! For an efficient use of these tables, first read `HowTo.pdf`.

T2.53B. Integrands involving rational functions of $(a+b\,x)$ and trigonometric functions on the interval $(0,\pi/2)$.

1. $\displaystyle\int_0^{\pi/2}\left(\frac{1}{x}-\cot x\right)\,dx=\ln\,\frac{\pi}{2}.$

2. $\displaystyle\int_0^{\pi/2}\frac{4x^2\,\cos x+(\pi-x)x}{\sin x}\,dx=\pi^2\,\ln\,2.$

3. $\displaystyle\int_0^{\pi/2}\frac{x\,dx}{1+\sin x}=\ln\,2.$

4. $\displaystyle\int_0^{\pi}\frac{x\,\cos x}{1+\sin x}\,dx=\pi\,\ln\,2-4\mathbf{G}.$

5. $\displaystyle\int_0^{\pi/2}\frac{x\,\cos x}{1+\sin x}\,dx=\pi\,\ln\,2-2\mathbf{G}.$

6. $\displaystyle\int_0^{\pi/2}\frac{x^2\,dx}{1-\cos x}=-\frac{\pi^2}{4}+\pi\,\ln\,2+4\mathbf{G}.$

7. $\displaystyle\int_0^{\pi/2}\frac{x^{p+1}\,dx}{1-\cos x}=-\left(\frac{\pi}{2}\right)^{p+1}+\left(\frac{\pi}{2}\right)^{p}(p+1)\left\{\frac{2}{p}-\sum_{k=1}^{\infty}\frac{1}{4^{2k-1}(p+2k)}\zeta(2k)\right\},\quad p>0.$

8. $\displaystyle\int_0^{\pi/2}\frac{x\,dx}{1+\cos x}=\frac{\pi}{2}-\ln\,2.$

9. $\displaystyle\int_0^{\pi/2}\frac{x\,\sin x\,dx}{1-\cos x}=\frac{\pi}{2}\,\ln\,2+2\mathbf{G}.$

10. $\displaystyle\int_0^{\pi/2} \frac{x\,\sin x}{1+\cos x}\,dx = -\frac{\pi}{2}\,\ln\,2 + 2\mathbf{G}.$

11. $\displaystyle\int_0^{\pi/2} \frac{x\,\cos x\,dx}{1+2a\,\sin x + a^2} = \frac{\pi}{2a}\,\ln\,(1+a) - \sum_{k=0}^{\infty}(-1)^k \frac{a^{2k}}{(2k+1)^2}, \quad a^2 < 1.$

12. $\displaystyle\int_0^{\pi/2} \frac{\cos x \pm \sin x}{\cos x \mp \sin x}x\,dx = \mp\frac{\pi}{4}\,\ln\,2 - \mathbf{G}.$

13. $\displaystyle\int_0^{\pi/2} \frac{x\,dx}{(\sin x + a\,\cos x)^2} = \frac{a}{1+a^2}\,\frac{\pi}{2} - \frac{\ln\,a}{1+a^2}, \quad a > 0.$

14. $\displaystyle\int_0^{\pi/2} \frac{x\,dx}{(\cos x \pm \sin x)\,\sin x} = \frac{\pi}{4}\,\ln\,2 + \mathbf{G}.$

15. $\displaystyle\int_0^{\pi/2} \frac{x\,\sin 2x\,dx}{1+a\,\cos^2 x} = \frac{\pi}{a}\,\ln\,\frac{1+\sqrt{1+a}}{2}, \quad a > -1,\, a \neq 0.$

16. $\displaystyle\int_0^{\pi/2} \frac{x\,\sin 2x\,dx}{1+a\,\sin^2 x} = \frac{\pi}{a}\,\ln\,\frac{2(1+a-\sqrt{1+a})}{2}, \quad a > -1,\, a \neq 0.$

17. $\displaystyle\int_0^{\pi/2} \frac{x\,\sin x\,dx}{\cos^2 t - \sin^2 x} = -2\,\csc t \sum_{k=0}^{\infty} \frac{\sin(2k+1)t}{(2k+1)^2}.$

18. $\displaystyle\int_0^{\pi/2} \ln\,\left(a - \sin^2 x\right)\,dx = -\pi\,\ln\,2 + i\,\pi\,\ln\,\arccos\sqrt{a}, \quad 0 < a < 1.$

19. p.v. $\displaystyle\int_0^{\pi/2} \ln\,\left(|a - \sin^2 x|\right)\,dx = -\pi\,\ln\,2, \quad 0 < a < 1.$

20. p.v. $\displaystyle\int_0^{\pi/2} \ln\,\left(|a - \cos^2 x|\right)\,dx = -\pi\,\ln\,2, \quad 0 < a < 1.$

21. $\displaystyle\int_0^{\pi/2} \frac{x\,\sin 2x\,dx}{a^2\,\cos^2 x + b^2\,\sin^2 x} = \frac{\pi}{a^2 - b^2}\,\ln\,\frac{a+b}{2b}, \quad a > 0,\ b > 0,\, a \neq b.$

22. $\displaystyle\int_0^{\pi/2} \frac{(1 - x\,\cot x)\,dx}{\sin^2 x} = \frac{\pi}{4}.$

23. $$\int_0^{\pi/2} \frac{x\ \cot x\, dx}{a^2\ \cos^2 x + b^2\ \sin^2 x} = \frac{\pi}{2a^2}\ \ln\ \frac{a+b}{b}, \quad a>0,\ b>0.$$

24. $$\int_0^{\pi/2} \frac{\left(\frac{\pi}{2}-x\right)\ \tan x\, dx}{a^2\ \cos^2 x + b^2\ \sin^2 x} = \frac{1}{2}\int_0^{\pi} \frac{\left(\frac{\pi}{2}-x\right)\ \tan x\, dx}{a^2\ \cos^2 x + b^2\ \sin^2 x} = \frac{\pi}{2b^2}\ \ln\ \frac{a+b}{a}, \quad a>0,\ b>0.$$

25. $$\int_0^{\pi/2} \frac{x\ \sin 2x\, dx}{(1+a\ \sin^2 x)(1+b\ \sin^2 x)} = \frac{\pi}{a-b}\ \ln\ \left\{\frac{1+\sqrt{1+b}}{1+\sqrt{1+a}} \cdot \frac{\sqrt{1+a}}{\sqrt{1+b}}\right\}, \quad a>0,\ b>0.$$

26. $$\int_0^{\pi/2} \frac{x\ \sin 2x\, dx}{(1+a\ \sin^2 x)(1+b\ \cos^2 x)} = \frac{\pi}{a+ab+b}\ \ln\ \frac{(1+\sqrt{1+n})\sqrt{1+a}}{1+\sqrt{1+a}}, \quad a>0,\ b>0.$$

27. $$\int_0^{\pi/2} \frac{x\ \sin 2x\, dx}{(1+a\ \cos^2 x)(1+b\ \cos^2\ x)} = \frac{\pi}{a-b}\ \ln\ \frac{1+\sqrt{1+a}}{1+\sqrt{1+b}}, \quad a>0,\ b>0.$$

28. $$\int_0^{\pi/2} \frac{x\ \sin 2x\, dx}{(1-\sin^2 t_1\ \cos^2 x)(1-\sin^2 t_2\ \cos^2 x)} = \frac{2\pi}{\cos^2 t_1 - \cos^2 t_2}\ \ln\ \frac{\cos(t_1/2)}{\cos(t_2/2)}, \quad -\pi<t_1<\pi,\ -\pi<t_2<\pi.$$
