! For an efficient use of these tables, first read `HowTo.pdf`.

T2.67A. Integrands involving product of powers of logarithm functions and rational functions on the interval $(0,1)$.

1. $$\int_0^1 (\ln x)^2 \frac{dx}{1+2x\cos t+x^2} = \frac{t(\pi^2-t^2)}{6\sin t}, \quad 0<t<\pi.$$

2. $$\int_0^1 \frac{(\ln x)^2\,dx}{x^2-x+1} = \frac{1}{2}\int_0^\infty \frac{(\ln x)^2\,dx}{x^2-x+1} = \frac{10\pi^3}{81\sqrt{3}}.$$

3. $$\int_0^1 \frac{(\ln x)^2\,dx}{x^2+x+1} = \frac{1}{2}\int_0^\infty \frac{(\ln x)^2\,dx}{x^2+x+1} = \frac{8\pi^3}{81\sqrt{3}}.$$

4. $$\int_0^1 (\ln x)^2 \frac{dx}{1+x^2} = \frac{\pi^3}{16}.$$

5. $$\int_0^1 (\ln x)^2 \frac{1+x^2}{1+x^4}\,dx = \frac{1}{2}\int_0^\infty (\ln x)^2 \frac{1+x^2}{1+x^4}\,dx = \frac{3\sqrt{2}}{64}\pi^3.$$

6. $$\int_0^1 (\ln x)^2 \frac{1-x}{1-x^6}\,dx = \frac{1}{36}\left(\frac{4\sqrt{3}\pi^3}{27} - \psi''\left(\frac{1}{3}\right)\right).$$

7. $$\int_0^1 (\ln x)^2 \frac{dx}{\sqrt{1-x^2}} = \frac{\pi}{2}\left[(\ln 2)^2 + \frac{\pi^2}{12}\right].$$

8. $$\int_0^1 (\ln x)^2\, \frac{x^n\,dx}{1+x} = (-1)^n\left(\frac{3}{2}\zeta(3) + 2\sum_{k=1}^n \frac{(-1)^k}{k^3}\right), \quad n=0,1,\ldots.$$

9. $$\int_0^1 (\ln x)^2 \frac{x^n\,dx}{1-x} = 2\left(\zeta(3) - \sum_{k=1}^n \frac{1}{k^3}\right), \quad n=0,1,\ldots.$$

10. $$\int_0^1 (\ln\, x)^2 \frac{x^{2n}\, dx}{1-x^2} = \frac{7}{4}\zeta(3) - 2\sum_{k=1}^{n} \frac{1}{(2k-1)^3}, \quad n = 0,\, 1, \ldots .$$

11. $$\int_0^1 (\ln\, x)^2 \frac{x^{2n}\, dx}{\sqrt{1-x^2}} = \frac{(2n-1)!!}{2\,(2n)!!}\,\pi \left\{\frac{\pi^2}{12} + \sum_{k=1}^{2n} \frac{(-1)^k}{k^2} + \left[\sum_{k=1}^{2n} \frac{(-1)^k}{k} + \ln\, 2\right]^2\right\}.$$

12. $$\int_0^1 (\ln\, x)^2 \frac{x^{2n+1}\, dx}{\sqrt{1-x^2}} = \frac{(2n)!!}{(2n+1)!!} \left\{-\frac{\pi^2}{12} - \sum_{k=1}^{2n+1} \frac{(-1)^k}{k^2} + \left[\sum_{k=1}^{2n+1} \frac{(-1)^k}{k} + \ln\, 2\right]^2\right\}.$$

13. $$\int_0^1 (\ln\, x)^2 x^{\mu-1}(1-x)^{\nu-1}\, dx = \mathrm{B}(\mu,\nu) \left\{[\psi(\mu) - \psi(\nu+\mu)]^2 + \psi'(\mu) - \psi'(\mu+\nu)\right\},$$
$$\Re\{\mu\} > 0,\ \Re\{\nu\} > 0.$$

14. $$\int_0^1 (\ln\, x)^2 \frac{1-x^{n+1}}{(1-x)^2}\, dx = 2(n+1)\,\zeta(3) - 2\sum_{k=1}^{n} \frac{n-k+1}{k^3}.$$

15. $$\int_0^1 (\ln\, x)^2 \frac{1+(-1)^n x^{n+1}}{(1+x)^2}\, dx = \frac{3}{2}(n+1)\,\zeta(3) - 2\sum_{k=1}^{n} (-1)^{k-1} \frac{n-k+1}{k^3}.$$

16. $$\int_0^1 (\ln\, x)^2 \frac{1-x^{2n+2}}{(1-x^2)^2}\, dx = \frac{7}{4}(n+1)\,\zeta(3) - 2\sum_{k=1}^{n} \frac{n-k+1}{(2k-1)^3}.$$

17. $$\int_0^1 (\ln\, x)^2 x^{p-1}(1-x^r)^{q-1}\, dx = \frac{1}{r^3}\mathrm{B}\left(\frac{p}{r},\, q\right) \left\{\psi'\left(\frac{p}{r}\right) - \psi'\left(\frac{p}{r}+q\right) + \left[\psi\left(\frac{p}{r}\right) - \psi\left(\frac{p}{r}+q\right)\right]^2\right\},$$
$$p > 0,\ q > 0,\ r > 0.$$

18. $$\int_0^1 (\ln\, x)^3 \frac{dx}{1+x} = -\frac{7}{120}\pi^4.$$

19. $$\int_0^1 (\ln\, x)^3 \frac{dx}{1-x} = -\frac{\pi^4}{15}.$$

20. $$\int_0^\infty (\ln\, x)^3 \frac{dx}{(x+a)(x-1)} = \frac{\left[\pi^2 + (\ln\, a)^2\right]^2}{4(a+1)}, \quad a > 0.$$

21. $\displaystyle\int_0^1 (\ln\,x)^3 \frac{x^n\,dx}{1+x} = (-1)^{n+1}\left[\frac{7\pi^4}{120} - 6\sum_{k=0}^{n-1}\frac{(-1)^k}{(k+1)^4}\right], \quad n = 1,\, 2, \ldots.$

22. $\displaystyle\int_0^1 (\ln\,x)^3 \frac{x^n\,dx}{1-x} = -\frac{\pi^4}{15} + 6\sum_{k=0}^{n-1}\frac{1}{(k+1)^4}, \quad n = 1,\, 2, \ldots.$

23. $\displaystyle\int_0^1 (\ln\,x)^3 \frac{x^{2n}\,dx}{1-x^2} = -\frac{\pi^4}{16} + 6\sum_{k=0}^{n-1}\frac{1}{(2k+1)^4}, \quad n = 1,\, 2, \ldots.$

24. $\displaystyle\int_0^1 (\ln\,x)^3 \frac{1-x^{n+1}}{(1-x)^2}\,dx = -\frac{(n+1)\pi^4}{15} + 6\sum_{k=1}^{n}\frac{n-k+1}{k^4}.$

25. $\displaystyle\int_0^1 (\ln\,x)^3 \frac{1+(-1)^n x^{n+1}}{(1+x)^2}\,dx = -\frac{7(n+1)\pi^4}{120} + 6\sum_{k=1}^{n}(-1)^{k-1}\frac{n-k+1}{k^4}.$

26. $\displaystyle\int_0^1 (\ln\,x)^3 \frac{1-x^{2n+2}}{(1-x^2)^2}\,dx = -\frac{(n+1)\pi^4}{16} + 6\sum_{k=1}^{n}\frac{n-k+1}{(2k-1)^4}.$

27. $\displaystyle\int_0^1 (\ln\,x)^4 \frac{dx}{1+x^2} = \frac{5\pi^5}{64}.$

28. $\displaystyle\int_0^1 (\ln\,x)^4 \frac{dx}{1+2x\cos t+x^2} = \frac{t(\pi^2-t^2)(7\pi^2-3t^2)}{30\sin t}, \quad |t| < \pi.$

29. $\displaystyle\int_0^1 (\ln\,x)^5 \frac{dx}{1+x} = -\frac{31\pi^6}{252}.$

30. $\displaystyle\int_0^1 (\ln\,x)^5 \frac{dx}{1-x} = -\frac{8\pi^6}{63}.$

31. $\displaystyle\int_0^1 (\ln\,x)^6 \frac{dx}{1+x^2} = \frac{61\pi^7}{256}.$

32. $\displaystyle\int_0^1 (\ln\,x)^7 \frac{dx}{1+x} = -\frac{127\pi^8}{240}.$

33. $\displaystyle\int_0^1 (\ln\, x)^7 \frac{dx}{1-x} = -\frac{8\pi^8}{15}.$

34. $\displaystyle\int_0^1 \frac{1-x}{1+x}\frac{dx}{\ln\, x} = \ln\, \frac{2}{\pi}.$

35. $\displaystyle\int_0^1 \frac{(1-x)^2}{1+x^2}\frac{dx}{\ln\, x} = \ln\, \frac{\pi}{4}.$

36. $$\int_0^1 \frac{(1-x)^2}{1+2x\cos\frac{mx}{n}+x^2}\,\frac{dx}{\ln\, x}$$
$$= \begin{cases} \dfrac{1}{\sin\frac{m\pi}{n}}\displaystyle\sum_{k=1}^{n-1}(-1)^k \sin\frac{km\pi}{n}\,\ln\,\frac{\left(\Gamma\left(\frac{n+k+1}{2n}\right)\right)^2\Gamma\left(\frac{k+2}{2n}\right)\Gamma\left(\frac{k}{2n}\right)}{\left(\Gamma\left(\frac{k+1}{2n}\right)\right)^2\Gamma\left(\frac{n+k}{2n}\right)\Gamma\left(\frac{n+k+2}{2n}\right)}, & m+n \text{ is odd,} \\ \dfrac{1}{\sin\frac{m\pi}{n}}\displaystyle\sum_{k=1}^{\lfloor (n-1)/2\rfloor}(-1)^k \sin\frac{km\pi}{n}\,\ln\,\frac{\left(\Gamma\left(\frac{n-k+1}{n}\right)\right)^2\Gamma\left(\frac{k+2}{n}\right)\Gamma\left(\frac{k}{n}\right)}{\left\{\Gamma\left(\frac{k+1}{n}\right)\right\}^2\Gamma\left(\frac{n-k}{n}\right)\Gamma\left(\frac{n-k+2}{n}\right)}, & m+n \text{ is even.} \end{cases}$$

37. $\displaystyle\int_0^1 \frac{1-x}{1+x}\,\frac{1}{1+x^2}\,\frac{dx}{\ln\, x} = -\frac{\ln\, 2}{2}.$

38. $\displaystyle\int_0^1 \frac{1-x}{1+x}\,\frac{x^2}{1+x^2}\,\frac{dx}{\ln\, x} = \ln\, \frac{2\sqrt{2}}{\pi}.$

39. $\displaystyle\int_0^1 (4-x)^p \frac{dx}{\ln\, x} = \sum_{k=1}^{\infty}(-1)^k \genfrac{}{}{0pt}{}{p}{k} \ln\,(1+k), \quad p \geq 1.$

40. $\displaystyle\int_0^1 \left(\frac{1-x^p}{1-x} - p\right)\frac{dx}{\ln\, x} = \ln\, \Gamma(p+1).$

41. $\displaystyle\int_0^1 \frac{x^{p-1}-x^{q-1}}{\ln\, x}\,dx = \ln\, \frac{p}{q}, \quad p>0,\, q>0.$

42. $\displaystyle\int_0^1 \frac{x^{p-1}-x^{q-1}}{\ln\, x}\,\frac{dx}{1+x} = \ln\, \frac{\Gamma\left(\frac{q}{2}\right)\Gamma\left(\frac{p+1}{2}\right)}{\Gamma\left(\frac{p}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}, \quad p>0,\, q>0.$

43. $\displaystyle\int_0^1 \frac{x^{p-1}-x^{-p}}{(1+x)\ln\, x}\,dx = \frac{1}{2}\int_0^\infty \frac{x^{p-1}-x^{-p}}{(1+x)\ln\, x}\,dx = \ln\,\left(\tan\frac{p\pi}{2}\right), \quad 0<p<1.$

44. $\displaystyle\int_0^1 (x^p - x^q)x^{r-1}\frac{dx}{\ln x} = \ln\frac{p+r}{r+q}, \quad r>0,\ p>0,\ q>0.$

45. $\displaystyle\int_0^1 \frac{x^p - x^q}{(1-ax)^n}\frac{dx}{x\ln x} = \ln\frac{p}{q} + \sum_{k=1}^{\infty}\binom{n+k-1}{k}a^k \ln\frac{p+k}{q+k}, \quad p>0,\ q>0,\ a^2<1.$

46. $\displaystyle\int_0^1 (x^p-1)(x^q-1)\frac{dx}{\ln x} = \ln\frac{p+q+1}{(p+1)(q+1)}, \quad p>-1,\ q>-1,\ p+q>-1.$

47. $\displaystyle\int_0^1 \frac{x^p - x^q}{1+x}\,\frac{1+x^{2n+1}}{x\ln x}\,dx = \ln\frac{\Gamma\left(\frac{p}{2}+n+1\right)\Gamma\left(\frac{q+1}{2}+n\right)\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q}{2}\right)}{\Gamma\left(\frac{q}{2}+n+1\right)\Gamma\left(\frac{p+1}{2}+n\right)\Gamma\left(\frac{q+1}{2}\right)\Gamma\left(\frac{p}{2}\right)},$

$p>0,\ q>0.$

48. $\displaystyle\int_0^1 \frac{x^p - x^q}{1-x}\,\frac{1-x^r}{\ln x}\,dx = \ln\frac{\Gamma(q+1)\Gamma(p+r+1)}{\Gamma(p+1)\Gamma(q+r+1)}, \quad p>-1,\ q>-1,\ p+r>-1,\ q+r>-1.$

49. $\displaystyle\int_0^1 \frac{x^{p-1} - x^{q-1}}{(1+x^r)\ln x}\,dx = \ln\frac{\Gamma\left(\frac{p+r}{2r}\right)\Gamma\left(\frac{q}{2r}\right)}{\Gamma\left(\frac{q+r}{2r}\right)\Gamma\left(\frac{p}{2r}\right)}, \quad p>0,\ q>0,\ r>0.$

50. $\displaystyle\int_0^1 \frac{1-x^{2p-2q}}{1+x^{2p}}\frac{x^{q-1}\,dx}{\ln x} = \ln\tan\frac{q\pi}{4p}, \quad 0<q<p.$

51. $\displaystyle\int_0^1 \frac{x^{p-1} - x^{q-1}}{1-x^{2n}}\,\frac{1-x^2}{\ln x}\,dx = \ln\frac{\Gamma\left(\frac{p+2}{2n}\right)\Gamma\left(\frac{q}{2n}\right)}{\Gamma\left(\frac{q+2}{2n}\right)\Gamma\left(\frac{p}{2n}\right)}, \quad p>0,\ q>0.$

52. $\displaystyle\int_0^1 \frac{x^{p-1} - x^{q-1}}{1+x^{2(2n+1)}}\frac{1+x^2}{\ln x}\,dx = \ln\frac{\Gamma\left(\frac{p+4n+4}{4(2n+1)}\right)\Gamma\left(\frac{q+2}{4(2n+1)}\right)\Gamma\left(\frac{p+4n+2}{4(2n+1)}\right)\Gamma\left(\frac{q}{4(2n+1)}\right)}{\Gamma\left(\frac{q+4n+4}{4(2n+1)}\right)\Gamma\left(\frac{p+2}{4(2n+1)}\right)\Gamma\left(\frac{q+4n+2}{4(2n+1)}\right)\Gamma\left(\frac{p}{4(2n+1)}\right)},$

$p>0,\ q>0.$

53. $\displaystyle\int_0^1 (1-x^p)(1-x^q)\frac{x^{r-1}\,dx}{\ln x} = \ln\frac{(p+q+r)r}{(p+r)(q+r)}, \quad p>0,\ q>0,\ r>0.$

54. $\displaystyle\int_0^1 (1-x^p)(1-x^q)\frac{x^{r-1}\,dx}{(1-x)\ln x} = \ln\frac{\Gamma(p+r)\Gamma(q+r)}{\Gamma(p+q+r)\Gamma(r)},$

$r>0,\ r+p>0,\ r+q>0,\ r+p+q>0.$

55. $$\int_0^1 (1-x^p)(1-x^q)(1-x^r)\frac{dx}{\ln x} = \ln \frac{(p+q+1)(q+r+1)(r+p+1)}{(p+q+r+1)(p+1)(q+1)(r+1)},$$
$$p>-1,\ q>-1,\ r>-1,\ p+q>-1,\ p+r>-1,\ q+r>-1,\ p+q+r>-1.$$

56. $$\int_0^1 (1-x^p)(1-x^q)(1-x^r)\frac{dx}{(1-x)\ln x} = \ln \frac{\Gamma(p+1)\Gamma(q+1)\Gamma(r+1)\Gamma(p+q+r+1)}{\Gamma(p+q+1)\Gamma(p+r+1)\Gamma(q+r+1)},$$
$$p>-1,\ q>-1,\ r>-1,\ p+q>-1,\ p+r>-1,\ q+r>-1,\ p+q+r>-1.$$

57. $$\int_0^1 (1-x^p)(1-x^q)(1-x^r)\frac{x^{s-1}\,dx}{\ln x} = \ln \frac{(p+q+s)(p+r+s)(q+r+s)s}{(p+s)(q+s)(r+s)(p+q+r+s)},$$
$$p>0,\ q>0,\ r>0,\ s>0.$$

58. $$\int_0^1 (1-x^p)(1-x^q)\frac{x^{s-1}\,dx}{(1-x^r)\ln x} = \ln \frac{\Gamma\left(\frac{p+s}{r}\right)\Gamma\left(\frac{q+s}{r}\right)}{\Gamma\left(\frac{s}{r}\right)\Gamma\left(\frac{p+q+s}{r}\right)},$$
$$p>0,\ q>0,\ r>0,\ s>0.$$

59. $$\int_0^1 (1-x^p)(1-x^q)(1-x^r)\frac{x^{s-1}\,dx}{(1-x)\ln x} = \ln \frac{\Gamma(p+s)\Gamma(q+s)\Gamma(r+s)\Gamma(p+q+r+s)}{\Gamma(p+q+s)\Gamma(p+r+s)\Gamma(q+r+s)\Gamma(s)},$$
$$p>0,\ q>0,\ r>0,\ s>0.$$

60. $$\int_0^1 (1-x^p)(1-x^q)(1-x^r)\frac{x^{s-1}\,dx}{(1-x^t)\ln x} = \ln \frac{\Gamma\left(\frac{p+s}{t}\right)\Gamma\left(\frac{q+s}{t}\right)\Gamma\left(\frac{r+s}{t}\right)\Gamma\left(\frac{p+q+r+s}{t}\right)}{\Gamma\left(\frac{p+q+s}{t}\right)\Gamma\left(\frac{q+r+s}{t}\right)\Gamma\left(\frac{p+r+s}{t}\right)\Gamma\left(\frac{s}{t}\right)},$$
$$p>0,\ q>0,\ r>0,\ s>0,\ t>0.$$

61. $$\int_0^1 \left\{\frac{x^p-x^{p+q}}{1-x}-q\right\}\frac{dx}{\ln x} = \ln \frac{\Gamma(p+q+1)}{\Gamma(p+1)}, \quad p>-1,\ p+q>-1.$$

62. $$\int_0^1 \left\{\frac{x^\mu-x}{x-1}-x(\mu-1)\right\}\frac{dx}{x\ln x} = \ln\Gamma(\mu), \quad \Re\{\mu\}>0.$$

63. $$\int_0^1 \left\{1-x-\frac{(1-x^p)(1-x^q)}{1-x}\right\}\frac{dx}{x\ln x} = \ln\{\mathrm{B}(p,q)\}, \quad p>0,\ q>0.$$

64. $$\int_0^1 \left\{\frac{x^{p-1}}{1-x}-\frac{x^{pq-1}}{1-x^q}-\frac{1}{x(1-x)}+\frac{1}{x(1-x^q)}\right\}\frac{dx}{\ln x} = q\ln p, \quad p>0.$$

65. $$\int_0^1 \left\{\frac{x^{q-1}}{1-x}-\frac{x^{pq-1}}{1-x^p}-\frac{p-1}{1-x^p}x^{p-1}-\frac{p-1}{2}x^{p-1}\right\}\frac{dx}{\ln x} = \frac{1-p}{2}\ln(2\pi)+\left(pq-\frac{1}{2}\right)\ln p,$$
$$p>0,\ q>0.$$

66. $$\int_0^1 \frac{(1-x^p)(1-x^q)-(1-x)^2}{x(1-x)\ln x}\,dx = \ln \mathrm{B}\,(p,\,q), \quad p>0,\, q>0.$$

67. $$\int_0^1 (x^p-1)^n \frac{dx}{\ln x} = \sum_{k=0}^{n} \genfrac{}{}{0pt}{}{n}{n-k} (-1)^{n-k} \ln(pk+1), \quad n>0,\, pn>-1.$$

68. $$\int_0^1 \frac{(1-x^p)^n}{1-x}\frac{dx}{\ln x} = \sum_{k=0}^{n} (-1)^{k-1} \ln \Gamma[(n-k)p+1], \quad n>1,\, pn>-1.$$

69. $$\int_0^1 (x^p-1)^n x^{q-1} \frac{dx}{\ln x} = \sum_{k=0}^{n} (-1)^k \genfrac{}{}{0pt}{}{n}{k} \ln[q+(n-k)p], \quad n>0,\, q>0,\, pn>-q.$$

70. $$\int_0^1 (1-x^p)^n x^{q-1} \frac{dx}{(1-x)\ln x} = \sum_{k=0}^{n} (-1)^{k-1} \ln \Gamma[(n-k)p+q], \quad n>1,\, q>0,\, pn>-q.$$

71. $$\int_0^1 (x^p-1)^n (x^q-1)^m \frac{x^{r-1}\,dx}{\ln x} = \sum_{j=0}^{n} (-1)^j \genfrac{}{}{0pt}{}{n}{j} \sum_{k=0}^{m} (-1)^k \genfrac{}{}{0pt}{}{m}{k} [r+(m-k)q+(n-j)p],$$
$$n \ge 0,\, m \ge 0,\, n+m>0,\, r>0,\, pn+qm+r>0.$$

72. $$\int_0^1 \frac{(x^p-x^q)(1-x^r)}{(\ln x)^2}\,dx = (p+1)\ln(p+1)-(q+1)\ln(q+1)$$
$$-(p+r+1)\ln(p+r+1)+(q+r+1)\ln(q+r+1),$$
$$p>-1,\, q>-1,\, p+r>-1,\, q+r>-1.$$

73. $$\int_0^1 (x^p-x^q)^2 \frac{dx}{(\ln x)^2} = (2p+1)\ln(2p+1)+(2q+1)\ln(2q+1)-2(p+q+1)\ln(p+q+1),$$
$$p>-\tfrac{1}{2},\, q>-\tfrac{1}{2}.$$

74. $$\int_0^1 (1-x^p)(1-x^q)(1-x^r)\frac{dx}{(\ln x)^2}$$
$$=(p+q+1)\ln(p+q+1)+(q+r+1)\ln(q+r+1)+(p+r+1)\ln(p+r+1)$$
$$-(p+1)\ln(p+1)-(q+1)\ln(q+1)-(r+1)\ln(r+1)-(p+q+r)\ln(p+q+r),$$
$$p>-1,\, q>-1,\, r>-1,\, p+q>-1,\, p+r>-1,\, q+r>-1,\, p+q+r>0.$$

75. $$\int_0^1 (1-x^p)^n x^{q-1} \frac{dx}{(\ln x)^2} = \frac{1}{2}\sum_{k=0}^{n} (-1)^k \binom{n}{k} (pk+q)^2 \ln(pk+q), \quad q>0,\, p>-\frac{q}{n}.$$

76. $$\int_0^1 (1-x^p)^n (1-x^q)^m x^{r-1} \frac{dx}{(\ln\, x)^2} = \sum_{j=0}^{n} (-1)^j \binom{n}{j} \sum_{k=0}^{m} (-1)^k \binom{m}{k}$$
$$\times [(m-k)q + (n-j)p + r] \ln\,[(m-k)q + (n-j)p + r],$$
$$r > 0,\ mq + r > 0,\ np + r > 0,\ mq + np + r > 0.$$

77. $$\int_0^1 [(q-r)x^{p-1} + (r-p)x^{q-1} + (p-q)x^{r-1}] \frac{dx}{(\ln\, x)^2} = (q-r)p \ln\, p + (r-p)q \ln\, q + (p-q)r \ln\, r,$$
$$p > 0,\ q > 0,\ r > 0.$$

78. $$\int_0^1 \left[\frac{x^{p-1}}{(p-q)(p-r)(p-s)} + \frac{x^{q-1}}{(q-p)(q-r)(q-s)} + \frac{x^{r-1}}{(r-p)(r-q)(r-s)} \right.$$
$$\left. + \frac{x^{s-1}}{(s-p)(s-q)(s-r)} \right] \frac{dx}{(\ln\, x)^2} = \frac{1}{2} \left[\frac{p^2 \ln\, p}{(p-q)(p-r)(p-s)} + \frac{q^2 \ln\, q}{(q-p)(q-r)(q-s)} \right.$$
$$\left. + \frac{r^2 \ln\, r}{(r-p)(r-q)(r-s)} + \frac{s^2\, \ln\, s}{(s-p)(s-q)(s-r)} \right], \quad p > 0,\ q > 0,\ r > 0,\ s > 0.$$

79. $$\int_0^1 \sqrt{\ln\, \frac{1}{x}}\, \frac{dx}{1+x^2} = \frac{\sqrt{\pi}}{2} \sum_{k=0}^{\infty} \frac{(-1)^k}{\sqrt{(2k+1)^3}}.$$

80. $$\int_0^1 \frac{dx}{\sqrt{\ln\, \frac{1}{x}}\, (1+x)^2} = \sqrt{\pi} \sum_{k=0}^{\infty} \frac{(-1)^k}{\sqrt{2k+1}}.$$

81. $$\int_0^1 \sqrt{\ln\, \frac{1}{x}}\, x^{p-1}\, dx = \frac{1}{2} \sqrt{\frac{\pi}{p^3}}, \quad p > 0.$$

82. $$\int_0^1 \frac{x^{p-1}}{\sqrt{\ln\, \frac{1}{x}}}\, dx = \sqrt{\frac{\pi}{p}}, \quad p > 0.$$

83. $$\int_0^1 \frac{\sin t - x^n \sin[(n+1)t] + x^{n+1} \sin nt}{1 - 2x \cos t + x^2} \frac{dx}{\sqrt{\ln\, \frac{1}{x}}} = \sqrt{\pi} \sum_{k=1}^{n} \frac{\sin kt}{\sqrt{k}}, \quad |t| < \pi.$$

84. $$\int_0^1 \frac{\cos t - x - x^{n-1} \cos nt + x^n\, \cos[(n-1)t]}{1 - 2x \cos t + x^2} \frac{dx}{\sqrt{\ln\, \frac{1}{x}}} = \sqrt{\pi} \sum_{k=1}^{n-1} \frac{\cos kt}{\sqrt{k}}, \quad |t| < \pi.$$

85. $\displaystyle\int_0^1 (\ln\, x)^{2n} \frac{dx}{1+x} = \frac{2^{2n}-1}{2^{2n}}\,(2n)!\,\zeta(2n+1).$

86. $\displaystyle\int_0^1 (\ln\, x)^{2n-1} \frac{dx}{1+x} = \frac{1-2^{2n-1}}{2n}\,\pi^{2n}|B_{2n}|, \quad n=1,2,\ldots.$

87. $\displaystyle\int_0^1 (\ln\, x)^{2n-1} \frac{dx}{1-x} = -\frac{1}{n}\,2^{2n-2}\pi^{2n}|B_{2n}|, \quad n=1,2,\ldots.$

88. $\displaystyle\int_0^1 (\ln\, x)^{p-1} \frac{dx}{1-x} = e^{i(p-1)\pi}\Gamma(p)\,\zeta(p), \quad p>1.$

89. $\displaystyle\int_0^1 (\ln\, x)^{n} \frac{dx}{1+x^2} = (-1)^n n! \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^{n+1}}.$

90. $\displaystyle\int_0^1 (\ln\, x)^{2n} \frac{dx}{1+x^2} = \frac{1}{2}\int_0^{\infty} (\ln\, x)^{2n} \frac{dx}{1+x^2} = \frac{\pi^{2n+1}}{2^{2n+2}}\,|E_{2n}|.$

91. $\displaystyle\int_0^1 (\ln\, x)^{2n} \frac{dx}{1-x^2} = \frac{2^{2n+1}-1}{2^{2n+1}}\,(2n)!\,\zeta(2n+1), \quad n=1,2,\ldots.$

92. $\displaystyle\int_0^1 (\ln\, x)^{2n-1} \frac{dx}{1-x^2} = \frac{1}{2}\int_0^{\infty} (\ln\, x)^{2n-1} \frac{dx}{1-x^2} = \frac{1-2^{2n}}{4n}\,\pi^{2n}\,|B_{2n}|, \quad n=1,2,\ldots.$

93. $\displaystyle\int_0^1 (\ln\, x)^{2n-1} \frac{x\,dx}{1-x^2} = -\frac{1}{4n}\,\pi^{2n}\,|B_{2n}|, \quad n=1,2,\ldots.$

94. $\displaystyle\int_0^1 (\ln\, x)^{2n} \frac{1+x^2}{(1-x^2)^2}\,dx = \frac{2^{2n}-1}{2}\,\pi^{2n}\,|B_{2n}|, \quad n=1,2,\ldots.$

95. $\displaystyle\int_0^1 (\ln\, x)^{2n+1} \frac{(\cos 2a\pi - x)\,dx}{1-2x\cos 2a\pi + x^2} = -(2n+1)! \sum_{k=1}^{\infty} \frac{\cos 2ak\pi}{k^{2n+2}}, \quad a \text{ not an integer}.$

96. $\displaystyle\int_0^1 (\ln\, x)^{n} \frac{x^{p-1}}{1-x^q}\,dx = -\frac{1}{q^{n+1}}\,\psi^{(n)}\left(\frac{p}{q}\right), \quad p>0,\ q>0.$

97. $\displaystyle\int_0^1 (\ln\, x)^{n} \frac{x^{p-1}}{1+x^q}\,dx = \frac{1}{q^{n+1}}\,\beta^{(n)}\left(\frac{p}{q}\right), \quad p>0,\ q>0.$

98. $$\int_0^1 \frac{[\ln\,(1/x)]^{q-1}\ dx}{1+2x\cos t+x^2} = \csc t\,\Gamma(q)\sum_{k=1}^{\infty}(-1)^{k-1}\,\frac{\sin kt}{k^q}, \quad |t|<\pi,\ q<1.$$

99. $$\int_0^1 \left(\ln\frac{1}{x}\right)^{q-1}\ \frac{(1+x)\,dx}{1+2x\cos t+x^2} = \sec\frac{t}{2}\,\Gamma(q)\sum_{k=1}^{\infty}(-1)^{k-1}\,\frac{\cos\left[\left(k-\frac{1}{2}\right)t\right]}{k^q}, \quad |t|<\pi,\ q<\tfrac{1}{2}.$$

100. $$\int_0^1 \left[\ln\left(\frac{1}{x}\right)\right]^{\mu}\ \frac{x^{\nu-1}\,dx}{1-2ax\cos t+x^2a^2} = \frac{\Gamma(\mu+1)}{a\sin t}\sum_{k=1}^{\infty}\frac{a^k\sin kt}{(\nu+k-1)^{\mu+1}},$$
$$a>0,\ \Re\{\mu\}>0,\ \Re\{\nu\}>0,\ -\pi<t<\pi.$$

101. $$\int_0^1 \left(\ln\frac{1}{x}\right)^{r-1}\ \frac{\cos\lambda-px}{1+p^2x^2-2px\cos\lambda}\,x^{q-1}\,dx = \Gamma(r)\sum_{k=1}^{\infty}\frac{p^{k-1}\cos k\lambda}{(q+k-1)^r}, \quad r>0,\ q>0.$$

102. $$\int_0^1 \left(\ln\frac{1}{x}\right)^{\mu-1}\ x^{\nu-1}\,dx = \frac{1}{\nu^{\mu}}\,\Gamma(\mu), \quad \Re\{\mu\}>0,\ \Re\{\nu\}>0.$$

103. $$\int_0^1 \left(\ln\frac{1}{x}\right)^{n-1/2} x^{\nu-1}\,dx = \frac{(2n-1)!!}{(2\nu)^n}\sqrt{\frac{\pi}{\nu}}, \quad \Re\{\nu\}>0.$$

104. $$\int_0^1 \left(\ln\frac{1}{x}\right)^{n-1}\ \frac{x^{\nu-1}}{1+x}\,dx = (n-1)!\sum_{k=0}^{\infty}\frac{(-1)^k}{(\nu+k)^n}, \quad \Re\{\nu\}>0.$$

105. $$\int_0^1 \left(\ln\frac{1}{x}\right)^{n-1}\ \frac{x^{\nu-1}}{1-x}\,dx = (n-1)!\,\zeta(n,\,\nu), \quad \Re\{\nu\}>0.$$

106. $$\int_0^1 \left(\ln\frac{1}{x}\right)^{\mu-1}\ (x-1)^n\left(a+\frac{nx}{x-1}\right)x^{a-1}\,dx = \Gamma(\mu)\sum_{k=0}^{n}\frac{(-1)^k n(n-1)\ldots(n-k+1)}{(a+n-k)^{\mu-1}k!},$$
$$\Re\{\mu\}>0.$$

107. $$\int_0^1 \left(\ln\frac{1}{x}\right)^{n-1}\ \frac{1-x^m}{1-x}\,dx = (n-1)!\sum_{k=1}^{m}\frac{1}{k^n}.$$

108. $$\int_0^1 \left(\ln\frac{1}{x}\right)^{\mu-1}\ \frac{x^{\nu-1}\,dx}{1-x^2} = \Gamma(\mu)\sum_{k=0}^{\infty}\frac{1}{(\nu+2k)^{\mu}} = \frac{1}{2^{\mu}}\Gamma(\mu)\,\zeta\left(\mu,\,\frac{\nu}{2}\right),$$
$$\Re\{\mu\}>0,\ \Re\{\nu\}>0.$$

109. $$\int_0^1 \frac{x^q - x^{-q}}{1-x^2}\left(\ln\frac{1}{x}\right)^p dx = \Gamma(p+1)\sum_{k=1}^{\infty}\left\{\frac{1}{(2k+q-1)^{p+1}} - \frac{1}{(2k-q-1)^{p+1}}\right\},$$
$$p > -1,\ q^2 < 1.$$

110. $$\int_0^1 \left(\ln\frac{1}{x}\right)^{r-1} \frac{x^{p-1}\,dx}{(1+x^q)^s} = \Gamma(r)\sum_{k=0}^{\infty}\binom{-s}{k}\frac{1}{(p+kq)^r}, \quad p>0,\ q>0,\ r>0,\ 0<s<r+2.$$

111. $$\int_0^1 \left(\ln\frac{1}{x}\right)^{n} (1+x^q)^m x^{p-1}\,dx = n!\sum_{k=0}^{m}\binom{m}{k}\frac{1}{(p+kq)^{n+1}}, \quad p>0,\ q>0.$$

112. $$\int_0^1 \left(\ln\frac{1}{x}\right)^{n} (1-x^q)^m x^{p-1}\,dx = n!\sum_{k=0}^{m}\binom{m}{k}\frac{(-1)^k}{(p+kq)^{n+1}}, \quad p>0,\ q>0.$$

113. $$\int_0^1 \left(\ln\frac{1}{x}\right)^{p-1} \frac{x^{q-1}\,dx}{1-ax^q} = \frac{1}{aq^p}\Gamma(p)\sum_{k=1}^{\infty}\frac{a^k}{k^p}, \quad p>0,\ q>0,\ a<1.$$

114. $$\int_0^1 \left(\ln\frac{1}{x}\right)^{2-1/n} (x^{p-1}-x^{q-1})\,dx = \frac{n}{n-1}\Gamma\left(\frac{1}{n}\right)\left(q^{1-1/n}-p^{1-1/n}\right), \quad q>p>0.$$

115. $$\int_0^1 \left(\ln\frac{1}{x}\right)^{2n-1} \frac{x^p - x^{-p}}{1-x^q}x^{q-1}\,dx = \frac{1}{p^{2n}}\sum_{k=n}^{\infty}\left(\frac{2p\pi}{q}\right)^k \frac{|B_{2k}|}{2k\,(2k-2n)!}, \quad p<\frac{q}{2}.$$

116. $$\int_0^1 \left[\left(\ln\frac{1}{x}\right)^{q-1} - x^{p-1}(1-x)^{q-1}\right] dx = \frac{\Gamma(q)}{\Gamma(p+q)}[\Gamma(p+q)-\Gamma(p)], \quad p>0,\ q>0.$$

117. $$\int_0^1 \left[x - \left(\frac{1}{1-\ln x}\right)^q\right]\frac{dx}{x\ln x} = -\psi(q), \quad q>0.$$