! For an efficient use of these tables, first read `HowTo.pdf`.

T2.63C. Integrands involving logarithm functions of complicated arguments, like $\left(1+a^2/x^2\right)$, $(1 \pm e^{-x})$, $\left(1+2e^{-x}\cos t+e^{-2x}\right)$ and others, on the interval $(0, \pi/4)$.

1. $\displaystyle\int_0^{\pi/4} \ln \sin x \, dx = -\frac{\pi}{4} \ln 2 - \frac{1}{2}\mathbf{G}.$

2. $\displaystyle\int_0^{\pi/4} \ln \cos x \, dx = -\frac{\pi}{4} \ln 2 + \frac{1}{2}\mathbf{G}.$

3. $\displaystyle\int_0^{\pi/4} \ln (\cos x - \sin x) \, dx = -\frac{\pi}{8} \ln 2 - \frac{1}{2}\mathbf{G}.$

4. $\displaystyle\int_0^{\pi/4} \ln (\cos x + \sin x) \, dx = -\frac{\pi}{8} \ln 2 + \frac{1}{2}\mathbf{G}.$

5. $\displaystyle\int_0^{\pi/4} \ln \tan x \, dx = -\mathbf{G}.$

6. $\displaystyle\int_0^{\pi/4} (\ln \tan x)^n \, dx = n!(-1)^n \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^{n+1}} = \frac{1}{2}\left(\frac{\pi}{2}\right)^{n+1} |E_n|, \quad n \text{ even}.$

7. $\displaystyle\int_0^{\pi/4} (\ln \tan x)^2 \, dx = \frac{\pi^3}{16}.$

8. $\displaystyle\int_0^{\pi/4} (\ln \tan x)^4 \, dx = \frac{5}{64}\pi^5.$

9. $\displaystyle\int_0^{\pi/4} \ln (1 + \tan x) \, dx = \frac{\pi}{8} \ln 2.$

10. $\displaystyle\int_0^{\pi/4} \ln(1-\tan x)\,dx = \frac{\pi}{8}\,\ln\,2 - \mathbf{G}.$

11. $\displaystyle\int_0^{\pi/4} \ln(1+\cot x)\,dx = \frac{\pi}{8}\,\ln\,2 + \mathbf{G}.$

12. $\displaystyle\int_0^{\pi/4} \ln(\cot x-1)\,dx = \frac{\pi}{8}\,\ln\,2.$

13. $\displaystyle\int_0^{\pi/4} \ln(\tan x+\cot x)\,dx = \frac{1}{2}\int_0^{\pi/2} \ln(\tan x+\cot x)\,dx = \frac{\pi}{2}\,\ln\,2.$

14. $\displaystyle\int_0^{\pi/4} \ln(\cot x-\tan x)\,dx = \frac{1}{2}\int_0^{\pi/2} \ln^2(\cot x-\tan x)\,dx = \frac{\pi}{2}\,\ln\,2.$

15. $\displaystyle\int_0^{\pi/4} \frac{dx}{\sqrt{\ln\,\cot x}}\,dx = \sqrt{\pi}\sum_{k=0}^{\infty}\frac{(-1)^k}{\sqrt{(2k+1)}}.$

16. $\displaystyle\int_0^{\pi/4} \ln\left(\sqrt{\tan x}+\sqrt{\cot x}\right)\,dx = \frac{1}{2}\int_0^{\pi/2} \ln\left(\sqrt{\tan x}+\sqrt{\cot x}\right)\,dx = \frac{\pi}{8}\,\ln\,2 + \frac{1}{2}\,\mathbf{G}.$

17. $\displaystyle\int_0^{\pi/4} \ln^2\left(\sqrt{\tan x}-\sqrt{\cot x}\right)\,dx = \frac{1}{2}\int_0^{\pi/2} \ln^2\left(\sqrt{\tan x}-\sqrt{\cot x}\right)\,dx = \frac{\pi}{4}\,\ln\,2 - \mathbf{G}.$
