! For an efficient use of these tables, first read `HowTo.pdf`.

T2.27D. Integrands of the form $\sqrt{\dfrac{x^2 \pm b^2}{(a^2 \pm x^2)^n}}$ for $n = 1, 3$ on the interval $(0, y)$.

Notation used: $\alpha = \arctan \dfrac{y}{b}$, $\gamma = \arcsin \dfrac{y}{b}\sqrt{\dfrac{a^2+b^2}{a^2+y^2}}$, $\eta = \arcsin \dfrac{y}{b}$,

$$q = \frac{\sqrt{a^2-b^2}}{a}, \quad r = \frac{b}{\sqrt{a^2+b^2}}, \quad t = \frac{b}{a}.$$

1. $$\int_0^y \sqrt{\frac{x^2+b^2}{(x^2+a^2)^3}}\,dx = \frac{1}{a}E(\alpha,\, q) - \frac{a^2-b^2}{a^2}\frac{y}{\sqrt{(a^2+y^2)(b^2+y^2)}}, \quad a > b,\, y > 0.$$

2. $$\int_0^y \sqrt{\frac{x^2+a^2}{(x^2+b^2)^3}}\,dx = \frac{a}{b^2}E(\alpha,\, q), \quad a > b,\, y > 0.$$

3. $$\int_0^y \sqrt{\frac{b^2-x^2}{(a^2+x^2)^3}}\,dx = \frac{\sqrt{a^2+b^2}}{a^2}E(\gamma,\, r) - \frac{1}{\sqrt{a^2+b^2}}F(\gamma,\, r), \quad b \geq y > 0.$$

4. $$\int_0^y \sqrt{\frac{x^2+a^2}{(b^2-x^2)^3}}\,dx = \frac{a^2}{b^2\sqrt{a^2+b^2}}F(\gamma,\, r) - \frac{\sqrt{a^2+b^2}}{b^2}E(\gamma,\, r) + \frac{(a^2+b^2)y}{b^2\sqrt{(a^2+y^2)(b^2-y^2)}},$$
$$b > y > 0.$$

5. $$\int_0^y \sqrt{\frac{b^2-x^2}{(a^2-x^2)^3}}\,dx = \frac{1}{a}\left\{F(\eta,\, t) - E(\eta,\, t) + \frac{y}{a}\sqrt{\frac{b^2-y^2}{a^2-y^2}}\right\}, \quad a > b \geq y > 0.$$

6. $$\int_0^y \sqrt{\frac{a^2-x^2}{(b^2-x^2)^3}}\,dx = \frac{a}{b^2}[F(\eta,\, t) - E(\eta,\, t)] + \frac{y}{b^2}\sqrt{\frac{a^2-y^2}{b^2-y^2}}, \quad a > b > y > 0.$$