! For an efficient use of these tables, first read `HowTo.pdf`.

T2.24D. Integrands of the form $\sqrt{\dfrac{d-x}{(a-x)(b-x)(c-x)^3}}$, $\sqrt{\dfrac{c-x}{(a-x)(b-x)(d-x)^3}}$, $\sqrt{\dfrac{b-x}{(a-x)(c-x)(d-x)^3}}$, and $\sqrt{\dfrac{a-x}{(b-x)(c-x)(d-x)^3}}$, and similar expressions with cubes of one of the factors in the denominator, on the intervals (y,d) and (d,y).

Notation used: $\alpha = \arcsin\sqrt{\dfrac{(a-c)(d-y)}{(a-d)(c-y)}}$, $\beta = \arcsin\sqrt{\dfrac{(a-c)(y-d)}{(c-d)(a-y)}}$,

$q = \sqrt{\dfrac{(b-c)(a-d)}{(a-c)(b-d)}}$, $r = \sqrt{\dfrac{(a-b)(c-d)}{(a-c)(b-d)}}$.

Then

1. $$\int_y^d \sqrt{\frac{d-x}{(a-x)(b-x)(c-x)^3}}\,dx = \frac{2}{b-c}\sqrt{\frac{b-d}{a-c}}[F(\alpha,\,q) - E(\alpha,\,q)], \quad a>b>c>d>y.$$

2. $$\int_d^y \sqrt{\frac{x-d}{(a-x)(b-x)(c-x)^3}}\,dx = \frac{-2}{b-c}\sqrt{\frac{b-d}{a-c}}E(\beta,\,r) + \frac{2}{b-c}\sqrt{\frac{(b-y)(y-d)}{(a-y)(c-y)}},$$
$$a>b>c\ge y>d.$$

3. $$\int_d^y \sqrt{\frac{b-x}{(a-x)(c-x)^3(x-d)}}\,dx = \frac{2}{c-d}\sqrt{\frac{b-d}{a-c}}[F(\beta,\,r) - E(\beta,\,r)] + \frac{2}{c-d}\sqrt{\frac{(b-y)(y-d)}{(a-y)(c-y)}},$$
$$a>b>c>y>d.$$

4. $$\int_y^d \sqrt{\frac{a-x}{(b-x)(c-x)^3(d-x)}}\,dx = \frac{2\sqrt{(a-c)(b-d)}}{(b-c)(c-d)}E(\alpha,\,q)$$
$$-\frac{a-b}{b-c}\frac{2}{\sqrt{(a-c)(b-d)}}F(\alpha,\,q), \quad a>b>c>d>y.$$

5. $$\int_d^y \sqrt{\frac{a-x}{(b-x)(c-x)^3(x-d)}}\,dx = \frac{2(a-d)}{(c-d)\sqrt{(a-c)(b-d)}}F(\beta,\,r) - 2\frac{\sqrt{(a-c)(b-d)}}{(b-c)(c-d)}E(\beta,\,r)$$
$$+2\frac{a-c}{(b-c)(c-d)}\sqrt{\frac{(b-y)(y-d)}{(a-y)(c-y)}}, \quad a>b>c>y>d.$$

6. $$\int_y^d \sqrt{\frac{d-x}{(a-x)(b-x)^3(c-x)}}\,dx = \frac{2\sqrt{(a-c)(b-d)}}{(a-b)(b-c)}E(\alpha,\,q) - \frac{2(c-d)}{(b-c)\sqrt{(a-c)(b-d)}}F(\alpha,\,q)$$
$$-\frac{2}{a-b}\sqrt{\frac{(a-y)(d-y)}{(b-y)(c-y)}}, \quad a>b>c>d>y.$$

7. $$\int_d^y \sqrt{\frac{x-d}{(a-x)(b-x)^3(c-x)}}\,dx = \frac{2\sqrt{(a-c)(b-d)}}{(a-b)(b-c)}E(\beta,\,r) - \frac{2(a-d)}{(a-b)\sqrt{(a-c)(b-d)}}F(\beta,\,r)$$
$$+\frac{2}{b-c}\sqrt{\frac{(c-y)(y-d)}{(a-y)(b-y)}}, \quad a>b>c\geq y>d.$$

8. $$\int_y^d \sqrt{\frac{c-x}{(a-x)(b-x)^3(d-x)}}\,dx = \frac{2}{a-b}\sqrt{\frac{a-c}{b-d}}E(\alpha,\,q) - \frac{2(b-c)}{(a-b)(b-d)}\sqrt{\frac{(a-y)(d-y)}{(b-y)(c-y)}}$$
$$a>b>c>d>y.$$

9. $$\int_d^y \sqrt{\frac{c-x}{(a-x)(b-x)^3(x-d)}}\,dx = \frac{2}{a-b}\sqrt{\frac{a-c}{b-d}}[F(\beta,\,r)-E(\beta,\,r)] + \frac{2}{b-d}\sqrt{\frac{(c-y)(y-d)}{(a-y)(b-y)}},$$
$$a>b>c\geq y>d.$$

10. $$\int_y^d \sqrt{\frac{a-x}{(b-x)^3(c-x)(d-x)}}\,dx = \frac{2}{b-c}\sqrt{\frac{a-c}{b-d}}[F(\alpha,\,q)-E(\alpha,\,q)] + \frac{2}{b-d}\sqrt{\frac{(a-y)(d-y)}{(b-y)(c-y)}},$$
$$a>b>c>d>y.$$

11. $$\int_d^y \sqrt{\frac{a-x}{(b-x)^3(c-x)(x-d)}}\,dx = \frac{2}{b-c}\sqrt{\frac{a-c}{b-d}}E(\beta,\,r) - \frac{2(a-b)}{(b-c)(b-d)}\sqrt{\frac{(y-d)(c-y)}{(a-y)(b-y)}},$$
$$a>b>c\geq y>d.$$

12. $$\int_y^d \sqrt{\frac{d-x}{(a-x)^3(b-x)(c-x)}}\,dx = \frac{2}{b-a}\sqrt{\frac{b-d}{a-c}}E(\alpha,\,q) + \frac{2}{a-b}\sqrt{\frac{(b-y)(d-y)}{(a-y)(c-y)}},$$
$$a>b>c>d>y.$$

13. $$\int_d^y \sqrt{\frac{x-d}{(a-x)^3(b-x)(c-x)}}\,dx = \frac{2}{a-b}\sqrt{\frac{b-d}{a-c}}[F(\beta,\,q)-E(\beta,\,q)], \quad a>b>c\geq y>d.$$

14. $$\int_y^d \sqrt{\frac{c-x}{(a-x)^3(b-x)(d-x)}}\,dx = \frac{2(c-d)}{(a-d)\sqrt{(a-c)(b-d)}}F(\alpha,\,q) - \frac{2\sqrt{(a-c)(b-d)}}{(a-b)(a-d)}E(\alpha,\,q)$$
$$+\frac{2(a-c)}{(a-b)(a-d)}\sqrt{\frac{(b-y)(d-y)}{(a-y)(c-y)}}, \quad a>b>c>d>y.$$

15. $$\int_d^y \sqrt{\frac{c-x}{(a-x)^3(b-x)(x-d)}}\,dx = \frac{2\sqrt{(a-c)(b-d)}}{(a-b)(a-d)}E(\beta,\,r)$$
$$-\frac{2(b-c)}{(a-b)\sqrt{(a-c)(b-d)}}F(\beta,\,r), \quad a>b>c\geq y>d.$$

16. $$\int_y^d \sqrt{\frac{b-x}{(a-x)^3(c-x)(d-x)}}\,dx = \frac{2}{a-d}\sqrt{\frac{b-d}{a-c}}[F(\alpha,\,q)-E(\alpha,\,q)] + \frac{2}{a-d}\sqrt{\frac{(b-y)(d-y)}{(a-y)(c-y)}},$$
$$a>b>c>d>y.$$

17. $$\int_d^y \sqrt{\frac{b-x}{(a-x)^3(c-x)(x-d)}}\,dx = \frac{2}{a-d}\sqrt{\frac{b-d}{a-c}}E(\beta,\,r), \quad a>b>c\geq y>d.$$