T1.34. Integrand involving hyperbolic functions, exponentials, and powers.

1. $\displaystyle\int e^{ax}\,\sinh(ax+c)\,dx = -\frac{1}{2}xe^{-c} + \frac{1}{4a}e^{2ax+c}.$
2. $\displaystyle\int e^{-ax}\,\sinh(ax+c)\,dx = \frac{1}{2}xe^{c} + \frac{1}{4a}e^{-(2ax+c)}.$
3. $\displaystyle\int e^{ax}\,\cosh(ax+c)\,dx = \frac{1}{2}xe^{-c} + \frac{1}{4a}e^{2ax+c}.$
4. $\displaystyle\int e^{-ax}\,\cosh(ax+c)\,dx = \frac{1}{2}xe^{c} - \frac{1}{4a}e^{-(2ax+c)}.$
5. $\displaystyle\int e^{ax}\,\sinh(bx+c)\,dx = \frac{e^{ax}}{a^2-b^2}[a\,\sinh(bx+c) - b\,\cosh(bx+c)], \quad a^2 \neq b^2.$
6. $\displaystyle\int e^{ax}\,\cosh(bx+c)\,dx = \frac{e^{ax}}{a^2-b^2}[a\,\cosh(bx+c) - b\,\sinh(bx+c)], \quad a^2 \neq b^2.$
7. $\displaystyle\int x^p e^{ax}\,\sinh\,ax\,dx = \frac{1}{2}\int x^p e^{2ax}\,dx - \frac{x^{p+1}}{2(p+1)}.$
8. $\displaystyle\int x^p e^{-ax}\,\sinh\,ax\,dx = \frac{x^{p+1}}{2(p+1)} - \frac{1}{2}\int x^p e^{-2ax}\,dx.$
9. $\displaystyle\int x^p e^{ax}\,\cosh\,ax\,dx = \frac{x^{p+1}}{2(p+1)} + \frac{1}{2}\int x^p e^{2ax}\,dx.$
10. $\displaystyle\int x^p e^{ax}\,\sinh\,bx\,dx = \frac{1}{2}\left\{\int x^p e^{(a+b)x}\,dx - \int x^p e^{(a-b)x}\,dx\right\}, \quad a^2 \neq b^2.$
11. $\displaystyle\int x^p e^{ax}\,\cosh\,bx\,dx = \frac{1}{2}\left\{\int x^p e^{(a+b)x}\,dx + \int x^p e^{(a-b)x}\,dx\right\}, \quad a^2 \neq b^2.$
12. $\displaystyle\int xe^{ax}\,\sinh\,ax\,dx = \frac{e^{2ax}}{4a}\left(x - \frac{1}{2a}\right) - \frac{x^2}{4}.$
13. $\displaystyle\int xe^{-ax}\,\sinh\,ax\,dx = \frac{e^{-2ax}}{4a}\left(x + \frac{1}{2a}\right) + \frac{x^2}{4}.$
14. $\displaystyle\int xe^{ax}\,\cosh\,ax\,dx = \frac{x^2}{4} + \frac{e^{2ax}}{4a}\left(x - \frac{1}{2a}\right).$
15. $\displaystyle\int xe^{-ax}\,\cosh\,ax\,dx = \frac{x^2}{4} - \frac{e^{-2ax}}{4a}\left(x + \frac{1}{2a}\right).$

16. $\int x^2 e^{ax} \sinh ax\,dx = \frac{e^{2ax}}{4a}\left(x^2 - \frac{x}{a} + \frac{1}{2a^2}\right) - \frac{x^3}{6}.$

17. $\int x^2 e^{-ax} \sinh ax\,dx = \frac{e^{-2ax}}{4a}\left(x^2 + \frac{x}{a} + \frac{1}{2a^2}\right) + \frac{x^3}{6}.$

18. $\int x^2 e^{ax} \cosh ax\,dx = \frac{x^3}{6} + \frac{e^{2ax}}{4a}\left(x^2 - \frac{x}{a} + \frac{1}{2a^2}\right).$

19. $\int xe^{ax} \sinh bx\,dx = \frac{e^{ax}}{a^2-b^2}\left[\left(ax - \frac{a^2+b^2}{a^2-b^2}\right)\sinh bx - \left(bx - \frac{2ab}{a^2-b^2}\right)\cosh bx\right],\ a^2 \neq b^2.$

20. $\int xe^{ax} \cosh bx\,dx = \frac{e^{ax}}{a^2-b^2}\left[\left(ax - \frac{a^2+b^2}{a^2-b^2}\right)\cosh bx - \left(bx - \frac{2ab}{a^2-b^2}\right)\sinh bx\right],\ a^2 \neq b^2.$

21. $\int x^2 e^{ax} \sinh bx\,dx = \frac{e^{ax}}{a^2-b^2}\left\{\left[ax^2 - \frac{2(a^2+b^2)}{a^2-b^2}x + \frac{2a(a^2+3b^2)}{(a^2-b^2)^2}\right]\sinh bx - \left[bx^2 - \frac{4ab}{a^2-b^2}x + \frac{2b(3a^2+b^2)}{(a^2-b^2)^2}\right]\cosh x\right\},\quad a^2 \neq b^2.$

22. $\int x^2 e^{ax} \cosh bx\,dx = \frac{e^{ax}}{a^2-b^2}\left\{\left[ax^2 - \frac{2(a^2+b^2)}{a^2-b^2}x + \frac{2a(a^2+3b^2)}{(a^2-b^2)^2}\right]\cosh bx - \left[bx^2 - \frac{4ab}{a^2-b^2}x + \frac{2b(3a^2+b^2)}{(a^2-b^2)^2}\right]\sinh x\right\},\quad a^2 \neq b^2.$

23. $\int e^{ax} \sinh ax \frac{dx}{x} = \frac{1}{2}\left[\mathrm{Ei}(2ax) - \ln x\right].$

24. $\int e^{-ax} \sinh ax \frac{dx}{x} = \frac{1}{2}\left[\ln x - \mathrm{Ei}(-2ax)\right].$

25. $\int e^{ax} \cosh ax \frac{dx}{x} = \frac{1}{2}\left[\ln x + \mathrm{Ei}(2ax)\right].$

26. $\int e^{ax} \sinh ax \frac{dx}{x^2} = -\frac{1}{2x}(e^{2ax} - 1) + a\,\mathrm{Ei}(2ax).$

27. $\int e^{-ax} \sinh ax \frac{dx}{x^2} = -\frac{1}{2x}(1 - e^{-2ax}) + a\,\mathrm{Ei}(-2ax).$

28. $\int e^{ax} \cosh ax \frac{dx}{x^2} = -\frac{1}{2x}(e^{2ax} + 1) + a\,\mathrm{Ei}(2ax).$

29. $\int e^{ax} \sinh bx \frac{dx}{x} = \frac{1}{2}\left\{\mathrm{Ei}[(a+b)x] - \mathrm{Ei}[(a-b)x]\right\},\quad a^2 \neq b^2.$

30. $\int e^{ax} \cosh bx \frac{dx}{x} = \frac{1}{2}\left\{\mathrm{Ei}[(a+b)x] + \mathrm{Ei}[(a-b)x]\right\},\quad a^2 \neq b^2.$

31. $\int e^{ax} \sinh bx \frac{dx}{x^2} = -\frac{e^{ax}\sinh bx}{2x} + \frac{1}{2}\left\{(a+b)\mathrm{Ei}[(a+b)x] - (a-b)\mathrm{Ei}[(a-b)x]\right\},\quad a^2 \neq b^2.$

32. $\int e^{ax} \cosh bx \frac{dx}{x^2} = -\frac{e^{ax} \cosh bx}{2x} + \frac{1}{2}\left\{(a+b)\mathrm{Ei}[(a+b)x] + (a-b)\mathrm{Ei}[(a-b)x]\right\}, \quad a^2 \neq b^2.$
