! For an efficient use of these tables, first read `HowTo.pdf`.

T2.56A. Integrands involving product of exponentials, trigonometric functions and powers of trigonometric functions on the interval $(0, \pi/2)$.

1. $$\int_0^{\pi/2} e^{2i\,\beta x}\,\sin^{2\mu}\,x\,\cos^{2\nu}x\,dx = \frac{1}{2^{2\mu+2\nu+1}}\Big\{\exp\left[i\pi\left(\beta-\nu-\tfrac{1}{2}\right)\right]\mathrm{B}\left(\beta-\mu-\nu,\,2\nu+1\right)$$
$$\times F(-2\mu,\,\beta-\mu-\nu;\,1+\beta-\mu+\nu;\,-1)+\exp\left[i\,\pi\left(\mu+\tfrac{1}{2}\right)\right]$$
$$\times \mathrm{B}(\beta-\mu-\nu,\,2\mu+1)F(-2\nu,\,\beta-\mu-\nu;\,1+\beta+\mu-\nu;\,-1)\Big\},$$
$$\Re\{\mu\}>-\tfrac{1}{2},\ \Re\{\nu\}>-\tfrac{1}{2}.$$

2. $$\int_0^{\pi/2} e^{i\,(\mu+\nu)x}\,\sin^{\mu-1}x\,\cos^{\nu-1}\,x\,dx = e^{i\,\mu\pi/2}\,\mathrm{B}(\mu,\,\nu)$$
$$= \frac{1}{2^{\mu+\nu-1}}\,e^{i\,\mu\pi/2}\left\{\frac{1}{\mu}F(1-\nu,\,1;\,\mu+1;\,-1)+\frac{1}{\nu}F(1-\mu,\,1;\,\nu+1;\,-1)\right\},$$
$$\Re\{\mu\}>0,\ \Re\{\nu\}>0.$$

3. $$\int_0^{\pi/2} e^{-px}\,\sin^{2m}\,x\,dx = \frac{(2m)!}{p(p^2+2^2)(p^2+4^2)\ldots[p^2+(2m)^2]}$$
$$\times\left\{1-\,e^{-p\,\pi/2}\left[1+\frac{p^2}{2!}+\frac{p^2(p^2+2^2)}{4!}+\cdots+\frac{p^2(p^2+2^2)\ldots[p^2+(2m-2)^2]}{(2m)!}\right]\right\}.$$

4. $$\int_0^{\pi/2} e^{-px}\,\sin^{2m+1}\,x\,dx = \frac{(2m+1)!}{(p^2+1^2)(p^2+3^2)\ldots[p^2+(2m+1)^2]}$$
$$\times\left\{1-p\,e^{p\,\pi/2}\left[1+\frac{p^2+1^2}{3!}+\cdots+\frac{(p^2+1^2)(p^2+3^2)\ldots[p^2+(2m-1)^2]}{(2m+1)!}\right]\right\}.$$

5. $$\int_0^{\pi/2} e^{-p\,x}\,\cos^{2m}\,x\,dx = \frac{(2m)!}{p(p^2+2^2)\ldots[p^2+(2m)^2]}$$
$$\times\left\{-\,e^{-p\frac{\pi}{2}}+1+\frac{p^2}{2!}+\frac{p^2(p^2+2^2)}{4!}+\cdots+\frac{p^2(p^2+2^2)\ldots[p^2+(2m-2)^2]}{(2m)!}\right\}.$$

6. $$\int_0^{\pi/2} e^{-p\,x}\,\cos^{2m+1}x\,dx = \frac{(2m+1)!}{(p^2+1^2)(p^2+3^2)\dots[p^2+(2m+1)^2]}$$
$$\times\left\{e^{-p\pi/2}+p\left[1+\frac{p^2+1^2}{3!}+\dots+\frac{(p^2+1)(p^2+3^2)\dots[p^2+(2m-1)^2]}{(2m+1)!}\right]\right\},$$
$$p\neq 0.$$

7. $$\int_{-\pi/2}^{\pi/2} e^{i\,\beta\,x}\,\cos^{\nu}x\,\left(\beta^2\,e^{i\,x}+\nu^2\,e^{-i\,x}\right)^{\mu}\,dx = \frac{\pi_2 F_1\left(-\mu,\,\frac{\beta}{2}-\frac{\nu}{2}-\frac{\mu}{2};\,1+\frac{\beta}{2}+\frac{\nu}{2}-\frac{\mu}{2};\,\frac{\beta^2}{\nu^2}\right)}{2^{\nu}(\nu+1)\,\mathrm{B}\left(1+\frac{\beta}{2}+\frac{\nu}{2}-\frac{\mu}{2},\,1-\frac{\beta}{2}+\frac{\nu}{2}+\frac{\mu}{2}\right)},$$
$$\Re\{\nu\}>-1,\ |\nu|>|\beta|.$$

8. $$\int_0^{\pi/2}\frac{\exp(-p\,\tan x)\,dx}{\sin 2x+a\,\cos 2x+a} = -\frac{1}{2}\,e^{ap}\,\mathrm{Ei}(-ap),\quad p>0.$$

9. $$\int_0^{\pi/2}\frac{\exp(-p\,\cot x)\,dx}{\sin 2x+a\,\cos 2x-a} = -\frac{1}{2}\,e^{-ap}\,\mathrm{Ei}(ap),\quad p>0.$$

10. $$\int_0^{\pi/2}\frac{\exp(-p\,\tan x)\,\sin 2x\,dx}{(1-a^2)-2a^2\,\cos 2x-(1+a^2)\,\cos^2 2x} = -\frac{1}{4}\left[e^{-ap}\,\mathrm{Ei}(ap)+\,e^{ap}\,\mathrm{Ei}(-ap)\right],\quad p>0.$$

11. $$\int_0^{\pi/2}\frac{\exp(-p\,\cot x)\,\sin 2x\,dx}{(1-a^2)+2a^2\,\cos 2x-(1+a^2)\,\cos^2 2x} = -\frac{1}{4}\left[e^{-ap}\,\mathrm{Ei}(ap)+\,e^{ap}\,\mathrm{Ei}(-ap)\right],\quad p>0.$$

12. $$\int_0^{\pi/2} e^{-2\beta\,\cot x}\,\cos^{\nu-1/2}x\,\sin^{-(\nu+1)}x\,\sin\left[\beta-\left(\nu-\frac{1}{2}\right)x\right]\,dx = \frac{\sqrt{\pi}}{2(2\beta)^{\nu}}\Gamma\left(\nu+\frac{1}{2}\right)J_{\nu}(\beta),$$
$$\Re\{\nu\}>-\tfrac{1}{2}.$$

13. $$\int_0^{\pi/2} e^{-2\beta\,\cot x}\,\cos^{\nu-1/2}x\,\sin^{-(\nu+1)}x\,\cos\left[\beta-\left(\nu-\frac{1}{2}\right)x\right]\,dx = \frac{\sqrt{\pi}}{2(2\beta)^{\nu}}\Gamma\left(\nu+\frac{1}{2}\right)Y_{\nu}(\beta),$$
$$\Re\{\nu\}>-\tfrac{1}{2}.$$

14. $$\int_0^{\pi/2}\frac{\cos^{\mu}x}{\sin^{2\mu+2}x}\,e^{i\,p\,(\beta-\mu x)-2\beta\,\cot x}\,dx = \frac{i\,p}{2}\sqrt{\frac{\pi}{2\beta}}\,(2\beta)^{-\mu}\Gamma(\mu+1)\,H^{(j)}_{\mu+1/2}(\beta),$$
$$j=1,2;\ p=(-1)^{j+1};\ \Re\{\beta\}>0,\ \Re\{\mu\}>-1.$$

15. $$\int_0^{\pi/2} \frac{\cos^\mu x \, \sin(\beta - \mu x)}{\sin^{2\mu+2} x} e^{-2\beta \cot x} \, dx = \frac{1}{2}\sqrt{\frac{\pi}{2\beta}}\,(2\beta)^{-\mu}\Gamma(\mu+1)J_{\mu+1/2}(\beta),$$
$$\Re\{\beta\} > 0,\ \Re\{\mu\} > -1.$$

16. $$\int_0^{\pi/2} \frac{\cos^\mu x \, \cos(\beta - \mu x)}{\sin^{2\mu+2} x}\, e^{-2\beta \cot x} \, dx = -\frac{1}{2}\sqrt{\frac{\pi}{2\beta}}\,(2\beta)^{-\mu}\Gamma(\mu+1)Y_{\mu+1/2}(\beta),$$
$$\Re\{\beta\} > 0,\ \Re\{\mu\} > -1.$$

17. $$\int_0^{\pi/2} \frac{\sin 2nx}{\sin^{2n+2} x} \cdot \frac{dx}{\exp(2\pi \cot x) - 1} = (-1)^{n-1}\frac{2n-1}{4(2n+1)}.$$

18. $$\int_0^{\pi/2} \frac{\sin 2nx}{\sin^{2n+2} x} \cdot \frac{dx}{\exp(\pi \cot x) - 1} = (-1)^{n-1}\frac{n}{2n+1}.$$

19. $$\int_0^{\pi/2} e^{-p^2 \tan x}\, \frac{\sin \frac{x}{2}\sqrt{\cos x}}{\sin 2x}\, dx = \left[C(p) - \frac{1}{2}\right]^2 + \left[S(p) - \frac{1}{2}\right]^2.$$