! For an efficient use of these tables, first read `HowTo.pdf`.

T2.47B. Integrands involving different forms of powers of trigonometric functions on the interval $(0, \pi/2)$.

1. $$\int_0^{\pi/2} \frac{\sin^{2\mu-1} x \cos^{2\nu-1} x\, dx}{(1-k^2 \sin^2 x)^\rho} = \frac{1}{2} \mathrm{B}(\mu, \nu)\, F(\rho, \mu; \mu+\nu; k^2), \quad \Re\{\mu\} > 0, \Re\{\nu\} > 0.$$

2. $$\int_0^{\pi/2} \frac{\sin^{2\mu-1} x \cos^{2\nu-1} x\, dx}{(1-k^2 \sin^2 x)^{\mu+\nu}} = \frac{\mathrm{B}(\mu, \nu)}{2(1-k^2)^\mu}, \quad \Re\{\mu\} > 0, \Re\{\nu\} > 0.$$

3. $$\int_0^{\pi/2} \frac{\sin^\mu x\, dx}{\cos^{\mu-3} x (1-k^2 \sin^2 x)^{\mu/2-1}}$$

$$= \frac{\Gamma\left(\frac{\mu+1}{2}\right)\Gamma\left(2-\frac{\mu}{2}\right)}{k^3\sqrt{\pi(\mu-1)(\mu-3)(\mu-5)}} \left\{ \frac{1+(\mu-3)k+k^2}{(1+k)^{\mu-3}} - \frac{1-(\mu-3)k+k^2}{(1-k)^{\mu-3}} \right\},$$

$$-1 < \Re\{\mu\} < 4.$$

4. $$\int_0^{\pi/2} \frac{\sin^{\mu+1} x\, dx}{\cos^\mu x (1-k^2 \sin^2 x)^{(\mu+1)/2}} = \frac{(1+k)^{-\mu} - (1+k)^{-\mu}}{2k\mu\sqrt{\pi}} \Gamma\left(1+\frac{\mu}{2}\right)\Gamma\left(\frac{1-\mu}{2}\right),$$

$$-2 < \Re\{\mu\} < 1.$$

5. $$\int_0^{\pi/2} \frac{\sin^\mu x \cos^\nu x}{(a-b\cos^2 x)^\rho}\, dx = \frac{1}{2a^\rho} \mathrm{B}\left(\frac{\mu+1}{2}, \frac{\nu+1}{2}\right) F\left(\frac{\nu+1}{2}, \rho; \frac{\mu+\nu}{2}+1; \frac{b}{a}\right),$$

$$\Re\{\mu\} > -1, \Re\{\nu\} > -1, a > |b| \geq 0.$$

6. $$\int_0^{\pi/2} (\sin^{\mu-1} x - \sin^{\nu-1} x) \frac{dx}{\cos x} = \int_0^{\pi/2} (\cos^{\mu-1} x - \cos^{\nu-1} x) \frac{dx}{\sin x} = \frac{1}{2}\left[\psi\left(\frac{\nu}{2}\right) - \psi\left(\frac{\mu}{2}\right)\right],$$

$$\Re\{\mu\} > 0, \Re\{\nu\} > 0.$$

7. $$\int_0^{\pi/2} (\sin^\mu x - \csc^\mu x) \frac{dx}{\cos x} = \int_0^{\pi/2} (\cos^\mu x - \sec^\mu x) \frac{dx}{\sin x} = -\frac{\pi}{2} \tan\frac{\mu\pi}{2}, \quad |\Re\{\mu\}| < 1.$$

8. $$\int_0^{\pi/2} \frac{\tan x\,dx}{\cos^{\mu} x + \sec^{\mu} x} = \int_0^{\pi/2} \frac{\cot x\,dx}{\sin^{\mu} x + \csc^{\mu} x} = \frac{\pi}{4\mu}.$$

9. $$\int_0^{\pi/2} \frac{\sin^{\mu-1} x + \sin^{\nu-1} x}{\cos^{\mu+\nu-1} x}\,dx = \int_0^{\pi/2} \frac{\cos^{\mu-1} x + \cos^{\nu-1} x}{\sin^{\mu+\nu-1} x}\,dx = \frac{\cos\left(\frac{\nu-\mu}{4}\pi\right)}{2\cos\left(\frac{\nu+\mu}{4}\pi\right)} \mathrm{B}\left(\frac{\mu}{2}, \frac{\nu}{2}\right),$$
$$\Re\{\mu\} > 0,\ \Re\{\nu\} > 0,\ \Re\{\mu+\nu\} < 2.$$

10. $$\int_0^{\pi/2} \frac{\sin^{\mu-1} x - \sin^{\nu-1} x}{\cos^{\mu+\nu-1} x}\,dx = \int_0^{\pi/2} \frac{\cos^{\mu-1} x - \cos^{\nu-1} x}{\sin^{\mu+\nu-1} x}\,dx = \frac{\sin\left(\frac{\nu-\mu}{4}\pi\right)}{2\sin\left(\frac{\nu+\mu}{4}\pi\right)} \mathrm{B}\left(\frac{\mu}{2}, \frac{\nu}{2}\right),$$
$$\Re\{\mu\} > 0,\ \Re\{\nu\} > 0,\ \Re\{\mu+\nu\} < 4.$$

11. $$\int_0^{\pi/2} \frac{\sin^{\mu} x + \sin^{\nu} x}{\sin^{\mu+\nu} x + 1} \cot x\,dx = \int_0^{\pi/2} \frac{\cos^{\mu} x + \cos^{\nu} x}{\cos^{\mu+\nu} x + 1} \tan x\,dx = \frac{\pi}{\mu+\nu} \sec\left(\frac{\mu-\nu}{\mu+\nu} \cdot \frac{\pi}{2}\right),$$
$$\Re\{\mu\} > 0,\ \Re\{\nu\} > 0.$$

12. $$\int_0^{\pi/2} \frac{\sin^{\mu} x - \sin^{\nu} x}{\sin^{\mu+\nu} x - 1} \cot x\,dx = \int_0^{\pi/2} \frac{\cos^{\mu} x - \cos^{\nu} x}{\cos^{\mu+\nu} x - 1} \tan x\,dx = \frac{\pi}{\mu+\nu} \tan\left(\frac{\mu-\nu}{\mu+\nu} \cdot \frac{\pi}{2}\right),$$
$$\Re\{\mu\} > 0,\ \Re\{\nu\} > 0.$$

13. $$\int_0^{\pi/2} \frac{\cos^{\mu} x + \sec^{\mu} x}{\cos^{\nu} x + \sec^{\nu} x} \tan x\,dx = \frac{\pi}{2\nu} \sec\left(\frac{\mu}{\nu} \cdot \frac{\pi}{2}\right), \quad |\Re\{\nu\}| > |\Re\{\mu\}|.$$

14. $$\int_0^{\pi/2} \frac{\cos^{\mu} x - \sec^{\mu} x}{\cos^{\nu} x - \sec^{\nu} x} \tan x\,dx = \frac{\pi}{2\nu} \tan\left(\frac{\mu}{\nu} \cdot \frac{\pi}{2}\right), \quad |\Re\{\nu\}| > |\Re\{\mu\}|.$$

15. $$\int_0^{\pi/2} \frac{1}{(\tan^{\mu} x + \cot^{\mu} x)^{\nu}} \cdot \frac{dx}{\tan x} = \int_0^{\pi/2} \frac{1}{(\tan^{\mu} x + \cot^{\mu} x)^{\nu}} \cdot \frac{dx}{\sin 2x}$$
$$= \frac{\sqrt{\pi}}{2^{2\nu+1}\mu} \frac{\Gamma(\nu)}{\Gamma\left(\nu + \frac{1}{2}\right)}, \quad \nu > 0.$$

16. $$\int_0^{\pi/2} \frac{(1+\tan x)^{\nu} - 1}{(1+\tan x)^{\mu+\nu}} \frac{dx}{\sin x\,\cos x} = \psi(\mu+\nu) - \psi(\mu), \quad \mu > 0,\ \nu > 0.$$

17. $$\int_0^{\pi/2} \frac{(\sin^{\mu} x + \csc^{\mu} x)\,\cot x\,dx}{\sin^{\nu} x - 2\cos t + \csc^{\nu} x} = \frac{\pi}{\nu} \csc t \csc\frac{\mu\pi}{\nu} \sin\frac{\mu t}{\nu}, \quad \mu < \nu.$$

18. $$\int_0^{\pi/2} \frac{\sin^\mu x - 2\cos t_1 + \csc^\mu x}{\sin^\nu x + 2\cos t_2 + \csc^\nu x} \cdot \cot x\, dx = \frac{\pi}{\nu} \csc t_2 \csc\frac{\mu\pi}{\nu} \sin\frac{\mu t_2}{\nu} - \frac{t_2}{\nu} \csc t_2 \cos t_1,$$
$\nu > \mu > 0$ or $\nu < \mu < 0$ or $\mu > 0$, $\nu < 0$ and $\mu + \nu < 0$ or $\mu < 0$, $\nu > 0$ and $\mu + \nu > 0$.
