

! For an efficient use of these tables, first read [HowTo.pdf](#).

**T1.15.** Integrand involving  $(a + bx)^{k/n}$ .

Notation used:  $X = a + bx$ .

$$1. \int \frac{dx}{\sqrt{X}} = \frac{2}{b} \sqrt{X}.$$

$$2. \int \frac{x dx}{\sqrt{X}} = \left( \frac{1}{3} X - a \right) \frac{2\sqrt{X}}{b^2}.$$

$$3. \int \frac{x^2 dx}{\sqrt{X}} = \left( \frac{1}{5} X^2 - \frac{2}{3} aX + a^2 \right) \frac{2\sqrt{X}}{b^3}.$$

$$4. \int \frac{dx}{\sqrt{X^3}} = -\frac{2}{b\sqrt{X}}.$$

$$5. \int \frac{x dx}{\sqrt{X^3}} = (X + a) \frac{2}{b^2 \sqrt{X}}.$$

$$6. \int \frac{x^2 dx}{\sqrt{X^3}} = \left( \frac{X^2}{3} - 2aX - a^2 \right) \frac{2}{b^3 \sqrt{X}}.$$

$$\int x^n X^{m+c/k} dx = \frac{k X^{m+1+c/k}}{b^{n+1}} \sum_{k=0}^n \frac{(-1)^k \binom{n}{k} X^{n-k} a^k}{k(n-k+m+1)+c}.$$

$$7. \int x^n \sqrt{X^{2m+1}} dx = \frac{2\sqrt{X^{2m+1}}}{b^{n+1}} \sum_{k=0}^n \frac{(-1)^k \binom{n}{k} X^{n-k} a^k}{2n-2k+2m+1}.$$

$$8. \int \frac{X^m dx}{x^n \sqrt{X}} = -\frac{X^m \sqrt{X}}{(n-1)ax^{n-1}} + \frac{2m-2n+3}{2(n-1)} \frac{b}{a} \int \frac{X^m dx}{x^{n-1} \sqrt{X}}.$$

$$\begin{aligned}
9. \int \frac{X^m dx}{x^n \sqrt{X}} &= -X^m \sqrt{X} \left\{ \frac{1}{(n-1)ax^{n-1}} \right. \\
&\quad + \sum_{k=1}^{n-2} \frac{(2m-2n+3)(2m-2n+5) \dots (2m-2n+2k+1)}{2^k(n-1)(n-2) \dots (n-k-1)x^{n-k-1}} \frac{b^k}{a^{k+1}} \left. \right\} \\
&\quad + \frac{(2m-2n+3)(2m-2n+5) \dots (2m-3)(2m-1)}{2^{n-1}(n-1)!x} \frac{b^{n-1}}{a^{n-1}} \int \frac{X^m dx}{x \sqrt{X}}.
\end{aligned}$$

$$10. \int \frac{X^m}{x \sqrt{X}} dx = \frac{2X^m}{(2m-1)\sqrt{X}} + a \int \frac{X^{m-1}}{x \sqrt{X}} dx.$$

$$11. \int \frac{X^m}{x \sqrt{X}} dx = \sum_{k=1}^m \frac{2a^{m-k} X^k}{(2k-1)\sqrt{X}} + a^m \int \frac{dx}{x \sqrt{X}}.$$

$$12. \int \frac{dx}{x \sqrt{X}} = \begin{cases} \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{X} - \sqrt{a}}{\sqrt{X} + \sqrt{a}} \right|, & a > 0, \\ \frac{2}{\sqrt{-a}} \arctan \frac{\sqrt{X}}{\sqrt{-a}}, & a < 0. \end{cases}$$

$$13. \int \frac{\sqrt{X} dx}{x} = 2\sqrt{X} + a \int \frac{dx}{x \sqrt{X}}.$$

$$14. \int \frac{\sqrt{X} dx}{x^2} = -\frac{\sqrt{X}}{x} + \frac{b}{2} \int \frac{dx}{x \sqrt{X}}.$$

$$15. \int \frac{\sqrt{X} dx}{x^3} = -\frac{\sqrt{X^3}}{2ax^2} + \frac{b\sqrt{X}}{4ax} - \frac{b^2}{8a} \int \frac{dx}{x \sqrt{X}}.$$

$$16. \int \frac{\sqrt{X^3} dx}{x} = \left( \frac{X}{3} + a \right) 2\sqrt{X} + a^2 \int \frac{dx}{x \sqrt{X}}.$$

$$17. \int \frac{\sqrt{X^3} dx}{x^2} = -\frac{\sqrt{X^5}}{ax} + \frac{3b}{2a} \int \frac{\sqrt{X^3} dx}{x}.$$

$$18. \int \frac{\sqrt{X^3} dx}{x^3} = -\left( \frac{1}{2ax^2} + \frac{b}{4a^2x} \right) \sqrt{X^5} + \frac{3b^2}{8a^2} \int \frac{\sqrt{X^3} dx}{x}.$$

$$19. \int \frac{dx}{x X^m \sqrt{X}} = \sum_{k=0}^{m-1} \frac{2}{(2k+1)a^{m-k} X^k \sqrt{X}} + \frac{1}{a^m} \int \frac{dx}{x \sqrt{X}}.$$

$$20. \int \frac{dx}{x^2 \sqrt{X}} = -\frac{\sqrt{X}}{ax} - \frac{b}{2a} \int \frac{dx}{x \sqrt{X}}.$$

$$21. \int \frac{dx}{x^3 \sqrt{X}} = \left( -\frac{1}{2ax^2} + \frac{3b}{4a^2 x} \right) \sqrt{X} + \frac{3b^2}{8a^2} \int \frac{dx}{x \sqrt{X}}.$$

$$22. \int \frac{dx}{x \sqrt{X^3}} = \frac{2}{a \sqrt{X}} + \frac{1}{a} \int \frac{dx}{x \sqrt{X}}.$$

$$23. \int \frac{dx}{x^2 \sqrt{X^3}} = \left( -\frac{1}{ax} - \frac{3b}{a^2} \right) \frac{1}{\sqrt{X}} - \frac{3b}{2a^2} \int \frac{dx}{x \sqrt{X}}.$$

$$24. \int \frac{dx}{x^3 \sqrt{X^3}} = \left( -\frac{1}{2ax^2} + \frac{5b}{4a^2 x} + \frac{15b^2}{4a^3} \right) \frac{1}{\sqrt{X}} + \frac{15b^2}{8a^3} \int \frac{dx}{x \sqrt{X}}.$$

$$25. \int X^{m+1/3} x^n dx = \left\{ \sum_{k=0}^n \frac{(-1)^k \binom{n}{k} X^{n-k} a^k}{3n-3k+3(m+1)+1} \right\} \frac{3 X^{(m+1)+1/3}}{b^{n+1}}.$$

$$26. \int \frac{x^n dx}{X^{m+2/3}} = \left\{ \sum_{k=0}^n \frac{(-1)^k \binom{n}{k} X^{n-k} a^k}{3n-3k-3(m-1)-2} \right\} \frac{3}{b^{n+1} X^{(m-1)+2/3}}.$$

$$27. \int X^{m+2/3} x^n dx = \left\{ \sum_{k=0}^n \frac{(-1)^k \binom{n}{k} X^{n-k} a^k}{3n-3k+3(m+1)+2} \right\} \frac{3 X^{(m+1)+2/3}}{b^{n+1}}.$$

$$28. \int \frac{x^n dx}{X^{m+1/3}} = \left\{ \sum_{k=0}^n \frac{(-1)^k \binom{n}{k} X^{n-k} a^k}{3n-3k-3(m-1)-1} \right\} \frac{3}{b^{n+1} X^{(m-1)+1/3}}.$$

$$29. \int \frac{X^n dx}{x^m x^{2/3}} = -\frac{X^{n+\frac{1}{3}}}{(m-1)ax^{m-1}} + \frac{3n-3m+4}{3(m-1)} \frac{b}{a} \int \frac{X^n dx}{x^{m-1} X^{2/3}}.$$

$$30. \int \frac{X^n dx}{x X^{2/3}} = \frac{3X^n}{(3n-2)X^{2/3}} + a \int \frac{X^{n-1} dx}{x X^{2/3}}.$$

$$31. \int \frac{dx}{x X^n X^{2/3}} = \frac{3X^{1/3}}{(3n-1)aX^n} + \frac{1}{a} \int \frac{X^{1/3} dx}{x X^n}.$$

$$32. \int \frac{dx}{x X^{2/3}} = \frac{1}{a^{2/3}} \left\{ \frac{3}{2} \ln \frac{X^{1/3} - a^{1/3}}{x^{1/3}} - \sqrt{3} \arctan \frac{\sqrt{3} X^{1/3}}{X^{1/3} + 2 a^{1/3}} \right\}.$$

$$33. \int \frac{X^{1/3} dx}{x} = 3 X^{1/3} + a \int \frac{dx}{x X^{2/3}}.$$

$$34. \int \frac{X^{1/3} dx}{x^2} = -\frac{X X^{1/3}}{ax} + \frac{b}{a} X^{1/3} + \frac{b}{3} \int \frac{dx}{x X^{2/3}}.$$

$$35. \int \frac{X^{1/3} dx}{x^3} = \left( -\frac{1}{2ax^2} + \frac{b}{3a^2x} \right) X X^{1/3} - \frac{b^2}{3a^2} X^{1/3} - \frac{b^2}{9a} \int \frac{dx}{x X^{2/3}}.$$

$$36. \int \frac{dx}{x^2 X^{2/3}} = -\frac{X^{1/3}}{ax} - \frac{2b}{3a} \int \frac{dx}{x X^{2/3}}.$$

$$37. \int \frac{dx}{x^3 X^{2/3}} = \left[ -\frac{1}{2ax^2} + \frac{5b}{6a^2x} \right] X^{1/3} + \frac{5b^2}{9a^2} \int \frac{dx}{x X^{2/3}}.$$

$$38. \int \frac{X^n dx}{x^m X^{2/3}} = -\frac{X^n X^{2/3}}{(m-1)ax^{m-1}} + \frac{3n-3m+5}{3(m-1)} \frac{b}{a} \int \frac{X^n dx}{x^{m-1} X^{1/3}}.$$

$$39. \int \frac{X^n dx}{x X^{1/3}} = \frac{3X^n}{(3n-1)X^{1/3}} + a \int \frac{X^{n-1} dx}{x X^{1/3}}.$$

$$40. \int \frac{dx}{x X^n X^{1/3}} = \frac{3X^{2/3}}{(3n-2)aX^n} + \frac{1}{a} \int \frac{X^{2/3} dx}{x X^n}.$$

$$41. \int \frac{dx}{x X^{1/3}} = \frac{1}{a^{2/3}} \left[ \frac{3}{2} \ln \frac{X^{1/3} - a^{1/3}}{x^{1/3}} + \sqrt{3} \arctan \frac{\sqrt{3} X^{1/3}}{X^{1/3} + 2 a^{1/3}} \right].$$

$$42. \int \frac{X^{2/3} dx}{x} = \frac{3}{2} X^{2/3} + a \int \frac{dx}{x X^{1/3}}.$$

$$43. \int \frac{X^{2/3} dx}{x^2} = -\frac{X^{5/3}}{ax} + \frac{b}{a}X^{2/3} + \frac{2b}{3} \int \frac{dx}{xX^{1/3}}.$$

$$44. \int \frac{X^{2/3} dx}{x^3} = \left[ -\frac{1}{2ax^2} + \frac{b}{6a^2x} \right] X^{5/3} - \frac{b^2}{6a^2}X^{2/3} - \frac{b^2}{9a} \int \frac{dx}{xX^{1/3}}.$$

$$45. \int \frac{dx}{x^2X^{1/3}} = -\frac{X^{2/3}}{ax} - \frac{b}{3a} \int \frac{dx}{xX^{1/3}}.$$

$$46. \int \frac{dx}{x^3X^{1/3}} = \left[ -\frac{1}{2ax^2} + \frac{2b}{3a^2x} \right] X^{1/3} + \frac{2b^2}{9a^2} \int \frac{dx}{xX^{1/3}}.$$

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