! For an efficient use of these tables, first read `HowTo.pdf`.

T2.50B. Integrands involving product and division of trigonometric functions by powers of $(a+b\,x)$ on the interval $(0,y)$.

1. $$\int_0^y x^{\nu-1}(y-x)^{\mu-1}\,\sin(ax)\,dx=\frac{y^{\mu+\nu-1}}{2i}\,\mathrm{B}\,(\mu,\,\nu)\left[{}_1F_1(\nu;\,\mu+\nu;\,i\,ay)-\,{}_1F_1(\nu;\,\mu+\nu;\,-i\,ay)\right],$$ $$a>0,\ \Re\{\mu\}>0,\ \Re\{\nu\}>-1,\ \nu\neq 0.$$

2. $$\int_0^y x^{\nu-1}(y-x)^{\mu-1}\,\cos(ax)\,dx=\frac{y^{\mu+\nu-1}}{2}\,\mathrm{B}(\mu,\,\nu)\left[\,{}_1F_1(\nu;\,\mu+\nu;\,i\,ay)+\,{}_1F_1(\nu;\,\mu+\nu;\,-i\,ay)\right],$$ $$a>0,\ \Re\{\mu\}>0,\ \Re\{\nu\}>0.$$

3. $$\int_0^y x^{\mu-1}(y-x)^{\mu-1}\,\sin(ax)\,dx=\sqrt{\pi}\left(\frac{y}{a}\right)^{\mu-1/2}\sin\frac{ay}{2}\Gamma(\mu)J_{\mu-1/2}\left(\frac{ay}{2}\right),\quad \Re\{\mu\}>0.$$

4. $$\int_0^y x^{\mu-1}(y-x)^{\mu-1}\,\cos(ax)\,dx=\sqrt{\pi}\left(\frac{y}{a}\right)^{\mu-1/2}\cos\frac{ay}{2}\Gamma(\mu)\,J_{\mu-1/2}\left(\frac{ay}{2}\right),\quad \Re\{\mu\}>0.$$

5. $$\int_0^y x^{2\nu-1}(y^2-x^2)^{\mu-1}\,\sin(ax)\,dx=\frac{a}{2}\,y^{2\mu+2\nu-1}\,\mathrm{B}\left(\mu,\,\nu+\frac{1}{2}\right)\,{}_1F_2\left(\nu+\frac{1}{2};\,\frac{3}{2},\,\mu+\nu+\frac{1}{2};\,-\frac{a^2y^2}{4}\right),$$ $$\Re\{\mu\}>0,\ \Re\{\nu\}>-\tfrac{1}{2}.$$

6. $$\int_0^y x^{2\nu-1}(y^2-x^2)^{\mu-1}\,\cos(ax)\,dx=\frac{1}{2}\,y^{2\mu+2\nu-2}\,\mathrm{B}\,(\mu,\,\nu)\,{}_1F_2\left(\nu;\,\frac{1}{2},\,\mu+\nu;\,-\frac{a^2y^2}{4}\right),$$ $$\Re\{\mu\}>0,\ \Re\{\nu\}>0.$$

7. $$\int_0^y (y^2-x^2)^{\nu-1/2}\,\sin(ax)\,dx=\frac{\sqrt{\pi}}{2}\left(\frac{2y}{a}\right)^{\nu}\Gamma\left(\nu+\frac{1}{2}\right)\mathbf{H}_\nu(ay),\quad a>0,\ y>0,\ \Re\{\nu\}>-\tfrac{1}{2}.$$

8. $$\int_0^y (y^2-x^2)^{\nu-1/2}\,\cos(ax)\,dx=\frac{\sqrt{\pi}}{2}\left(\frac{2y}{a}\right)^{\nu}\Gamma\left(\nu+\frac{1}{2}\right)J_\nu(ay),\quad a>0,\ y>0,\ \Re\{\nu\}>-\tfrac{1}{2}.$$

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9. $$\int_0^y x(y^2-x^2)^{\nu-1/2}\,\sin(ax)\,dx = \frac{\sqrt{\pi}}{2} y \left(\frac{2y}{a}\right)^\nu \Gamma\left(\nu+\frac{1}{2}\right) J_{\nu+1}(ay),$$
$$a>0,\ y>0,\ \Re\{\nu\}>-\tfrac{1}{2}.$$

10. $$\int_0^y x(y^2-x^2)^{\nu-1/2}\,\cos(ax)\,dx = -\frac{y^{\nu+1}}{a^\nu} s_{\nu-1,\nu+1}(ay)$$
$$= \frac{1}{2}\left(\nu+\frac{1}{2}\right)^{-1} y^{2\nu+1} - \frac{\sqrt{\pi}}{2} y \left(\frac{2y}{a}\right)^\nu \Gamma\left(\nu+\frac{1}{2}\right) \mathbf{H}_{\nu+1}(ay),$$
$$a>0,\ y>0,\ \Re\{\nu\}>-\tfrac{1}{2}.$$

11. $$\int_0^y \frac{(x+i\sqrt{y^2-x^2})^\nu + (x-i\sqrt{y^2-x^2})^\nu}{\sqrt{y^2-x^2}}\,\sin(ax)\,dx = \frac{\pi}{2} y^\nu \csc\frac{\nu\pi}{2}\left[\mathbf{J}_\nu(ay) - \mathbf{J}_{-\nu}(ay)\right],$$
$$a>0,\ y>0.$$

12. $$\int_0^y \frac{(x+i\sqrt{y^2-x^2})^\nu + (x-i\sqrt{y^2-x^2})^\nu}{\sqrt{y^2-x^2}}\,\cos(ax)\,dx = \frac{\pi}{2} y^\nu \sec\frac{\nu\pi}{2}\left[\mathbf{J}_\nu(ay) + \mathbf{J}_{-\nu}(ay)\right],$$
$$a>0,\ y>0,\ |\Re\{\nu\}|<1.$$