! For an efficient use of these tables, first read `HowTo.pdf`.

T3.42A. Integrands involving product of exponentials and trigonometric functions of linear and quadratic arguments on the interval $(0,\infty)$.

1. $$\int_0^\infty e^{-\gamma x}\cos ax^2(\cos\gamma x-\sin\gamma x)\,dx=\sqrt{\frac{\pi}{8a}}\exp\left(-\frac{\gamma^2}{2a}\right),\quad a>0,\ \Re\{\gamma\}\ge|\Im\{\gamma\}|.$$

2. $$\int_0^\infty e^{-\beta x^2}\sin ax^2\,dx=\sqrt{\frac{\pi}{8}}\sqrt{\frac{\sqrt{\beta^2+a^2}-\beta}{\beta^2+a^2}}=\frac{1}{2}\int_{-\infty}^\infty e^{-\beta x^2}\sin ax^2\,dx$$
$$=\frac{\sqrt{\pi}}{2\left(\beta^2+a^2\right)^{1/4}}\sin\left(\frac{1}{2}\arctan\frac{a}{\beta}\right),\quad \Re\{\beta\}>0,\ a>0.$$

3. $$\int_0^\infty e^{-\beta x^2}\cos ax^2\,dx=\frac{1}{2}\int_{-\infty}^\infty e^{-\beta x^2}\cos ax^2\,dx=\sqrt{\frac{\pi}{8}}\sqrt{\frac{\sqrt{\beta^2+a^2}+\beta}{\beta^2+a^2}}$$
$$=\frac{1}{2}\int_{-\infty}^\infty e^{-\beta x^2}\sin ax^2\,dx=\frac{\sqrt{\pi}}{2\left(\beta^2+a^2\right)^{1/4}}\cos\left(\frac{1}{2}\arctan\frac{a}{\beta}\right),\quad \Re\{\beta\}>0,\ a>0.$$

4. $$\int_0^\infty e^{-\beta x^2}\sin ax^2\cos bx\,dx=-\frac{1}{2}\sqrt{\frac{\pi}{\beta^2+a^2}}e^{-A\beta}(B\sin Aa-C\cos Aa)$$
$$=\frac{\sqrt{\pi}}{2\left(\beta^2+a^2\right)^{1/4}}\exp\left(-\frac{\beta b^2}{4(\beta^2+a^2)}\right)\sin\left\{\frac{1}{2}\arctan\frac{a}{\beta}-\frac{ab^2}{4(\beta^2+a^2)}\right\},$$
$a>0, b>0; \Re\{\beta\}>0$, ($\Re\{\beta\}>|\Im\{a\}|$ for complex a), and
$$A=\frac{b^2}{4(a^2+\beta^2)},\quad B=\sqrt{\tfrac{1}{2}(\sqrt{\beta^2+a^2}+\beta)},\quad C=\sqrt{\tfrac{1}{2}(\sqrt{\beta^2+a^2}-\beta)}.$$

5. $$\int_0^\infty e^{-\beta x^2}\,\cos ax^2\,\cos bx\,dx = \frac{1}{2}\sqrt{\frac{\pi}{\beta^2+a^2}}e^{-A\beta}(B\,\cos Aa + C\,\sin Aa)$$

$$= \frac{\sqrt{\pi}}{2\,(\beta^2+a^2)^{1/4}}\,\exp\left(-\frac{\beta b^2}{4(\beta^2+a^2)}\right)\,\cos\left\{\frac{1}{2}\,\arctan\frac{a}{\beta} - \frac{ab^2}{4(\beta^2+a^2)}\right\},$$

$a>0, b>0; \Re\{\beta\}>0$, ($\Re\{\beta\} > |\Im\{a\}|$ for complex a), and

$$A = \frac{b^2}{4(a^2+\beta^2)},\quad B = \sqrt{\tfrac{1}{2}(\sqrt{\beta^2+a^2}+\beta)},\quad C = \sqrt{\tfrac{1}{2}(\sqrt{\beta^2+a^2}-\beta)}.$$

6. $$\int_0^\infty e^{-\beta x^4}\,\sin bx^2\,dx = \frac{\pi}{4}\sqrt{\frac{b}{2\beta}}\,\exp\left(-\frac{b^2}{8\beta}\right)I_{1/4}\left(\frac{b^2}{8\beta}\right),\quad \Re\{\beta\}>0,\ b>0.$$

7. $$\int_0^\infty e^{-\beta x^4}\,\cos bx^2\,dx = \frac{\pi}{4}\sqrt{\frac{b}{2\beta}}\,\exp\left(-\frac{b^2}{8\beta}\right)\,I_{-1/4}\left(\frac{b^2}{8\beta}\right),\quad \Re\{\beta\}>0,\ b>0.$$

8. $$\int_0^\infty e^{-p^2/x^2}\,\sin 2a^2x^2\,dx = \frac{1}{2}\int_{-\infty}^\infty e^{-p^2/x^2}\,\sin 2a^2x^2\,dx = \frac{\sqrt{\pi}}{4a}e^{-2ap}(\cos 2ap + \sin 2ap),$$

$$a>0,\ b>0.$$

9. $$\int_0^\infty e^{-p^2/x^2}\,\cos 2a^2x^2\,dx = \frac{1}{2}\int_{-\infty}^\infty e^{-p^2/x^2}\,\cos 2a^2x^2\,dx = \frac{\sqrt{\pi}}{4a}e^{-2ap}(\cos 2ap - \sin 2ap),$$

$$a>0,\ b>0.$$

10. $$\int_0^\infty e^{-\left(\beta x^2+\gamma/x^2\right)}\,\sin ax^2\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a^2+\beta^2}}e^{-2u\sqrt{\gamma}}\left[v\,\cos(2v\sqrt{\gamma}) + u\,\sin(2v\sqrt{\gamma})\right],$$

where $u = \sqrt{\dfrac{\sqrt{a^2+\beta^2}+\beta}{2}},\quad v = \sqrt{\dfrac{\sqrt{a^2+\beta^2}-\beta}{2}};\quad \Re\{\beta\}>0,\ \Re\{\gamma\}>0.$

11. $$\int_0^\infty e^{-\left(\beta x^2+\gamma/x^2\right)}\,\cos ax^2\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a^2+\beta^2}}e^{-2u\sqrt{\gamma}}\left[u\,\cos(2v\sqrt{\gamma}) - v\,\sin(2v\sqrt{\gamma})\right],$$

where $u = \sqrt{\dfrac{\sqrt{a^2+\beta^2}+\beta}{2}},\quad v = \sqrt{\dfrac{\sqrt{a^2+\beta^2}-\beta}{2}};\quad \Re\{\beta\}>0,\ \Re\{\gamma\}>0.$

12. $$\int_0^\infty \exp\left[-\left(p^2x^2+\frac{q^2}{x^2}\right)\right]\,\sin\left(a^2x^2+\frac{b^2}{x^2}\right)\,dx = \frac{\sqrt{\pi}}{2r}e^{-2rs\,\cos(A+B)}\,\sin\left\{A + 2rs\,\sin(A+B)\right\},$$

where $r = \left(a^4+p^4\right)^{1/4},\quad s = \left(b^4+q^4\right)^{1/4},\quad A = \dfrac{1}{2}\,\arctan\dfrac{a^2}{p^2},\quad B = \dfrac{1}{2}\,\arctan\dfrac{b^2}{q^2}.$

13. $$\int_0^\infty \exp\left[-\left(p^2x^2+\frac{q^2}{x^2}\right)\right]\cos\left(a^2x^2+\frac{b^2}{x^2}\right)dx=\frac{\sqrt{\pi}}{2r}e^{-2rs\cos(A+B)}\cos\{A+2rs\sin(A+B)\},$$

where $r=\left(a^4+p^4\right)^{1/4}$, $s=\left(b^4+q^4\right)^{1/4}$, $A=\frac{1}{2}\arctan\frac{a^2}{p^2}$, $B=\frac{1}{2}\arctan\frac{b^2}{q^2}$.

14. $$\int_0^\infty\left[e^{-x}\cos(p\sqrt{x})+pe^{-x^2}\sin px\right]dx=1.$$

15. $$\int_0^\infty e^{-p/x}\sin^2\frac{a}{x}\,dx=a\arctan\frac{2a}{p}+\frac{p}{4}\ln\frac{p^2}{p^2+4a^2},\quad a>0,\ p>0.$$