

! For an efficient use of these tables, first read [HowTo.pdf](#).

**T3.08A.** Integrands of the form  $\frac{1}{\sqrt{x^n(x-1)^n(x^2-x+1)^m}}$ ,  $n = 1, 3, 5$ ;  $m = 1, 3$ , on the intervals  $(y, \infty)$ .

Notation used:  $\alpha = \arcsin \frac{1}{\sqrt{y^2 - y + 1}}$ .

$$1. \int_y^\infty \frac{dx}{\sqrt{x(x-1)(x^2-x+1)}} = F\left(\alpha, \frac{\sqrt{3}}{2}\right), \quad y \geq 1.$$

$$2. \int_y^\infty \frac{dx}{\sqrt{x^3(x-1)^3(x^2-x+1)}} = \frac{2(2y-1)}{\sqrt{y(y-1)(y^2-y+1)}} - 4E\left(\alpha, \frac{\sqrt{3}}{2}\right), \quad y > 1.$$

$$3. \int_y^\infty \frac{(2x-1)^2 dx}{\sqrt{x^3(x-1)^3(x^2-x+1)}} = 4\left[F\left(\alpha, \frac{\sqrt{3}}{2}\right) - E\left(\alpha, \frac{\sqrt{3}}{2}\right) + \frac{2y-1}{2\sqrt{y(y-1)(y^2-y+1)}}\right],$$

$y > 1.$

$$4. \int_y^\infty \frac{dx}{\sqrt{x(x-1)(x^2-x+1)^3}} = \frac{4}{3}\left[F\left(\alpha, \frac{\sqrt{3}}{2}\right) - E\left(\alpha, \frac{\sqrt{3}}{2}\right)\right], \quad y \geq 1.$$

$$5. \int_y^\infty \frac{(2x-1)^2 dx}{\sqrt{x(x-1)(x^2-x+1)^3}} = 4E\left(\alpha, \frac{\sqrt{3}}{2}\right), \quad y > 1.$$

$$6. \int_y^\infty \sqrt{\frac{x(x-1)}{(x^2-x+1)^3}} dx = \frac{4}{3}E\left(\alpha, \frac{\sqrt{3}}{2}\right) - \frac{1}{3}F\left(\alpha, \frac{\sqrt{3}}{2}\right), \quad y > 1.$$

$$7. \int_y^\infty \frac{dx}{(2x-1)^2} \sqrt{\frac{x(x-1)}{x^2-x+1}} = \frac{1}{3}\left[F\left(\alpha, \frac{\sqrt{3}}{2}\right) - E\left(\alpha, \frac{\sqrt{3}}{2}\right)\right] + \frac{1}{2(2y-1)}\sqrt{\frac{y(y-1)}{y^2-y+1}},$$

$y > 1.$

$$8. \int_y^\infty \frac{dx}{(2x-1)^2} \sqrt{\frac{x^2-x+1}{x(x-1)}} = E\left(\alpha, \frac{\sqrt{3}}{2}\right) - \frac{3}{2(2y-1)} \sqrt{\frac{y(y-1)}{y^2-y+1}}, \quad y > 1.$$

$$9. \int_y^\infty \frac{dx}{(2x-1)^2 \sqrt{x(x-1)(x^2-x+1)}} = \frac{4}{3} E\left(\alpha, \frac{\sqrt{3}}{2}\right) - \frac{1}{3} F\left(\alpha, \frac{\sqrt{3}}{2}\right) - \frac{2}{2y-1} \sqrt{\frac{y(y-1)}{y^2-y+1}}, \quad y > 1.$$

$$10. \int_y^\infty \frac{dx}{\sqrt{x^5(x-1)^5(x^2-x+1)}} = \frac{40}{3} E\left(\alpha, \frac{\sqrt{3}}{2}\right) - \frac{4}{3} F\left(\alpha, \frac{\sqrt{3}}{2}\right) - \frac{2(2y-1)(9y^2-9y-1)}{3\sqrt{y^3(y-1)^3(y^2-y+1)}}, \quad y > 1.$$

$$11. \int_y^\infty \frac{dx}{\sqrt{x(x-1)(x^2-x+1)^5}} = \frac{44}{27} F\left(\alpha, \frac{\sqrt{3}}{2}\right) - \frac{56}{27} E\left(\alpha, \frac{\sqrt{3}}{2}\right) + \frac{2(2y-1)\sqrt{y(y-1)}}{9\sqrt{(y^2-y+1)^3}}, \quad y > 1.$$

$$12. \int_y^\infty \frac{dx}{(2x-1)^4 \sqrt{x(x-1)(x^2-x+1)}} = \frac{16}{27} E\left(\alpha, \frac{\sqrt{3}}{2}\right) - \frac{1}{27} F\left(\alpha, \frac{\sqrt{3}}{2}\right) - \frac{8(5y^2-5y+2)}{9(2y-1)^3} \sqrt{\frac{y(y-1)}{y^2-y+1}}, \quad y > 1.$$


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