! For an efficient use of these tables, first read `HowTo.pdf`.

T2.30A. Integrands with the fourth roots of polynomials in the denominator, and their product with rational functions on the interval $(0, y)$.

Notation ysed: $\alpha = \arccos \dfrac{1}{(y^2+1)^{1/4}}, \quad \beta = \arccos \left(1 - y^2\right)^{1/4}.$

Then

1. $$\int_0^y \frac{dx}{(x^2+1)^{1/4}} = \sqrt{2}\left[F\left(\alpha, \frac{1}{\sqrt{2}}\right) - 2E\left(\alpha, \frac{1}{\sqrt{2}}\right)\right] + \frac{2y}{(y^2+1)^{1/4}}, \quad y > 0.$$

2. $$\int_0^y \frac{dx}{(1-x^2)^{1/4}} = \sqrt{2}\left[2E\left(\beta, \frac{1}{\sqrt{2}}\right) - F\left(\beta, \frac{1}{\sqrt{2}}\right)\right], \quad 0 < y \le 1.$$

3. $$\int_0^y \frac{x^2\,dx}{(1-x^2)^{1/4}} = \frac{2\sqrt{2}}{5}\left[2E\left(\beta, \frac{1}{\sqrt{2}}\right) - F\left(\beta, \frac{1}{\sqrt{2}}\right)\right] - \frac{2y}{5}\left(1-y^2\right)^{3/4}, \quad 0 < y \le 1.$$

4. $$\int_0^y \frac{dx}{(x^2+1)^{3/4}} = \sqrt{2}F\left(\alpha, \frac{1}{\sqrt{2}}\right), \quad y > 0.$$

5. $$\int_0^y \frac{dx}{(1-x^2)^{3/4}} = \sqrt{2}F\left(\beta, \frac{1}{\sqrt{2}}\right), \quad 0 < y \le 1.$$

6. $$\int_0^y \frac{x^2\,dx}{(1-x^2)^{3/4}} = \frac{2\sqrt{2}}{3}F\left(\beta, \frac{1}{\sqrt{2}}\right) - \frac{2}{3}y\left(1-y^2\right)^{1/4}, \quad 0 < y \le 1.$$

7. $$\int_0^y \frac{dx}{(x^2+1)^{5/4}} = 2\sqrt{2}E\left(\alpha, \frac{1}{\sqrt{2}}\right) - \sqrt{2}F\left(\alpha, \frac{1}{\sqrt{2}}\right), \quad y > 0.$$

8. $$\int_0^y \frac{x^2\,dx}{(x^2+1)^{5/4}} = 2\sqrt{2}\left[F\left(\alpha, \frac{1}{\sqrt{2}}\right) - 2E\left(\alpha, \frac{1}{\sqrt{2}}\right)\right] + \frac{2y}{(y^2+1)^{1/4}}, \quad y > 0.$$

9. $$\int_0^y \frac{x^2\,dx}{(x^2+1)^{7/4}} = \frac{1}{3\sqrt{2}}F\left(\alpha,\,\frac{1}{\sqrt{2}}\right) - \frac{y}{6\,(y^2+1)^{3/4}}, \quad y>0.$$

10. $$\int_0^y \frac{1+\sqrt{x^2+1}}{(x^2+1)(x^2+1)^{1/4}}\,dx = 2\sqrt{2}E\left(\alpha,\,\frac{1}{\sqrt{2}}\right), \quad y>0$$

11. $$\int_0^y \frac{dx}{(1+\sqrt{1-x^2})(1-x^2)^{1/4}} = \sqrt{2}\left[F\left(\beta,\,\frac{1}{\sqrt{2}}\right) - E\left(\beta,\,\frac{1}{\sqrt{2}}\right)\right] + \frac{y\big(1-y^2\big)^{1/4}}{1+\sqrt{1-y^2}},$$
$$0<y\le 1.$$

12. $$\int_0^y \frac{1-\sqrt{1-x^2}}{1+\sqrt{1-x^2}}\,\frac{dx}{(1-x^2)^{3/4}} = \sqrt{2}\left[2E\left(\beta,\,\frac{1}{\sqrt{2}}\right) - F\left(\beta,\,\frac{1}{\sqrt{2}}\right)\right] - \frac{2y\big(1-y^2\big)^{1/4}}{1+\sqrt{1-y^2}},$$
$$0<y\le 1.$$