! For an efficient use of these tables, first read `HowTo.pdf`.

T3.28A. Integrands involving algebraic functions of exponentials and powers of $(a+b\,x)$ on the interval $(0,\infty)$.

1. $$\int_0^\infty xe^{-x}\sqrt{1-e^{-x}}\,dx = \frac{4}{3}\left(\frac{4}{3}-\ln 2\right).$$

2. $$\int_0^\infty xe^{-x}\sqrt{1-e^{-2x}}\,dx = \frac{\pi}{4}\left(\frac{1}{2}+\ln 2\right).$$

3. $$\int_0^\infty \frac{x\,dx}{\sqrt{e^x-1}} = 2\pi\,\ln 2.$$

4. $$\int_0^\infty \frac{x^2\,dx}{\sqrt{e^x-1}} = 4\pi\left\{(\ln 2)^2+\frac{\pi^2}{12}\right\}.$$

5. $$\int_0^\infty \frac{xe^{-x}\,dx}{\sqrt{e^x-1}} = \frac{\pi}{2}[2\,\ln 2-1].$$

6. $$\int_0^\infty \frac{xe^{-x}\,dx}{\sqrt{e^{2x}-1}} = 1-\ln 2.$$

7. $$\int_0^\infty \frac{xe^{-2x}\,dx}{\sqrt{e^x-1}} = \frac{3}{4}\pi\left(\ln 2-\frac{7}{12}\right).$$

8. $$\int_0^\infty \frac{xe^x}{a^2e^x-(a^2-b^2)}\,\frac{dx}{\sqrt{e^x-1}} = \frac{2\pi}{ab}\,\ln\left(1+\frac{b}{a}\right), \quad ab>0.$$

9. $$\int_0^\infty \frac{xe^x\,dx}{[a^2e^x-(a^2+b^2)]\sqrt{e^x-1}} = \frac{2\pi}{ab}\,\arctan\frac{b}{a}, \quad ab>0.$$

10. $$\int_0^\infty \frac{xe^{-2nx}\,dx}{\sqrt{e^{2x}+1}} = \frac{(2n-1)!!}{(2n)!!}\,\frac{\pi}{2}\left\{\ln 2+\sum_{k=1}^{2n}\frac{(-1)^k}{k}\right\}.$$

11. $\displaystyle\int_0^\infty \frac{xe^{-(2n-1)x}\,dx}{\sqrt{e^{2x}-1}} = -\frac{(2n-2)!!}{(2n-1)!!}\left\{\ln 2 + \sum_{k=1}^{2n-1}\frac{(-1)^k}{k}\right\}.$

12. $\displaystyle\int_0^\infty \frac{x^2e^x\,dx}{\sqrt{(e^x-1)^3}} = 8\pi\,\ln 2.$

13. $\displaystyle\int_0^\infty \frac{x^3e^x\,dx}{\sqrt{(e^x-1)^3}} = 24\pi\left[(\ln 2)^2 + \frac{\pi^2}{12}\right].$

14. $\displaystyle\int_0^\infty \frac{x\,dx}{\sqrt[3]{e^{3x}-1}} = \frac{\pi}{3\sqrt{3}}\left[\ln 3 + \frac{\pi}{3\sqrt{3}}\right].$

15. $\displaystyle\int_0^\infty \frac{x\,dx}{\sqrt[3]{(e^{3x}-1)^2}} = \frac{\pi}{3\sqrt{3}}\left[\ln 3 - \frac{\pi}{3\sqrt{3}}\right].$

16. $\displaystyle\int_0^\infty xe^{-x}(1-e^{-2x})^{n-1/2}\,dx = \frac{(2n-1)!!}{4\cdot(2n)!!}\pi\left[\gamma_e + \psi(n+1) + 2\,\ln 2\right].$

17. $\displaystyle\int_0^{\ln 2} xe^x(e^x-1)^{p-1}\,dx = \frac{1}{p}\left[\ln 2 + \sum_{k=0}^{\infty}\frac{(-1)^{k-1}}{p+k+1}\right], \quad p > -1.$

18. $\displaystyle\int_0^\infty x^{2n}e^{-px^2}\,dx = \frac{(2n-1)!!}{2(2p)^n}\sqrt{\frac{\pi}{p}}, \quad p > 0,\ n = 0,\,1,\ldots.$

19. $\displaystyle\int_0^\infty x^{2n+1}e^{-px^2}\,dx = \frac{n!}{2p^{n+1}}, \quad p > 0.$

20. $\displaystyle\int_0^\infty x^{\nu-1}e^{-\beta x^2-\gamma x}\,dx = (2\beta)^{-\nu/2}\Gamma(\nu)\,\exp\left(\frac{\gamma^2}{8\beta}\right)D_{-\nu}\left(\frac{\gamma}{\sqrt{2\beta}}\right), \quad \Re\{\beta\} > 0,\ \Re\{\nu\} > 0.$

21. $\displaystyle\int_0^\infty xe^{-\mu x^2-2\nu x}\,dx = \frac{1}{2\mu} - \frac{\nu}{2\mu}\sqrt{\frac{\pi}{\mu}}\,e^{\nu^2/\mu}\left[1 - \operatorname{erf}\left(\frac{\nu}{\sqrt{\mu}}\right)\right], \quad |\arg\nu| < \frac{\pi}{2},\ \Re\{\mu\} > 0.$

22. $\displaystyle\int_0^\infty x^2e^{-\mu x^2-2\nu x}\,dx = -\frac{\nu}{2\mu^2} + \sqrt{\frac{\pi}{\mu^5}}\frac{2\nu^2+\mu}{4}\,e^{\nu^2/\mu}\left[1 - \operatorname{erf}\left(\frac{\nu}{\sqrt{\mu}}\right)\right],$

$|\arg\nu| < \frac{\pi}{2},\ \Re\{\mu\} > 0.$

23. $\displaystyle\int_0^\infty \frac{e^{-x^2}-e^{-x}}{x}\,dx = \frac{1}{2}\gamma_e.$

24. $$\int_0^\infty \frac{e^{-\mu x^2} - e^{-\nu x^2}}{x^2}\, dx = \sqrt{\pi}(\sqrt{\nu} - \sqrt{\mu}), \quad \Re\{\mu\} > 0,\, \Re\{\nu\} > 0.$$

25. $$\int_0^\infty (1 + 2\beta x^2) e^{-\mu x^2}\, dx = \frac{\mu + \beta}{2}\sqrt{\frac{\pi}{\mu^3}}, \quad \Re\{\mu\} > 0.$$

26. $$\int_0^\infty \frac{e^{-\mu^2 x^2}}{x^2 + \beta^2}\, dx = [1 - \operatorname{erf}(\beta\mu)]\, \frac{\pi}{2\beta} e^{\beta^2\mu^2}, \quad \Re\{\beta\} > 0,\ |\arg\mu| < \frac{\pi}{4}.$$

27. $$\int_0^\infty \frac{x^2 e^{-\mu^2 x^2}}{x^2 + \beta^2}\, dx = \frac{\sqrt{\pi}}{2\mu} - \frac{\pi\beta}{2} e^{\mu^2\beta^2}[1 - \operatorname{erf}(\beta\mu)], \quad \Re\{\beta\} > 0,\ |\arg\mu| < \frac{\pi}{4}.$$

28. $$\int_0^\infty \left(e^{-x^2} - \frac{1}{1 + x^2}\right) \frac{dx}{x} = -\frac{1}{2}\gamma_e.$$

29. $$\int_0^\infty \frac{x e^{-\mu x^2}\, dx}{\sqrt{a^2 + x^2}} = \frac{1}{2}\sqrt{\frac{\pi}{\mu}}\, e^{a^2\mu}\ \left[1 - \operatorname{erf}(a\sqrt{u})\right], \quad \Re\{\mu\} > 0,\ a > 0.$$

30. $$\int_0^\infty e^{-\mu x^4 - 2\nu x^2}\, dx = \frac{1}{4}\sqrt{\frac{2\nu}{\mu}}\, \exp\left(\frac{\nu^2}{2\mu}\right) K_{1/4}\left(\frac{\nu^2}{2\mu}\right), \quad \Re\{\mu\} \geq 0.$$

31. $$\int_0^\infty \frac{e^{-x^4} - e^{-x}}{x}\, dx = \frac{3}{4}\gamma_e.$$

32. $$\int_0^\infty \frac{e^{-x^4} - e^{-x^2}}{x^2}\, dx = \frac{1}{4}\gamma_e.$$

33. $$\int_0^\infty x^{\nu-1}(x+\gamma)^{\mu-1}\, e^{-\beta/x}\, dx = \beta^{(\nu-1)/2}\gamma^{(\nu-1)/2+\mu}\, \Gamma(1-\mu-\nu)\, e^{\beta/(2\gamma)}\, W_{(\nu-1)/2+\mu,\, -\nu/2}\left(\frac{\beta}{\gamma}\right),$$
$$|\arg\gamma| < \pi,\ \Re\{1 - \mu\} > \Re\{\nu\} > 0.$$

34. $$\int_0^\infty x^{\nu-1}\, e^{-\beta/x - \gamma x}\, dx = 2\left(\frac{\beta}{\gamma}\right)^{\nu/2} K_\nu(2\sqrt{\beta\gamma}), \quad \Re\{\beta\} > 0,\ \Re\{\gamma\} > 0.$$

35. $$\int_0^\infty x^{\nu-1}\,\exp\left[\frac{i\mu}{2}\left(x-\frac{\beta^2}{x}\right)\right]\,dx = 2\beta^\nu\,e^{i\,\nu\pi/2}K_{-\nu}(\beta\mu),$$
$$\Im\{\mu\}>0,\ \Im\{\beta^2\mu\}<0;\ \text{here } K_{-\nu}\equiv K_\nu.$$

36. $$\int_0^\infty x^{\nu-1}\,\exp\left[\frac{i\mu}{2}\left(x+\frac{\beta^2}{x}\right)\right]\,dx = i\pi\beta^\nu\,e^{-i\,\nu\pi/2}\,H_{-\nu}^{(1)}(\beta\mu),\quad \Im\{\mu\}>0,\ \Im\{\beta^2\mu\}>0.$$

37. $$\int_0^\infty x^{\nu-1}\,\exp\left(-x-\frac{\mu^2}{4x}\right)\,dx = 2\left(\frac{\mu}{2}\right)^\nu K_{-\nu}(\mu),$$
$$|\arg\mu|<\frac{\pi}{2},\ \Re\{\mu^2\}>0;\ \text{here}\ K_{-\nu}\equiv K_\nu.$$

38. $$\int_0^\infty \frac{x^{\nu-1}\,e^{-\beta/x}}{x+\gamma}\,dx = \gamma^{\nu-1}\,e^{\beta/\gamma}\Gamma(1-\nu)\Gamma\left(\nu,\,\frac{\beta}{\gamma}\right),\quad |\arg\gamma|<\pi,\ \Re\{\beta\}>0,\Re\{\nu\}<1.$$

39. $$\int_0^\infty \left(\exp\left(-\frac{a}{x^2}\right)-1\right)e^{-\mu x^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{\mu}}\,\left[\exp(-2\sqrt{a\mu})-1\right],\quad \Re\{\mu\}>0,\Re\{a\}>0.$$

40. $$\int_0^\infty x^2\,\exp\left(-\frac{a}{x^2}-\mu x^2\right)\,dx = \frac{1}{4}\sqrt{\frac{\pi}{\mu^3}}(1+2\sqrt{a\mu})\,\exp(-2\sqrt{a\mu}),\quad \Re\{\mu\}>0,\Re\{a\}>0.$$

41. $$\int_0^\infty \exp\left(-\frac{a}{x^2}-\mu x^2\right)\frac{dx}{x^2} = \frac{1}{2}\sqrt{\frac{\pi}{a}}\,\exp(-2\sqrt{a\mu}),\quad \Re\{\mu\}>0,\,a>0.$$

42. $$\int_0^\infty \exp\left[-\frac{1}{2a}\left(x^2+\frac{1}{x^2}\right)\right]\frac{dx}{x^4} = \sqrt{\frac{a\pi}{2}}(1+a)e^{-1/a},\quad a>0.$$

43. $$\int_0^\infty \exp(-x^n)\,x^{(m+1/2)n-1}\,dx = \frac{(2m-1)!!}{2^m n}\sqrt{\pi}.$$

44. $$\int_0^\infty \left\{\exp(-x^2)-\frac{1}{1+x^{2^n}}\right\}\frac{dx}{x} = -\frac{1}{2}\gamma_e.$$

45. $$\int_0^\infty \left\{\exp(-x^{2^n})-\frac{1}{1+x^2}\right\}\frac{dx}{x} = -2^{-n}\gamma_e.$$

46. $$\int_0^\infty \{\exp(-x^{2^n})-e^{-x}\}\frac{dx}{x} = (1-2^{-n})\gamma_e.$$

47. $$\int_0^\infty [\exp(-\nu x^p) - \exp(-\mu x^p)]\,\frac{dx}{x} = \frac{1}{p}\,\ln\,\frac{\mu}{\nu}, \quad \Re\{\mu\} > 0,\ \Re\{\nu\} > 0.$$

48. $$\int_0^\infty [\exp(-x^p) - \exp(-x^q)]\,\frac{dx}{x} = \frac{p-q}{pq}\,\gamma_e, \quad p > 0,\ q > 0.$$

49. $$\int_0^\infty x^{\nu-1}\,\exp(-\mu x^p)\,dx = \frac{1}{p}\,\mu^{-\nu/p}\,\Gamma\left(\frac{\nu}{p}\right), \quad \Re\{\mu\} > 0, \Re\{\nu\} > 0,\ p > 0.$$

50. $$\int_0^\infty x^{\nu-1}[1 - \exp(-\mu x^p)]\,dx = -\frac{1}{|p|}\mu^{-\nu/p}\Gamma\left(\frac{\nu}{p}\right),$$
$$\Re\{\mu\} > 0 \,\text{and}\, -p < \Re\{\nu\} < 0, \quad \text{for}\, p > 0,\ 0 < \Re\{\nu\} < -p, \quad \text{for}\, p < 0.$$

51. $$\int_0^\infty x^{\nu-1}\,\exp(-\beta x^p - \gamma x^{-p})\,dx = \frac{2}{p}\left(\frac{\gamma}{\beta}\right)^{\nu/(2p)} K_{\nu/p}(2\sqrt{\beta\gamma}), \quad \Re\{\beta\} > 0, \Re\{\gamma\} > 0.$$

52. $$\int_0^\infty \frac{x^{\nu-1}\,\exp(-\beta\sqrt{1+x})}{\sqrt{1+x}}\,dx = \frac{2}{\sqrt{\pi}}\left(\frac{\beta}{2}\right)^{1/2-\nu} \Gamma(\nu)\,K_{1/2-\nu}(\beta), \quad \Re\{\beta\} > 0, \Re\{\nu\} > 0.$$

53. $$\int_0^\infty \frac{x^{\nu-1}\,\exp(i\mu\sqrt{1+x^2})}{\sqrt{1+x^2}}\,dx = i\frac{\sqrt{\pi}}{2}\left(\frac{\mu}{2}\right)^{(1-\nu)/2}\Gamma\left(\frac{\nu}{2}\right)\,H^{(1)}_{(1-\nu)/2}(\mu), \quad \Im\{\mu\} > 0, \Re\{\nu\} > 2.$$

54. $$\int_0^\infty \exp(nx - \beta\,\sinh x)\,dx = \frac{1}{2}[S_n(\beta) - \pi\,\mathbf{E}_n(\beta) - \pi N_n(\beta)], \quad \Re\{\beta\} > 0.$$

55. $$\int_0^\infty \exp(-nx - \beta\,\sinh x)\,dx = (-1)^{n+1}\frac{1}{2}[S_n(\beta) + \pi\mathbf{E}_n(\beta) + \pi N_n(\beta)], \quad \Re\{\beta\} > 0.$$

56. $$\int_0^\infty \exp(-\nu x - \beta\,\sinh x)\,dx = \frac{\pi}{\sin\nu\pi}[\mathbf{J}_\nu(\beta) - J_\nu(\beta)], \quad \Re\{\beta\} > 0.$$

57. $$\int_0^\infty \left[\left(1 + \frac{a}{qx}\right)^{qx} - \left(1 + \frac{a}{px}\right)^{px}\right]\,\frac{dx}{x} = (e^a - 1)\,\ln\,\frac{q}{p}, \quad p > 0,\ q > 0.$$