! For an efficient use of these tables, first read `HowTo.pdf`.

T3.28B. Integrands involving algebraic functions of exponentials and powers of $(a+b\,x)$ on the interval $(-\infty,\infty)$.

1. $$\int_{-\infty}^{\infty}\frac{xe^{x}\,dx}{(a+e^{x})^{n+3/2}}=\frac{2}{(2n+1)a^{n+1/2}}\left[\ln(4a)-3\gamma_e-2\psi(2n)-\psi(n)\right].$$

2. $$\int_{-\infty}^{\infty}\frac{x\,dx}{(a^2e^{x}+e^{-x})^{\mu}}=\frac{-1}{2a^{\mu}}\mathrm{B}\left(\frac{\mu}{2},\,\frac{\mu}{2}\right)\ln a,\quad a>0,\ \Re\{\mu>0\}.$$

3. $$\int_{-\infty}^{\infty}\frac{xe^{x}\,dx}{(a+e^{x})^{\nu+1}}=\begin{cases}\dfrac{1}{\nu a^{\nu}}\left[\ln a-\gamma_e-\psi(\nu)\right], & a>0,\\[2mm] \dfrac{1}{\nu a^{\nu}}\left[\ln a-\displaystyle\sum_{k=1}^{\nu-1}\frac{1}{k}\right], & a>0,\end{cases}\qquad \nu=1,\,2,\ldots.$$

4. $$\int_{-\infty}^{\infty}(x+ai)^{2n}\,e^{-x^2}\,dx=\frac{(2n-1)!!\,\sqrt{\pi}}{2^{n}}\sum_{k=0}^{n}(-1)^{k}\frac{(2a)^{2k}\,n!}{(2k)!(n-k)!}.$$

5. $$\int_{-\infty}^{\infty}x^{n}e^{-px^2+2qx}\,dx=\begin{cases}\dfrac{1}{2^{n-1}p}\sqrt{\dfrac{\pi}{p}}\,\dfrac{d^{n-1}}{dq^{n-1}}\left(qe^{q^2/p}\right), & p>0,\\[2mm] n!\,e^{q^2/p}\sqrt{\dfrac{\pi}{p}}\left(\dfrac{q}{p}\right)^{n}\displaystyle\sum_{k=0}^{\lfloor n/2\rfloor}\frac{1}{(n-2k)!(k)!}\left(\frac{p}{4q^2}\right)^{k}, & p>0.\end{cases}$$

6. $$\int_{-\infty}^{\infty}(ix)^{\nu}e^{-\beta^2x^2-iqx}\,dx=2^{-\nu/2}\sqrt{\pi}\,\beta^{-\nu-1}\,\exp\left(-\frac{q^2}{8\beta^2}\right)D_{\nu}\left(\frac{q}{\beta\sqrt{2}}\right),$$
$$\Re\{\beta^2\}>0,\ \Re\{\nu\}>-1;\ \arg(i\,x)=\frac{\pi}{2}\,\mathrm{sgn}\,x.$$

7. $$\int_{-\infty}^{\infty}x^{n}\,\exp\left[-(x-\beta)^2\right]\,dx=(2i)^{-n}\sqrt{\pi}\,H_n(i\beta).$$

8. $$\int_{-\infty}^{\infty} x e^{-px^2+2qx}\,dx = \frac{q}{p}\sqrt{\frac{\pi}{p}}\,\exp\left(\frac{q^2}{p}\right), \quad \Re\{p\} > 0.$$

8. $$\int_{-\infty}^{\infty} x^2 e^{-\mu x^2+2\nu x}\,dx = \frac{1}{2\mu}\sqrt{\frac{\pi}{\mu}}\left(1+2\frac{\nu^2}{\mu}\right) e^{\nu^2/\mu}, \quad |\arg \nu| < \pi,\ \Re\{\mu\} > 0.$$

9. $$\int_{-\infty}^{\infty} \frac{\exp(-a|x|)}{x-u}\,dx = \frac{\operatorname{sgn} u}{\pi}\left[\exp(a|u|)\operatorname{Ei}(-a|u|) - \exp(-a|u|)\operatorname{Ei}(a|u|)\right], \quad a > 0.$$

10. $$\int_{-\infty}^{\infty} \frac{\operatorname{sgn} x\,\exp(-a|x|)}{x-u}\,dx = -\left[\exp(a|u|)\operatorname{Ei}(-a|u|) - \exp(-a|u|)\operatorname{Ei}(a|u|)\right], \quad a > 0.$$

11. $$\int_{-\infty}^{\infty} x e^{x}\,\exp(-\mu e^{x})\,dx = -\frac{1}{\mu}(\gamma_e + \ln\,\mu), \quad \Re\{\mu\} > 0.$$

12. $$\int_{-\infty}^{\infty} x e^{x}\,\exp(-\mu e^{2x})\,dx = -\frac{1}{4}[\gamma_e + \ln\,(4\mu)]\sqrt{\frac{\pi}{\mu}}, \quad \Re\{\mu\} > 0.$$

13. $$\int_{-\infty}^{\infty} \frac{\exp(\nu\,\operatorname{arcsinh} x - i\,ax)}{\sqrt{1+x^2}}\,dx = \begin{cases} 2\,\exp\left(-\dfrac{i\,\nu\pi}{2}\right) K_\nu(a) & \text{for } a > 0, \\ 2\,\exp\left(\dfrac{i\,\nu\pi}{2}\right) K_\nu(-a) & \text{for } a < 0, \end{cases} \quad |\Re\{\nu\}| < 1.$$