! For an efficient use of these tables, first read `HowTo.pdf`.

T3.43A. Integrands involving product of trigonometric functions, exponentials and powers of x and $1/x$ on the interval $(0,\infty)$.

1. $\displaystyle\int_0^\infty e^{-px}\,\sin qx\frac{dx}{x}=\arctan\frac{q}{p},\quad p>0.$

2. $\displaystyle\int_0^\infty e^{-px}\,\cos qx\frac{dx}{x}=\infty.$

3. $\displaystyle\int_0^\infty e^{-px}\,\cos px\frac{x\,dx}{b^4+x^4}=\frac{\pi}{4b^2}\,\exp(-bp\sqrt{2}),\quad p>0,b>0.$

4. $\displaystyle\int_0^\infty e^{-px}\,\cos px\frac{x\,dx}{b^4-x^4}=\frac{\pi}{4b^2}e^{-bp}\,\sin bp,\quad p>0,b>0.$

5. $\displaystyle\int_0^\infty e^{-\beta x}(1-\cos ax)\frac{dx}{x}=\frac{1}{2}\,\ln\,\frac{a^2+\beta^2}{\beta^2},\quad \Re\{\beta\}>0.$

6. $\displaystyle\int_0^\infty x^{\mu-1}e^{-\beta x}\,\sin\delta x\,dx=\frac{\Gamma(\mu)}{(\beta^2+\delta^2)^{\mu/2}}\,\sin\left(\mu\,\arctan\frac{\delta}{\beta}\right),\quad \Re\{\mu\}>-1,\Re\{\beta\}>|\Im\{\delta\}|.$

7. $\displaystyle\int_0^\infty x^{\mu-1}e^{-\beta x}\,\cos\delta x\,dx=\frac{\Gamma(\mu)}{(\delta^2+\beta^2)\mu/2}\,\cos\left(\mu\,\arctan\frac{\delta}{\beta}\right),\quad \Re\{\mu\}>0,\Re\{\beta\}>|\Im\{\delta\}|.$

8. $\displaystyle\int_0^\infty x^{\mu-1}\,\exp(-ax\,\cos t)\,\sin(ax\,\sin t)\,dx=\Gamma(\mu)a^{-\mu}\,\sin(\mu t),\quad \Re\{\mu\}>-1,\ a>0,|t|<\frac{\pi}{2}.$

9. $\displaystyle\int_0^\infty x^{\mu-1}\,\exp(-ax\,\cos t)\,\cos(ax\,\sin t)\,dx=\Gamma(\mu)a^{-\mu}\,\cos(\mu t),\quad \Re\{\mu\}>-1;a>0,|t|<\frac{\pi}{2}.$

10. $\displaystyle\int_0^\infty x^{p-1}e^{-qx}\,\sin(qx\,\tan t)\,dx=\frac{1}{q^p}\Gamma(p)\,\cos^p t\,\sin pt,\quad |t|<\frac{\pi}{2},q>0.$

11. $$\int_0^\infty x^{p-1}e^{-qx}\,\cos(qx\,\tan t)\,dx = \frac{1}{q^p}\Gamma(p)\,\cos^p(t)\,\cos pt, \quad |t|<\frac{\pi}{2}, q>0.$$

12. $$\int_0^\infty x^n e^{-\beta x}\,\sin bx\,dx = n!\left(\frac{\beta}{\beta^2+b^2}\right)^{n+1}\sum_{0\le 2k\le n}(-1)^k\binom{n+1}{2k+1}\left(\frac{b}{\beta}\right)^{2k+1} = (-1)^n\frac{\partial^n}{\partial\beta^n}\left(\frac{b}{b^2+\beta^2}\right),$$
$$\Re\{\beta\}>0, b>0.$$

13. $$\int_0^\infty x^n e^{-\beta x}\,\cos bx\,dx = n!\left(\frac{\beta}{\beta^2+b^2}\right)^{n+1}\sum_{0\le 2k\le n+1}(-1)^k\binom{n+1}{2k}\left(\frac{b}{\beta}\right)^{2k} = (-1)^n\frac{\partial^n}{\partial\beta^n}\left(\frac{\beta}{b^2+\beta^2}\right),$$
$$\Re\{\beta\}>0, b>0.$$

14. $$\int_0^\infty x^{n-1/2}e^{-\beta x}\,\sin bx\,dx = (-1)^n\sqrt{\frac{\pi}{2}}\frac{d^n}{d\beta^n}\left(\frac{\sqrt{\sqrt{\beta^2+b^2}-\beta}}{\sqrt{\beta^2+b^2}}\right), \quad \Re\{\beta\}>0, b>0.$$

15. $$\int_0^\infty x^{n-1/2}e^{-\beta x}\,\cos bx\,dx = (-1)^n\sqrt{\frac{\pi}{2}}\frac{d^n}{d\beta^n}\left(\frac{\sqrt{\sqrt{\beta^2+b^2}+\beta}}{\sqrt{\beta^2+b^2}}\right), \quad \Re\{\beta\}>0, b>0.$$

16. $$\int_0^\infty (e^{-\beta x}\,\sin ax - e^{-\gamma x}\,\sin bx)\frac{dx}{x^r}$$
$$= \Gamma(1-r)\left\{(b^2+\gamma^2)\frac{r-1}{2}\,\sin\left[(r-1)\,\arctan\frac{b}{\gamma}\right] - \left(a^2+\beta^2\right)\frac{r-1}{2}\,\sin\left[(r-1)\,\arctan\frac{a}{\beta}\right]\right\},$$
$$\Re\{\beta\}>0, \Re\{\gamma\}>0, r<2, r\neq 1.$$

17. $$\int_0^\infty (e^{-\beta x}\,\cos ax - e^{-\gamma x}\,\cos bx)\frac{dx}{x^r}$$
$$= \Gamma(1-r)\left\{(a^2+\beta^2)\frac{r-1}{2}\,\cos\left[(r-1)\,\arctan\frac{a}{\beta}\right] - \left(b^2+\gamma^2\right)\frac{r-1}{2}\,\cos\left[(r-1)\,\arctan\frac{b}{\gamma}\right]\right\},$$
$$\Re\{\beta\}>0, \Re\{\gamma\}>0, r<2, r\neq 1.$$

18. $$\int_0^\infty (ae^{-\beta x}\,\sin bx - be^{-\gamma x}\,\sin ax)\frac{dx}{x^2} = ab\left[\frac{1}{2}\,\ln\,\frac{a^2+\gamma^2}{b^2+\beta^2} + \frac{\gamma}{a}\,\operatorname{arccot}\frac{\gamma}{a} - \frac{\beta}{b}\,\operatorname{arccot}\frac{\beta}{b}\right],$$
$$\Re\{\beta\}>0, \Re\{\gamma\}>0.$$

19. $$\int_0^\infty e^{-px}\ \sin^{2m+1}\ ax\frac{dx}{x} = \frac{(-1)^m}{2^{2m}}\sum_{k=0}^{m}(-1)^k\binom{2m+1}{k}\ \arctan\frac{(2m-2k+1)a}{p},$$
$$m = 0, 1, \ldots;\ p > 0.$$

20. $$\int_0^\infty e^{-px}\ \sin^{2m}\ ax\frac{dx}{x} = \frac{(-1)^{m+1}}{2^{2m}}\sum_{k=0}^{m-1}(-1)^k\binom{2m}{k}\ \ln\ \left[p^2+(2m-2k)^2a^2\right]-\frac{1}{2^{2m}}\binom{2m}{m}\ \ln\ p,$$
$$m = 1, 2, \ldots;\ p > 0.$$

21. $$\int_0^\infty e^{-\beta x}\ \sin\gamma x\ \sin ax\frac{dx}{x} = \frac{1}{4}\ \ln\ \frac{\beta^2+(a+\gamma)^2}{\beta^2+(a-\gamma)^2},\quad \Re\{\beta\} > |\Im\{\gamma\}|,\ a > 0.$$

22. $$\int_0^\infty e^{-px}\ \sin ax\ \sin bx\frac{dx}{x^2} = \frac{a}{2}\ \arctan\frac{2pb}{p^2+a^2-b^2}+\frac{b}{2}\ \arctan\frac{2pa}{p^2+b^2-a^2}+\frac{p}{4}\ \ln\ \frac{p^2+(a-b)^2}{p^2+(a+b)^2},$$
$$p > 0.$$

23. $$\int_0^\infty e^{-px}\ \sin ax\ \cos bx\frac{dx}{x} = \frac{1}{2}\ \arctan\frac{2pa}{p^2-a^2+b^2}+s\,\frac{\pi}{2},$$
$$a \geq 0, p > 0,\ s = \begin{cases} 0 & \text{for } p^2-a^2+b^2 \geq 0, \\ s = 1 & \text{for } p^2-a^2+b^2 < 0. \end{cases}$$

24. $$\int_0^\infty e^{-\beta x}\,(\sin ax - \sin bx)\frac{dx}{x} = \arctan\frac{(a-b)\beta}{ab+\beta^2},\quad \Re\{\beta\} > 0.$$

25. $$\int_0^\infty e^{-\beta x}(\cos ax - \cos bx)\frac{dx}{x} = \frac{1}{2}\ \ln\ \frac{b^2+\beta^2}{a^2+\beta^2},\quad \Re\{\beta\} > 0.$$

26. $$\int_0^\infty e^{-\beta x}(\cos ax - \cos bx)\frac{dx}{x^2} = \frac{\beta}{2}\ \ln\ \frac{a^2+\beta^2}{b^2+\beta^2}+b\ \arctan\frac{b}{\beta}-a\ \arctan\frac{a}{\beta},\quad \Re\{p\} > 0.$$

27. $$\int_0^\infty e^{-\beta x}(\sin^2\ ax - \sin^2 bx)\frac{dx}{x^2} = a\ \arctan\frac{2a}{p}-b\ \arctan\frac{2b}{p}-\frac{p}{4}\ \ln\ \frac{p^2+4a^2}{p^2+4b^2},\quad p > 0.$$

28. $$\int_0^\infty e^{-\beta x}(\cos^2\ ax - \cos^2 bx)\frac{dx}{x^2} = -a\ \arctan\frac{2a}{p}+b\ \arctan\frac{2b}{p}+\frac{p}{4}\ \ln\ \frac{p^2+4a^2}{p^2+4b^2},\quad p > 0.$$

29. $$\int_0^\infty e^{-px}\sin ax\sin bx\sin cx\frac{dx}{x} = -\frac{1}{4}\ \arctan\frac{a+b+c}{p}+\frac{1}{4}\ \arctan\frac{a+b-c}{p}+\frac{1}{4}\ \arctan\frac{a-b+c}{p}$$
$$+\frac{1}{4}\ \arctan\frac{-a+b+c}{p},\quad p > 0.$$

30. $$\int_0^\infty e^{-\beta x}\,\sin^2\,ax\,\sin bx\frac{dx}{x} = \frac{1}{2}\,\arctan\frac{b}{p} - \frac{1}{2}\left[\frac{1}{2}\arctan\frac{2pb}{p^2+4a^2-b^2} + s\frac{\pi}{2}\right],$$
$$s = \begin{cases} 1 & \text{for } p^2+4a^2-b^2<0, \\ 0 & \text{for } p^2+4a^2-b^2\geq 0. \end{cases}$$

31. $$\int_0^\infty e^{-\beta x}\,\sin^2\,ax\,\cos bx\frac{dx}{x} = \frac{1}{8}\,\ln\,\frac{[p^2+(2a+b)^2][p^2+(2a-b)^2]}{(p^2+b^2)^2}, \quad p>0.$$

32. $$\int_0^\infty e^{-\beta x}\,\sin ax\,\cos^2 bx\frac{dx}{x} = \frac{1}{2}\,\arctan\frac{a}{p}\frac{1}{2}\left[\arctan\frac{2pb}{p^2+4a^2-b^2} + s\frac{\pi}{2}\right],$$
$$s = \begin{cases} 1 & \text{for } p^2+4b^2-a^2<0, \\ 0 & \text{for } p^2+4b^2-a^2\geq 0. \end{cases}$$

33. $$\int_0^\infty e^{-px}\,\sin^2\,ax\,\sin bx\,\sin cx\frac{dx}{x} = \frac{1}{8}\ln\,\frac{p^2+(b+c)^2}{p^2+(b-c)^2}$$
$$+\frac{1}{16}\,\ln\,\frac{[p^2+(2a-b+c)^2][p^2+(2a+b-c)^2]}{[p^2+(2a+b+c)^2][p^2+(2a-b-c)^2]}, \quad p>0.$$

34. $$\int_0^\infty (1-e^{-x})\,\cos x\frac{dx}{x} = \ln\sqrt{2}.$$

35. $$\int_0^\infty \frac{e^{-\gamma x}-e^{-\beta x}}{x}\,\sin bx\,dx = \arctan\frac{b\,(\beta-\gamma)}{b^2+\beta\gamma}, \quad \Re\{\beta\}>0,\ \Re\{\gamma\}\geq 0.$$

36. $$\int_0^\infty \frac{e^{-\gamma x}-e^{-\beta x}}{x}\,\cos bx\,dx = \frac{1}{2}\,\ln\,\frac{b^2+\beta^2}{b^2+\gamma^2}, \quad \Re\{\beta\}>0,\ \Re\{\gamma\}\geq 0.$$

37. $$\int_0^\infty \frac{e^{-\gamma x}-e^{-\beta x}}{x}\,\sin bx\,dx = \frac{b}{2}\,\ln\,\frac{b^2+\beta^2}{b^2+\gamma^2} + \beta\,\arctan\frac{b}{\beta} - \gamma\,\arctan\frac{b}{\gamma}, \quad \Re\{\beta\}>0,\ \Re\{\gamma\}>0.$$

38. $$\int_0^\infty \frac{x}{e^{\beta x}-1}\,\cos bx\,dx = \frac{1}{2b^2} - \frac{\pi^2}{2\beta^2}\,\operatorname{csch}^2\frac{b\pi}{\beta}, \quad \Re\{\beta\}>0.$$

39. $$\int_0^\infty \left(\frac{1}{e^x-1} - \frac{1}{x}\right)\cos bx\,dx = \ln b - \frac{1}{2}[\psi(i\,b)+\psi(-i\,b)], \quad b>0.$$

40. $$\int_0^\infty \frac{1-\cos ax}{e^{2\pi x}-1}\cdot\frac{dx}{x} = \frac{a}{4} + \frac{1}{2}\,\ln\,\frac{1-e^{-a}}{a}, \quad a>0.$$

41. $$\int_0^\infty (e^{-\beta x} - e^{-\gamma x}\,\cos ax)\frac{dx}{x} = \frac{1}{2}\,\ln\,\frac{a^2+\gamma^2}{\beta^2}, \quad \Re\{\beta\}>0, \Re\{\gamma\}>0.$$

42. $$\int_0^\infty \frac{\cos px - e^{-px}}{b^4+x^4}\frac{dx}{x} = \frac{\pi}{2b^4}\,e^{-bp/\sqrt{2}}\sin\left(\frac{bp}{\sqrt{2}}\right), \quad p>0.$$

43. $$\int_0^\infty \left(\frac{1}{e^x-1} - \frac{\cos x}{x}\right)dx = \gamma_e.$$

44. $$\int_0^\infty \left(ae^{-px} - \frac{e^{-qx}}{x}\,\sin ax\right)\frac{dx}{x} = \frac{a}{2}\,\ln\,\frac{a^2+q^2}{p^2} + q\,\arctan\frac{a}{q} - a, \quad p>0, q>0.$$

45. $$\int_0^\infty \frac{x^{2m}\,\sin bx}{e^x-1}\,dx = (-1)^m\frac{\partial^{2m}}{\partial b^{2m}}\left[\frac{\pi}{2}\,\coth\,b\pi - \frac{1}{2b}\right], \quad b>0.$$

46. $$\int_0^\infty \frac{x^{2m+1}\,\cos bx}{e^x-1}\,dx = (-1)^m\frac{\partial^{2m+1}}{\partial b^{2m+1}}\left[\frac{\pi}{2}\,\coth\,b\pi - \frac{1}{2b}\right], \quad b>0.$$

47. $$\int_0^\infty \frac{x^{2m}\,\sin bx\,dx}{e^{(2n+1)cx}-e^{(2n-1)cx}} = (-1)^m\frac{\partial^{2m}}{\partial b^{2m}}\left[\frac{\pi}{4c}\,\tanh\,\frac{b\pi}{2c} - \sum_{k=1}^{n}\frac{b}{b^2+(2k-1)^2c^2}\right], \quad b>0.$$

48. $$\int_0^\infty \frac{x^{2m+1}\,\cos bx\,dx}{e^{(2n+1)cx}-e^{(2n-1)cx}} = (-1)^m\frac{\partial^{2m+1}}{\partial b^{2m+1}}\left[\frac{\pi}{4c}\,\tanh\,\frac{b\pi}{2c} - \sum_{k=1}^{n}\frac{b}{b^2+(2k-1)^2c^2}\right], \quad b>0.$$

49. $$\int_0^\infty \frac{x^{2m}\,\sin bx\,dx}{e^{(2n-2)cx}} = (-1)^m\frac{\partial^{2m}}{\partial b^{2m}}\left[\frac{\pi}{4c}\,\coth\,\frac{b\pi}{2c} - \frac{1}{2b} - \sum_{k=1}^{n-1}\frac{b}{b^2+(2k)^2c^2}\right], \quad b>0, c>0.$$

50. $$\int_0^\infty \frac{x^{2m+1}\,\cos bx\,dx}{e^{2ncx}-e^{(2n-2)cx}} = (-1)^m\frac{\partial^{2m+1}}{\partial b^{2m+1}}\left[\frac{\pi}{4c}\,\coth\,\frac{b\pi}{2c} - \frac{1}{2b} - \sum_{k=1}^{n-1}\frac{b}{b^2+(2k)^2c^2}\right], \quad b>0, c>0.$$

51. $$\int_0^\infty \frac{\cos ax - \cos bx}{e^{(2m+1)px}-e^{(2m-1)px}}\frac{dx}{x} = \frac{1}{2}\,\ln\,\frac{\cosh\frac{b\pi}{2p}}{\cosh\frac{a\pi}{2p}} - \frac{1}{2}\sum_{k=1}^{m}\,\ln\,\frac{b^2+(2k-1)^2p^2}{a^2+(2k-1)^2p^2}, \quad p>0.$$

52. $$\int_0^\infty \frac{\cos ax - \cos bx}{e^{2mpx}-e^{(2m-2)px}}\frac{dx}{x} = \frac{1}{2}\,\ln\,\frac{a\,\sinh\frac{b\pi}{2p}}{b\,\sinh\frac{a\pi}{2p}} - \frac{1}{2}\sum_{k=1}^{m-1}\,\ln\,\frac{b^2+4k^2p^2}{a^2+4k^2p^2}, \quad p>0.$$

53. $$\int_0^\infty \frac{\sin x\,\sin bx}{1-e^x}\cdot\frac{dx}{x} = \frac{1}{4}\,\ln\,\frac{(b+1)\,\sinh[(b-1)\pi]}{(b-1)\,\sinh[(b+1)\pi]}, \quad b^2\neq 1.$$

54. $\displaystyle\int_0^\infty \frac{\sin^2 ax}{1-e^x}\cdot\frac{dx}{x} = \frac{1}{4}\ln\frac{2a\pi}{\sinh 2a\pi}.$

55. $\displaystyle\int_0^\infty xe^{-p^2x^2}\sin ax\,dx = \frac{a\sqrt{\pi}}{4p^3}\exp\left(-\frac{a^2}{4p^2}\right).$

56. $\displaystyle\int_0^\infty xe^{-p^2x^2}\cos ax\,dx = \frac{1}{2p^2} - \frac{a}{4p^3}\sum_{k=0}^\infty \frac{(-1)^k k!}{(2k+1)!}\left(\frac{a}{p}\right)^{2k+1}, \quad a>0.$

57. $\displaystyle\int_0^\infty x^2e^{-p^2x^2}\sin ax\,dx = \frac{a}{4p^4} + \frac{2p^2-a^2}{8p^5}\sum_{k=0}^\infty \frac{(-1)^k k!}{(2k+1)!}\left(\frac{a}{p}\right)^{2k+1}, \quad a>0.$

58. $\displaystyle\int_0^\infty x^2e^{-p^2x^2}\cos ax\,dx = \sqrt{\pi}\frac{2p^2-a^2}{8p^5}\exp\left(-\frac{a^2}{4p^2}\right).$

59. $\displaystyle\int_0^\infty x^3e^{-p^2x^2}\sin ax\,dx = \sqrt{\pi}\frac{6ap^2-a^3}{16p^7}\exp\left(-\frac{a^2}{4p^2}\right).$

60. $\displaystyle\int_0^\infty e^{-p^2x^2}\sin ax\frac{dx}{x} = \frac{a\sqrt{\pi}}{2p}\sum_{k=0}^\infty \frac{(-1)^k}{k!(2k+1)}\left(\frac{a}{2p}\right)^{2k} = \frac{\pi}{2}\operatorname{erf}\left(\frac{a}{2p}\right).$

61. $\displaystyle\int_0^\infty x^{\mu-1}e^{-\beta x^2}\sin\gamma x\,dx = \frac{\gamma\,e^{-\gamma^2/(4\beta)}}{2\beta^{(\mu+1)/2}}\Gamma\left(\frac{1+\mu}{2}\right)\,{}_1F_1\left(1-\frac{\mu}{2};\,\frac{3}{2};\,\frac{\gamma^2}{4\beta}\right),$
$\Re\{\beta\}>0, \Re\{\mu\}>-1.$

62. $\displaystyle\int_0^\infty x^{\mu-1}e^{-\beta x^2}\cos ax\,dx = \frac{1}{2}\beta^{-\mu/2}\Gamma(\mu/2)e^{-a^2/4\beta}\,{}_1F_1\left(-\mu/2+1/2;\,1/2;\,a^2/4\beta\right),$
$\Re\{\beta\}>0,\ \Re\{\mu\}>0,\ a>0.$

63. $$\int_0^\infty x^{2n}e^{-\beta^2x^2}\cos ax\,dx = (-1)^n\frac{\sqrt{\pi}}{2^{n+1}\beta^{2n+1}}\exp\left(-\frac{a^2}{8\beta^2}\right)D_{2n}\left(\frac{a}{\beta\sqrt{2}}\right)$$
$$= (-1)^n\frac{\sqrt{\pi}}{(2\beta)^{2n+1}}\exp\left(-\frac{a^2}{4\beta^2}\right)H_{2n}\left(\frac{a}{2\beta}\right), \quad |\arg\beta|<\frac{\pi}{4},\ a>0.$$

64. $$\int_0^\infty x^{2n+1}e^{-\beta^2x^2}\sin ax\,dx = (-1)^n\frac{\sqrt{\pi}}{2^{n+3/2}\beta^{2n+2}}\exp\left(-\frac{a^2}{8\beta^2}\right)D_{2n+1}\left(\frac{a}{\beta\sqrt{2}}\right)$$
$$= (-1)^n\frac{\sqrt{\pi}}{(2\beta)^{2n+2}}\exp\left(-\frac{a^2}{4\beta^2}\right)H_{2n+1}\left(\frac{a}{2\beta}\right), \quad |\arg\beta|<\frac{\pi}{4},\ a>0.$$

65. $$\int_0^\infty x^{\mu-1} e^{-\gamma x-\beta x^2} \sin ax\, dx$$
$$= -\frac{i}{2(2\beta)^{\mu/2}} \exp\left(\frac{\gamma^2-a^2}{8\beta}\right)\Gamma(\mu)\left\{\exp\left(-\frac{ia\gamma}{4\beta}\right)D_{-\mu}\left(\frac{\gamma-ia}{\sqrt{2\beta}}\right)-\exp\left(\frac{ia\gamma}{4\beta}\right)D_{-\mu}\left(\frac{\gamma+ia}{\sqrt{2\beta}}\right)\right\},$$
$$\Re\{\mu\}>-1,\ \Re\{\beta\}>0,\ a>0.$$

66. $$\int_0^\infty x^{\mu-1} e^{-\gamma x-\beta x^2} \cos ax\, dx$$
$$= \frac{1}{2(2\beta)^{\frac{\mu}{2}}} \exp\left(\frac{\gamma^2-a^2}{8\beta}\right)\Gamma(\mu)\left\{\exp\left(-\frac{ia\gamma}{4\beta}\right)D_{-\mu}\left(\frac{\gamma-ia}{\sqrt{2\beta}}\right)+\exp\left(\frac{ia\gamma}{4\beta}\right)D_{-\mu}\left(\frac{\gamma+ia}{\sqrt{2\beta}}\right)\right\},$$
$$\Re\{\mu\}>0,\ \Re\{\beta\}>0,\ a>0.$$

67. $$\int_0^\infty x e^{-\gamma x-\beta x^2} \sin ax\, dx = \frac{i\sqrt{\pi}}{8\sqrt{\beta^3}}\left\{(\gamma-ia)\exp\left[-\frac{(\gamma-ia)^2}{4\beta}\right]\left[1-\operatorname{erf}\left(\frac{\gamma-ia}{2\sqrt{\beta}}\right)\right]\right.$$
$$\left.-(\gamma+ia)\exp\left[-\frac{(\gamma+ia)^2}{4\beta}\right]\left[1-\operatorname{erf}\left(\frac{\gamma+ia}{2\sqrt{\beta}}\right)\right]\right\},\quad \Re\{\beta\}>0,\ a>0.$$

68. $$\int_0^\infty x e^{-\gamma x-\beta x^2} \cos ax\, dx = -\frac{\sqrt{\pi}}{8\sqrt{\beta^3}}\left\{(\gamma-ia)\ \exp\left[\frac{(\gamma-ia)^2}{4\beta}\right]\left[1-\operatorname{erf}\left(\frac{\gamma-ia}{2\sqrt{\beta}}\right)\right]\right.$$
$$\left.+(\gamma+ia)\ \exp\left[\frac{(\gamma+ia)^2}{4\beta}\right]\left[1-\operatorname{erf}\left(\frac{\gamma+ia}{2\sqrt{\beta}}\right)\right]\right\}+\frac{1}{2\beta},\quad \Re\{\beta\}>0,\ a>0.$$

69. $$\int_0^\infty e^{-\beta x^2} \sin ax\, \frac{x\,dx}{\gamma^2+x^2} = -\frac{\pi}{4}e^{\beta\gamma^2}\left[2\sinh a\gamma+e^{-\gamma a}\operatorname{erf}\left(\gamma\sqrt{\beta}-\frac{a}{2\sqrt{\beta}}\right)-e^{\gamma a}\operatorname{erf}\left(\gamma\sqrt{\beta}+\frac{a}{2\sqrt{\beta}}\right)\right],$$
$$\Re\{\beta\}>0,\ \Re\{\gamma\}>0,\ a>0.$$

70. $$\int_0^\infty e^{-\beta x^2} \cos ax\, \frac{dx}{\gamma^2+x^2} = \frac{\pi}{4\gamma}e^{\beta\gamma^2}\left[2\cosh a\gamma-e^{-\gamma a}\operatorname{erf}\left(\gamma\sqrt{\beta}-\frac{a}{2\sqrt{\beta}}\right)-e^{\gamma a}\operatorname{erf}\left(\gamma\sqrt{\beta}+\frac{a}{2\sqrt{\beta}}\right)\right],$$
$$\Re\{\beta\}>0,\ \Re\{\gamma\}>0,\ a>0.$$

71. $$\int_0^\infty x^\nu e^{-x^2/2}\cos\left(\beta x-\nu\frac{\pi}{2}\right)\,dx = \sqrt{\frac{\pi}{2}}e^{-\beta^2/4}D_\nu(\beta),\quad \Re\{\nu\}>-1$$

72. $$\int_0^\infty x^{\mu-1}\exp\left(\frac{-\beta^2}{4x}\right)\sin ax\, dx$$
$$= \frac{i}{2^\mu}\beta^\mu a^{-\mu/2}\left[e^{-i\mu\pi/4}K_\mu\left(\beta e^{i\pi/4}\sqrt{a}\right)-e^{i\mu\pi/4}K_\mu\left(\beta e^{-i\pi/4}\sqrt{a}\right)\right],$$
$$\Re\{\beta\}>0,\ \Re\{\mu\}<1,\ a>0.$$

73. $$\int_0^\infty x^{\mu-1}\,\exp\left(\frac{-\beta^2}{4x}\right)\cos ax\,dx$$
$$= \frac{1}{2^\mu}\beta^\mu a^{-\mu/2}\left[e^{-i\,\mu\pi/4}\,K_\mu\left(\beta e^{i\,\pi/4}\sqrt{a}\right) + e^{i\,\mu\pi/4}\,K_\mu\left(\beta e^{-i\,\pi/4}\sqrt{a}\right)\right],$$
$$\Re\{\beta\} > 0,\ \Re\{\mu\} < 1,\ a > 0.$$

74. $$\int_0^\infty x e^{-p^2x^2}\,\tan ax\,dx = \frac{a\sqrt{\pi}}{p^3}\sum_{k=1}^{\infty}(-1)^k k\,\exp\left(-\frac{a^2k^2}{p^2}\right),\quad p > 0.$$

75. $$\int_0^\infty \exp(-\beta\sqrt{\gamma^2+x^2})\,\sin ax\frac{x\,dx}{\sqrt{\gamma^2+x^2}} = \frac{a\gamma}{\sqrt{a^2+\beta^2}}\,K_1(\gamma\sqrt{a^2+\beta^2}),$$
$$\Re\{\beta\} > 0,\ \Re\{\gamma\} > 0,\ a > 0.$$

76. $$\int_0^\infty \exp[-\beta\sqrt{\gamma^2+x^2}]\,\cos ax\frac{dx}{\sqrt{\gamma^2+x^2}} = K_0(\gamma\sqrt{a^2+\beta^2}),\quad \Re\{\beta\} > 0,\ \Re\{\gamma\} > 0,\ a > 0.$$

77. $$\int_0^\infty \frac{\sqrt{\sqrt{\gamma^2+x^2}-\gamma}\,\exp(-\beta\sqrt{\gamma^2+x^2})}{\sqrt{\gamma^2+x^2}}\,\sin ax\,dx = \sqrt{\frac{\pi}{2}}\frac{a\,\exp(-\gamma\sqrt{a^2+\beta^2})}{\sqrt{\beta^2+a^2}\sqrt{\beta+\sqrt{a^2+\beta^2}}},$$
$$\Re\{\beta\} > 0,\ \Re\{\gamma\} > 0,\ a > 0.$$

78. $$\int_0^\infty \frac{x\,\exp(-\beta\sqrt{\gamma^2+x^2})}{\sqrt{\gamma^2+x^2}\sqrt{\sqrt{\gamma^2+x^2}-\gamma}}\,\cos ax\,dx = \sqrt{\frac{\pi}{2}}\frac{\sqrt{\beta+\sqrt{a^2+\beta^2}}}{\sqrt{a^2+\beta^2}}\,\exp\left[-\gamma\sqrt{a^2+\beta^2}\right],$$
$$\Re\{\beta\} > 0,\ \Re\{\gamma\} > 0,\ a > 0.$$

79. $$\int_0^\infty e^{-\tan^2 x}\frac{\sin x}{\cos^2 x}\frac{dx}{x} = \frac{\sqrt{\pi}}{2}.$$

80. $$\int_0^\infty x e^{-\beta x}\,\sin ax^2\,\sin\beta x\,dx = \frac{\beta}{4}\sqrt{\frac{\pi}{2a^3}}\,e^{-\beta^2/(2a)},\quad |\arg\beta| < \frac{\pi}{4},\ a > 0.$$

81. $$\int_0^\infty x e^{-\beta x}\,\cos ax^2\,\cos\beta x\,dx = \frac{\beta}{4}\sqrt{\frac{\pi}{2a^3}}\,e^{-\beta^2/(2a)},\quad a > 0,\ \Re\{\beta\} > |\Im\{\beta|.$$

82. $$\int_0^\infty x e^{-px}\,\cos(2x^2+px)\,dx = 0,\quad p > 0.$$

83. $\int_0^\infty xe^{-px} \cos(2x^2 - px)\, dx = \frac{p\sqrt{\pi}}{8} \exp\left(-\frac{1}{4}p^2\right), \quad p > 0.$

84. $\int_0^\infty x^2 e^{-px} \left[\sin(2x^2 + px) + \cos(2x^2 + px)\right]\, dx = 0, \quad p > 0.$

85. $\int_0^\infty x^2 e^{-px} \left[\sin(2x^2 - px) - \cos(2x^2 - px)\right]\, dx = \frac{\sqrt{\pi}}{16}(2 - p^2) \exp\left(-\frac{1}{4}p^2\right).$

86. $\int_0^\infty x^{\mu-1} e^{-x} \cos(x + ax^2)\, dx = \frac{e^{1/(4a)}\Gamma(\mu)}{(2a)^{\mu/2}} \cos\frac{\mu\pi}{4} D_{-\mu}\left(\frac{1}{\sqrt{a}}\right), \quad \Re\{\mu\} > 0,\ a > 0.$

87. $\int_0^\infty x^{\mu-1} e^{-x} \sin(x + ax^2)\, dx = \frac{e^{1/(4a)}\Gamma(\mu)}{(2a)^{\mu/2}} \sin\frac{\mu\pi}{4} D_{-\mu}\left(\frac{1}{\sqrt{a}}\right), \quad \Re\{\mu\} > -1, a > 0.$

88. $$\int_0^\infty e^{-\beta x^2} \sin ax^4\, dx = -\frac{\pi}{8}\sqrt{\frac{\beta}{a}}\left[J_{1/4}\left(\frac{\beta^2}{8a}\right)\cos\left(\frac{\beta^2}{8a} + \frac{\pi}{8}\right) + Y_{1/4}\left(\frac{\beta^2}{8a}\right)\sin\left(\frac{\beta^2}{8a} + \frac{\pi}{8}\right)\right],$$
$$\Re\{\beta\} > 0,\ a > 0.$$

89. $$\int_0^\infty e^{-\beta x^2} \cos ax^4\, dx = \frac{\pi}{8}\sqrt{\frac{\beta}{a}}\left[J_{1/4}\left(\frac{\beta^2}{8a}\right)\sin\left(\frac{\beta^2}{8a} + \frac{\pi}{8}\right) - Y_{1/4}\left(\frac{\beta^2}{8a}\right)\cos\left(\frac{\beta^2}{8a} + \frac{\pi}{8}\right)\right],$$
$$\Re\{\beta\} > 0,\ a > 0.$$

90. $\int_0^\infty e^{-p^2x^4 + q^2x^2} \left[2px \cos(2pqx^3) + q \sin(2pqx^3)\right]\, dx = \frac{\sqrt{\pi}}{2}.$

91. $\int_0^\infty e^{-p^2x^4 + q^2x^2} \left[2px \sin(2pqx^3) - q \cos(2pqx^3)\right]\, dx = 0.$

92. $$\int_0^\infty \exp\left(-px^2 - \frac{q}{x^2}\right) \sin\left(ax^2 + \frac{b}{x^2}\right)\frac{dx}{x^2} = \frac{1}{2}\int_{-\infty}^\infty \exp\left(-px^2 - \frac{q}{x^2}\right) \sin\left(ax^2 + \frac{b}{x^2}\right)\frac{dx}{x^2}$$
$$= \frac{\sqrt{\pi}}{2s} \exp[-2rs \cos(A + B)] \sin[A + 2rs \sin(A + B)], \quad p \geq 0,\ q \geq 0,$$

where $r = \left(a^2 + p^2\right)^{1/4}, \quad s = \left(b^2 + q^2\right)^{1/4}, \quad A = \arctan\frac{a}{p}, \quad B = \arctan\frac{b}{q}.$

93. $$\int_0^\infty \exp\left(-px^2-\frac{q}{x^2}\right)\,\cos\left(ax^2+\frac{b}{x^2}\right)\frac{dx}{x^2}=\frac{1}{2}\int_{-\infty}^\infty \exp\left(-px^2-\frac{q}{x^2}\right)\,\cos\left(ax^2+\frac{b}{x^2}\right)\frac{dx}{x^2}$$
$$=\frac{\sqrt{\pi}}{2s}\,\exp[-2rs\,\cos(A+B)]\,\cos[A+2rs\,\sin(A+B)],\quad p\ge 0,\ q\ge 0,$$
where $r=\left(a^2+p^2\right)^{1/4},\quad s=\left(b^2+q^2\right)^{1/4},\quad A=\arctan\dfrac{a}{p},\quad B=\arctan\dfrac{b}{q}.$

94. $$\int_0^\infty e^{-\beta^2/x^2}\,\sin a^2x^2\frac{dx}{x^2}=\frac{\sqrt{\pi}}{2\beta}e^{-\sqrt{2}a\beta}\,\sin(\sqrt{2}a\beta),\quad \Re\{\beta\}>0,\ a>0.$$

95. $$\int_0^\infty e^{-\beta^2/x^2}\,\cos a^2x^2\frac{dx}{x^2}=\frac{\sqrt{\pi}}{2\beta}e^{-\sqrt{2}a\beta}\,\cos(\sqrt{2}a\beta),\quad \Re\{\beta\}>0,\ a>0.$$

96. $$\int_0^\infty x^2e^{-\beta x^2}\,\cos ax^2\,dx=\frac{\sqrt{\pi}}{4\sqrt[4]{(a^2+\beta^2)^3}}\,\cos\left(\frac{3}{2}\,\arctan\frac{a}{\beta}\right),\quad \Re\{\beta\}>0.$$

97. $$\int_0^\infty \exp\left[-\beta\sqrt{\gamma^4+x^4}\right]\,\sin ax^2\frac{dx}{\sqrt{\gamma^4+x^4}}$$
$$=\sqrt{\frac{a\pi}{8}}\,I_{1/4}\left[\frac{\gamma^2}{2}(\sqrt{\beta^2+a^2}-\beta)\right]K_{1/4}\left[\frac{\gamma^2}{4}(\sqrt{\beta^2+a^2}+\beta)\right],$$
$$\Re\{\beta\}>0,\ |\arg\gamma|<\frac{\pi}{4},\ a>0.$$

98. $$\int_0^\infty \exp[-\beta\sqrt{\gamma^4+x^4}]\,\cos ax^2\frac{dx}{\sqrt{\gamma^4+x^4}}$$
$$=\sqrt{\frac{a\pi}{8}}\,I_{-1/4}\left[\frac{\gamma^2}{2}(\sqrt{\beta^2+a^2}-\beta)\right]K_{1/4}\left[\frac{\gamma^2}{4}(\sqrt{\beta^2+a^2}+\beta)\right],$$
$$\Re\{\beta\}>0,\ |\arg\gamma|<\frac{\pi}{4},\ a>0.$$

99. $$\int_0^\infty \exp(p\,\cos ax)\,\sin(p\,\sin ax)\frac{dx}{x}=\frac{\pi}{2}(e^p-1),\quad p>0,a>0.$$

100. $$\int_0^\infty \exp(p\,\cos ax)\,\sin(p\,\sin ax+bx)\frac{x\,dx}{c^2+x^2}=\frac{\pi}{2}\,\exp(-cb+pe^{-ac}),$$
$$a>0,b>0,c>0,p>0.$$

101. $$\int_0^\infty \exp(p\,\cos ax)\,\cos(p\,\sin ax+bx)\frac{dx}{c^2+x^2}=\frac{\pi}{2c}\,\exp(-cb+pe^{-ac}),$$
$$a>0, b>0, c>0, p>0.$$

102. $$\int_0^\infty \exp(p\,\cos x)\,\sin(p\,\sin x+nx)\frac{dx}{x}=\frac{\pi}{2}e^p,\quad p>0.$$

103. $$\int_0^\infty \exp(p\,\cos x)\,\sin(p\,\sin x)\,\cos nx\frac{dx}{x}=\frac{p^n}{n!}\cdot\frac{\pi}{4}+\frac{\pi}{2}\sum_{k=n+1}^{\infty}\frac{p^k}{k!},\quad p>0.$$

104. $$\int_0^\infty \exp(p\,\cos x)\,\cos(p\,\sin x)\,\sin nx\frac{dx}{x}=\frac{\pi}{2}\sum_{k=0}^{n-1}\frac{p^k}{k!}+\frac{p^n}{n!}\frac{\pi}{4},\quad p>0.$$

105. $$\int_0^\infty \exp(p\,\cos ax)\,\sin(p\,\sin ax)\,\csc ax\frac{dx}{b^2+x^2}=\frac{\pi[e^p-\exp(pe^{-ab})]}{2b\,\sinh ab},\quad a>0, b>0, p>0.$$

106. $$\int_0^\infty [1-\exp(p\,\cos ax)\,\cos(p\,\sin ax)]\,\csc ax\frac{x\,dx}{b^2+x^2}=\frac{\pi[e^p-\exp(pe^{-ab})]}{2\,\sinh ab},$$
$$a>0, b>0, p>0.$$

107. $$\int_0^\infty \exp(p\,\cos ax)\,\sin(p\,\sin ax+ax)\,\csc ax\frac{dx}{b^2+x^2}=\frac{\pi[e^p-\exp(pe^{-ab}-ab)]}{2b\,\sinh ab},$$
$$a>0, b>0, p>0.$$

108. $$\int_0^\infty \exp(p\,\cos ax)\,\cos(p\,\sin ax+ax)\,\csc ax\frac{x\,dx}{b^2+x^2}=\frac{\pi[e^p-\exp(pe^{-ab}-ab)]}{2\,\sinh ab},$$
$$a>0, b>0, p>0.$$

109. $$\int_0^\infty \exp(p\,\cos ax)\,\sin(p\,\sin ax)\frac{x\,dx}{b^2-x^2}=\frac{\pi}{2}[1-\exp(p\,\cos ab)\,\cos(p\,\sin ab)],\quad p>0, a>0.$$

110. $$\int_0^\infty \exp(p\,\cos ax)\,\cos(p\,\sin ax)\frac{dx}{b^2-x^2}=\frac{\pi}{2b}\,\exp(p\,\cos ab)\,\sin(p\,\sin ab),$$
$$a>0, b>0, p>0.$$

111. $$\int_0^\infty \exp(p\,\cos ax)\,\sin(p\,\sin ax)\,\tan ax\frac{dx}{b^2+x^2} = \frac{\pi}{2b}\,\tanh ab\,\left[\exp\left(pe^{-ab}\right)-e^p\right],$$
$$a>0, b>0, p>0.$$

112. $$\int_0^\infty \exp(p\,\cos ax)\,\sin(p\,\sin ax)\,\cot ax\frac{dx}{b^2+x^2} = \frac{\pi}{2b}\,\coth\,ab\,[e^p-\exp(pe^{-ab})],$$
$$a>0, b>0, p>0.$$

113. $$\int_0^\infty \exp(p\cos ax)\sin(p\sin ax)\csc ax\frac{dx}{b^2-x^2} = \frac{\pi}{2b}\csc ab\,[e^p-\exp(p\cos ab)\cos(p\sin ab)],$$
$$a>0, b>0, p>0.$$

114. $$\int_0^\infty [1-\exp(p\,\cos ax)\,\cos(p\,\sin ax)]\,\csc ax\frac{x\,dx}{b^2-x^2} = -\frac{\pi}{2}\,\exp(p\,\cos ab)\,\sin(p\,\sin ab)\,\csc ab,$$
$$a>0, b>0, p>0.$$

115. $$\int_0^\infty \frac{\sin\left(\beta\,\arctan\frac{x}{\gamma}\right)}{(\gamma^2+x^2)^{\beta/2}}\cdot\frac{dx}{e^{2\pi x}-1} = \frac{1}{2}\zeta(\beta,\,\gamma)-\frac{1}{4\gamma^\beta}-\frac{\gamma^{1-\beta}}{2(\beta-1)},\quad \Re\{\beta\}>1, \Re\{\gamma\}>0.$$

116. $$\int_0^\infty \frac{\sin(\beta\,\arctan x)}{(1+x^2)^{\beta/2}}\cdot\frac{dx}{e^{2\pi x}+1} = \frac{1}{2(\beta-1)}-\frac{\zeta(\beta)}{2^\beta},\quad \Re\{\beta\}>1.$$

117. $$\int_0^\infty (1+x^2)^{\beta-1/2}e^{-px^2}\,\cos[2px+(2\beta-1)\,\arctan x]\,dx = \frac{e^{-p}}{2p^\beta}\,\sin\pi\beta\,\Gamma(\beta),\quad \Re\{\beta\}>0, p>0.$$

118. $$\int_0^\infty e^{-x^2}(2x\,\cos x-\sin x)\,\sin x\frac{dx}{x^2} = \sqrt{\pi}\,\frac{e-1}{2e}.$$