! For an efficient use of these tables, first read `HowTo.pdf`.

T3.26A. Integrands involving exponentials and arbitrary powers of $(a + b\,x)$ on the interval $(0, \infty)$.

1. $$\int_0^\infty x^{\nu-1} e^{-\mu x}\,dx = \frac{1}{\mu^\nu}\Gamma(\nu), \quad \Re\{\mu > 0, \Re\{\nu\} > 0.$$

2. $$\int_0^\infty x^{\nu-1} e^{-(p+iq)x}\,dx = \Gamma(\nu)(p^2+q^2)^{-\nu/2} \exp\left(-i\,\nu \arctan\frac{q}{p}\right),$$
$$p > 0,\ \Re\{\nu\} > 0 \text{ or } p = 0,\ 0 < \Re\{\nu\} < 1.$$

3. $$\int_0^\infty (1+x)^{-\nu} e^{-\mu x}\,dx = \mu^{\nu/2-1} e^{\mu/2} W_{-\nu/2,\,(1-\nu)/2}(\mu), \quad \Re\{\mu\} > 0.$$

4. $$\int_0^\infty (x+\beta)^{\nu} e^{-\mu x}\,dx = \mu^{-\nu-1} e^{\beta\mu}\Gamma(\nu+1,\,\beta\mu), \quad |\arg\beta| < \pi,\ \Re\{\mu\} > 0.$$

5. $$\int_0^\infty x^{\nu-1}(x+\beta)^{-\nu+1/2} e^{-\mu x}\,dx = 2^{\nu-1/2}\Gamma(\nu)\mu^{-1/2} e^{\beta\mu/2} D_{1-2\nu}(\sqrt{2\beta\mu}),$$
$$|\arg\beta| < \pi,\ \Re\{\nu\} > 0,\ \Re\{\mu\} \geq 0,\ \mu \neq 0.$$

6. $$\int_0^\infty x^{\nu-1}(x+\beta)^{-\nu-1/2} e^{-\mu x}\,dx = 2^{\nu}\Gamma(\nu)\beta^{-1/2} e^{\beta\mu/2} D_{-2\nu}(\sqrt{2\beta\mu}),$$
$$|\arg\beta| < \pi,\ \Re\{\nu\} > 0,\ \Re\{\mu\} \geq 0.$$

7. $$\int_0^\infty x^{\nu-1}(x+\beta)^{\nu-1} e^{-\mu x}\,dx = \frac{1}{\sqrt{\pi}}\left(\frac{\beta}{\mu}\right)^{\nu-1/2} e^{\beta\mu/2}\Gamma(\nu)\,K_{1/2-\nu}\left(\frac{\beta\mu}{2}\right),$$
$$|\arg\beta| < \pi,\ \Re\{\mu\} > 0,\ \Re\{\nu\} > 0.$$

8. $$\int_0^\infty \frac{x^{\nu-1} e^{-\mu x}}{x+\beta}\,dx = \beta^{\nu-1} e^{\beta\mu}\Gamma(\nu)\Gamma(1-\nu,\,\beta\mu), \quad |\arg\beta| < \pi,\ \Re\{\mu\} > 0,\ \Re\{\nu\} > 0.$$

9. $$\int_0^\infty x^{\alpha-1}(x+a)^{\alpha-1}\,e^{-px}\,dx = \frac{1}{\sqrt{\pi}}\,\Gamma(\alpha)\left(\frac{p}{a}\right)^{1/2-\alpha} e^{pa/2}\,K_{\alpha-1/2}(pa/2),$$
$$\Re\{\alpha\}>0,\ \Re\{p\}>0,\ |\arg a|<\pi.$$

10. $$\int_0^\infty (x^2+u^2)^{\nu-1}e^{-\mu x}\,dx = \frac{\sqrt{\pi}}{2}\left(\frac{2u}{\mu}\right)^{\nu-1/2}\Gamma(\nu)\left[\mathbf{H}_{\nu-1/2}(u\mu)-Y_{\nu-1/2}(u,\,\mu)\right],$$
$$|\arg u|<\pi,\ \Re\{\mu\}>0.$$

11. $$\int_0^\infty (2\beta x+x^2)^{\nu-1}e^{-\mu x}\,dx = \frac{1}{\sqrt{\pi}}\left(\frac{2\beta}{\mu}\right)^{\nu-1/2} e^{\beta\mu}\,\Gamma(\nu)\,K_{\nu-1/2}(\beta\mu),$$
$$|\arg\beta|<\pi;\ \Re\{\nu\}>0,\ \Re\{\mu\}>0.$$

12. $$\int_0^\infty (x^2+ix)^{\nu-1}e^{-\mu x}\,dx = -\frac{i\sqrt{\pi}e^{i\mu/2}}{2\mu^{\nu-1/2}}\Gamma(\nu)\,H^{(2)}_{\nu-1/2}\left(\frac{\mu}{2}\right), \quad \Re\{\mu\}>0,\ \Re\{\nu\}>0.$$

13. $$\int_0^\infty (x^2-ix)^{\nu-1}e^{-\mu x}\,dx = \frac{i\sqrt{\pi}e^{-i\mu/2}}{2\mu^{\nu-1/2}}\Gamma(\nu)\,H^{(1)}_{\nu-1/2}\left(\frac{\mu}{2}\right), \quad \Re\{\mu\}>0,\ \Re\{\nu\}>0.$$

14. $$\int_0^\infty x^{2\nu-1}(u^2+x^2)^{\rho-1}e^{-\mu x}\,dx = \frac{u^{2\nu+2\rho-2}}{2\sqrt{\pi}\Gamma(1-\rho)}\,G^{31}_{13}\left(\frac{\mu^2u^2}{4}\Big|^{1-\nu}_{1-\rho-\nu,\,0,\,1/2}\right),$$
$$|\arg u|<\frac{\pi}{2},\ \Re\{\mu\}>0,\ \Re\{\nu\}>0.$$

15. $$\int_0^\infty \frac{x^\nu e^{-\mu x}}{\beta^2+x^2}\,dx = \frac{1}{2}\Gamma(\nu)\,\beta^{\nu-1}\left[\exp\left(i\mu\beta+i\frac{(\nu-1)\pi}{2}\right)\right.$$
$$\left.\times\Gamma(1-\nu,\,i\beta\mu)+\exp\left(-i\beta\mu-i\frac{(\nu-1)\pi}{2}\right)\Gamma\left(1-\nu,\,-i\beta\mu\right)\right],$$
$$\Re\{\beta\}>0,\ \Re\{\mu\}>0,\ \Re\{\nu\}>-1.$$

16. $$\int_0^\infty \frac{x^{\nu-1}e^{-\mu x}\,dx}{1+x^2} = \pi\csc(\nu\pi)V_\nu(2\mu,\,0), \quad \Re\{\mu\}>0,\ \Re\{\nu\}>0.$$

17. $$\int_0^\infty \left[\left(\sqrt{x+2\beta}+\sqrt{x}\right)^{2\nu}-\left(\sqrt{x+2\beta}-\sqrt{x}\right)^{2\nu}\right]e^{-\mu x}\,dx = 2^{\nu+1}\frac{\nu}{\mu}\beta^\nu e^{\beta\mu}\,K_\nu(\beta\mu),$$
$$|\arg\beta|<\pi,\ \Re\{\mu\}>0.$$

18. $$\int_0^\infty (x+\sqrt{1+x^2})^\nu e^{-\mu x}\,dx = \frac{1}{\mu}S_{1,\,\nu}(\mu)+\frac{\nu}{\mu}S_{0,\,\nu}(\mu), \quad \Re\{\mu\}>0.$$

19. $$\int_0^\infty (\sqrt{1+x^2}-x)^\nu e^{-\mu x}\,dx = \frac{1}{\mu}S_{1,\,\nu}(\mu) - \frac{\nu}{\mu}S_{0,\,\nu}(\mu), \quad \Re\{\mu\} > 0.$$

20. $$\int_0^\infty \frac{(x+\sqrt{1+x^2})^\nu}{\sqrt{1+x^2}} e^{-\mu x}\,dx = \pi \csc \nu\pi[\mathbf{J}_{-\nu}(\mu) - J_{-\nu}(\mu)], \quad \Re\{\mu\} > 0.$$

21. $$\int_0^\infty \frac{(\sqrt{1+x^2}-x)^\nu}{\sqrt{1+x^2}} e^{-\mu x}\,dx = S_{0,\,\nu}(\mu) - \nu S_{-1,\,\nu}(\mu), \quad \Re\{\mu\} > 0.$$

22. $$\int_0^\infty \frac{(x+\sqrt{x^2+4\beta^2})^{2\nu}}{\sqrt{x^3+4\beta^2 x}} e^{-\mu x}\,dx$$
$$= \frac{\sqrt{\mu\pi^3}}{2^{2\nu+3/2}\beta^{2\nu}}[J_{\nu+1/4}(\beta\mu)Y_{\nu-1/4}(\beta\mu) - J_{\nu-1/4}(\beta\mu)Y_{\nu+1/4}(\beta\mu)], \quad \Re\{\beta\} > 0,\ \Re\{\mu\} > 0.$$

23. $$\int_0^\infty \frac{(1+\sqrt{1+x^2})^{\nu+1/2}}{x^{\nu+1}\sqrt{1+x^2}} e^{-\mu x}\,dx = \sqrt{2}\Gamma(-\nu)D_\nu(\sqrt{2i\mu})D_\nu(\sqrt{-2i\mu}), \quad \Re\{\mu\} \ge 0,\ \Re\{\nu\} < 0.$$

24. $$\int_0^\infty \frac{(x+\sqrt{x^2+1})^\nu + \cos\nu\pi(x+\sqrt{x^2+1})^{-\nu}}{\sqrt{x^2+1}}\, e^{-\mu x}\,dx = -\pi[\mathbf{E}_\nu(\mu) + Y_\nu(\mu)], \quad \Re\{\mu\} > 0.$$