T1.28. Integrand involving products of trigonometric functions and powers of $(a + b\,x)$.

Notation used: $X = a + bx$ (in Formulas 26-37 and 58-69).

1. $$\int x \, \sin ax \, dx = \frac{1}{a^2} \sin ax - \frac{x}{a} \cos ax.$$

2. $$\int x^2 \, \sin ax \, dx = \frac{2x}{a^2} \sin ax + \frac{2 - a^2x^2}{a^3} \cos ax$$

3. $$\int x^3 \, \sin ax \, dx = \frac{3a^2x^2 - 6}{a^4} \sin ax + \frac{6x - a^2x^3}{a^3} \cos ax$$

4. $$\int x^n \, \sin ax \, dx = \begin{cases} -\dfrac{1}{a} x^n + \dfrac{n}{a} \displaystyle\int x^{n-1} \cos ax \, dx, \\ \qquad \text{or} \\ \cos ax \displaystyle\sum_{k=0}^{\lfloor n/2 \rfloor} \frac{(-1)^k n!}{(n-2k)!} \frac{x^{n-2k}}{a^{2k+1}} + \sin ax \sum_{k=0}^{\lfloor (n-1)/2 \rfloor} \frac{(-1)^k n!}{(m-2k-1)!} \frac{x^{m-2k-1}}{a^{2k+2}}, \\ \qquad \text{or} \\ -\displaystyle\sum_{k=0}^{n} k! \binom{n}{k} \frac{x^{n-k}}{a^{k+1}} \cos\left(ax + \frac{1}{2}k\pi\right). \end{cases}$$

5. $$\int x^{2n} \, \sin x \, dx = (2n)! \left\{ \sum_{k=0}^{n} (-1)^{k+1} \frac{x^{2n-2k}}{(2n-2k)!} \cos x + \sum_{k=0}^{n-1} (-1)^k \frac{x^{2n-2k-1}}{(2n-2k-1)!} \sin x \right\}.$$

6. $$\int x^{2n+1} \, \sin x \, dx = (2n+1)! \left\{ \sum_{k=0}^{n} (-1)^{k+1} \frac{x^{2n-2k+1}}{(2n-2k+1)!} \cos x + \sum_{k=0}^{n} (-1)^k \frac{x^{2n-2k}}{(2n-2k)!} \sin x \right\}.$$

7. $$\int x \, \cos ax \, dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax.$$

8. $$\int x^2 \, \cos ax \, dx = \frac{2x}{a^2} \cos ax + \frac{a^2x^2 - 2}{a^3} \sin ax$$

9. $$\int x^3 \cos ax\,dx = \frac{3a^2x^2-6}{a^4}\cos ax + \frac{a^2x^3-6x}{a^3}\sin ax$$

10. $$\int x^n \cos ax\,dx = \begin{cases} \dfrac{x^n}{a}\sin ax - \dfrac{n}{a}\displaystyle\int x^{n-1}\sin ax\,dx, \\ \text{or} \\ \sin ax \displaystyle\sum_{k=0}^{\lfloor n/2 \rfloor} \frac{(-1)^k n!}{(n-2k)!}\frac{x^{n-2k}}{a^{2k+1}} + \cos ax \sum_{k=0}^{\lfloor (n-1)/2 \rfloor} \frac{(-1)^k n!}{(m-2k-1)!}\frac{x^{m-2k-1}}{a^{2k+2}}, \\ \text{or} \\ \displaystyle\sum_{k=0}^{n} k!\binom{n}{k}\frac{x^{n-k}}{a^{k+1}}\sin\left(ax+\frac{1}{2}k\pi\right). \end{cases}$$

11. $$\int x^{2n}\cos x\,dx = (2n)!\left\{\sum_{k=0}^{n}(-1)^k\frac{x^{2n-2k}}{(2n-2k)!}\sin x + \sum_{k=0}^{n-1}(-1)^k\frac{x^{2n-2k-1}}{(2n-2k-1)!}\cos x\right\}.$$

12. $$\int x^{2n+1}\cos x\,dx = (2n+1)!\left\{\sum_{k=0}^{n}(-1)^k\frac{x^{2n-2k+1}}{(2n-2k+1)!}\sin x + \sum_{k=0}^{n}\frac{x^{2n-2k}}{(2n-2k)!}\cos x\right\}.$$

13. $$\int x^m \sin^n x\,dx = \frac{x^{m-1}\sin^{n-1}x}{n^2}\{m\sin x - nx\cos x\} + \frac{n-1}{n}\int x^m\sin^{n-2}x\,dx - \frac{m(m-1)}{n^2}\int x^{m-2}\sin^n x\,dx.$$

14. $$\int x^m \cos^n x\,dx = \frac{x^{m-1}\cos^{n-1}x}{n^2}\{m\cos x + nx\sin x\} + \frac{n-1}{n}\int x^m\cos^{n-2}x\,dx - \frac{m(m-1)}{n^2}\int x^{m-2}\cos^n x\,dx.$$

15. $$\int x^n\sin^{2m}x\,dx = \binom{2m}{m}\frac{x^{n+1}}{2^{2m}(n+1)} + \frac{(-1)^m}{2^{2m-1}}\sum_{k=0}^{m-1}(-1)^k\binom{2m}{k}\int x^n\cos(2m-2k)x\,dx.$$

16. $$\int x^n\sin^{2m+1}x\,dx = \frac{(-1)^m}{2^{2m}}\sum_{k=0}^{m}(-1)^k\binom{2m+1}{k}\int x^n\sin(2m-2k+1)x\,dx.$$

17. $$\int x^n\cos^{2m}x\,dx = \binom{2m}{m}\frac{x^{n+1}}{2^{2m}(n+1)} + \frac{1}{2^{2m-1}}\sum_{k=0}^{m-1}\binom{2m}{k}\int x^n\cos(2m-2k)x\,dx.$$

18. $$\int x^n \cos^{2m+1} x\,dx = \frac{1}{2^{2m}}\sum_{k=0}^{m}\binom{2m+1}{k}\int x^n \cos(2m-2k+1)x\,dx.$$

19. $$\int x^{\nu-1} \sin\beta x\,dx = \frac{i}{2}(i\beta)^{-\nu}\gamma(\nu,\,i\beta x) - \frac{i}{2}(-i\beta)^{-\nu}\gamma(\nu,\,-i\beta x), \quad \Re\,\nu > -1,\ x > 0.$$

20. $$\int x^{\nu-1} \sin ax\,dx = -\frac{1}{2a^\nu}\left\{\exp\left[\frac{\pi i}{2}(\nu-1)\right]\Gamma(\nu,\,-iax) + \exp\left[\frac{\pi i}{2}(1-\nu)\right]\Gamma(\nu,\,iax)\right\},$$
$$\Re\,\nu < 1,\ a > 0,\ x > 0.$$

21. $$\int x^{\nu-1} \cos\beta x\,dx = \frac{1}{2}\{(i\beta)^{-\nu}\gamma(\nu,\,i\beta x) + (-i\beta)^{-\nu}\gamma(\nu,\,-i\beta x)\}, \quad \Re\,\nu > 0,\ x > 0.$$

22. $$\int x^{\nu-1} \cos ax\,dx = -\frac{1}{2a^\nu}\left\{\exp\left(i\nu\frac{\pi}{2}\right)\Gamma(\nu,\,-iax) + \exp\left(-i\nu\frac{\pi}{2}\right)\Gamma(\nu,\,iax)\right\}.$$

23. $$\int x^r \sin^p x \cos^q x\,dx = \begin{cases} \dfrac{1}{(p+q)^2}\Big[(p+q)x^r \sin^{p+1} x \cos^{q-1} x \\ \quad + r x^{r-1} \sin^p x \cos^q x - r(r-1)\displaystyle\int x^{r-2} \sin^p x \cos^q x\,dx \\ \quad - rp\displaystyle\int x^{r-1} \sin^{p-1} x \cos^{q-1} x\,dx + (q-1)(p+q)\displaystyle\int x^r \sin^p x \cos^{q-2} x\,dx\Big], \\ \quad \text{or} \\ \dfrac{1}{(p+q)^2}\Big[-(p+q)x^r \sin^{p-1} x \cos^{q+1} x + r x^{r-1} \sin^p x \cos^q x \\ \quad - r(r-1)\displaystyle\int x^{r-2} \sin^p x \cos^q x\,dx + rq\displaystyle\int x^{r-1} \sin^{p-1} x \cos^{q-1} x\,dx \\ \quad + (p-1)(p+q)\displaystyle\int x^r \sin^{p-2} x \cos^q x\,dx\Big]. \end{cases}$$

24. $$\int P_n(x) \sin mx\,dx = -\frac{\cos mx}{m}\sum_{k=0}^{[n/2]}(-1)^k\frac{P_n^{(2k)}(x)}{m^{2k}} + \frac{\sin mx}{m}\sum_{k=1}^{[(n+1)/2]}(-1)^{k-1}\frac{P_n^{(2k-1)}(x)}{m^{2k-1}},$$

25. $$\int P_n(x) \cos mx\,dx = \frac{\sin mx}{m}\sum_{k=0}^{[n/2]}(-1)^k\frac{P_n^{(2k)}(x)}{m^{2k}} + \frac{\cos mx}{m}\sum_{k=1}^{[(n+1)/2]}(-1)^{k-1}\frac{P_n^{(2k-1)}(x)}{m^{2k-1}},$$

where in these two formulas $P_n(x) \in \mathcal{P}_n$ and $P_n^{(k)}(x)$ is its k th derivative with respect to x .

26. $$\int X \sin kx\,dx = -\frac{1}{k}X \cos kx + \frac{b}{k^2} \sin kx.$$

27. $$\int X \ \cos kx \, dx = \frac{1}{k} X \ \sin kx + \frac{b}{k^2} \ \cos kx.$$

28. $$\int X^2 \ \sin kx \, dx = \frac{1}{k} \left(\frac{2b^2}{k^2} - X^2 \right) \cos kx + \frac{2bX}{k^2} \ \sin kx.$$

29. $$\int X^2 \ \cos kx \, dx = \frac{1}{k} \left(X^2 - \frac{2b^2}{k^2} \right) \sin kx + \frac{2bX}{k^2} \ \cos kx.$$

30. $$\int X^3 \ \sin kx \, dx = \frac{X}{k} \left(\frac{6b^2}{k^2} - X^2 \right) \cos kx + \frac{3b}{k^2} \left(X^2 - \frac{2b^2}{k^2} \right) \sin kx.$$

31. $$\int X^3 \ \cos kx \, dx = \frac{X}{k} \left(X^2 - \frac{6b^2}{k^2} \right) \sin kx + \frac{3b}{k^2} \left(X^2 - \frac{2b^2}{k^2} \right) \cos kx.$$

32. $$\int X^4 \ \sin kx \, dx = -\frac{1}{k} \left(X^4 - \frac{12b^2}{k^2} X^2 + \frac{24b^4}{k^4} \right) \cos kx + \frac{4bX}{k^2} \left(X^2 - \frac{6b^2}{k^2} \right) \sin kx.$$

33. $$\int X^4 \ \cos kx \, dx = \frac{1}{k} \left(X^4 - \frac{12b^2}{k^2} X^2 + \frac{24b^4}{k^4} \right) \sin kx + \frac{4bX}{k^2} \left(X^2 - \frac{6b^2}{k^2} \right) \cos kx.$$

34. $$\int X^5 \ \sin kx \, dx = \frac{5b}{k^2} \left(X^4 - \frac{12b^2}{k^2} X^2 + \frac{24b^4}{k^4} \right) \sin kx - \frac{X}{k} \left(X^4 - \frac{20b^2}{k^2} X^2 + \frac{120b^4}{k^4} \right) \cos kx.$$

35. $$\int X^5 \ \cos kx \, dx = \frac{5b}{k^2} \left(X^4 - \frac{12b^2}{k^2} X^2 + \frac{24b^4}{k^4} \right) \cos kx + \frac{X}{k} \left(X^4 - \frac{20b^2}{k^2} X^2 + \frac{120b^4}{k^4} \right) \sin kx.$$

36. $$\int X^6 \ \sin kx \, dx = \frac{6bX}{k^2} \left(X^4 - \frac{20b^2}{k^2} X^2 + \frac{120b^4}{k^4} \right) \sin kx - \frac{1}{k} \left(X^6 - \frac{30b^2}{k^2} z_1^4 + \frac{360b^4}{k^4} X^2 - \frac{720b^6}{k^6} \right) \cos kx.$$

37. $$\int X^6 \ \cos kx \, dx = \frac{6bX}{k^2} \left(X^4 - \frac{20b^2}{k^2} X^2 + \frac{120b^4}{k^4} \right) \cos kx + \frac{1}{k} \left(X^6 - \frac{30b^2}{k^2} X^4 + \frac{360b^4}{k^4} X^2 - \frac{720b^6}{k^6} \right) \sin kx.$$

38. $$\int x^n \sin^2 x\,dx = \frac{x^{n+1}}{2(n+1)} + \frac{n!}{4}\left\{\sum_{k=0}^{[n/2]} \frac{(-1)^{k+1} x^{n-2k}}{2^{2k}(n-2k)!}\sin 2x + \sum_{k=0}^{[(n-1)/2]} \frac{(-1)^{k+1} x^{n-2k-1}}{2^{2k+1}(n-2k-1)!}\cos 2x\right\}.$$

39. $$\int x^n \cos^2 x\,dx = \frac{x^{n+1}}{2(n+1)} - \frac{n!}{4}\left\{\sum_{k=0}^{[n/2]} \frac{(-1)^{k+1} x^{n-2k}}{2^{2k}(n-2k)!}\sin 2x + \sum_{k=0}^{[(n-1)/2]} \frac{(-1)^{k+1} x^{n-2k-1}}{2^{2k+1}(n-2k-1)!}\cos 2x\right\}.$$

40. $$\int x \sin^2 x\,dx = \frac{x^2}{4} - \frac{x}{4}\sin 2x - \frac{1}{8}\cos 2x.$$

41. $$\int x^2 \sin^2 x\,dx = \frac{x^3}{6} - \frac{x}{4}\cos 2x - \frac{1}{4}\left(x^2 - \frac{1}{2}\right)\sin 2x.$$

42. $$\int x \cos^2 x\,dx = \frac{x^2}{4} + \frac{x}{4}\sin 2x + \frac{1}{8}\cos 2x.$$

43. $$\int x^2 \cos^2 x\,dx = \frac{x^3}{6} + \frac{x}{4}\cos 2x + \frac{1}{4}\left(x^2 - \frac{1}{2}\right)\sin 2x.$$

44. $$\int x^n \sin^3 x\,dx = \frac{n!}{4}\left\{\sum_{k=0}^{[n/2]} \frac{(-1)^k x^{n-2k}}{(n-2k)!}\left(\frac{\cos 3x}{3^{2k+1}} - 3\cos x\right) - \sum_{k=0}^{[(n-1)/2]} (-1)^k \frac{x^{n-2k-1}}{(n-2k-1)!}\left(\frac{\sin 3x}{3^{2k+2}} - 3\sin x\right)\right\}.$$

45. $$\int x^n \cos^3 x\,dx = \frac{n!}{4}\left\{\sum_{k=0}^{[n/2]} \frac{(-1)^k x^{n-2k}}{(n-2k)!}\left(\frac{\sin 3x}{3^{2k+1}} + 3\sin x\right) + \sum_{k=0}^{[(n-1)/2]} (-1)^k \frac{x^{n-2k-1}}{(n-2k-1)!}\left(\frac{\cos 3x}{3^{2k+2}} + 3\cos x\right)\right\}.$$

46. $$\int x \sin^3 x\,dx = \frac{3}{4}\sin x - \frac{1}{36}\sin 3x - \frac{3}{4}x\cos x + \frac{x}{12}\cos 3x.$$

47. $$\int x^2 \sin^3 x\,dx = -\left(\frac{3}{4}x^2 + \frac{3}{2}\right)\cos x + \left(\frac{x^2}{12} + \frac{1}{54}\right)\cos 3x + \frac{3}{2}x\sin x - \frac{x}{18}\sin 3x.$$

48. $$\int x \cos^3 x\,dx = \frac{3}{4}\cos x + \frac{1}{36}\cos 3x + \frac{3}{4}x\sin x + \frac{x}{12}\sin 3x.$$

49. $$\int x^2 \cos^3 x\,dx = \left(\frac{3}{4}x^2 - \frac{3}{2}\right)\sin x + \left(\frac{x^2}{12} - \frac{1}{54}\right)\sin 3x + \frac{3}{2}x\cos x + \frac{x}{18}\cos 3x.$$

50. $$\int \frac{\sin^q x}{x^p}\,dx = -\frac{\sin^{q-1} x[(p-2)\sin x + qx\cos x]}{(p-1)(p-2)x^{p-1}} - \frac{q^2}{(p-1)(p-2)}\int \frac{\sin^q x\,dx}{x^{p-2}} + \frac{q(q-1)}{(p-1)(p-2)}\int \frac{\sin^{q-2} x\,dx}{x^{p-2}}, \quad p \neq 1,\ p \neq 2.$$

51. $$\int \frac{\cos^q x}{x^p}\,dx = -\frac{\cos^{q-1} x[(p-2)\cos x - qx\sin x]}{(p-1)(p-2)x^{p-1}} - \frac{q^2}{(p-1)(p-2)}\int \frac{\cos^q x\,dx}{x^{p-2}} + \frac{q(q-1)}{(p-1)(p-2)}\int \frac{\cos^{q-2} x\,dx}{x^{p-2}}, \quad p \neq 1,\ p \neq 2.$$

52. $$\int \frac{\sin x\,dx}{x^p} = -\frac{\sin x}{(p-1)x^{p-1}} + \frac{1}{p-1}\int \frac{\cos x\,dx}{x^{p-1}} = -\frac{\sin x}{(p-1)x^{p-1}} - \frac{\cos x}{(p-1)(p-2)x^{p-2}} - \frac{1}{(p-1)(p-2)}\int \frac{\sin x\,dx}{x^{p-2}}, \quad p > 2.$$

53. $$\int \frac{\cos x\,dx}{x^p} = -\frac{\cos x}{(p-1)x^{p-1}} - \frac{1}{p-1}\int \frac{\sin x\,dx}{x^{p-1}} = -\frac{\cos x}{(p-1)x^{p-1}} + \frac{\sin x}{(p-1)(p-2)x^{p-2}} - \frac{1}{(p-1)(p-2)}\int \frac{\cos x\,dx}{x^{p-2}}, \quad p > 2.$$

54. $$\int \frac{\sin x\,dx}{x^{2n}} = \frac{(-1)^{n+1}}{x(2n-1)!}\left\{\sum_{k=0}^{n-2}\frac{(-1)^k(2k+1)!}{x^{2k+1}}\cos x + \sum_{k=0}^{n-1}\frac{(-1)^{k+1}(2k)!}{x^{2k}}\sin x\right\} + \frac{(-1)^{n+1}}{(2n-1)!}\,\mathrm{ci}\,(x).$$

55. $$\int \frac{\sin x}{x^{2n+1}} dx = \frac{(-1)^{n+1}}{x(2n)!}\left\{\sum_{k=0}^{n-1} \frac{(-1)^{k+1}(2k)!}{x^{2k}} \cos x + \sum_{k=0}^{n-1} \frac{(-1)^{k+1}(2k+1)!}{x^{2k+1}} \sin x\right\} + \frac{(-1)^n}{(2n)!} \operatorname{si}(x).$$

56. $$\int \frac{\cos x\, dx}{x^{2n}}\, dx = \frac{(-1)^{n+1}}{x(2n-1)!}\left\{\sum_{k=0}^{n-1} \frac{(-1)^{k+1}(2k)!}{x^{2k}} \cos x - \sum_{k=0}^{n-2} \frac{(-1)^{k}(2k+1)!}{x^{2k+1}} \sin x\right\} + \frac{(-1)^n}{(2n-1)!} \operatorname{si}(x).$$

57. $$\int \frac{\cos x\, dx}{x^{2n+1}} = \frac{(-1)^{n+1}}{x(2n)!}\left\{\sum_{k=0}^{n-1} \frac{(-1)^{k+1}(2k+1)!}{x^{2k+1}} \cos x - \sum_{k=0}^{n-1} \frac{(-1)^{k+1}(2k)!}{x^{2k}} \sin x\right\} + \frac{(-1)^n}{(2n)!} \operatorname{ci}(x).$$

58. $$\int \frac{\sin kx}{X}\, dx = \frac{1}{b}\left[\cos\frac{ka}{b} \operatorname{si}(kX/b) - \sin\frac{ka}{b} \operatorname{ci}(kX/b)\right].$$

59. $$\int \frac{\cos kx}{X}\, dx = \frac{1}{b}\left[\cos\frac{ka}{b} \operatorname{ci}(kX/b) + \sin\frac{ka}{b} \operatorname{si}(kX/b)\right].$$

60. $$\int \frac{\sin kx}{X^2}\, dx = -\frac{1}{b}\frac{\sin kx}{X} + \frac{k}{b}\int \frac{\cos kx}{X}\, dx.$$

61. $$\int \frac{\cos kx}{X^2}\, dx = -\frac{1}{b}\frac{\cos kx}{X} - \frac{k}{b}\int \frac{\sin kx}{X}\, dx.$$

62. $$\int \frac{\sin kx}{X^3}\, dx = -\frac{\sin kx}{2bX^2} - \frac{k\, \cos kx}{2b^2 X} - \frac{k^2}{2b^2}\int \frac{\sin kx}{X}\, dx.$$

63. $$\int \frac{\cos kx}{X^3}\, dx = -\frac{\cos kx}{2bX^2} + \frac{k\, \sin kx}{2b^2 X} - \frac{k^2}{2b^2}\int \frac{\cos kx}{X}\, dx.$$

64. $$\int \frac{\sin kx}{X^4}\, dx = -\frac{\sin kx}{3bX^3} - \frac{k\, \cos kx}{6b^2X^2} + \frac{k^2\, \sin kx}{6b^2 X} - \frac{k^3}{6b^3}\int \frac{\cos kx}{X}\, dx.$$

65. $$\int \frac{\cos kx}{X^4}\,dx = -\frac{\cos kx}{3bX^3} + \frac{k\,\sin kx}{6b^2X^2} + +\frac{k^2\,\cos kx}{6b^3X} + \frac{k^3}{6b^3}\int \frac{\sin kx}{X}\,dx.$$

66. $$\int \frac{\sin kx}{X^5}\,dx = -\frac{\sin kx}{4bX^4} - \frac{k\,\cos kx}{12b^2X^3} + \frac{k^2\,\sin kx}{24b^3X^2} + \frac{k^3\,\cos kx}{24b^4X}\frac{k^4}{24b^4}\int \frac{\sin kx}{X}\,dx.$$

67. $$\int \frac{\cos kx}{X^5}\,dx = -\frac{\cos kx}{4bX^4} + \frac{k\,\sin kx}{12b^2X^3} + \frac{k^2\,\cos kx}{24b^3X^2} - \frac{k^3\,\sin kx}{24b^4X} + \frac{k^4}{24b^4}\int \frac{\cos kx}{X}\,dx.$$

68. $$\int \frac{\sin kx}{X^6}\,dx = -\frac{\sin kx}{5bX^5} - \frac{k\,\cos kx}{20b^2X^4} + \frac{k^2\,\sin kx}{60b^3X^3} + \frac{k^3\,\cos kx}{120b^4X^2} - \frac{k^4\,\sin kx}{120b^5X} + \frac{k^5}{120b^5}\int \frac{\cos kx}{X}\,dx.$$

69. $$\int \frac{\cos kx}{X^6}\,dx = -\frac{\cos kx}{5bX^5} + \frac{k\,\sin kx}{20b^2X^4} + \frac{k^2\,\cos kx}{60b^3X^3} - \frac{k^3\,\sin kx}{120b^4X^2} - \frac{k^4\,\cos kx}{120b^5X} - \frac{k^5}{120b^5}\int \frac{\sin kx}{X}\,dx.$$

70. $$\int \frac{\sin^{2m}\,x}{x}\,dx = \binom{2m}{m}\frac{\ln\,x}{2^{2m}} + \frac{(-1)^m}{2^{2m-1}}\sum_{k=0}^{m-1}(-1)^k\binom{2m}{k}\,\mathrm{ci}\,[(2m-2k)x].$$

71. $$\int \frac{\sin^{2m+1}\,x}{x}\,dx = \frac{(-1)^m}{2^{2m}}\sum_{k=0}^{m}(-1)^k\binom{2m+1}{k}\,\mathrm{si}\,[(2m-2k+1)x].$$

72. $$\int \frac{\cos^{2m}\,x}{x}\,dx = \binom{2m}{m}\frac{\ln\,x}{2^{2m}} + \frac{1}{2^{2m-1}}\sum_{k=0}^{m-1}\binom{2m}{k}\,\mathrm{ci}\,[(2m-2k)x].$$

73. $$\int \frac{\cos^{2m+1}\,x}{x}\,dx = \frac{1}{2^{2m}}\sum_{k=0}^{m}\binom{2m+1}{k}\,\mathrm{ci}\,[(2m-2k+1)x].$$

74. $$\int \frac{\sin^{2m}\,x}{x^2}\,dx = -\binom{2m}{m}\frac{1}{2^{2m}x}$$
$$+\frac{(-1)^m}{2^{2m-1}}\sum_{k=0}^{m-1}(-1)^{k+1}\binom{2m}{k}\left\{\frac{\cos(2m-2k)x}{x} + (2m-2k)\,\mathrm{si}\,[(2m-2k)x]\right\}.$$

75. $$\int \frac{\sin^{2m+1}\,x}{x^2}\,dx = \frac{(-1)^m}{2^{2m}}\sum_{k=0}^{m}(-1)^{k+1}\binom{2m+1}{k}$$
$$\times\left\{\frac{\sin(2m-2k+1)x}{x} - (2m-2k+1)\,\mathrm{ci}\,[(2m-2k+1)x]\right\}.$$

76. $$\int \frac{\cos^{2m} x}{x^2}\,dx = -\binom{2m}{m}\frac{1}{2^{2m}x} - \frac{1}{2^{2m-1}}\sum_{k=0}^{m-1}\binom{2m}{k}\left\{\frac{\cos(2m-2k)x}{x} + (2m-2k)\operatorname{si}[(2m-2k)x]\right\}.$$

77. $$\int \frac{\cos^{2m+1} x}{x^2} = -\frac{1}{2^{2m}}\sum_{k=0}^{m}\binom{2m+1}{k}\left\{\frac{\cos(2m-2k+1)x}{x} + (2m-2k+1)\operatorname{si}[(2m-2k+1)x]\right\}.$$

78. $$\int \frac{x^p\,dx}{\sin^q x} = -\frac{x^{p-1}[p\sin x + (q-2)x\cos x]}{(q-1)(q-2)\sin^{q-1} x} + \frac{q-2}{q-1}\int \frac{x^p\,dx}{\sin^{q-2} x} + \frac{p(p-1)}{(q-1)(q-2)}\int \frac{x^{p-2}\,dx}{\sin^{q-2} x}.$$

79. $$\int \frac{x^p\,dx}{\cos^q x} = -\frac{x^{p-1}[p\cos x - (q-2)x\sin x]}{(q-1)(q-2)\cos^{q-1} x} + \frac{q-2}{q-1}\int \frac{x^p\,dx}{\cos^{q-2} x} + \frac{p(p-1)}{(q-1)(q-2)}\int \frac{x^{p-2}\,dx}{\cos^{q-2} x}.$$

80. $$\int \frac{x^n}{\sin x}\,dx = \frac{x^n}{n} + \sum_{k=1}^{\infty}(-1)^{k+1}\frac{2(2^{2k-1}-1)}{(n+2k)(2k)!}B_{2k}x^{n+2k}, \quad |x| < \pi,\ n > 0.$$

81. $$\int \frac{dx}{x^n\sin x} = -\frac{1}{nx^n} - [1+(-1)^n](-1)^{\frac{n}{2}}\frac{2^{n-1}-1}{n!}B_n \ln x - \sum_{\substack{k=1\\ k\neq n/2}}^{\infty}(-1)^k\frac{2(2^{2n}-1)}{(2k-n)\,(2k)!}B_{2k}x^{2k-n}, \quad n > 1,\ |x| > \pi.$$

82. $$\int \frac{x^n\,dx}{\cos x} = \sum_{k=0}^{\infty}\frac{|E_{2k}|x^{n+2k+1}}{(n+2k+1)(2k)!}, \quad |x| < \frac{\pi}{2},\ n > 0.$$

83. $$\int \frac{dx}{x^n\cos x} = \frac{1}{2}[1-(-1)^n]\frac{|E_{n-1}|}{(n-1)!}\ln x + \sum_{\substack{k=0\\ k\neq(n-1)/2}}^{\infty}\frac{|E_{2k}|x^{2k-n+1}}{(2k-n+1)\,(2k)!}, \quad |x| < \frac{\pi}{2}.$$

84. $$\int \frac{x^n\,dx}{\sin^2 x} = -x^n\cot x + \frac{n}{n-1}x^{n-1} + n\sum_{k=1}^{\infty}(-1)^k\frac{2^{2k}x^{n+2k-1}}{(n+2k-1)(2k)!}B_{2k}, \quad |x| < \pi,\ n > 1.$$

85. $$\int \frac{dx}{x^n\sin^2 x} = -\frac{\cot x}{x^n} + \frac{n}{(n+1)x^{n+1}} - [1-(-1)^n](-1)^{\frac{n+1}{2}}\frac{2^n n}{(n+1)!}B_{n+1}\ln x$$

$$-\frac{n}{2^{n+1}} \sum_{\substack{k=1 \\ k\neq(n+1)/2}}^{\infty} \frac{(-1)^k (2x)^{2k}}{(2k-n-1)(2k)!} B_{2k}, \quad |x| < \pi.$$

86. $$\int \frac{x^n\,dx}{\cos^2 x} = x^n \tan x + n \sum_{k=1}^{\infty} (-1)^k \frac{2^{2k}(2^{2k}-1)x^{n+2k-1}}{(n+2k-1)\,(2k)!} B_{2k}, \quad n > 1,\ |x| < \frac{\pi}{2}.$$

87. $$\int \frac{dx}{x^n \cos^2 x} = \frac{\tan x}{x^n} - [1-(-1)^n](-1)^{\frac{n+1}{2}} \frac{2^n n}{(n+1)!}(2^{n+1}-1)B_{n+1} \ln x$$
$$-\frac{n}{x^{n+1}} \sum_{\substack{k=1 \\ k\neq(n+1)/2}}^{\infty} \frac{(-1)^k (2^{2k}-1)(2x)^{2k}}{(2k-n-1)(2k)!} B_{2k}, \quad |x| < \frac{\pi}{2}.$$

88. $$\int \frac{x\,dx}{\sin^{2n} x} = -\sum_{k=0}^{n-1} \frac{(2n-2)(2n-4)\dots(2n-2k+2)}{(2n-1)(2n-3)\dots(2n-2k+3)} \frac{\sin x + (2n-2k)x \cos x}{(2n-2k+1)(2n-2k) \sin^{2n-2k+1} x}$$
$$+\frac{2^{n-1}(n-1)!}{(2n-1)!!}(\ln \sin x - x \cot x).$$

89. $$\int \frac{x\,dx}{\sin^{2n+1} x} = -\sum_{k=0}^{n-1} \frac{(2n-1)(2n-3)\dots(2n-2k+1)}{2n(2n-2)\dots(2n-2k+2)} \frac{\sin x + (2n-2k-1)x \cos x}{(2n-2k)(2n-2k-1) \sin^{2n-2k} x}$$
$$+\frac{(2n-1)!!}{2^n n!} \int \frac{x\,dx}{\sin x}.$$

90. $$\int \frac{x\,dx}{\cos^{2n} x} = \sum_{k=0}^{n-1} \frac{(2n-2)(2n-4)\dots(2n-2k+2)}{(2n-1)(2n-3)\dots(2n-2k+3)} \frac{(2n-2k)x \sin x - \cos x}{(2n-2k+1)(2n-2k) \cos^{2n-2k+1} x}$$
$$+\frac{2^{n-1}(n-1)!}{(2n-1)!!}(x \tan x + \ln \cos x).$$

91. $$\int \frac{x\,dx}{\cos^{2n+1} x} = \sum_{k=0}^{n-1} \frac{(2n-1)(2n-3)\dots(2n-2k+1)}{2n(2n-2)\dots(2n-2k+2)} \frac{(2n-2k+1)x \sin x - \cos x}{(2n-2k)(2n-2k-1) \cos^{2n-2k} x}$$
$$+\frac{(2n-1)!!}{2^n n!} \int \frac{x\,dx}{\cos x}.$$

92. $$\int \frac{x\,dx}{\sin x} = x + \sum_{k=1}^{\infty} (-1)^{k+1} \frac{2(2^{2k-1}-1)}{(2k+1)!} B_{2k} x^{2k+1}.$$

93. $$\int \frac{x\,dx}{\cos x} = \sum_{k=0}^{\infty} \frac{|E_{2k}| x^{2k+2}}{(2k+2)(2k)!}.$$

94. $$\int \frac{x\,dx}{\sin^2 x} = -x \cot x + \ln \sin x.$$

95. $$\int \frac{x\,dx}{\cos^2 x} = x \tan x + \ln \cos x.$$

96. $$\int \frac{x\,dx}{\sin^3 x} = -\frac{\sin x + x \cos x}{2 \sin^2 x} + \frac{1}{2} \int \frac{x}{\sin x}\,dx.$$

97. $$\int \frac{x\,dx}{\cos^3 x} = \frac{x \sin x - \cos x}{2 \cos^2 x} + \frac{1}{2} \int \frac{x\,dx}{\cos x}.$$

98. $$\int \frac{x\,dx}{\sin^4 x} = -\frac{x \cos x}{3 \sin^3 x} - \frac{1}{6 \sin^2 x} - \frac{2}{3} x \cot x + \frac{2}{3} \ln(\sin x).$$

99. $$\int \frac{x\,dx}{\cos^4 x} = \frac{x \sin x}{3 \cos^3 x} - \frac{1}{6 \cos^2 x} + \frac{2}{3} x \tan x - \frac{2}{3} \ln(\cos x).$$

100. $$\int \frac{x\,dx}{\sin^5 x} = -\frac{x \cos x}{4 \sin^4 x} - \frac{1}{12 \sin^3 x} - \frac{3x \cos x}{8 \sin^2 x} - \frac{3}{8 \sin x} + \frac{3}{8} \int \frac{x\,dx}{\sin x}.$$

101. $$\int \frac{x\,dx}{\cos^5 x} = \frac{x \sin x}{4 \cos^4 x} - \frac{1}{12 \cos^3 x} + \frac{3x \sin x}{8 \cos^2 x} - \frac{3}{8 \cos x} + \frac{3}{8} \int \frac{x\,dx}{\cos x}.$$

102. $$\int x^p \frac{\sin^{2m} x}{\cos^n x}\,dx = \sum_{k=0}^{m} (-1)^k \binom{m}{k} \int \frac{x^p\,dx}{\cos^{n-2k} x}.$$

103. $$\int x^p \frac{\sin^{2m+1} x}{\cos^n x}\,dx = \sum_{k=0}^{m} (-1)^k \binom{m}{k} \int \frac{x^p \sin x}{\cos^{n-2k} x}.$$

104. $$\int x^p \frac{\sin x\,dx}{\cos^n x} = \frac{x^p}{(n-1) \cos^{n-1} x} - \frac{p}{n-1} \int \frac{x^{p-1}}{\cos^{n-1} x}\,dx, \quad n > 1.$$

105. $$\int x^p \frac{\cos^{2m} x}{\sin^n x}\,dx = \sum_{k=0}^{m} (-1)^k \binom{m}{k} \int \frac{x^p\,dx}{\sin^{n-2k} x}.$$

106. $\displaystyle\int x^p \frac{\cos^{2m+1}\ x}{\sin^n\ x}\,dx = \sum_{k=0}^{m}(-1)^k \binom{m}{k} \int \frac{x^p\ \cos x}{\sin^{n-2k}\ x}\,dx.$

107. $\displaystyle\int x^p \frac{\cos x}{\sin^n\ x} = -\frac{x^p}{(n-1)\ \sin^{n-1}\ x} + \frac{p}{n-1}\int \frac{x^{p-1}\,dx}{\sin^{n-1}\ x}, \quad n > 1.$

108. $\displaystyle\int \frac{x\ \cos x}{\sin^2\ x}\,dx = -\frac{x}{\sin x} + \ln\ \tan\ \frac{x}{2}.$

109. $\displaystyle\int \frac{x\ \sin x}{\cos^2\ x}\,dx = \frac{x}{\cos x} - \ln\ \tan\left(\frac{x}{2}+\frac{\pi}{4}\right).$

110. $\displaystyle\int x^p\ \tan x\,dx = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{2^{2k}(2^{2k-1}-1)}{(p+2k)\,(2k)!}B_{2k}x^{p+2k}, \quad p \geq -1,\ |x| < \frac{\pi}{2}.$

111. $\displaystyle\int x^p\ \cot x\,dx = \sum_{k=0}^{\infty}(-1)^k \frac{2^{2k}B_{2k}}{(p+2k)(2k)!}x^{p+2k}, \quad p \geq 1,\ |x| < \pi.$

112. $\displaystyle\int x^p\ \tan^2\ x\,dx = x\ \tan x + \ln\ \cos\ x - \frac{x^2}{2}.$

113. $\displaystyle\int x\ \cot^2\ x\,dx = -x\ \cot x + \ln\ \sin\ x - \frac{x^2}{2}.$

114. $\displaystyle\int \frac{x^n\ \cos x\,dx}{(a+b\ \sin x)^m} = -\frac{x^n}{(m-1)b(a+b\ \sin x)^{m-1}} + \frac{n}{(m-1)b}\int \frac{x^{n-1}\,dx}{(a+b\ \sin x)^{m-1}}, \quad m \neq 1.$

115. $\displaystyle\int \frac{x^n\ \sin x\,dx}{(a+b\ \cos x)^m} = \frac{x^n}{(m-1)b(a+b\ \cos x)^{m-1}} - \frac{n}{(m-1)b}\int \frac{x^{n-1}\,dx}{(a+b\ \cos x)^{m-1}}, \quad m \neq 1.$

116. $\displaystyle\int \frac{x\,dx}{1+\sin x} = -x\ \tan\left(\frac{\pi}{4}-\frac{x}{2}\right) + 2\ \ln\ \cos\ \left(\frac{\pi}{4}-\frac{x}{2}\right).$

117. $\displaystyle\int \frac{x\,dx}{1-\sin x} = x\ \cot\left(\frac{\pi}{4}-\frac{x}{2}\right) + 2\ \ln\ \sin\ \left(\frac{\pi}{4}-\frac{x}{2}\right).$

118. $\displaystyle\int \frac{x\,dx}{1+\cos x} = x\ \tan\frac{x}{2} + 2\ \ln\ \cos\ \frac{x}{2}.$

119. $\displaystyle\int \frac{x\,dx}{1-\cos x} = -x\,\cot\frac{x}{2} + 2\,\ln\,\cos\,\frac{x}{2}.$

120. $\displaystyle\int \frac{x\,\cos x}{(1+\sin x)^2}\,dx = -\frac{x}{1+\sin x} + \tan\left(\frac{x}{2}-\frac{\pi}{4}\right).$

121. $\displaystyle\int \frac{x\,\cos x}{(1-\sin x)^2}\,dx = \frac{x}{1-\sin x} + \tan\left(\frac{x}{2}+\frac{\pi}{4}\right).$

122. $\displaystyle\int \frac{x\,\sin x}{(1+\cos x)^2}\,dx = \frac{x}{1+\cos x} - \tan\frac{x}{2}.$

123. $\displaystyle\int \frac{x\,\sin x}{(1-\cos x)^2}\,dx = -\frac{x}{1-\cos x} - \cot\frac{x}{2}.$

124. $\displaystyle\int \frac{x+\sin x}{1+\cos x}\,dx = x\,\tan\frac{x}{2}.$

125. $\displaystyle\int \frac{x-\sin x}{1-\cos x}\,dx = -x\,\cot\frac{x}{2}.$

126, $\displaystyle\int \frac{x^2dx}{[(ax-b)\,\sin x+(a+bx)\,\cos x]^2} = \frac{x\,\sin x+\cos x}{b[(ax-b)\,\sin x+(a+bx)\,\cos x]}.$

127. $\displaystyle\int \frac{dx}{[a+(ax+b)\,\tan x]^2} = \frac{\tan x}{a[a+(ax+b)\,\tan x]}.$

128. $\displaystyle\int \frac{x\,dx}{\cos(x+t)\,\cos(x-t)} = \csc\,2t\left\{x\,\ln\,\frac{\cos(x-t)}{\cos(x+t)} - L(x+t) + L(x-t)\right\},$

$t \neq n\pi,\ |x| < \left|\frac{\pi}{2} - |t_0|\right|$, where $t_0 \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

129. $\displaystyle\int \frac{\sin x}{\sqrt{x}}\,dx = \sqrt{2\pi}\,S(\sqrt{x}).$

130. $\displaystyle\int \frac{\cos x}{\sqrt{x}}\,dx = \sqrt{2\pi}\,C(\sqrt{x}).$

131. $\displaystyle\int \frac{x\,\sin x\,\cos x}{\Delta}\,dx = -\frac{x\Delta}{k^2} + \frac{1}{k^2}E(x,\,k).$

132. $\displaystyle\int \frac{x\,\sin^3 x\,\cos x}{\Delta}\,dx = -\frac{k'^2}{9k^4}F(x,\,k) + \frac{2k^2+5}{9k^4}E(x,\,k) - \frac{1}{9k^4}[3(3-\Delta^2)x + k^2\,\sin x\,\cos x]\,\Delta.$

133. $\displaystyle\int \frac{x\,\sin x\,\cos^3 x}{\Delta}\,dx = -\frac{k'^2}{9k^4}F(x,\,k) + \frac{7k^2-5}{9k^4}E(x,\,k) - \frac{1}{9k^4}[3(\Delta^2-3k'^2)x - k^2\,\sin x\,\cos x]\,\Delta.$

134. $\displaystyle\int \frac{x\,\sin x\,dx}{\Delta^3}\,dx = -\frac{x\,\cos x}{k'^2\Delta} + \frac{1}{kk'^2}\,\arcsin(k\,\sin x).$

135. $\displaystyle\int \frac{x\,\cos x\,dx}{\Delta^3} = \frac{x\,\sin x}{\Delta} + \frac{1}{k}\,\ln(k\,\cos x + \Delta).$

136. $\displaystyle\int \frac{x\,\sin x\,\cos x\,dx}{\Delta^3} = \frac{x}{k^2\Delta} - \frac{1}{k^2}F(x,\,k).$

137. $\displaystyle\int \frac{x\,\sin^3 x\,\cos x\,dx}{\Delta^3} = x\frac{2-k^2\,\sin^2 x}{k^4\Delta} - \frac{1}{k^4}[E(x,\,k) + F(x,\,k)].$

138. $\displaystyle\int \frac{x\,\sin x\,\cos^3 x\,dx}{\Delta^3} = x\frac{k^2\,\sin^2 x + k^2 - 2}{k^4\Delta} + \frac{k'^2}{k^4}F(x,\,k) + \frac{1}{k^4}E(x,\,k).$

139. $\displaystyle\int x^p\,\sin x^2\,dx = -\frac{x^{p-1}}{2}\,\cos x^2 + \frac{p-1}{2}\int x^{p-2}\,\cos x^2\,dx.$

140. $\displaystyle\int x^p\,\cos x^2\,dx = \frac{x^{p-1}}{2}\,\sin x^2 - \frac{p-1}{2}\int x^{p-2}\,\sin x^2\,dx.$

141. $$\int x^n\,\sin x^2\,dx = (n-1)!!\Bigg\{\sum_{k=1}^{s}(-1)^k\left[\frac{x^{n-4k+3}\,\cos x^2}{2^{2k-1}(n-4k+3)!!} - \frac{x^{n-4k+1}\,\sin x^2}{2^{2k}(n-4k+1)!!}\right] + \frac{(-1)^s}{2^{2s}(n-4s-1)!!}\int x^{n-4s}\,\sin x^2\,dx\Bigg\},\quad s = \lfloor n/4\rfloor.$$

142. $$\int x^n\,\cos x^2\,dx = (n-1)!!\Bigg\{\sum_{k=1}^{s}(-1)^{k-1}\left[\frac{x^{n-4k+3}\,\sin x^2}{2^{2k-1}(n-4k+3)!!} + \frac{x^{n-4k+1}\,\cos x^2}{2^{2k}(n-4k+1)!!}\right] + \frac{(-1)^s}{2^{2s}(n-4s-1)!!}\int x^{n-4s}\,\cos x^2\,dx\Bigg\},\quad s = \lfloor n/4\rfloor.$$

143. $\displaystyle\int x\,\sin x^2\,dx = -\frac{\cos x^2}{2}.$

144. $\displaystyle\int x\,\cos x^2\,dx = -\frac{\sin x^2}{2}.$

145. $\displaystyle\int x^2\,\sin x^2\,dx = -\frac{x}{2}\,\cos x^2 + \frac{1}{2}\sqrt{\frac{\pi}{2}}C(x).$

146. $\displaystyle\int x^2\,\cos x^2\,dx = \frac{x}{2}\,\sin x^2 - \frac{1}{2}\sqrt{\frac{\pi}{2}}S(x).$

147. $\displaystyle\int x^3\,\sin x^2\,dx = -\frac{x^2}{2}\,\cos x^2 + \frac{1}{2}\,\sin x^2.$

148. $\displaystyle\int x^3\,\cos x^2\,dx = \frac{x^2}{2}\,\sin x^2 + \frac{1}{2}\,\cos x^2.$
