! For an efficient use of these tables, first read `HowTo.pdf`.

T2.53C. Integrands involving rational functions of $(a+b\,x)$ and trigonometric functions on the interval $(0,\pi)$.

1. $$\int_0^{\pi} \frac{\left(\frac{\pi}{2}-x\right)\,\cos x}{1-\sin x}\,dx = 2\int_0^{\pi/2} \frac{\left(\frac{\pi}{2}-x\right)\,\cos x}{1-\sin x}\,dx = \pi\,\ln 2 + 4\mathbf{G}.$$

2. $$\int_0^{\pi} \frac{x^2\,dx}{1-\cos x} = 4\pi\,\ln 2.$$

3. $$\int_0^{\pi} \frac{x\,\sin x\,dx}{1-\cos x} = 2\pi\,\ln 2.$$

4. $$\int_0^{\pi} \frac{x-\sin x}{1-\cos x}\,dx = \frac{\pi}{2} + \int_0^{\pi/2} \frac{x-\sin x}{1-\cos x}\,dx = 2.$$

5. $$\int_0^{\pi} \frac{x\,\sin x\,dx}{1-2a\,\cos x+a^2} = \begin{cases} \dfrac{\pi}{a}\,\ln(1+a), & a^2<1, \\ \dfrac{\pi}{a}\,\ln\left(1+\dfrac{1}{a}\right), & a^2<1, \end{cases} \qquad a \neq 0.$$

6. $$\int_0^{\pi} \frac{x\,dx}{1+a^2+2a\,\cos x} = \frac{\pi^2}{2(1-a^2)} + \frac{4}{(1-a^2)}\sum_{k=0}^{\infty} \frac{a^{2k+1}}{(2k+1)^2}, \qquad a^2<1.$$

7. $$\int_0^{\pi} \frac{x\,\sin x\,dx}{a+b\,\cos x} = \frac{\pi}{b}\,\ln\frac{a+\sqrt{a^2-b^2}}{2(a-b)}, \qquad a>|b|>0.$$

8. $$\int_0^{\pi} \frac{a\,\cos x+b}{(a+b\,\cos x)^2}x^2\,dx = \frac{2\pi}{b}\,\ln\frac{2(a-b)}{a+\sqrt{a^2-b^2}}, \qquad a>|b|>0.$$

9. $$\int_0^{\pi} \frac{\sin x}{1-\cos t_1\,\cos x}\cdot\frac{x\,dx}{1-\cos t_2\,\cos x} = \pi\,\csc\frac{t_1+t_2}{2}\,\csc\frac{t_1-t_2}{2}\,\ln\frac{1+\tan(t_1/2)}{1+\tan(t_2/2)}.$$

10. $$\int_0^\pi \frac{x\,\sin x\,dx}{a+b\,\cos^2 x} = \begin{cases} \dfrac{\pi}{\sqrt{ab}}\ \arctan\sqrt{\dfrac{b}{a}}, & a>0,\ b>0, \\ \dfrac{\pi}{2\sqrt{-ab}}\ \ln\dfrac{\sqrt{a}+\sqrt{-b}}{\sqrt{a}-\sqrt{-b}}, & a>-b>0. \end{cases}$$

11. $$\int_0^\pi \frac{x\,dx}{a^2-\cos^2 x} = \begin{cases} \dfrac{\pi^2}{2a\sqrt{a^2-1}}, & a^2>1, \\ 0, & \text{p.v. for } 0<a^2<1; \text{ divergent if } a^2=0. \end{cases}$$

12. $$\int_0^\pi \frac{x\,\sin x\,dx}{a^2-\cos^2 x} = \frac{\pi}{2a}\ \ln\left|\frac{1+a}{1-a}\right|, \quad 0<a^2<1; \quad \text{divergent for } a^2=0.$$

13. $$\int_0^\pi \frac{x\,\sin 2x\,dx}{a^2-\cos^2 x} = \begin{cases} \pi\ \ln\{4(1-a^2)\}, & \text{p.v. for } 0\le a^2<1, \\ 2\pi\ \ln[2(1-a^2+a\sqrt{a^2-1})], & a^2>1; \text{ divergent for } |a|=1. \end{cases}$$

14. $$\int_0^\pi \frac{x\,\sin x\,dx}{1-\cos^2 t\,\sin^2 x} = \pi(\pi-2t)\ \csc 2t.$$

15. $$\int_0^\pi \frac{x\,\cos x\,dx}{\cos^2 t-\cos^2 x} = 4\ \csc t\sum_{k=0}^\infty \frac{\sin(2k+1)t}{(2k+1)^2}.$$

16. $$\int_0^\pi \frac{x\,\sin x\,dx}{\tan^2 t+\cos^2 x} = \frac{\pi}{2}(\pi-2t)\ \cot t.$$

17. $$\int_0^\pi \frac{x\,dx}{a^2\,\cos^2 x+b^2\,\sin^2 x} = \frac{1}{4}\int_0^{2\pi} \frac{x\,dx}{a^2\,\cos^2 x+b^2\,\sin^2 x} = \frac{\pi^2}{2ab}, \quad a>0,\ b>0.$$

18. $$\int_0^\pi \frac{x\,\sin 2x\,dx}{a^2\,\cos^2 x+b^2\,\sin^2 x} = \frac{2\pi}{a^2-b^2}\ \ln\frac{a+b}{2a}, \quad a>0,\ b>0,\ a\ne b.$$

19. $$\int_0^\pi \frac{x^2\,\sin 2x}{(a^2-\cos^2 x)^2}\,dx = \pi^2\frac{\sqrt{a^2-1}-a}{a(a^2-1)}, \quad a>1.$$

20. $$\int_0^\pi \frac{(a^2-1-\sin^2 x)\,\cos x}{(a^2-\cos^2 x)^2}x^2\,dx = \frac{\pi}{a}\ \ln\left|\frac{1-a}{1+a}\right|, \quad a^2>1.$$

21. $$\int_0^\pi \frac{a\,\cos 2x-\sin^2 x}{(a+\sin^2 x)^2}x^2\,dx = -2\pi\ \ln\left[2(-a+\sqrt{a}\,\sqrt{a+1}\,\right], \quad a>0,$$

and $a<-1$ provided the two square roots are not combined as $\sqrt{a\,(a+1)}$.

22. $$\int_0^\pi \frac{a\,\cos 2x + \sin^2 x}{(a - \sin^2 x)^2} x^2\, dx = 2\pi\, \ln\left[2\left(-a - \sqrt{a}\,\sqrt{a-1}\,\right)\right], \quad a > 1,$$

and $a < -1$ provided the two square roots are not combined as $\sqrt{a\,(a-1)}$.