

! For an efficient use of these tables, first read [HowTo.pdf](#).

T1.14. Integrand involving $a + bx^k$ and \sqrt{x} .

Notation used: $X_1 = a + bx$.

$$1. \int \frac{dx}{X_1 \sqrt{x}} = \begin{cases} \frac{2}{\sqrt{ab}} \arctan \sqrt{\frac{bx}{a}}, & ab > 0, \\ \frac{1}{\sqrt{-ab}} \ln \frac{a - bx + 2\sqrt{-xab}}{X_1}, & ab < 0 \end{cases}.$$

$$2. \int \frac{dx}{X_1^2 \sqrt{x}} = \frac{\sqrt{x}}{aX_1} + \frac{1}{2a} \int \frac{dx}{X_1 \sqrt{x}}.$$

$$3. \int \frac{dx}{X_1^3 \sqrt{x}} = \left(\frac{1}{2aX_1^2} + \frac{3}{4a^2 X_1} \right) \sqrt{x} + \frac{3}{8a^2} \int \frac{dx}{X_1 \sqrt{x}}.$$

$$4. \int \frac{\sqrt{x} dx}{X_1} = \frac{2\sqrt{x}}{b} - \frac{a}{b} \int \frac{dx}{X_1 \sqrt{x}}.$$

$$5. \int \frac{x\sqrt{x} dx}{X_1} = \left(\frac{x}{3b} - \frac{a}{b^2} \right) 2\sqrt{x} + \frac{a^2}{b^2} \int \frac{dx}{X_1 \sqrt{x}}.$$

$$6. \int \frac{x^2 \sqrt{x} dx}{X_1} = \left(\frac{x^2}{5b} - \frac{xa}{3b^2} + \frac{a^2}{b^3} \right) 2\sqrt{x} - \frac{a^3}{b^3} \int \frac{dx}{X_1 \sqrt{x}}.$$

$$7. \int \frac{\sqrt{x} dx}{X_1^2} = -\frac{\sqrt{x}}{bX_1} + \frac{1}{2b} \int \frac{dx}{X_1 \sqrt{x}}.$$

$$8. \int \frac{x\sqrt{x} dx}{X_1^2} = \frac{2x\sqrt{x}}{bX_1} - \frac{3a}{b} \int \frac{\sqrt{x} dx}{X_1^2}.$$

$$9. \int \frac{x^2 \sqrt{x} dx}{X_1^2} = \left(\frac{x^2}{3b} - \frac{5ax}{3b^2} \right) \frac{2\sqrt{x}}{X_1} + \frac{5a^2}{b^2} \int \frac{\sqrt{x} dx}{X_1^2}.$$

$$10. \int \frac{\sqrt{x} dx}{X_1^3} = \left(-\frac{1}{2bX_1^2} + \frac{1}{4abX_1} \right) \sqrt{x} + \frac{1}{8ab} \int \frac{dx}{X_1 \sqrt{x}}.$$

$$\begin{aligned}
11. \int \frac{x\sqrt{x} dx}{X_1^3} &= -\frac{2x\sqrt{x}}{bX_1^2} + \frac{3a}{b} \int \frac{\sqrt{x} dx}{X_1^3}. \\
12. \int \frac{x^2\sqrt{x} dx}{X_1^3} &= \left(\frac{x^2}{b} + \frac{5ax}{b^2}\right) \frac{2\sqrt{x}}{X_1^2} - \frac{15a^2}{b^2} \int \frac{\sqrt{x} dx}{X_1^3}. \\
13. \int \frac{x^m\sqrt{x}}{X_1} dx &= 2\sqrt{x} \sum_{k=0}^m \frac{(-1)^k a^k x^{m-k}}{(2m-2k+1)b^{k+1}} + (-1)^{m+1} \frac{a^{m+1}}{b^{m+1}} \int \frac{dx}{X_1\sqrt{x}}.
\end{aligned}$$

Notation used: $X_2 = a + bx^2$, $\alpha = \left(\frac{a}{b}\right)^{1/4}$, $\alpha' = \left(-\frac{a}{b}\right)^{1/4}$.

$$\begin{aligned}
14. \int \frac{dx}{X_2\sqrt{x}} &= \begin{cases} \frac{1}{b\alpha^3\sqrt{2}} \left[\ln \frac{x + \alpha\sqrt{2x} + \alpha^2}{\sqrt{X_2}} + \arctan \frac{\alpha\sqrt{2x}}{\alpha^2 - x} \right], & \frac{a}{b} > 0, \\ \frac{1}{2b\alpha'^3} \left(\ln \frac{\alpha' - \sqrt{x}}{\alpha' + \sqrt{x}} - 2 \arctan \frac{\sqrt{x}}{\alpha'} \right), & \frac{a}{b} < 0. \end{cases} \\
15. \int \frac{\sqrt{x} dx}{X_2} &= \begin{cases} \frac{1}{b\alpha\sqrt{2}} \left[-\ln \frac{x + \alpha\sqrt{2x} + \alpha^2}{\sqrt{X_2}} + \arctan \frac{\alpha\sqrt{2x}}{\alpha^2 - x} \right], & \frac{a}{b} > 0, \\ \frac{1}{2b\alpha'} \left[\ln \frac{\alpha' - \sqrt{x}}{\alpha' + \sqrt{x}} + 2 \arctan \frac{\sqrt{x}}{\alpha'} \right], & \frac{a}{b} < 0. \end{cases} \\
16. \int \frac{x\sqrt{x} dx}{X_2} &= \frac{2\sqrt{x}}{b} - \frac{a}{b} \int \frac{dx}{X_2\sqrt{x}}. \\
17. \int \frac{x^2\sqrt{x} dx}{X_2} &= \frac{2x\sqrt{x}}{3b} - \frac{a}{b} \int \frac{\sqrt{x} dx}{X_2}. \\
18. \int \frac{x}{X_2^2\sqrt{x}} &= \frac{\sqrt{x}}{2aX_2} + \frac{3}{4a} \int \frac{dx}{X_2\sqrt{x}}. \\
19. \int \frac{\sqrt{x} dx}{X_2^2} &= \frac{x\sqrt{x}}{2aX_2} + \frac{1}{4a} \int \frac{\sqrt{x} dx}{X_2}. \\
20. \int \frac{x\sqrt{x} dx}{X_2^2} &= -\frac{\sqrt{x}}{2bX_2} + \frac{1}{4b} \int \frac{dx}{X_2\sqrt{x}}. \\
21. \int \frac{x^2\sqrt{x} dx}{X_2^2} &= -\frac{x\sqrt{x}}{2bX_2} + \frac{3}{4b} \int \frac{\sqrt{x} dx}{X_2}. \\
22. \int \frac{dx}{X_2^3\sqrt{x}} &= \left(\frac{1}{4aX_2^2} + \frac{7}{16a^2X_2} \right) \sqrt{x} + \frac{21}{32a^2} \int \frac{dx}{X_2\sqrt{x}}. \\
23. \int \frac{\sqrt{x} dx}{X_2^3} &= \left(\frac{1}{4aX_2^2} + \frac{5}{16a^2X_2} \right) x\sqrt{x} + \frac{5}{32a^2} \int \frac{\sqrt{x} dx}{X_2}. \\
24. \int \frac{x\sqrt{x} dx}{X_2^3} &= \frac{(bx^2 - 3a)\sqrt{x}}{16abX_2^2} + \frac{3}{32ab} \int \frac{dx}{X_2\sqrt{x}}.
\end{aligned}$$

$$25. \int \frac{x^2 \sqrt{x} \, dx}{X_2^3} = -\frac{2x\sqrt{x}}{5bX_2^2} + \frac{3a}{5b} \int \frac{\sqrt{x} \, dx}{X_2^3}.$$

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