! For an efficient use of these tables, first read `HowTo.pdf`.

T2.49A. Integrands involving trigonometric functions of arguments containing trigonometric and hyperbolic functions on the interval $(0, \pi)$.

1. $$\int_0^\pi \cos(z \cosh x) \sin^{2\mu} x\, dx = \sqrt{\pi} \left(\frac{2}{z}\right)^\mu \Gamma\left(\mu + \frac{1}{2}\right) I_\mu(z), \quad \Re\{z\} > 0,\ \Re\{\mu\} > -\tfrac{1}{2}.$$

2. $$\int_0^\pi \sin(z \sin x) \sin ax\, dx = \begin{cases} \sin a\pi\ s_{0,\,a}(z), \\ \qquad \text{or} \\ \sin a\pi \displaystyle\sum_{k=1}^\infty \frac{(-1)^{k-1} z^{2k-1}}{(1^2 - a^2)(3^2 - a^2)\ldots[(2k-1)^2 - a^2]}, \quad a > 0. \end{cases}$$

3. $$\int_0^\pi \sin(z \sin x) \sin nx\, dx = \frac{1}{2}\int_{-\pi}^\pi \sin(z \sin x) \sin nx\, dx$$
$$= \int_0^{\pi/2} \sin(z \sin x) \sin nx\, dx = [1 - (-1)^n]\, \frac{\pi}{2} J_n(z), \quad n = 0, \pm 1, \pm 2, \ldots.$$

4. $$\int_0^\pi \sin(z \sin x) \cos ax\, dx = \begin{cases} (1 + \cos a\pi)\, s_{0,\,a}(z), \\ \qquad \text{or} \\ (1 + \cos a\pi) \displaystyle\sum_{k=1}^\infty \frac{(-1)^{k-1} z^{2k-1}}{(1^2 - a^2)(3^2 - a^2)\ldots[(2k-1)^2 - a^2]}, \quad a > 0. \end{cases}$$

5. $$\int_0^\pi \sin(z \sin x) \cos[(2n+1)x]\, dx = 0.$$

6. $$\int_0^\pi \cos(z \sin x) \sin ax\, dx = \begin{cases} -a(1 - \cos a\pi)\, s_{-1,a}(z), \\ \qquad \text{or} \\ -a(1 - \cos a\pi) \left\{ -\dfrac{1}{a^2} + \displaystyle\sum_{k=1}^\infty \frac{(-1)^{k-1} z^{2k}}{a^2(2^2 - a^2)(4^2 - a^2)\ldots[(2k)^2 - a^2]} \right\}, \quad a > 0. \end{cases}$$

7. $\displaystyle\int_0^\pi \cos(z\,\sin x)\,\sin 2nx\,dx = 0.$

8. $$\int_0^\pi \cos(z\,\sin x)\,\cos ax\,dx = \begin{cases} -a\,\sin a\pi\, s_{-1,a}(z), \\ \quad \text{or} \\ -a\,\sin a\pi \left\{ -\dfrac{1}{a^2} + \displaystyle\sum_{k=1}^{\infty} \dfrac{(-1)^{k-1} z^{2k}}{a^2(2^2-a^2)(4^2-a^2)\ldots[(2k)^2-a^2]} \right\}, \end{cases} \quad a > 0.$$

9. $$\int_0^\pi \cos(z\,\sin x)\,\cos nx\,dx = \frac{1}{2}\int_{-\pi}^{\pi} \cos(z\,\sin x)\,\cos nx\,dx$$
$$= [1+(-1)^n] \int_0^{\pi/2} \cos(z\,\sin x)\,\cos nx\,dx = [1+(-1)^n]\,\frac{\pi}{2} J_n(z).$$

10. $$\int_0^\pi \sin(z\,\cos x)\,\cos nx\,dx = \frac{1}{2}\int_{-\pi}^{\pi} \sin(z\,\cos x)\,\cos nx\,dx = \pi\,\sin\frac{n\pi}{2} J_n(z).$$

11. $$\int_0^\pi \cos(z\,\cos x)\,\cos nx\,dx = \frac{1}{2}\int_{-\pi}^{\pi} \cos(z\,\cos x)\,\cos nx\,dx = \pi\,\cos\frac{n\pi}{2} J_n(z).$$

12. $$\int_0^\pi \cos(z\,\cos x)\,\sin^{2\mu} x\,dx = \sqrt{\pi}\left(\frac{2}{z}\right)^{\mu} \Gamma\left(\mu + \frac{1}{2}\right) J_\mu(z), \quad \Re\{\mu\} > -\tfrac{1}{2}.$$

13. $\displaystyle\int_0^\pi \sin(\nu x - z\,\sin x)\,dx = \pi\,\mathbf{E}_\nu(z).$

14. $\displaystyle\int_0^\pi \cos(nx - z\,\sin x)\,dx = \pi\,J_n(z).$

15. $\displaystyle\int_0^\pi \cos(\nu x - z\,\sin x)\,dx = \pi\,\mathbf{J}_\nu(z).$
