T1.33. Integrand involving hyperbolic functions and powers $(a + b\,x)$.

Notation used: $X = a + bx$.

1. $\displaystyle\int x \sinh x\, dx = x \cosh x - \sinh x.$

2. $\displaystyle\int x^2 \sinh x\, dx = (x^2 + 2) \cosh x - 2x \sinh x.$

3. $\displaystyle\int x \cosh x\, dx = x \sinh x - \cosh x.$

4. $\displaystyle\int x^2 \cosh x\, dx = (x^2 + 2) \sinh x - 2x \cosh x.$

5. $\displaystyle\int x^n \sinh^{2m} x\, dx = (-1)^m \binom{2m}{m} \frac{x^{n+1}}{2^{2m}(n+1)} + \frac{1}{2^{2m-1}} \sum_{k=0}^{m-1} (-1)^k \binom{2m}{k} \int x^n \cosh(2m-2k)x\, dx.$

6. $\displaystyle\int x^n \sinh^{2m+1} x\, dx = \frac{1}{2^{2m}} \sum_{k=0}^{m} (-1)^k \binom{2m+1}{k} \int x^n \sinh(2m-2k+1)x\, dx.$

7. $\displaystyle\int x^n \cosh^{2m} x\, dx = \binom{2m}{m} \frac{x^{n+1}}{2^{2m}(n+1)} + \frac{1}{2^{2m-1}} \sum_{k=0}^{m-1} \binom{2m}{k} \int x^n \cosh(2m-2k)x\, dx.$

8. $\displaystyle\int x^n \cosh^{2m+1} x\, dx = \frac{1}{2^{2m}} \sum_{k=0}^{m} \binom{2m+1}{k} \int x^n \cosh(2m-2k+1)x\, dx.$

9. $\displaystyle\int x^r \sinh^p x \cosh^q x\,dx$

$$= \begin{cases} \dfrac{1}{(p+q)^2}\Big[(p+q)x^r \sinh^{p-1} x \cosh^{q-1} x \\ -rx^{r-1} \sinh^p x \cosh^q x + r(r+1)\displaystyle\int x^{r-2} \sinh^p x \cosh^q x\,dx \\ +rp\displaystyle\int x^{r-1} \sinh^{p-1} x \cosh^{q-1} x\,dx + (q-1)(p+q)\displaystyle\int x^r \sinh^p x \cosh^{q-2} x\,dx\Big], \\ \quad\text{or} \\ \dfrac{1}{(p+q)^2}\Big[(p+q)x^r \sinh^{p-1} x \cosh^{q+1} x \\ -rx^{r-1} \sinh^p x \cosh^q x + r(r-1)\displaystyle\int x^{r-2} \sinh^p x \cosh^q x\,dx \\ -rq\displaystyle\int x^{r-1} \sinh^{p-1} x \cosh^{q-1} x\,dx - (p-1)(p+q)\displaystyle\int x^r \sinh^{p-2} x \cosh^q x\,dx\Big]. \end{cases}$$

10. $\displaystyle\int X \sinh kx\,dx = \frac{1}{k}X \cosh kx - \frac{b}{k^2}\sinh kx.$

11. $\displaystyle\int X \cosh kx\,dx = \frac{1}{k}X \sinh kx - \frac{b}{k^2}\cosh kx.$

12. $\displaystyle\int X^2 \sinh kx\,dx = \frac{1}{k}\left(X^2 + \frac{2b^2}{k^2}\right)\cosh kx - \frac{2bX}{k^2}\sinh kx.$

13. $\displaystyle\int X^2 \cosh kx\,dx = \frac{1}{k}\left(X^2 + \frac{2b^2}{k^2}\right)\sinh kx - \frac{2bX}{k^2}\cosh kx.$

14. $\displaystyle\int X^3 \sinh kx\,dx = \frac{X}{k}\left(X^2 + \frac{6b^2}{k^2}\right)\cosh kx - \frac{3b}{k^2}\left(X^2 + \frac{2b^2}{k^2}\right)\sinh kx.$

15. $\displaystyle\int X^3 \cosh kx\,dx = \frac{X}{k}\left(X^2 + \frac{6b^2}{k^2}\right)\sinh kx - \frac{3b}{k^2}\left(X^3 + \frac{2b^2}{k^2}\right)\cosh kx.$

16. $\displaystyle\int X^4 \sinh kx\,dx = \frac{1}{k}\left(X^4 + \frac{12b^2}{k^2}X^2 + \frac{24b^4}{k^4}\right)\cosh kx - \frac{4bX}{k^2}\left(X^2 + \frac{6b^2}{k^2}\right)\sinh kx.$

17. $\displaystyle\int X^4 \cosh kx\,dx = \frac{1}{k}\left(X^4 + \frac{12b^2}{k^2}X^2 + \frac{24b^4}{k^4}\right)\sinh kx - \frac{4bX}{k^2}\left(X^2 + \frac{6b^2}{k^2}\right)\cosh kx.$

18. $\displaystyle\int X^5 \sinh kx\,dx = \frac{X}{k}\left(X^4 + \frac{20b^2}{k^2}X^2 + 120\frac{b^4}{k^4}\right)\cosh kx - \frac{5b}{k^2}\left(X^4 + 12\frac{b^2}{k^2}X^2 + 24\frac{b^4}{k^4}\right)\sinh kx.$

19. $\displaystyle\int X^5 \cosh kx\,dx = \frac{X}{k}\left(X^4 + 20\frac{b^2}{k^2}X^2 + 120\frac{b^4}{k^4}\right)\sinh kx - \frac{5b}{k^2}\left(X^4 + 12\frac{b^2}{k^2}X^2 + 24\frac{b^4}{k^4}\right)\cosh kx.$

20. $$\int X^6 \sinh kx\,dx = \frac{1}{k}\left(X^6 + 30\frac{b^2}{k^2}X^4 + 360\frac{b^4}{k^4}X^2 + 720\frac{b^6}{k^6}\right)\cosh kx - \frac{6bX}{k^2}\left(X^4 + 20\frac{b^2}{k^2}X^2 + 120\frac{b^4}{k^4}\right)\sinh kx.$$

21. $$\int X^6 \cosh kx\,dx = \frac{1}{k}\left(X^6 + 30\frac{b^2}{k^2}X^4 + 360\frac{b^4}{k^4}X^2 + 720\frac{b^6}{k^6}\right)\sinh kx - \frac{6bX}{k^2}\left(X^4 + 20\frac{b^2}{k^2}X + 120\frac{b^4}{k^4}\right)\cosh kx.$$

22. $$\int x^n \sinh x\,dx = x^n \cosh x - n\int x^{n-1}\cosh x\,dx = x^n \cosh x - nx^{n-1}\sinh x + n(n-1)\int x^{n-2}\sinh x\,dx.$$

23. $$\int x^n \cosh x\,dx = x^n \sinh x - n\int x^{n-1}\sinh x\,dx = x^n \sinh x - nx^{n-1}\cosh x + n(n-1)\int x^{n-2}\cosh x\,dx.$$

24. $$\int x^{2n}\sinh x\,dx = (2n)!\left\{\sum_{k=0}^{n}\frac{x^{2k}}{(2k)!}\cosh x - \sum_{k=1}^{n}\frac{x^{2k-1}}{(2k-1)!}\sinh x\right\}.$$

25. $$\int x^{2n+1}\sinh x\,dx = (2n+1)!\sum_{k=0}^{n}\left\{\frac{x^{2k+1}}{(2k+1)!}\cosh x - \frac{x^{2k}}{(2k)!}\sinh x\right\}.$$

26. $$\int x^{2n}\cosh x\,dx = (2n)!\left\{\sum_{k=1}^{n}\frac{x^{2k}}{(2k)!}\sinh x - \sum_{k=1}^{n}\frac{x^{2k-1}}{(2k-1)!}\cosh x\right\}.$$

27. $$\int x^{2n+1}\cosh x\,dx = (2n+1)!\sum_{k=0}^{n}\left\{\frac{x^{2k+1}}{(2k+1)!}\sinh x - \frac{x^{2k}}{(2k)!}\cosh x\right\}.$$

28.1. $$\int x^n \sinh^2 x\,dx = -\frac{x^{n+1}}{2(n+1)} + \frac{n!}{4}\sum_{k=0}^{[n/2]}\left\{\frac{x^{n-2k}}{2^{2k}(n-2k)!}\sinh 2x - \frac{x^{n-2k-1}}{2^{2k+1}(n-2k-1)!}\cosh 2x\right\}.$$

29. $$\int x^n \cosh^2 x\,dx = \frac{x^{n+1}}{2(n+1)} + \frac{n!}{4}\sum_{k=0}^{[n/2]}\left\{\frac{x^{n-2k}}{2^{2k}(n-2k)!}\sinh 2x - \frac{x^{n-2k-1}}{2^{2k+1}(n-2k-1)!}\cosh 2x\right\}.$$

30. $$\int x \sinh^2 x\,dx = \frac{1}{4}x\sinh 2x - \frac{1}{8}\cosh 2x - \frac{x^2}{4}.$$

31. $$\int x^2 \sinh^2 x\,dx = \frac{1}{4}\left(x^2 + \frac{1}{2}\right)\sinh 2x - \frac{x}{4}\cosh 2x - \frac{x^3}{6}.$$

32. $$\int x \cosh^2 x\,dx = \frac{x}{4}\sinh 2x - \frac{1}{8}\cosh 2x + \frac{x^2}{4}.$$

33. $\displaystyle\int x^2 \cosh^2 x\,dx = \frac{1}{4}\left(x^2+\frac{1}{2}\right)\sinh 2x - \frac{x}{4}\cosh 2x + \frac{x^3}{6}.$

34. $\displaystyle\int x^n \sinh^3 x\,dx$

$$= \frac{n!}{4}\sum_{k=0}^{[n/2]}\left\{\frac{x^{n-2k}}{(n-2k)!}\left(\frac{\cosh 3x}{3^{2k+1}} - 3\cosh x\right) - \frac{x^{n-2k-1}}{(n-2k-1)!}\left(\frac{\sinh 3x}{3^{2k+2}} - 3\sinh x\right)\right\}.$$

35. $\displaystyle\int x^n \cosh^3 x\,dx$

$$= \frac{n!}{4}\sum_{k=0}^{[n/2]}\left\{\frac{x^{n-2k}}{(n-2k)!}\left(\frac{\sinh 3x}{3^{2k+1}} + 3\sinh x\right) - \frac{x^{n-2k-1}}{(n-2k-1)!}\left(\frac{\cosh 3x}{3^{2k+2}} + 3\cosh x\right)\right\}.$$

36. $\displaystyle\int x\sinh^3 x\,dx = \frac{3}{4}\sinh x - \frac{1}{36}\sinh 3x - \frac{3}{4}x\cosh x - \frac{x}{12}\cosh 3x.$

37. $\displaystyle\int x^2\sinh^3 x\,dx = -\left(\frac{3x^2}{4}+\frac{3}{2}\right)\cosh x + \left(\frac{x^2}{12}+\frac{1}{54}\right)\cosh 3x + \frac{3x}{2}\sinh x - \frac{x}{18}\sinh 3x.$

38. $\displaystyle\int x\cosh^3 x\,dx = -\frac{3}{4}\cosh x - \frac{1}{36}\cosh 3x + \frac{3}{4}x\sinh x + \frac{x}{12}\sinh 3x.$

39. $\displaystyle\int x^2\cosh^3 x\,dx = \left(\frac{3}{4}x^2+\frac{3}{2}\right)\sinh x + \left(\frac{x^2}{12}+\frac{1}{54}\right)\sinh 3x - \frac{3}{2}x\cosh x - \frac{x}{18}\cosh 3x.$

40. $\displaystyle\int\frac{\sinh^q x}{x^p}\,dx = -\frac{(p-2)\sinh^q x + qx\sinh^{q-1}x\cosh x}{(p-1)(p-2)x^{p-1}}$

$$+\frac{q(q-1)}{(p-1)(p-2)}\int\frac{\sinh^{q-2}x}{x^{p-2}}\,dx + \frac{q^2}{(p-1)(p-2)}\int\frac{\sinh^q x}{x^{p-2}}\,dx,\quad p>2.$$

41. $\displaystyle\int\frac{\cosh^q x}{x^p}\,dx = -\frac{(p-2)\cosh^q x + qx\cosh^{q-1}x\sinh x}{(p-1)(p-2)x^{p-1}}$

$$-\frac{q(q-1)}{(p-1)(p-2)}\int\frac{\cosh^{q-2}x}{x^{p-2}}\,dx + \frac{q^2}{(p-1)(p-2)}\int\frac{\cosh^q x}{x^{p-2}}\,dx,\quad p>2.$$

42. $\displaystyle\int\frac{\sinh x}{x^{2n}}\,dx = -\frac{1}{x(2n-1)!}\left\{\sum_{k=0}^{n-2}\frac{(2k+1)!}{x^{2k+1}}\cosh x + \sum_{k=0}^{n-1}\frac{(2k)!}{x^{2k}}\sinh x\right\} + \frac{1}{(2n-1)!}\mathrm{chi}(x).$

43. $\displaystyle\int\frac{\sinh x}{x^{2n+1}}\,dx = -\frac{1}{x(2n)!}\left\{\sum_{k=0}^{n-1}\frac{(2k)!}{x^{2k}}\cosh x + \sum_{k=0}^{n-1}\frac{(2k+1)!}{x^{2k+1}}\sinh x\right\} + \frac{1}{(2n)!}\mathrm{shi}(x).$

44. $\displaystyle\int\frac{\cosh x}{x^{2n}}\,dx = -\frac{1}{x(2n-1)!}\left\{\sum_{k=0}^{n-2}\frac{(2k+1)!}{x^{2k+1}}\cosh x + \sum_{k=0}^{n-1}\frac{(2k)!}{x^{2k}}\cosh x\right\}+\frac{1}{(2n-1)!}\sinh\mathrm{shi}(x).$

45. $$\int \frac{\cosh\, x}{x^{2n+1}}\, dx = -\frac{1}{(2n)!x}\left\{\sum_{k=0}^{n-1}\frac{(2k)!}{x^{2k}}\sinh\, x + \sum_{k=0}^{n-1}\frac{(2k+1)!}{x^{2k+1}}\cosh\, x\right\} + \frac{1}{(2n)!}\cosh\operatorname{chi}(x).$$

46. $$\int \frac{\sinh^{2m} x}{x}\, dx = \frac{1}{2^{2m-1}}\sum_{k=0}^{m-1}(-1)^k\binom{2m}{k}\operatorname{chi}(2m-2k)x + \frac{(-1)^m}{2^{2m}}\binom{2m}{m}\ln\, x.$$

47. $$\int \frac{\sinh^{2m+1} x}{x}\, dx = \frac{1}{2^{2m}}\sum_{k=0}^{m}(-1)^k\binom{2m+1}{k}\operatorname{shi}(2m-2k+1)x.$$

48. $$\int \frac{\cosh^{2m} x}{x}\, dx = \frac{1}{2^{2m-1}}\sum_{k=0}^{m-1}\binom{2m}{k}\cosh\operatorname{chi}(2m-2k)x + \frac{1}{2^{2m}}\binom{2m}{m}\ln\ x.$$

49. $$\int \frac{\cosh^{2m+1} x}{x}\, dx = \frac{1}{2^{2m}}\sum_{k=0}^{m}\binom{2m+1}{k}\cosh\operatorname{chi}(2m-2k+1)x.$$

50. $$\int \frac{\sinh^{2m} x}{x^2}\, dx = \frac{(-1)^{m-1}}{2^{2m}x}\binom{2m}{m}$$
$$+\frac{1}{2^{2m-1}}\sum_{k=0}^{m-1}(-1)^{k+1}\binom{2m}{k}\left\{\frac{\cosh(2m-2k)x}{x} - (2m-2k)\operatorname{shi}(2m-2k)x\right\}.$$

51. $$\int \frac{\sinh^{2m+1} x}{x^2}\, dx = \frac{1}{2^{2m}}\sum_{k=0}^{m}(-1)^{k+1}\binom{2m+1}{k}$$
$$\times\left\{\frac{\sinh(2m-2k+1)x}{x} - (2m-2k+1)\cosh\operatorname{chi}(2m-2k+1)x\right\}.$$

52. $$\int \frac{\cosh^{2m} x}{x^2}\, dx = -\frac{1}{2^{2m}x}\binom{2m}{m}$$
$$-\frac{1}{2^{2m-1}}\sum_{k=0}^{m-1}\binom{2m}{k}\left\{\frac{\cosh(2m-2k)x}{x} - (2m-2k)\operatorname{shi}(2m-2k)x\right\}.$$

53. $$\int \frac{\cosh^{2m+1} x}{x^2}\, dx$$
$$= -\frac{1}{2^{2m}}\sum_{k=0}^{m}\binom{2m+1}{k}\left\{\frac{\cosh(2m-2k+1)x}{x} - (2m-2k+1)\operatorname{shi}(2m-2k+1)x\right\}.$$

54. $$\int \frac{\sinh kx}{a+bx}\, dx = \frac{1}{b}\left[\cosh\frac{ka}{b}\operatorname{shi}(u) - \sinh\frac{ka}{b}\operatorname{chi}(u)\right]$$
$$= \frac{1}{2b}\left[\exp\left(-\frac{ka}{b}\right)\operatorname{Ei}(u) - \exp\left(\frac{ka}{b}\right)\operatorname{Ei}(-u)\right], \quad u = \frac{k}{b}(a+bx).$$

55. $$\int \frac{\cosh\,kx}{a+bx}\,dx = \frac{1}{b}\left[\cosh\frac{ka}{b}\cosh\operatorname{chi}(u) - \sinh\frac{ka}{b}\sinh\operatorname{shi}(u)\right]$$
$$= \frac{1}{2b}\left[\exp\left(-\frac{ka}{b}\right)\operatorname{Ei}(u) + \exp\left(\frac{ka}{b}\right)\operatorname{Ei}(-u)\right], \quad u = \frac{k}{b}(a+bx).$$

56. $$\int \frac{\sinh\,kx}{(a+bx)^2}\,dx = -\frac{1}{b}\,\frac{\sinh\,kx}{a+bx} + \frac{k}{b}\int \frac{\cosh\,kx}{a+bx}\,dx.$$

57. $$\int \frac{\cosh\,kx}{(a+bx)^2}\,dx = -\frac{1}{b}\,\frac{\cosh\,kx}{a+bx} + \frac{k}{b}\int \frac{\sinh\,kx}{a+bx}\,dx.$$

58. $$\int \frac{\sinh\,kx}{(a+bx)^3}\,dx = -\frac{\sinh\,kx}{2b(a+bx)^2} - \frac{k\,\cosh\,kx}{2b^2(a+bx)} + \frac{k^2}{2b^2}\int \frac{\sinh\,kx}{a+bx}\,dx.$$

59. $$\int \frac{\cosh\,kx}{(a+bx)^3}\,dx = -\frac{\cosh\,kx}{2b(a+bx)^2} - \frac{k\,\sinh\,kx}{2b^2(a+bx)} + \frac{k^2}{2b^2}\int \frac{\cosh\,kx}{a+bx}\,dx.$$

60. $$\int \frac{\sinh\,kx}{(a+bx)^4}\,dx = -\frac{\sinh\,kx}{3b(a+bx)^3} - \frac{k\,\cosh\,kx}{6b^2(a+bx)^2} - \frac{k^2\sinh\,kx}{6b^3(a+bx)} + \frac{k^3}{6b^3}\int \frac{\cosh\,kx}{a+bx}\,dx.$$

61. $$\int \frac{\cosh\,kx}{(a+bx)^4}\,dx = -\frac{\cosh\,kx}{3b(a+bx)^3} - \frac{k\,\sinh\,kx}{6b^2(a+bx)^2} - \frac{k^2\cosh\,kx}{6b^3(a+bx)} + \frac{k^3}{6b^3}\int \frac{\sinh\,kx}{a+bx}\,dx.$$

62. $$\int \frac{\sinh\,kx}{(a+bx)^5}\,dx = -\frac{\sinh kx}{4b(a+bx)^4} - \frac{k\,\cosh\,kx}{12b^2(a+bx)^3} - \frac{k^2\,\sinh\,kx}{24b^3(a+bx)^2}$$
$$-\frac{k^3\cosh\,kx}{24b^4(a+bx)} + \frac{k^4}{24b^4}\int \frac{\sinh\,kx}{a+bx}\,dx.$$

63. $$\int \frac{\cosh\,kx}{(a+bx)^5}\,dx = -\frac{\cosh\,kx}{4b(a+bx)^4} - \frac{k\,\sinh\,kx}{12b^2(a+bx)^3} - \frac{k^2\cosh\,kx}{24b^3(a+bx)^2}$$
$$-\frac{k^3\sinh\,kx}{24b^4(a+bx)} + \frac{k^4}{24b^4}\int \frac{\cosh\,kx}{a+bx}\,dx.$$

64. $$\int \frac{\sinh\,kx}{(a+bx)^6}\,dx = -\frac{\sinh\,kx}{5b(a+bx)^5} - \frac{k\,\cosh\,kx}{20b^2(a+bx)^4} - \frac{k^2\sinh\,kx}{60b^3(a+bx)^3} - \frac{k^3\cosh\,kx}{120b^4(a+bx)^2}$$
$$-\frac{k^4\sinh\,kx}{120b^5(a+bx)} + \frac{k^5}{120b^5}\int \frac{\cosh\,kx}{a+bx}\,dx.$$

65. $$\int \frac{\cosh\,kx}{(a+bx)^6}\,dx = -\frac{\cosh\,kx}{5b(a+bx)^5} - \frac{k\,\sinh\,kx}{20b^2(a+bx)^4} - \frac{k^2\cosh\,kx}{60b^3(a+bx)^3} - \frac{k^3\sinh\,kx}{120b^4(a+bx)^2}$$
$$-\frac{k^4\cosh\,kx}{120b^5(a+bx)} + \frac{k^5}{120b^5}\int \frac{\sinh\,kx}{a+bx}\,dx.$$

66. $$\int \frac{x^p\,dx}{\sinh^q x} = \frac{-px^{p-1}\sinh\,x - (q-2)x^p\,\cosh\,x}{(q-1)(q-2)\sinh^{q-1}x} + \frac{p(p-1)}{(q-1)(q-2)}\int \frac{x^{p-2}}{\sinh^{q-2}x}\,dx$$
$$-\frac{q-2}{q-1}\int \frac{x^p\,dx}{\sinh^{q-2}x}, \quad q > 2.$$

67. $$\int \frac{x^p\,dx}{\cosh^q x} = \frac{px^{p-1}\cosh\,x + (q-2)x^p\,\sinh\,x}{(q-1)(q-2)\cosh^{q-1}x} - \frac{p(p-1)}{(q-1)(q-2)}\int \frac{x^{p-2}\,dx}{\cosh^{q-2}x} + \frac{q-2}{q-1}\int \frac{x^p\,dx}{\cosh^{q-2}x}, \quad q > 2.$$

68. $$\int \frac{x^n}{\sinh\,x}\,dx = \sum_{k=0}^{\infty} \frac{(2-2^{2k})B_{2k}}{(n+2k)(2k)!}x^{n+2k}, \quad |x| < \pi,\ n > 0.$$

69. $$\int \frac{x^n}{\cosh\,x}\,dx = \sum_{k=0}^{\infty} \frac{E_{2k}x^{n+2k+1}}{(n+2k+1)(2k)!}, \quad |x| < \frac{\pi}{2},\ n \geq 0.$$

70. $$\int \frac{dx}{x^n\,\sinh\,x} = -[1+(-1)^n]\frac{2^{n-1}-1}{n!}B_n\,\ln\,x + \sum_{\substack{k=0\\ k\neq n/2}}^{\infty} \frac{2-2^{2k}}{(2k-n)(2k)!}B_{2k}x^{2k-n}, \quad |x| < \pi,\ n \geq 1.$$

71. $$\int \frac{dx}{x^n\cosh\,x} = \sum_{\substack{k=0\\ k\neq (n-1)/2}}^{\infty} \frac{E_{2k}}{(2k-n+1)(2k)!}x^{2k-n+1} + \frac{1}{2}[1+(-1)^n]^n + \frac{E_{n-1}}{(n-1)!}\ln\,x, \quad |x| < \frac{\pi}{2}.$$

72. $$\int \frac{x^n}{\sinh^2 x}\,dx = -x^n\,\coth\,x + n\sum_{k=0}^{\infty} \frac{2^{2k}B_{2k}}{(n+2k-1)(2k)!}x^{n+2k-1}, \quad n > 1,\ |x| < \pi.$$

73. $$\int \frac{x^n}{\cosh^2 x}\,dx = x^n\,\tanh\,x - n\sum_{k=1}^{\infty} \frac{2^{2k}(2^{2k}-1)B_{2k}}{(n+2k-1)(2k)!}x^{n+2k-1} \qquad n > 1,\ |x| < \frac{\pi}{2}.$$

74. $$\int \frac{dx}{x^n\,\sinh^2 x} = -\frac{\coth\,x}{x^n} - [1-(-1)^n]\frac{2^n n}{(n+1)!}B_{n+1}\,\ln\,x - \frac{n}{x^{n+1}} \sum_{\substack{k=0\\ k\neq (n+1)/2}}^{\infty} \frac{B_{2k}}{(2k-n-1)(2k)!}(2x)^{2^k}, \quad |x| < \pi.$$

75. $$\int \frac{dx}{x^n\,\cosh^2 x} = \frac{\tanh\,x}{x^n} + [1-(-1)^n] - \frac{2n(2^{n+1}-1)n}{(n+1)!}B_{n+1}\,\ln\,x + \frac{n}{x^{n+1}} \sum_{\substack{k=1\\ k\neq (n+1)/2}}^{\infty} \frac{(2^{2k}-1)B_{2k}}{(2k-n-1)(2k)!}(2x)^{2^k}, \quad |x| < \frac{\pi}{2}.$$

76. $$\int \frac{x}{\sinh^{2n}x}\,dx = \sum_{k=1}^{n-1}(-1)^k \frac{(2n-2)(2n-4)\ldots(2n-2k+2)}{(2n-1)(2n-3)\ldots(2n-2k+1)} \times \left\{\frac{x\,\cosh\,x}{\sinh^{2n-2k+1}x} + \frac{1}{(2n-2k)\sinh^{2n-2k}x}\right\} + (-1)^{n-1}\frac{(2n-2)!!}{(2n-1)!!}\int \frac{x\,dx}{\sinh^2 x}.$$

77. $$\int \frac{x}{\sinh^{2n-1} x}\,dx = \sum_{k=1}^{n-1}(-1)^k \frac{(2n-3)(2n-5)\dots(2n-2k+1)}{(2n-2)(2n-4)\dots(2n-2k)} \times \left\{\frac{x\cosh x}{\sinh^{2n-2k} x} + \frac{1}{(2n-2k-1)\sinh^{2n-2k-1} x}\right\} + (-1)^{n-1}\frac{(2n-3)!!}{(2n-2)!!}\int \frac{x\,dx}{\sinh x}.$$

78. $$\int \frac{x}{\cosh^{2n} x}\,dx = \sum_{k=1}^{n-1} \frac{(2n-2)(2n-4)\dots(2n-2k+2)}{(2n-1)(2n-3)\dots(2n-2k+1)} \times \left\{\frac{x\sinh x}{\cosh^{2n-2k+1} x} + \frac{1}{(2n-2k)\cosh^{2n-2k} x}\right\} + \frac{(2n-2)!!}{(2n-1)!!}\int \frac{x\,dx}{\cosh^2 x}.$$

79. $$\int \frac{x}{\cosh^{2n-1} x}\,dx = \sum_{k=1}^{n-1} \frac{(2n-3)(2n-5)\dots(2n-2k+1)}{(2n-2)(2n-4)\dots(2n-2k)} \times \left\{\frac{x\sinh x}{\cosh^{2n-2k} x} + \frac{1}{(2n-2k-1)\cosh^{2n-2k-1} x}\right\} + \frac{(2n-3)!!}{(2n-2)!!}\int \frac{x\,dx}{\cosh x}.$$

80. $$\int \frac{x\,dx}{\sinh x} = \sum_{k=0}^{\infty} \frac{2-2^{2k}}{(2k+1)(2k)!} B_{2k} x^{2k+1}, \quad |x| < \pi.$$

81. $$\int \frac{x\,dx}{\cosh x} = \sum_{k=0}^{\infty} \frac{E_{2k} x^{2k+2}}{(2k+2)(2k)!}, \quad |x| < \frac{\pi}{2}.$$

82. $$\int \frac{x\,dx}{\sinh^2 x} = -x\coth x + \ln\sinh x.$$

83. $$\int \frac{x\,dx}{\cosh^2 x} = x\tanh x - \ln\cosh x.$$

84. $$\int \frac{x\,dx}{\sinh^3 x} = -\frac{x\cosh x}{2\sinh^2 x} - \frac{1}{2\sinh x} - \frac{1}{2}\int \frac{x\,dx}{\sinh x}.$$

85. $$\int \frac{x\,dx}{\cosh^3 x} = \frac{x\sinh x}{2\cosh^2 x} + \frac{1}{2\cosh x} + \frac{1}{2}\int \frac{x\,dx}{\cosh x}.$$

86. $$\int \frac{x\,dx}{\sinh^4 x} = -\frac{x\cosh x}{3\sinh^3 x} - \frac{1}{6\sinh^2 x} + \frac{2}{3}x\coth x - \frac{2}{3}\ln\sinh x.$$

87. $$\int \frac{x\,dx}{\cosh^4 x} = \frac{x\sinh x}{3\cosh^3 x} + \frac{1}{6\cosh^2 x} + \frac{2}{3}x\tanh x - \frac{2}{3}\ln\cosh x.$$

88. $$\int \frac{x\,dx}{\sinh^5 x} = -\frac{x\cosh x}{4\sinh^4 x} - \frac{1}{12\sinh^3 x} + \frac{3x\cosh x}{8\sinh^2 x} + \frac{3}{8\sinh x} + \frac{3}{8}\int \frac{x\,dx}{\sinh x}.$$

89. $$\int \frac{x\,dx}{\cosh^5 x} = \frac{x\sinh x}{4\cosh^4 x} + \frac{1}{12\cosh^3 x} + \frac{3x\sinh x}{8\cosh^2 x} + \frac{3}{8\cosh x} + \frac{3}{8}\int \frac{x\,dx}{\cosh x}.$$

90. $$\int \frac{x^n \cosh x\,dx}{(a+b\sinh x)^m} = -\frac{x^n}{(m-1)b(a+b\sinh x)^{m-1}} + \frac{n}{(m-1)b}\int \frac{x^{n-1}\,dx}{(a+b\sinh x)^{m-1}}, \quad m \neq 1.$$

91. $$\int \frac{x^n \sinh x\,dx}{(a+b\cosh x)^m} = -\frac{x^n}{(m-1)b(a+b\cosh x)^{m-1}} + \frac{n}{(m-1)b}\int \frac{x^{n-1}\,dx}{(a+b\cosh x)^{m-1}}, \ m \neq 1.$$

92. $$\int \frac{x\,dx}{1+\cosh x} = x\tanh\frac{x}{2} - 2\ln\cosh\frac{x}{2}.$$

93. $$\int \frac{x\,dx}{1-\cosh x} = x\coth\frac{x}{2} - 2\ln\sinh\frac{x}{2}.$$

94. $$\int \frac{x\sinh x\,dx}{(1+\cosh x)^2} = -\frac{x}{1+\cosh x} + \tanh\frac{x}{2}.$$

95. $$\int \frac{x\sinh x\,dx}{(1-\cosh x)^2} = \frac{x}{1-\cosh x} - \coth\frac{x}{2}.$$

96. $$\int \frac{x\,dx}{\cosh 2x - \cos 2t} = \frac{1}{2\sin 2t}\left[L(u+t) - L(u-t) - 2L(t)\right]$$

where $u = \arctan(\tanh x\,\cot t), \quad t \neq \pm n\pi.$

97. $$\int \frac{x\cosh x\,dx}{\cosh 2x - \cos 2t} = \frac{1}{2\sin t}\Bigg[L\left(\frac{u+t}{2}\right) - L\left(\frac{u-t}{2}\right) + L\left(\pi - \frac{v+t}{2}\right) + L\left(\frac{v-t}{2}\right) - 2L\left(\frac{t}{2}\right) - 2L\left(\frac{\pi-t}{2}\right)\Bigg],$$

where $u = 2\arctan\left(\tanh\frac{x}{2}\cot\frac{t}{2}\right), \quad v = 2\arctan\left(\coth\frac{x}{2}\cot\frac{t}{2}\right), \quad t \neq \pm n\pi.$

98. $$\int x^p \frac{\sinh^{2m} x}{\cosh^n x}\,dx = \sum_{k=0}^{m} (-1)^{m+k}\binom{m}{k}\int \frac{x^p\,dx}{\cosh^{n-2k} x}.$$

99. $$\int x^p \frac{\sinh^{2m+1} x}{\cosh^n x}\,dx = \sum_{k=0}^{m} (-1)^{m+k}\binom{m}{k}\int x^p \frac{\sinh x}{\cosh^{n-2k} x}\,dx, \quad n > 1.$$

100. $$\int x^p \frac{\sinh x}{\cosh^n x}\,dx = -\frac{x^p}{(n-1)\cosh^{n-1} x} + \frac{p}{n-1}\int \frac{x^{p-1}\,dx}{\cosh^{n-1} x}, \quad n > 1.$$

101. $$\int x^p \frac{\cosh^{2m} x}{\sinh^n x}\,dx = \sum_{k=0}^{m} \binom{m}{k}\int \frac{x^p\cosh x}{\sinh^{n-2k} x}.$$

102. $$\int x^p \frac{\cosh^{2m+1} x}{\sinh^n x}\,dx = \sum_{k=0}^{m} \binom{m}{k}\int \frac{x^p\cosh x}{\sinh^{n-2k} x}\,dx.$$

103. $$\int x^p \frac{\cosh x}{\sinh^n x}\,dx = -\frac{x^p}{(n-1)\sinh^{n-1} x} + \frac{p}{n-1}\int \frac{x^{p-1}\,dx}{\sinh^{n-1} x}, \quad n > 1.$$

104. $\displaystyle\int x^p \tanh x\,dx = \sum_{k=1}^{\infty} \frac{2^{2k}(2^{2k}-1)B_{2k}}{(2k+p)(2k)!} x^{p+2k}, \quad p > -1,\ |x| < \frac{\pi}{2}.$

105. $\displaystyle\int x^p \coth x\,dx = \sum_{k=0}^{\infty} \frac{2^{2k}B_{2k}}{(p+2k)(2k)!} x^{p+2k}, \quad p \geq +1,\ |x| < \pi.$

106. $\displaystyle\int \frac{x \cosh x}{\sinh^2 x}\,dx = \ln \tanh \frac{x}{2} - \frac{x}{\sinh x}.$

107. $\displaystyle\int \frac{x \sinh x}{\cosh^2 x}\,dx = -\frac{x}{\cosh x} + \arctan(\sinh x).$