

! For an efficient use of these tables, first read [HowTo.pdf](#).

T2.12D. Integrands of the form $\frac{x^n}{\sqrt{(a-x)(b-x)(c-x)(d-x)}}$, $n = 0, 1$;
and $\frac{1}{x\sqrt{(a-x)(b-x)(c-x)(d-x)}}$ and $\frac{1}{(p-x)\sqrt{(a-x)(b-x)(c-x)(d-x)}}$
on the intervals (y, d) and (d, y) .

Notation used: $\alpha = \arcsin \sqrt{\frac{(a-c)(d-y)}{(a-d)(c-y)}}$, $\beta = \arcsin \sqrt{\frac{(a-c)(y-d)}{(c-d)(a-y)}}$,

$$q = \sqrt{\frac{(b-c)(a-d)}{(a-c)(b-d)}}, \quad r = \sqrt{\frac{(a-b)(c-d)}{(a-c)(b-d)}}.$$

$$1. \int_y^d \frac{dx}{\sqrt{(a-x)(b-x)(c-x)(d-x)}} = \frac{2}{\sqrt{(a-c)(b-d)}} F(\alpha, q), \quad a > b > c > d > y.$$

$$2. \int_d^y \frac{dx}{\sqrt{(a-x)(b-x)(c-x)(x-d)}} = \frac{2}{\sqrt{(a-c)(b-d)}} F(\beta, r), \quad a > b > c \geq y > d.$$

$$3. \int_y^d \frac{x dx}{\sqrt{(a-x)(b-x)(c-x)(d-x)}} = \frac{2}{\sqrt{(a-c)(b-d)}} \left\{ (d-c) \Pi \left(\alpha, \frac{a-d}{a-c}, q \right) + c F(\alpha, q) \right\},$$

$$a > b > c > d > y.$$

$$4. \int_d^y \frac{x dx}{\sqrt{(a-x)(b-x)(c-x)(x-d)}} = \frac{2}{\sqrt{(a-c)(b-d)}} \left\{ (d-a) \Pi \left(\beta, \frac{d-c}{a-c}, r \right) + a F(\beta, r) \right\},$$

$$a > b > c \geq y > d.$$

$$5. \int_y^d \frac{dx}{x\sqrt{(a-x)(b-x)(c-x)(d-x)}} = \frac{2}{cd\sqrt{(a-c)(b-d)}} \left\{ (c-d) \Pi \left(\alpha, \frac{c(a-d)}{d(a-c)}, q \right) + d F(\alpha, q) \right\}, \quad a > b > c > d > y.$$

$$\begin{aligned}
6. \int_d^y \frac{dx}{x\sqrt{(a-x)(b-x)(c-x)(x-d)}} \\
= \frac{2}{ad\sqrt{(a-c)(b-d)}} \left\{ (a-d)\Pi\left(\beta, \frac{a(d-c)}{d(a-c)}, r\right) + dF(\beta, r) \right\}, \quad a > b > c \geq y > d.
\end{aligned}$$

$$\begin{aligned}
7. \int_y^d \frac{dx}{(p-x)\sqrt{(a-x)(b-x)(c-x)(d-x)}} &= \frac{2}{(p-c)(p-d)\sqrt{(a-c)(b-d)}} \\
&\times \left[(d-c)\Pi\left(\alpha, \frac{(a-d)(p-c)}{(a-c)(p-d)}, q\right) + (p-d)F(\alpha, q) \right], \quad a > b > c > d > y, \quad p \neq d.
\end{aligned}$$

$$\begin{aligned}
8. \int_d^y \frac{dx}{(p-x)\sqrt{(a-x)(b-x)(c-x)(x-d)}} &= \frac{2}{(p-a)(p-d)\sqrt{(a-c)(b-d)}} \\
&\times \left[(d-a)\Pi\left(\beta, \frac{(d-c)(p-a)}{(a-c)(p-d)}, r\right) + (p-d)F(\beta, r) \right], \quad a > b > c \geq y > d, \quad p \neq d.
\end{aligned}$$
