! For an efficient use of these tables, first read `HowTo.pdf`.

T2.64A. Integrands involving logarithm functions and rational functions on the interval $(0,1)$.

1. $\displaystyle\int_0^1 \frac{\ln x}{1+x}\,dx = -\frac{\pi^2}{12}.$

2. $\displaystyle\int_0^1 \frac{\ln x}{1-x}\,dx = -\frac{\pi^2}{6}.$

3. $\displaystyle\int_0^1 \frac{x\ln x}{1-x}\,dx = 1-\frac{\pi^2}{6}.$

4. $\displaystyle\int_0^1 \frac{1+x}{1-x}\ln x\,dx = 1-\frac{\pi^2}{3}.$

5. $\displaystyle\int_0^\infty \frac{\ln x\,dx}{(x+a)^2} = \frac{\ln a}{a}, \quad 0<a<1.$

6. $\displaystyle\int_0^1 \frac{\ln x}{(1+x)^2}\,dx = -\ln 2.$

7. $\displaystyle\int_0^1 \frac{\ln x}{1+x^2}\,dx = -\int_1^\infty \frac{\ln x}{1+x^2}\,dx = -\mathbf{G}.$

8. $\displaystyle\int_0^1 \frac{\ln x\,dx}{1-x^2} = -\frac{\pi^2}{8}.$

9. $\displaystyle\int_0^1 \frac{x\ln x}{1+x^2}\,dx = -\frac{\pi^2}{48}.$

10. $\displaystyle\int_0^1 \frac{x\ln x}{1-x^2}\,dx = -\frac{\pi^2}{24}.$

11. $\displaystyle\int_0^1 \ln x \frac{1-x^{2n+2}}{(1-x^2)^2}\,dx = -\frac{(n+1)\pi^2}{8} + \sum_{k=1}^{n} \frac{n-k+1}{(2k-1)^2}.$

12. $\displaystyle\int_0^1 \ln x \frac{1+(-1)^n x^{n+1}}{(1+x)^2}\,dx = -\frac{(n+1)\pi^2}{12} - \sum_{k=1}^{n} (-1)^k \frac{n-k+1}{k^2}.$

13. $\displaystyle\int_0^1 \ln x \frac{1-x^{n+1}}{(1-x)^2}\,dx = -\frac{(n+1)\pi^2}{6} + \sum_{k=1}^{n} \frac{n-k+1}{k^2}.$

14. $\displaystyle\int_0^1 \frac{\ln x\, dx}{1+x+x^2} = \frac{2}{9}\left[\frac{2\pi^2}{3} - \psi'\left(\frac{1}{3}\right)\right].$

15. $\displaystyle\int_0^1 \frac{\ln x\, dx}{1-x+x^2} = \frac{1}{3}\left[\frac{2\pi^2}{3} - \psi'\left(\frac{1}{3}\right)\right].$

16. $\displaystyle\int_0^1 \frac{x\ln x\, dx}{1+x+x^2} = -\frac{1}{9}\left[\frac{7\pi^2}{6} - \psi'\left(\frac{1}{3}\right)\right].$

17. $\displaystyle\int_0^1 \frac{x\ln x\, dx}{1-x+x^2} = \frac{1}{6}\left[\frac{5\pi^2}{6} - \psi'\left(\frac{1}{3}\right)\right].$

18. $\displaystyle\int_0^1 \frac{x\ln x\, dx}{(1+x^2)^2} = -\frac{1}{4}\ln 2.$

19. $\displaystyle\int_0^1 \frac{x^2\ln x\, dx}{(1-x^2)(1+x^4)} = -\frac{\pi^2}{16(2+\sqrt{2})}.$

20. $\displaystyle\int_0^1 \ln x \frac{x^{m-1}+x^{n-m-1}}{1-x^n}\,dx = -\frac{\pi^2}{n^2\sin^2\left(\frac{m}{n}\pi\right)}, \quad n>m.$

21. $\displaystyle\int_0^1 \left\{\frac{1+(p-1)\ln x}{1-x} + \frac{x\ln x}{(1-x)^2}\right\} x^{p-1}\,dx = -1+\psi'(p), \quad p>0.$

22. $\displaystyle\int_0^1 \left[\frac{1}{1-x} + \frac{x\ln x}{(1-x)^2}\right] dx = \frac{\pi^2}{6} - 1.$

23. $\displaystyle\int_0^1 \frac{x^{a-1}}{1-x^b}\ln x\, dx = -\sum_{n=0}^{\infty} \frac{1}{(a+b\,n)^2}.$

24. $\displaystyle\int_0^1 \frac{x^{a-1}}{1+x^b}\,\ln\,x\,dx = -\sum_{n=0}^{\infty}\frac{(-1)^n}{(a+b\,n)^2}.$