! For an efficient use of these tables, first read `HowTo.pdf`.

T2.42A. Integrands involving powers of trigonometric functions on the interval $(0, \pi/4)$.

1. $$\int_0^{\pi/4} \tan^\mu x\, dx = \frac{1}{2}\beta\left(\frac{\mu+1}{2}\right), \quad \Re\{\mu\} > -1.$$

2. $$\int_0^{\pi/4} \tan^{2n} x\, dx = (-1)^n \frac{\pi}{4} + \sum_{k=0}^{n-1} \frac{(-1)^k}{2n-2k-1}.$$

3. $$\int_0^{\pi/4} \tan^{2n+1} x\, dx = (-1)^n \frac{\ln 2}{2} + \sum_{k=0}^{n-1} \frac{(-1)^k}{2n-2k}.$$

4. $$\int_0^{\pi/4} \tan^\mu x \sin^2 x\, dx = \frac{1+\mu}{4}\beta\left(\frac{\mu+1}{2}\right) - \frac{1}{4}, \quad \Re\{\mu\} > -1.$$

5. $$\int_0^{\pi/4} \tan^\mu x \cos^2 x\, dx = \frac{1-\mu}{4}\beta\left(\frac{\mu+1}{2}\right) + \frac{1}{4}, \quad \Re\{\mu\} > -1.$$

6. $$\int_0^{\pi/4} \frac{\sin^p x}{\cos^{p+2} x}\, dx = \frac{1}{p+1}, \quad p > -1.$$

7. $$\int_0^{\pi/4} \frac{\cos^{n-1/2} 2x}{\cos^{2n+1} x}\, dx = \frac{(2n-1)!!}{2\cdot(2n)!!}\,\pi.$$

8. $$\int_0^{\pi/4} \frac{\cos^\mu 2x}{\cos^{2(\mu+1)} x} + dx = 2^{2\mu}\,\mathrm{B}\,(\mu+1,\, \mu+1), \quad \Re\{\mu\} > -1.$$

9. $$\int_0^{\pi/4} \frac{\sin^{2\mu-2} x}{\cos^\mu 2x}\, dx = 2^{1-2\mu}\,\mathrm{B}\,(2\mu-1,\, 1-\mu) = \frac{\Gamma\left(\mu-\frac{1}{2}\right)\Gamma(1-\mu)}{2\sqrt{\pi}}, \quad \tfrac{1}{2} < \Re\{\mu\} < 1.$$

10. $$\int_0^{\pi/4} \frac{\sin^{2n-1} x \cos^p 2x}{\cos^{2p+2n+1} x}\, dx = \begin{cases} \dfrac{(n-1)!}{2} \cdot \dfrac{\Gamma(p+1)}{\Gamma(p+n+1)}, \\ \qquad \text{or} \\ \dfrac{(n-1)!}{2(p+n)(p+n-1)\dots(p+1)}, \\ \qquad \text{or} \\ \dfrac{1}{2}\,\mathrm{B}\,(n,\, p+1), \quad p > -1. \end{cases}$$

11. $$\int_0^{\pi/4} \frac{\sin^{2n} x \cos^p 2x}{\cos^{2p+2n+2} x}\, dx = \frac{1}{2}\,\mathrm{B}\left(n + \frac{1}{2},\, p+1\right), \quad p > -1.$$

12. $$\int_0^{\pi/4} \frac{\sin^{2n-1} x \cos^{m-1/2} 2x}{\cos^{2n+2m} x}\, dx = \frac{(2n-2)!!(2m-1)!!}{(2n+2m-1)!!}.$$

13. $$\int_0^{\pi/4} \frac{\sin^{2n} x \cos^{m-1/2} 2x}{\cos^{2n+2m+1} x} + \, dx = \frac{(2n-1)!!(2m-1)!!}{(2n+2m)!!} \cdot \frac{\pi}{2}.$$

14. $$\int_0^{\pi/4} \frac{\sin^{2n-1} x}{\cos^{2n+2} x} \sqrt{\cos 2x}\, dx = \frac{(2n-2)!!}{(2n+1)!!}.$$

15. $$\int_0^{\pi/4} \frac{\sin^{2n} x}{\cos^{2n+3} x} \sqrt{\cos 2x}\, dx = \frac{(2n-1)!!}{(2n+2)!!} \cdot \frac{\pi}{2}.$$