$\int \frac{dx}{a+b\,\cos^2\,x} = \begin{cases} \dfrac{-\,\text{sgn}\ a}{\sqrt{a(a+b)}}\ \arctan\left(\sqrt{\dfrac{a+b}{a}}\ \cot x\right), & \dfrac{b}{a} > -1, \\ \dfrac{-\,\text{sgn}\ a}{\sqrt{-a(a+b)}}\ \text{arctanh}\left(\sqrt{-\dfrac{a+b}{a}}\ \cot x\right), & \dfrac{b}{a} < -1,\ \cos^2\,x < -\dfrac{a}{b}, \\ \dfrac{-\,\text{sgn}\ a}{\sqrt{-a(a+b)}}\ \text{arccoth}\left(\sqrt{-\dfrac{a+b}{a}}\ \cot x\right), & \dfrac{b}{a} < -1,\ \cos^2\,x > -\dfrac{a}{b}. \end{cases}$$

46. $$\int \frac{dx}{1+\sin^2\,x} = \frac{1}{\sqrt{2}}\ \arctan(\sqrt{2}\ \tan x).$$

47. $$\int \frac{dx}{1-\sin^2\,x} = \tan x.$$

48. $$\int \frac{dx}{1+\cos^2\,x} = -\frac{1}{\sqrt{2}}\ \arctan(\sqrt{2}\ \cot x).$$

49. $$\int \frac{dx}{1-\cos^2\,x} = -\cot x.$$

50. $$\int \frac{dx}{(a+b\,\sin^2\,x)^2} = \frac{1}{2a(a+b)}\left[(2a+b)\int \frac{dx}{a+b\,\sin^2\,x} + \frac{b\,\sin x\,\cos x}{a+b\,\sin^2\,x}\right].$$

51. $$\int \frac{dx}{(a+b\,\cos^2\,x)^2} = \frac{1}{2a(a+b)}\left[(2a+b)\int \frac{dx}{a+b\,\cos^2\,x} - \frac{b\,\sin x\,\cos x}{a+b\,\cos^2\,x}\right].$$

52. $$\int \frac{dx}{(a+b\,\sin^2\,x)^3} = \begin{cases} \dfrac{1}{8pa^3}\left[\left(3+\dfrac{2}{p^2}+\dfrac{3}{p^4}\right)\arctan(p\,\tan x) + \left(3+\dfrac{2}{p^2}-\dfrac{3}{p^4}\right)\dfrac{p\,\tan x}{1+p^2\,\tan^2\,x}\right. \\ \left. +\left(1-\dfrac{2}{p^2}-\dfrac{1}{p^2}\ \tan^2\,x\right)\dfrac{2p\,\tan x}{\left(1+p^2\,\tan^2\,x\right)^2}\right] \quad p^2 = 1+\dfrac{b}{a} > 0, \\ \dfrac{1}{8qa^3}\left[\left(3-\dfrac{2}{q^2}+\dfrac{3}{q^4}\right) + \left(3-\dfrac{2}{q^2}-\dfrac{3}{q^4}\right)\dfrac{q\,\tan x}{1-q^2\,\tan^2\,x}\right. \\ \left. +\left(1+\dfrac{2}{q^2}+\dfrac{1}{q^2}\ \tan^2\,x\right)\dfrac{2q\,\tan x}{\left(1-q^2\,\tan^2\,x\right)^2}\right], \quad q^2 = -1-\dfrac{b}{a} > 0,\ \sin^2\,x < -\dfrac{a}{b}; \end{cases}$$

and for $\sin^2 x > -a/b$, replace $\text{arctanh}(q\,\tan x)$ with $\text{arccoth}\,(q\,\tan x)$.

53. $$\int \frac{dx}{(a+b\cos^2 x)^3} = \begin{cases} -\dfrac{1}{8pa^3}\left[\left(3+\dfrac{2}{p^2}+\dfrac{3}{p^4}\right)\arctan(p\cot x)+\left(3+\dfrac{2}{p^2}-\dfrac{3}{p^4}\right)\dfrac{p\cot x}{1+p^2\cot^2 x}\right. \\ \left.+\left(1-\dfrac{2}{p^2}-\dfrac{1}{p^2}\cot^2 x\right)\dfrac{2p\cot x}{\left(1+p^2\cot^2 x\right)^2}\right], \quad p^2 = 1+\dfrac{b}{a} > 0, \\ -\dfrac{1}{8qa^3}\left[\left(3-\dfrac{2}{q^2}+\dfrac{3}{q^4}\right)\operatorname{arctanh}(q\cot x)+\left(3-\dfrac{2}{q^2}-\dfrac{3}{q^4}\right)\dfrac{q\cot x}{1-q^2\cot^2 x}\right. \\ \left.+\left(1+\dfrac{2}{q^2}+\dfrac{1}{q^2}\cot^2 x\right)\dfrac{2p\cot x}{\left(1-q^2\cot^2 x\right)^2}\right], \quad q^2 = -1-\dfrac{b}{a} < 0,\ \cos^2 x < -\dfrac{a}{b}; \end{cases}$$

and for $\cos^2 x > -a/b$, replace $\operatorname{arctanh}(q\cot x)$ with $\operatorname{arccoth}(q\cot x)$.

54. $$\int \frac{\tan x\,dx}{1+m^2\tan^2 x} = \frac{\ln(\cos^2 x + m^2\sin^2 x)}{2(m^2-1)}.$$

55. $$\int \frac{\tan\alpha - \tan x}{\tan\alpha + \tan x}\,dx = \sin 2\alpha\,\ln\sin(x+\alpha) - x\cos 2\alpha.$$

56. $$\int \frac{\tan x\,dx}{a+b\tan x} = \frac{1}{a^2+b^2}\{bx - a\ln(a\cos x + b\sin x)\}.$$

57. $$\int \frac{dx}{a+b\tan^2 x} = \frac{1}{a-b}\left[x - \sqrt{\frac{b}{a}}\arctan\left(\sqrt{\frac{b}{a}}\tan x\right)\right].$$

58. $$\int \frac{dx}{\sqrt{a+b\tan^2 cx}} = \begin{cases} \dfrac{1}{c\sqrt{a-b}}\arcsin\left(\sqrt{\dfrac{(a-b}{a}}\sin cx\right), \quad (4k-1)\pi/2 < x \le (4k+1)\pi/2, \\ \dfrac{-1}{c\sqrt{a-b}}\arcsin\left(\sqrt{\dfrac{(a-b}{a}}\sin cx\right), \quad (4k+1)\pi/2 < x \le (4k+3)\pi/2;\ k\in \mathrm{N}. \end{cases}$$