

! For an efficient use of these tables, first read [HowTo.pdf](#).

**T1.06.** Integrand involving  $a + bx^3 \equiv X_3$ , and  $a^3 \pm x^3$ .

Notation used:  $A = \left(\frac{a}{b}\right)^{1/3}$ .

1.  $\int \frac{dx}{X_3} = \frac{A}{3a} \left\{ \frac{1}{2} \ln \frac{(x+A)^2}{x^2 - Ax + A^2} + \sqrt{3} \arctan \frac{x\sqrt{3}}{2A - x} \right\},$   
 $= \frac{A}{3a} \left\{ \frac{1}{2} \ln \frac{(x+A)^2}{x^2 - Ax + A^2} + \sqrt{3} \arctan \frac{2x - A}{A\sqrt{3}} \right\}.$
2.  $\int \frac{x dx}{X_3} = -\frac{1}{3bA} \left\{ \frac{1}{2} \ln \frac{(x+A)^2}{x^2 - Ax + A^2} - \sqrt{3} \arctan \frac{2x - A}{A\sqrt{3}} \right\}.$
3.  $\int \frac{x^n dx}{X_3^m} = \frac{x^{n-2}}{X_3^{m-1}(n+1-3m)b} - \frac{(n-2)a}{b(n+1-3m)} \int \frac{x^{n-3} dx}{X_3^m}.$
4.  $\int \frac{x^2 dx}{X_3} = \frac{1}{3b} \ln(1 + x^3 A^{-3}) = \frac{1}{3b} \ln X_3.$
5.  $\int \frac{x^3 dx}{X_3} = \frac{x}{b} - \frac{a}{b} \int \frac{dx}{X_3}.$
6.  $\int \frac{x^4 dx}{X_3} = \frac{x^2}{2b} - \frac{a}{b} \int \frac{x dx}{X_3}.$
7.  $\int \frac{dx}{X_3^2} = \frac{x}{3aX_3} + \frac{2}{3a} \int \frac{dx}{X_3}.$
8.  $\int \frac{x dx}{X_3^2} = \frac{x^2}{3aX_3} + \frac{1}{3a} \int \frac{x dx}{X_3}.$
9.  $\int \frac{x^2 dx}{X_3^2} = -\frac{1}{3bX_3}.$
10.  $\int \frac{x^3 dx}{X_3^2} = -\frac{x}{3bX_3} + \frac{1}{3b} \int \frac{dx}{X_3}.$
11.  $\int \frac{dx}{xX_3} = \frac{1}{3a} \ln \frac{x^3}{X_3}.$

12.  $\int \frac{dx}{x^2 X_3} = -\frac{1}{ax} - \frac{b}{a} \int \frac{x dx}{X_3}.$
13.  $\int \frac{dx}{x^3 X_3} = -\frac{1}{2ax^2} - \frac{b}{a} \int \frac{dx}{X_3}.$
14.  $\int \frac{dx}{x X_3^2} = \frac{1}{3a X_3} + \frac{1}{3a^2} \ln \frac{x^3}{X_3}.$
15.  $\int \frac{dx}{x^2 X_3^2} = -\left[ \frac{1}{ax} + \frac{4bx^2}{3a^2} \right] \frac{1}{X_3} - \frac{4b}{3a^2} \int \frac{x dx}{X_3}.$
16.  $\int \frac{dx}{x^3 X_3^2} = -\left[ \frac{1}{2ax^2} + \frac{5bx}{6a^2} \right] \frac{1}{X_3} - \frac{5b}{3a^2} \int \frac{dx}{X_3}.$
17.  $\int \frac{dx}{x^n X_3^m} = \begin{cases} -\frac{1}{(n-1)ax^{n-1}X_3^{m-1}} - \frac{b(3m+n-4)}{a(n-1)} \int \frac{dx}{x^{n-3}X_3^m}, \\ \text{or} \\ \frac{1}{3a(m-1)x^{n-1}X_3^{m-1}} + \frac{n+3m-4}{3a(m-1)} \int \frac{dx}{x^n X_3^{m-1}}. \end{cases}$
18.  $\int \frac{dx}{a^3 \pm x^3} = \pm \frac{1}{6a^2} \ln \left( \frac{(a \pm x)^3}{(a^3 \pm x^3)} \right) + \frac{1}{a^2 \sqrt{3}} \arctan \frac{2x \mp a}{a \sqrt{3}}.$
19.  $\int \frac{dx}{(a^3 \pm x^3)^2} = \frac{x}{3a^3(a^3 \pm x^3)} + \frac{2}{3a^3} \int \frac{dx}{a^3 \pm x^3}.$
20.  $\int \frac{dx}{(a^3 \pm x^3)^{n+1}} = \frac{1}{3na^3} \left[ \frac{x}{(a^3 \pm x^3)^n} + (3n-1) \int \frac{dx}{(a^3 \pm x^3)^n} \right], \quad n \neq 0.$
21.  $\int \frac{x dx}{a^3 \pm x^3} = \frac{1}{6a} \ln \left( \frac{a^3 \pm x^3}{(a^3 \pm x^3)^3} \right) \pm \frac{1}{a \sqrt{3}} \arctan \frac{2x \mp a}{a \sqrt{3}}.$
22.  $\int \frac{x dx}{(a^3 \pm x^3)^2} = \frac{x^2}{3a^3(a^3 \pm x^3)} + \frac{1}{3a^3} \int \frac{x dx}{a^3 \pm x^3}.$
23.  $\int \frac{x dx}{(a^3 \pm x^3)^{n+1}} = \frac{1}{3na^3} \left[ \frac{x^2}{(a^3 \pm x^3)^n} + (3n-2) \int \frac{x dx}{(a^3 \pm x^3)^n} \right], \quad n \neq 0.$
24.  $\int \frac{x^2 dx}{a^3 \pm x^3} = \pm \frac{1}{3} \ln (a^3 \pm x^3).$
25.  $\int \frac{x^2 dx}{(a^3 \pm x^3)^{n+1}} = \mp \frac{1}{3n(a^3 \pm x^3)^n}, \quad n \neq 0.$
26.  $\int \frac{dx}{x(a^3 \pm x^3)} = \frac{1}{3a^3} \ln \frac{x^3}{a^3 \pm x^3}.$
27.  $\int \frac{dx}{x(a^3 \pm x^3)^2} = \frac{1}{3a^3(a^3 \pm x^3)} + \frac{1}{3a^6} \ln \frac{x^3}{a^3 \pm x^3}.$

$$28. \int \frac{dx}{x(a^3 \pm x^3)^{n+1}} = \frac{1}{3na^3(a^3 \pm x^3)^n} + \frac{1}{a^3} \int \frac{dx}{x(a^3 \pm x^3)^n} \Big], \quad n \neq 0.$$

$$29. \int \frac{dx}{x^2(a^3 \pm x^3)} = -\frac{1}{a^3x} \mp \frac{1}{a^3} \int \frac{x dx}{a^3 \pm x^3}.$$

$$30. \int \frac{dx}{x^2(a^3 \pm x^3)^{n+1}} = \frac{1}{a^3} \int \frac{dx}{x^2(a^3 \pm x^3)^n} \mp \frac{1}{a^3} \int \frac{x dx}{(a^3 \pm x^3)^{n+1}}.$$

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