

! For an efficient use of these tables, first read [HowTo.pdf](#).

T2.09C. Integrands of the form $\sqrt{\frac{\pm(a-x)}{(b-x)^n(c-x)}}$, $\sqrt{\frac{\pm(b-x)}{(a-x)^n(c-x)}}$ and $\sqrt{\frac{\pm(c-x)}{(a-x)^n(b-x)}}$ for $n = 1, 3$, on the intervals (y, c) and (c, y) .

Notation used: $\beta = \arcsin \sqrt{\frac{c-y}{b-y}}$, $\gamma = \arcsin \sqrt{\frac{y-c}{b-c}}$, $p = \sqrt{\frac{a-b}{a-c}}$, $q = \sqrt{\frac{b-c}{a-c}}$.

$$1. \int_y^c \sqrt{\frac{a-x}{(b-x)(c-x)}} dx = 2\sqrt{a-c} [F(\beta, p) - E(\beta, p)] + 2\sqrt{\frac{(a-y)(c-y)}{b-y}}, \quad a > b > c > y.$$

$$2. \int_c^y \sqrt{\frac{a-x}{(b-x)(x-c)}} dx = 2\sqrt{a-c} E(\gamma, q) \quad a > b \geq y > c.$$

$$3. \int_y^c \sqrt{\frac{b-x}{(a-x)(c-x)}} dx = \frac{2(b-c)}{\sqrt{a-c}} F(\beta, p) - 2\sqrt{a-c} E(\beta, p) + 2\sqrt{\frac{(a-y)(c-y)}{b-y}}, \quad a > b > c > y.$$

$$4. \int_c^y \sqrt{\frac{b-x}{(a-x)(x-c)}} dx = 2\sqrt{a-c} E(\gamma, q) - \frac{2(a-b)}{\sqrt{a-c}} F(\gamma, q), \quad a > b \geq y > c.$$

$$5. \int_y^c \sqrt{\frac{c-x}{(a-x)(b-x)}} dx = -2\sqrt{a-c} E(\beta, p) + 2\sqrt{\frac{(a-y)(c-y)}{b-y}}, \quad a > b > c > y.$$

$$6. \int_c^y \sqrt{\frac{x-c}{(a-x)(b-x)}} dx = 2\sqrt{a-c} [F(\gamma, q) - E(\gamma, q)], \quad a > b \geq y > c.$$

$$7. \int_y^c \sqrt{\frac{a-x}{(b-x)^3(c-x)}} dx = \frac{2\sqrt{a-c}}{b-c} E(\beta, p) \quad a > b > c > y.$$

$$8. \int_c^y \sqrt{\frac{a-x}{(b-x)^3(x-c)}} dx = \frac{2\sqrt{a-c}}{b-c} [F(\gamma, q) - E(\gamma, q)] + \frac{2}{b-c} \sqrt{\frac{(a-y)(y-c)}{b-y}},$$

$$a > b > y > c.$$

$$9. \int_y^c \sqrt{\frac{b-x}{(a-x)^3(c-x)}} dx = \frac{2}{\sqrt{a-c}} E(\beta, p) - \frac{2(a-b)}{a-c} \sqrt{\frac{c-y}{(a-y)(b-y)}}, \quad a > b > c > y.$$

$$10. \int_c^y \sqrt{\frac{b-x}{(a-x)^3(x-c)}} dx = \frac{2}{\sqrt{a-c}} [F(\gamma, q) - E(\gamma, q)]$$

$$+ \frac{2}{a-c} \sqrt{\frac{(b-y)(y-c)}{a-y}}, \quad a > b \geq y > c.$$

$$11. \int_y^c \sqrt{\frac{c-x}{(a-x)^3(b-x)}} dx = \frac{2\sqrt{a-c}}{a-b} E(\beta, p) - \frac{2(b-c)}{(a-b)\sqrt{a-c}} F(\beta, p)$$

$$- 2\sqrt{\frac{c-y}{(a-y)(b-y)}}, \quad a > b > c > y.$$

$$12. \int_c^y \sqrt{\frac{x-c}{(a-x)^3(b-x)}} dx = \frac{2\sqrt{a-c}}{a-b} E(\gamma, q) - \frac{2}{\sqrt{a-c}} F(\gamma, q)$$

$$- \frac{2}{a-b} \sqrt{\frac{(b-y)(y-c)}{a-y}}, \quad a > b \geq y > c.$$

$$13. \int_y^c \sqrt{\frac{c-x}{(a-x)(b-x)^3}} dx = \frac{2\sqrt{a-c}}{a-b} [F(\beta, p) - E(\beta, p)], \quad a > b > c > y.$$

$$14. \int_c^y \sqrt{\frac{x-c}{(a-x)(b-x)^3}} dx = -\frac{2\sqrt{a-c}}{a-b} E(\gamma, q) + \frac{2}{a-b} \sqrt{\frac{(a-y)(y-c)}{b-y}}, \quad a > b > y > c.$$
