! For an efficient use of these tables, first read `HowTo.pdf`.

T2.54A. Integrands involving powers of trigonometric functions and powers of $(a + b\,x^n)$ for $n = 1, 2, 3, 4$, on the interval $(0, \pi/4)$.

1. $$\int_0^{\pi/4} \frac{x^2\,dx}{\sin^2 x} = -\frac{\pi^2}{16} + \frac{\pi}{4}\ln 2 + \mathbf{G}.$$

2. $$\int_0^{\pi/4} \frac{x^2\,dx}{\cos^2 x} = \frac{\pi^2}{16} + \frac{\pi}{4}\ln 2 - \mathbf{G}.$$

3. $$\int_0^{\pi/4} \frac{x^{p+1}}{\sin^2 x}\,dx = -\left(\frac{\pi}{4}\right)^{p+1} + (p+1)\left(\frac{\pi}{4}\right)^{p}\left\{\frac{1}{p} - \frac{1}{2}\sum_{k=1}^{\infty}\frac{1}{4^{2k-1}(p+2k)}\zeta(2k)\right\}, \quad p > 0.$$

4. $$\int_0^{\pi/4} \frac{x\,\sin^{p-1} x}{\cos^{p+1} x}\,dx = \frac{\pi}{4p} - \frac{1}{2p}\beta\left(\frac{p+1}{2}\right), \quad p > -1.$$

5. $$\int_0^{\pi/4} \frac{x\,\sin^{2m-1} x}{\cos^{2m+1} x}\,dx = \frac{\pi}{8m}(1 - \cos m\pi) + \frac{1}{2m}\sum_{k=0}^{m-1}\frac{(-1)^{k-1}}{2m-2k-1}.$$

6. $$\int_0^{\pi/4} \frac{x\,\sin^{2m} x}{\cos^{2m+2} x}\,dx = \frac{1}{2(2m+1)}\left[\frac{\pi}{2} + (-1)^{m-1}\ln 2 + \sum_{k=0}^{m-1}\frac{(-1)^{k-1}}{m-k}\right].$$

7. $$\int_0^{\pi/4} x\,\tan^2 x\,dx = \frac{\pi}{4} - \frac{\pi^3}{32} - \frac{1}{2}\ln 2.$$

8. $$\int_0^{\pi/4} x\,\tan^3 x\,dx = \frac{\pi}{4} - \frac{1}{2} + \frac{\pi}{8}\ln 2 - \frac{1}{2}\mathbf{G}.$$

9. $$\int_0^{\pi/4} \frac{x^2\,\tan x}{\cos^2 x}\,dx = \frac{1}{2}\ln 2 - \frac{\pi}{4} + \frac{\pi^2}{16}.$$

10. $\displaystyle\int_0^{\pi/4} \frac{x^2\ \tan^2 x}{\cos^2 x}\,dx = \frac{1}{3}\left(1 - \frac{\pi}{4}\ \ln 2 - \frac{\pi}{2} + \frac{\pi^2}{16} + \mathbf{G}\right).$