! For an efficient use of these tables, first read `HowTo.pdf`.

T2.23D. Integrands of the form $\sqrt{\dfrac{d-x}{(a-x)(b-x)(c-x)}}$, $\sqrt{\dfrac{c-x}{(a-x)(b-x)(d-x)}}$, $\sqrt{\dfrac{b-x}{(a-x)(c-x)(d-x)}}$, and $\sqrt{\dfrac{a-x}{(b-x)(c-x)(d-x)}}$ on the intervals (y,d) and (d,y).

Notation used $\quad \alpha = \arcsin\sqrt{\dfrac{(a-c)(d-y)}{(a-d)(c-y)}}, \quad \beta = \arcsin\sqrt{\dfrac{(a-c)(y-d)}{(c-d)(a-y)}},$

$$q = \sqrt{\frac{(b-c)(a-d)}{(a-c)(b-d)}}, \quad r = \sqrt{\frac{(a-b)(c-d)}{(a-c)(b-d)}}.$$

1. $$\int_y^d \sqrt{\frac{d-x}{(a-x)(b-x)(c-x)}}\,dx = \frac{2(c-d)}{\sqrt{(a-c)(b-d)}}\left\{\Pi\left(\alpha,\, \frac{a-d}{a-c},\, q\right) - F(\alpha,\, q)\right\},$$
$$a > b > c > d > y.$$

2. $$\int_d^y \sqrt{\frac{x-d}{(a-x)(b-x)(c-x)}}\,dx = \frac{2(d-a)}{\sqrt{(a-c)(b-d)}}\left\{\Pi\left(\beta,\, \frac{d-c}{a-c},\, r\right) - F(\beta,\, r)\right\},$$
$$a > b > c \geq y > d.$$

3. $$\int_y^d \sqrt{\frac{c-x}{(a-x)(b-x)(d-x)}}\,dx = \frac{2(c-d)}{\sqrt{(a-c)(b-d)}}\Pi\left(\alpha,\, \frac{a-d}{a-c},\, q\right), \quad a > b > c > d > y.$$

4. $$\int_d^y \sqrt{\frac{c-x}{(a-x)(b-x)(x-d)}}\,dx = \frac{2}{\sqrt{(a-c)(b-d)}}\left[(a-d)\Pi\left(\beta,\, \frac{d-c}{a-c},\, r\right) - (a-c)F(\beta,\, r)\right],$$
$$a > b > c \geq y > d.$$

5. $$\int_y^d \sqrt{\frac{b-x}{(a-x)(c-x)(d-x)}}\,dx = \frac{2}{\sqrt{(a-c)(b-d)}}\left[(c-d)\Pi\left(\alpha,\, \frac{a-d}{a-c},\, q\right) + (b-c)F(\alpha,\, q)\right],$$
$$a > b > c > d > y.$$

6. $$\int_d^y \sqrt{\frac{b-x}{(a-x)(c-x)(x-d)}}\,dx = \frac{2}{\sqrt{(a-c)(b-d)}}\left[(a-d)\Pi\left(\beta,\,\frac{d-c}{a-c},\,r\right) - (a-b)F(\beta,\,r)\right],$$
$$a > b > c \geq y > d.$$

7. $$\int_y^d \sqrt{\frac{a-x}{(b-x)(c-x)(d-x)}}\,dx = \frac{2}{\sqrt{(a-c)(b-d)}}\left[(c-d)\Pi\left(\alpha,\,\frac{a-d}{a-c},\,q\right) + (a-c)F(\alpha,\,q)\right],$$
$$a > b > c > d > y.$$

8. $$\int_d^y \sqrt{\frac{a-x}{(b-x)(c-x)(x-d)}}\,dx = \frac{2(a-d)}{\sqrt{(a-c)(b-d)}}\Pi\left(\beta,\,\frac{d-c}{a-c},\,r\right), \quad a > b > c \geq y > d.$$