! For an efficient use of these tables, first read `HowTo.pdf`.

T3.46A. Integrands involving trigonometric and hyperbolic functions and exponentials on the interval $(0, \infty)$.

1. $$\int_0^\infty \sin ax\, \sinh^\nu \gamma x\, e^{-\beta x}\, dx = -\frac{i\,\Gamma(\nu+1)}{2^{\nu+2}\,\gamma} \left\{ \frac{\Gamma\left(\frac{\beta-\nu\gamma-i\,a}{2\gamma}\right)}{\Gamma\left(\frac{\beta+\nu\gamma-i\,a}{2\gamma}+1\right)} - \frac{\Gamma\left(\frac{\beta-\nu\gamma+i\,a}{2\gamma}\right)}{\Gamma\left(\frac{\beta+\nu\gamma+i\,a}{2\gamma}+1\right)} \right\},$$
$$\Re\{\nu\} > 2,\ \Re\{\gamma\} > 0,\ |\Re\{\gamma\nu\}| < \Re\{\beta\}.$$

2. $$\int_0^\infty \cos ax\, \sinh^\nu \gamma x\, e^{-\beta x}\, dx = \frac{\Gamma(\nu+1)}{2^{\nu+2}\,\gamma} \left\{ \frac{\Gamma\left(\frac{\beta-\nu\gamma-i\,a}{2\gamma}\right)}{\Gamma\left(\frac{\beta+\nu\gamma-i\,a}{2\gamma}+1\right)} - \frac{\Gamma\left(\frac{\beta-\nu\gamma+i\,a}{2\gamma}\right)}{\Gamma\left(\frac{\beta+\nu\gamma+i\,a}{2\gamma}+1\right)} \right\},$$
$$\Re\{\nu\} > -1,\ \Re\{\gamma\} > 0,\ |\Re\{\gamma\nu\}| < \Re\{\beta\}.$$

3. $$\int_0^\infty e^{-\beta x}\,\frac{\sin ax}{\sinh \gamma x}\, dx = \sum_{k=1}^\infty \frac{2a}{a^2 + [\beta + (2k-1)\,\gamma]^2}$$
$$= \frac{1}{2i\,\gamma}\left[\psi\left(\frac{\beta+\gamma+i\,a}{2\gamma}\right) - \psi\left(\frac{\beta+\gamma-i\,a}{2\gamma}\right)\right], \quad \Re\{\beta\} > |\Re\{\gamma\}|.$$

4. $$\int_0^\infty e^{-x}\,\frac{\sin ax}{\sinh x}\, dx = \frac{\pi}{2}\coth\frac{a\,\pi}{2} - \frac{1}{a}.$$

5. $$\int_0^\infty \frac{\sin ax\, \sinh \beta x}{e^{\gamma x}-1}\, dx = -\frac{a}{2\,(a^2+\beta^2)} + \frac{\pi}{2\gamma}\,\frac{\sinh\frac{2\pi\,a}{\gamma}}{\cosh\frac{2\pi\,a}{\gamma} - \cos\frac{2\pi\beta}{\gamma}}$$
$$+\frac{i}{2\gamma}\left[\psi\left(\frac{\beta}{\gamma} + i\,\frac{a}{\gamma} + 1\right) - \psi\left(\frac{\beta}{\gamma} - i\,\frac{a}{\gamma}+1\right)\right], \quad \Re\{\gamma\} < |\Re\{\beta\}|,\ a > 0$$

6. $$\int_0^\infty \frac{\sin ax\, \cosh \beta x}{e^{\gamma x}-1}\, dx = -\frac{a}{2\,(a^2+\beta^2)} + \frac{\pi}{2\gamma}\,\frac{\sinh\frac{2\pi\,a}{\gamma}}{\cosh\frac{2\pi\,a}{\gamma} - \cos\frac{2\pi\beta}{\gamma}}, \quad \Re\{\gamma\} < |\Re\{\beta\}|,\ a > 0$$

7. $$\int_0^\infty \frac{\sin ax \, \cosh \beta x}{e^{\gamma x}+1}\,dx = \frac{a}{2\left(a^2+\beta^2\right)} - \frac{\pi}{\gamma}\,\frac{\sinh\frac{2\pi\,a}{\gamma}\,\cos\frac{\beta\pi}{\gamma}}{\cosh\frac{2\pi\,a}{\gamma}-\cos\frac{2\pi\beta}{\gamma}}, \quad \Re\{\gamma\} < |\Re\{\beta\}|,\ a>0$$

8. $$\int_0^\infty \frac{\cos ax \, \sinh \beta x}{e^{\gamma x}-1}\,dx = \frac{\beta}{2\left(a^2+\beta^2\right)} - \frac{\pi}{2\gamma}\,\frac{\sin\frac{2\pi\beta}{\gamma}}{\cosh\frac{2\pi\,a}{\gamma}-\cos\frac{2\pi\beta}{\gamma}}, \quad \Re\{\gamma\} < |\Re\{\beta\}|,\ a>0$$

9. $$\int_0^\infty \frac{\cos ax \, \sinh \beta x}{e^{\gamma x}+1}\,dx = -\frac{\beta}{2\left(a^2+\beta^2\right)} + \frac{\pi}{\gamma}\,\frac{\cosh\frac{\pi\,a}{\gamma}\,\sin\frac{2\pi\beta}{\gamma}}{\cosh\frac{2\pi\,a}{\gamma}-\cos\frac{2\pi\beta}{\gamma}}, \quad \Re\{\gamma\} < |\Re\{\beta\}|,\ a>0$$

10. $$\int_0^\infty e^{-\beta x^2}\,(\cosh x - \sin x)\,dx = \sqrt{\frac{\pi}{\beta}}\,\sinh\frac{1}{4\beta}, \quad \Re\{\beta\}>0.$$

11. $$\int_0^\infty e^{-\beta x^2}\,(\cosh x + \sin x)\,dx = \sqrt{\frac{\pi}{\beta}}\,\cosh\frac{1}{4\beta}, \quad \Re\{\beta\}>0.$$

12. $$\int_0^\infty e^{-\beta x^2}\,(\sinh x - \cos x)\,dx = \frac{1}{2}\sqrt{\frac{\pi}{\beta}}\,\exp\left(-\frac{1}{4\beta}\right)\left[\exp\left(\frac{1}{2\beta}\right)\operatorname{erf}\left(\frac{1}{2\sqrt{\beta}}\right)-1\right], \quad \Re\{\beta\}>0.$$

13. $$\int_0^\infty e^{-\beta x^2}\,(\sinh x + \cos x)\,dx = \frac{1}{2}\sqrt{\frac{\pi}{\beta}}\,\exp\left(-\frac{1}{4\beta}\right)\left[\exp\left(\frac{1}{2\beta}\right)\operatorname{erf}\left(\frac{1}{2\sqrt{\beta}}\right)+1\right], \quad \Re\{\beta\}>0.$$

14. $$\int_0^\infty \sin ax^2\,\cosh 2\gamma x\,e^{-\beta x^2}\,dx = \frac{1}{2}\left(\frac{\pi^2}{a^2+\beta^2}\right)^{1/4}\exp\left(-\frac{\beta\gamma^2}{a^2+\beta^2}\right)\sin\left(\frac{a\gamma^2}{a^2+\beta^2}+\frac{1}{2}\arctan\frac{a}{\beta}\right), \quad \Re\{\beta\}>0.$$

15. $$\int_0^\infty \cos ax^2\,\cosh 2\gamma x\,e^{-\beta x^2}\,dx = \frac{1}{2}\left(\frac{\pi^2}{a^2+\beta^2}\right)^{1/4}\exp\left(-\frac{\beta\gamma^2}{a^2+\beta^2}\right)\cos\left(\frac{a\gamma^2}{a^2+\beta^2}+\frac{1}{2}\arctan\frac{a}{\beta}\right), \quad \Re\{\beta\}>0.$$

16. $$\int_0^\infty (\sinh x^2 + \sin x^2)\,e^{-\beta x^4}\,dx = \frac{\sqrt{2}\pi}{4\sqrt{\beta}}\,I_{1/4}\left(\frac{1}{8\beta}\right)\cosh\frac{1}{8\beta}, \quad \Re\{\beta\}>0.$$

17. $$\int_0^\infty (\sinh x^2 - \sin x^2)\,e^{-\beta x^4}\,dx = \frac{\sqrt{2}\pi}{4\sqrt{\beta}}\,I_{1/4}\left(\frac{1}{8\beta}\right)\sinh\frac{1}{8\beta}, \quad \Re\{\beta\}>0.$$

18. $$\int_0^\infty (\cosh x^2 + \cos x^2)\, e^{-\beta x^4}\, dx = \frac{\sqrt{2}\pi}{4\sqrt{\beta}}\, I_{-1/4}\left(\frac{1}{8\beta}\right) \cosh\frac{1}{8\beta}, \quad \Re\{\beta\} > 0.$$

19. $$\int_0^\infty (\cosh x^2 - \cos x^2)\, e^{-\beta x^4}\, dx = \frac{\sqrt{2}\pi}{4\sqrt{\beta}}\, I_{-1/4}\left(\frac{1}{8\beta}\right) \sinh\frac{1}{8\beta}, \quad \Re\{\beta\} > 0.$$

20. $$\int_0^\infty \sin 2x^2\, \sinh 2x^2\, e^{-\beta x^4}\, dx = \frac{\pi}{(128\beta^2)^{1/4}}\, J_{-1/4}\left(\frac{1}{\beta}\right) \cos\left(\frac{1}{\beta} + \frac{\pi}{4}\right), \quad \Re\{\beta\} > 0.$$

21. $$\int_0^\infty \sin 2x^2\, \cosh 2x^2\, e^{-\beta x^4}\, dx = \frac{\pi}{(128\beta^2)^{1/4}}\, J_{1/4}\left(\frac{1}{\beta}\right) \cos\left(\frac{1}{\beta} - \frac{\pi}{4}\right), \quad \Re\{\beta\} > 0.$$

22. $$\int_0^\infty \cos 2x^2\, \sinh 2x^2\, e^{-\beta x^4}\, dx = \frac{\pi}{(128\beta^2)^{1/4}}\, J_{1/4}\left(\frac{1}{\beta}\right) \sin\left(\frac{1}{\beta} - \frac{\pi}{4}\right), \quad \Re\{\beta\} > 0.$$

23. $$\int_0^\infty \cos 2x^2\, \cosh 2x^2\, e^{-\beta x^4}\, dx = \frac{\pi}{(128\beta^2)^{1/4}}\, J_{-1/4}\left(\frac{1}{\beta}\right) \sin\left(\frac{1}{\beta} + \frac{\pi}{4}\right), \quad \Re\{\beta\} > 0.$$

24. $$\int_0^\infty (\sin 2x^2\, \cosh 2x^2 + \cos 2x^2\, \sinh 2x^2)\, e^{-\beta x^4}\, dx = \frac{\pi}{(32\beta^2)^{1/4}}\, J_{1/4}\left(\frac{1}{\beta}\right) \cos\left(\frac{1}{\beta}\right), \quad \Re\{\beta\} > 0.$$

25. $$\int_0^\infty (\sin 2x^2\, \cosh 2x^2 - \cos 2x^2\, \sinh 2x^2)\, e^{-\beta x^4}\, dx = \frac{\pi}{(32\beta^2)^{1/4}}\, J_{1/4}\left(\frac{1}{\beta}\right) \sin\left(\frac{1}{\beta}\right), \quad \Re\{\beta\} > 0.$$

26. $$\int_0^\infty (\cos 2x^2\, \cosh 2x^2 + \sin 2x^2\, \sinh 2x^2)\, e^{-\beta x^4}\, dx = \frac{\pi}{(32\beta^2)^{1/4}}\, J_{-1/4}\left(\frac{1}{\beta}\right) \cos\left(\frac{1}{\beta}\right), \quad \Re\{\beta\} > 0.$$

27. $$\int_0^\infty (\cos 2x^2\, \cosh 2x^2 - \sin 2x^2\, \sinh 2x^2)\, e^{-\beta x^4}\, dx = \frac{\pi}{(32\beta^2)^{1/4}}\, J_{-1/4}\left(\frac{1}{\beta}\right) \sin\left(\frac{1}{\beta}\right), \quad \Re\{\beta\} > 0.$$