! For an efficient use of these tables, first read `HowTo.pdf`.

T2.42B. Integrands involving powers of trigonometric functions on the interval $(0, \pi/2)$.

1. $$\int_0^{\pi/2} \sin^{\mu-1} x\, dx = \int_0^{\pi/2} \cos^{\mu-1} x\, dx = 2^{\mu-2}\, \mathrm{B}\left(\frac{\mu}{2}, \frac{\mu}{2}\right).$$

2. $$\int_0^{\pi/2} \sin^{3/2} x\, dx = \int_0^{\pi/2} \cos^{3/2} x\, dx = \frac{1}{6\sqrt{2\pi}} \left[\Gamma\left(\frac{1}{4}\right)\right]^2.$$

3. $$\int_0^{\pi/2} \sin^{2m} x\, dx = \int_0^{\pi/2} \cos^{2m} x\, dx = \frac{(2m-1)!!}{(2m)!!}\, \frac{\pi}{2}.$$

4. $$\int_0^{\pi/2} \sin^{2m+1} x\, dx = \int_0^{\pi/2} \cos^{2m+1} x\, dx = \frac{(2m)!!}{(2m+1)!!}.$$

5. $$\int_0^{\pi/2} \sin^{\mu-1} x\, \cos^{\nu-1} x\, dx = \frac{1}{2}\, \mathrm{B}\left(\frac{\mu}{2}, \frac{\nu}{2}\right), \quad \Re\{\mu\} > 0,\, \Re\{\nu\} > 0.$$

6. $$\int_0^{\pi/2} \sqrt{\sin x}\, dx = \sqrt{\frac{2}{\pi}} \left[\Gamma\left(\frac{3}{4}\right)\right]^2.$$

7. $$\int_0^{\pi/2} \frac{dx}{\sqrt{\sin x}} = \frac{1}{2\sqrt{2\pi}} \left[\Gamma\left(\frac{1}{4}\right)\right]^2.$$

8. $$\int_0^{\pi/2} \tan^{\pm\mu} x\, dx = \frac{\pi}{2} \sec \frac{\mu\pi}{2}, \quad |\Re\{\mu\}| < 1.$$

9. $$\int_0^{\pi/2} \tan^{\mu-1} x\, \cos^{2\nu-2} x\, dx = \frac{1}{2}\, \mathrm{B}\left(\frac{\mu}{2}, \nu - \frac{\mu}{2}\right), \quad 0 < \Re\{\mu\} < 2\,\Re\{\nu\}.$$

10. $$\int_0^{\pi/2} \cot^{\mu-1} x\, \sin^{2\nu-2} x\, dx = \frac{1}{2}\, \mathrm{B}\left(\frac{\mu}{2}, \nu - \frac{\mu}{2}\right), \quad 0 < \Re\{\mu\} < 2\,\Re\{\nu\}.$$

11. $$\int_0^{\pi/2} \frac{\sin^{\mu-1/2} x}{\cos^{2\mu-1} x}\, dx = \frac{1}{2}\left\{\frac{\Gamma\left(\frac{\mu}{2}+\frac{1}{4}\right)\Gamma(1-\mu)}{\Gamma\left(\frac{5}{4}-\frac{\mu}{2}\right)}\right\}, \quad -\tfrac{1}{2} < \Re\{\mu\} < 1.$$

12. $$\int_0^{\pi/2} \frac{\cos^{\mu-1/2} x}{\sin^{2\mu-1} x}\, dx = \frac{1}{2}\left\{\frac{\Gamma\left(\frac{\mu}{2}+\frac{1}{4}\right)\Gamma(1-\mu)}{\Gamma\left(\frac{5}{4}-\frac{\mu}{2}\right)}\right\}, \quad -\tfrac{1}{2} < \Re\{\mu\} < 1.$$

13. $$\int_0^{\pi/2} \left(\frac{\sin ax}{\sin x}\right)^2 dx = \frac{a\pi}{2} - \frac{1}{2}\sin \pi a[2a\beta(a)-1], \quad a > 0.$$

14. $$\int_0^{\pi/2} \frac{\tan^{\mu} x}{\cos^{\mu} x}\, dx = \frac{\Gamma(\mu)\Gamma\left(\frac{1}{2}-\mu\right)}{2^{\mu}\sqrt{\pi}} \sin\frac{\mu\pi}{2}, \quad -1 < \Re\{\mu\} < \tfrac{1}{2}.$$

15. $$\int_0^{\pi/2} \frac{\cot^{\mu} x}{\sin^{\mu} x}\, dx = \frac{\Gamma(\mu)\Gamma\left(\frac{1}{2}-\mu\right)}{2^{\mu}\sqrt{\pi}} \sin\frac{\mu\pi}{2}, \quad -1 < \Re\{\mu\} < \tfrac{1}{2}.$$

16. $$\int_0^{\pi/2} \sec^{2p+1} x \frac{d \sin^{2p} x}{dx}\, dx = \frac{1}{\sqrt{\pi}}\Gamma(p+1)\Gamma\left(\frac{1}{2}-p\right), \quad \tfrac{1}{2} > p > 0.$$
