! For an efficient use of these tables, first read `HowTo.pdf`.

T3.53A. Integrands involving product of powers of logarithm functions and rational functions on the interval $(0,\infty)$.

1. $$\int_0^\infty (\ln x)^2 \frac{dx}{(x-1)(x+a)} = \frac{\left[\pi^2 + (\ln a)^2\right] \ln a}{3(1+a)}, \quad a > 0.$$

2. $$\int_0^\infty (\ln x)^2 \frac{dx}{(1-x)^2} = \frac{2}{3}\pi^2.$$

3. $$\int_0^\infty (\ln x)^2 \frac{x^{\mu-1}}{1+x}\, dx = \frac{\pi^3(2-\sin^2 \mu\pi)}{\sin^3 \mu\pi}, \quad 0 < \Re\{\mu\} < 1.$$

4. $$\int_0^\infty (\ln x)^2 \frac{x^{p-1}\, dx}{x^2+2x\cos t+1} = \frac{\pi \sin(1-p)t}{\sin t \sin p\pi} \{\pi^2 - t^2 + 2\pi \cot p\pi [\pi \cot p\pi + t\cot(1-p)t]\},$$ $$0 < t < \pi,\ 0 < p < 2\ (p \neq 1).$$

5. $$\int_0^\infty (\ln x)^4 \frac{dx}{(x-1)(x+a)} = \frac{\ln a \left[\pi^2 + (\ln a)^2\right] \left[7\pi^2 + 3(\ln a)^2\right]}{15(1+a)}, \quad a > 0.$$

6. $$\int_0^\infty (\ln x)^5 \frac{dx}{(x-1)(x+a)} = \frac{\left[\pi^2 + (\ln a)^2\right]^2 \left[3\pi^2 + (\ln a)^2\right]}{6(1+a)}, \quad a > 0.$$

7. $$\int_0^\infty \frac{x^{p-1} - x^{q-1}}{(1+x^r)\,\ln x}\, dx = \ln\left(\tan\frac{p\pi}{2r} \cot\frac{q\pi}{2r}\right) 0 < p < r,\ 0 < q < r.$$

8. $$\int_0^\infty \frac{x^{p-1} - x^{q-1}}{(1-x^r)\,\ln x}\, dx = \ln\left(\frac{\sin\frac{p\pi}{r}}{\sin\frac{q\pi}{r}}\right), \quad 0 < p < r, 0 < q < r.$$

9. $$\int_0^\infty \frac{x^{p-1} - x^{q-1}}{1+x^{2(2n+1)}} \frac{1+x^2}{\ln x}\, dx = \ln\left\{\tan\frac{p\pi}{4(2n+1)} \tan\frac{(p+2)\pi}{4(2n+1)} \cot\frac{q\pi}{4(2n+1)} \cot\frac{(q+2)\pi}{4(2n+1)}\right\},$$ $$0 < p < 4n, 0 < q < 4n.$$

10. $$\int_0^\infty \frac{x^{p-1} - x^{q-1}}{1 - x^{2n}} \frac{1 - x^2}{\ln x}\, dx = \ln \frac{\sin \frac{p\pi}{2n} \, \sin \frac{(q+2)\pi}{2n}}{\sin \frac{q\pi}{2n} \, \sin \frac{(p+2)\pi}{2n}}, \quad 0 < p < 2n,\, 0 < q < 2n.$$

11. $$\int_0^\infty (1 - x^p)(1 - x^q) \frac{x^{s-1}\, dx}{(1 - x^{p+q+2s}) \ln x} = 2 \int_0^1 (1 - x^p)(1 - x^q) \frac{x^{s-1}\, dx}{(1 - x^{p+q+2s}) \ln x}$$
$$= 2 \ln \left\{ \sin \frac{s\pi}{p+q+2s} \csc \frac{(p+s)\pi}{p+q+2s} \right\}, \quad s > 0,\ s + p > 0,\ s + p + q > 0.$$

12. $$\int_0^\infty \frac{(\ln x)^{2n+1}}{1 + bx + x^2}\, dx = 0, \quad |b| < 2.$$

13. $$\int_0^\infty (\ln x)^{2n} \frac{dx}{1 - x^2} = 0.$$

14. $$\int_0^\infty (\ln x)^n \frac{x^{\nu-1}\, dx}{a^2 + 2ax \cos t + x^2} = -\pi \csc t \, \frac{d^n}{d\nu^n} \left[a^{\nu-2} \frac{\sin(\nu - 1)t}{\sin \nu\pi} \right],$$
$$a > 0,\ 0 < \Re\{\nu\} < 2,\ 0 < |t| < \pi.$$