# Errata for Optimal Control for Chemical Engineers

**Page xvi, line 9**

$$y'' \equiv \frac{\mathrm{d}y'}{\mathrm{d}x} \equiv \frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$$

**Page 1, line 9** "An **optimal control** is a function that optimizes ..."

**Page 7**

$$I = y(L) = y(0) + \int_{y(0)}^{y(L)} \mathrm{d}y = y_0 + \int_0^L \frac{\mathrm{d}y}{\mathrm{d}z}\,\mathrm{d}z \tag{1.12}$$

**Page 8**

$$\frac{\partial T}{\partial t} = -v\frac{\partial T}{\partial z} + \frac{h}{\rho C_\mathrm{p}}[T_\mathrm{w}(t) - T] \tag{1.13}$$

**Page 9**

$$\frac{\mathrm{d}T_\mathrm{s}}{\mathrm{d}z} = \frac{h}{v\rho C_\mathrm{p}}[\theta - T_\mathrm{s}] \tag{1.15}$$

**Page 12, Figure 1.7** Replace the labels "monomer:" and "polymer:" by "initiator:" and "monomer:", respectively.

**Page 39, line 9,11**

$$I(y_0 + \alpha h) = I(y) = \frac{\alpha h_1 h_2}{h_1 + h_2}$$

$$\delta I(y_0; h) \quad = \quad \frac{\mathrm{d}}{\mathrm{d}\alpha} I(y_0 + \alpha h)_{\alpha=0} \quad = \quad \frac{h_1 h_2}{h_1 + h_2}$$

**Page 41, line 3**

- $\delta y$ and $\delta y'$ are the functions $h(x)$ and $h'(x)$, respectively.

**Page 41, last sentence of Example 2.9** "Upon substituting $\delta y = -x$ and the corresponding ..."

**Page 66**

$$\frac{\partial H}{\partial \lambda} \equiv H_\lambda \quad = \quad \dot{y} \tag{3.28}$$

$$\frac{\partial H}{\partial y} \equiv H_y \quad = \quad -\dot{\lambda} \tag{3.29}$$

$$\frac{\partial H}{\partial u} \equiv H_u \quad = \quad 0 \tag{3.30}$$

**Page 67, line 6**

1. the state equation, Equation (3.25), obtained from $\dot{x} = H_\lambda$ and the initial condition $x(0) = 0$;

**Page 67, 2nd last equation**

$$J = \int_0^{t_\mathrm{f}} \left( F + \sum_{i=1}^{n} \lambda_i G_i \right) \mathrm{d}t \equiv \int_0^{t_\mathrm{f}} f(\mathbf{y}, \dot{\mathbf{y}}, \mathbf{u}, \boldsymbol{\lambda})\, \mathrm{d}t$$

**Page 90, the line before Equation (4.2)** Remove the extra "the" before "constraint $K(y) = k_0, \ldots$"

**Page 101, the equation before Equation (4.13)**

$$\delta K_i = \lim_{\Delta t_i \to 0} \left[ -\frac{\delta y_{i+1} - \delta y_i}{\Delta t_i} + \underbrace{g_y\big|_{t_i} \delta y_i + g_u\big|_{t_i} \delta u_i}_{\delta g_i} \right] \Delta t_i$$

**Page 103, 2nd last line of Example 4.3** "exists at least one set of variations $(\delta \dot{x}, \delta x, \delta u)$ at each $t$ ..."

**Page 106, line 13–17** For consistency, use $\mu$ instead of $\nu$ as follows:

$$M(\hat{\mathbf{y}}, \mathbf{u}) = J(\hat{\mathbf{y}}, \mathbf{u}) + \int_0^{t_\mathrm{f}} \sum_{i=1}^{l} \mu_i h_i(\hat{\mathbf{y}}, \mathbf{u})\, \mathrm{d}t = J + \int_0^{t_\mathrm{f}} \boldsymbol{\mu}^\top \mathbf{h}(\hat{\mathbf{y}}, \mathbf{u})\, \mathrm{d}t$$

where $\hat{\mathbf{y}}$ denotes the state vector that satisfies state equations for any admissible control vector $\mathbf{u}$ and $\boldsymbol{\mu}$ is the vector of time dependent Lagrange multipliers

$$\begin{bmatrix} \mu_1(t) & \mu_2(t) & \ldots & \mu_l(t) \end{bmatrix}^\top$$

**Page 108, line 12**

$$J[\hat{u}(k_0 + \Delta k_0)] = J[\hat{u}(k_0)] + \mathrm{d}J[\hat{u}(k_0); \hat{u}_{k_0} \Delta k_0] + \mathrm{d}J[\hat{u}(k_0); \epsilon] + \epsilon_1(\Delta k_0 \delta u)$$

**Page 111, 2nd last line**

$$\mu \;\geq\; 0, \qquad \underbrace{\mu(\hat{u} - c\hat{y}) \;=\; 0 \quad \text{at} \quad t_\mathrm{f}}_{\text{complementary slackness condition}}$$

**Page 114, line 9**

$$M = J + \int_0^{t_\mathrm{f}} \left\{ \mu_1 \underbrace{\left[ \frac{x_2}{x_1} - a\left( \frac{1}{b^{u_2 - c}} - 1 \right) \right]}_{K_1} + \mu_2 \underbrace{(-u_2 + u_\mathrm{min})}_{K_2} + \mu_3 \underbrace{(u_2 - u_\mathrm{max})}_{K_3} \right\} \mathrm{d}t$$

**Page 124, line 16**

$$= \int_0^{t_\mathrm{f}} (H_y + \dot{\lambda}) \delta y\, \mathrm{d}t - \Big[ \lambda \delta y \Big]_0^{t_\mathrm{f}} + \int_0^{t_\mathrm{f}} H_u \delta u\, \mathrm{d}t + \int_0^{t_\mathrm{f}} (-\dot{y} + H_\lambda) \delta \lambda\, \mathrm{d}t$$

**Page 154, line 14,15** Sum the terms containing $\delta u_i$ separately as follows:

$$\delta J = \int_0^{t_\mathrm{f}} \left[ \sum_{i=1}^{n} \left( \frac{\partial H}{\partial y_i}\delta y_i + \frac{\partial H}{\partial \lambda_i}\delta\lambda_i - \lambda_i \delta\dot{y}_i - \dot{y}_i \delta\lambda_i \right) \right] \mathrm{d}t + \int_0^{t_\mathrm{f}} \left[ \sum_{i=1}^{m} \frac{\partial H}{\partial u_i}\delta u_i \right] \mathrm{d}t + \left[ H - \boldsymbol{\lambda}^\top \dot{\mathbf{y}} \right]_{t_\mathrm{f}} \delta t_\mathrm{f}$$

**Page 158, the equation after Equation (6.11)**

$$\boldsymbol{\mu} = \begin{bmatrix} \mu_1 & \mu_2 & \ldots & \mu_l \end{bmatrix}^\top$$

**Page 160, 3rd equation**

$$\begin{bmatrix} \dot{\lambda}_1 \\ \dot{\lambda}_2 \\ \dot{\lambda}_3 \end{bmatrix} = - \underbrace{\begin{bmatrix} 2(\lambda_1 - \lambda_2)a_1 y_1 e^{E_1/T} \\ (1 + \lambda_2 - \lambda_3)a_2 e^{E_2/T} \\ (-1 + \lambda_3)a_3 e^{E_3/T} \end{bmatrix}}_{-H_\mathbf{y}}$$

**Page 166, line 8** Remove explicit $t$ from the arguments as follows:

$$f_i(\mathbf{y}, \mathbf{u}) \le 0, \quad i = 1, 2, \ldots, l \quad \text{or} \quad \mathbf{f}(\mathbf{y}, \mathbf{u}) \le \mathbf{0}$$

**Page 167, last equation**

$$\begin{bmatrix} \dot{\lambda}_1 \\ \dot{\lambda}_2 \\ \dot{\lambda}_3 \end{bmatrix} = - \underbrace{\begin{bmatrix} 2(\lambda_1 - \lambda_2)a_1 y_1 e^{E_1/T} \\ (1 + \lambda_2 - \lambda_3)a_2 e^{E_2/T} \\ (-1 + \lambda_3)a_3 e^{E_3/T} \end{bmatrix}}_{-L_\mathbf{y}}, \quad \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix}_{t_\mathrm{f}} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

**Page 168, last equation of Example 6.9** Remove the subscript $t = 0$ as follows:

$$\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \ge \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \text{and} \qquad \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}^\top \underbrace{\begin{bmatrix} -T + T_\mathrm{min} \\ T - T_\mathrm{max} \end{bmatrix}}_{L_{\boldsymbol{\mu}}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

**Page 169, 1st equation** Remove explicit $t$ from the argument as follows:

$$\int_0^{t_\mathrm{f}} F_i(\mathbf{y}, \mathbf{u}) \, \mathrm{d}t = k_i, \quad i = 1, 2, \ldots, l$$

**Page 170, Example 6.10** For consistency, use $(y - y^*)$ instead of $(y^* - y)$.

**Page 176, 3rd equation from bottom**

$$H_u = 1 + \lambda_1 = 0$$

**Page 177, 2nd equation**

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix}_{t_{1-}} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix}_{t_{1+}} + \underbrace{\begin{bmatrix} \dfrac{\partial q_1}{\partial y_1} & \dfrac{\partial q_1}{\partial y_2} & \dfrac{\partial q_1}{\partial y_3} & \dfrac{\partial q_1}{\partial y_4} \\ \dfrac{\partial q_2}{\partial y_1} & \dfrac{\partial q_2}{\partial y_2} & \dfrac{\partial q_2}{\partial y_3} & \dfrac{\partial q_2}{\partial y_4} \end{bmatrix}}_{\mathbf{q}_{\mathbf{y}(t_1)}}^{\top} \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}$$

**Page 178, Example of Section 6.7** For notational consistency in case of partial time derivatives, use $c_t$, $\delta c_t$, $\lambda_t$ and $\delta\lambda_t$ instead of $\dot{c}$, $\delta\dot{c}$, $\dot{\lambda}$ and $\delta\dot{\lambda}$, respectively.

**Page 182**

$$\lambda_z(L, t) = 0, \quad 0 \le t \le t_\mathrm{f} \tag{6.26}$$

**Page 182, 1st item of Bibliography**

K.J. Arrow, L. Hurwicz, and H. Uzawa. Constraint qualifications in nonlinear programming. *Nav. Res. Logist. Q.*, 8(2), 1961.

**Page 204, 2nd and 3rd equations**

$$\boldsymbol{\lambda}^{\top}(1) = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix}_{\sigma=1}^{\top} = \underbrace{\begin{bmatrix} y_4 - b_1 y_2 \\ y_4 - b_2 y_3 \end{bmatrix}_{\sigma=1}^{\top}}_{\mathbf{q}^{\top}} \underbrace{\begin{bmatrix} \alpha^r & 0 \\ 0 & \alpha^r \end{bmatrix}}_{\mathbf{W}} \underbrace{\begin{bmatrix} 0 & -b_1 & 0 & 1 \\ 0 & 0 & -b_2 & 1 \end{bmatrix}}_{\mathbf{q}_{\mathbf{y}(1)}}$$

$$= \alpha^r \begin{bmatrix} 0 \\ -b_1(y_4 - b_1 y_2) \\ -b_2(y_4 - b_2 y_3) \\ 2y_4 - b_1 y_2 - b_2 y_3 \end{bmatrix}^{\top}$$

**Page 208, line 1,2** "and the final costates, $\lambda_4(1) = 0$ and"

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix}^\top_{\sigma=1} = \underbrace{\begin{bmatrix} y_1 - y_{1,\mathrm{f}} \\ y_2 - y_{2,\mathrm{f}} \\ y_3 - y_{3,\mathrm{f}} \end{bmatrix}^\top_{\sigma=1}}_{\mathbf{q}^\top} \underbrace{\begin{bmatrix} \alpha^r & 0 & 0 \\ 0 & \alpha^r & 0 \\ 0 & 0 & \alpha^r \end{bmatrix}}_{\mathbf{W}} = \alpha^r \begin{bmatrix} y_1 - y_{1,\mathrm{f}} \\ y_2 - y_{2,\mathrm{f}} \\ y_3 - y_{3,\mathrm{f}} \end{bmatrix}^\top_{\sigma=1}$$

**Page 212, line 15,16** "... the optimal objective functional was $-4.46$, which corresponds to the final product concentration of 4.46 g/cm$^3$."

**Page 215, 3rd line from bottom** "increased from 60 to 64.1 min."

**Page 219, 2nd last paragraph, line 4,5** "... the value of $E$ was $4.0 \times 10^{-7}$."

**Page 227, 1st two equations**

$$\frac{\mathrm{d}y_{\lambda_0}}{\mathrm{d}t} = -\left[\frac{\lambda(y - y_\mathrm{f})}{2} + u_\mathrm{s}\right] y_{\lambda_0} + \left[(y - y_\mathrm{f})\lambda_{\lambda_0} + \lambda y_{\lambda_0}\right] \frac{(y_\mathrm{f} - y)}{2} - 2ky_{\lambda_0}$$

$$\frac{\mathrm{d}\lambda_{\lambda_0}}{\mathrm{d}t} = -2y_{\lambda_0} + \left[\frac{\lambda(y - y_\mathrm{f})}{2} + u_\mathrm{s} + 2ky\right]\lambda_{\lambda_0} + \left[(y - y_\mathrm{f})\lambda_{\lambda_0} + (\lambda + 4k)y_{\lambda_0}\right]\frac{\lambda}{2}$$

**Page 227, 3rd line after Results** "shows the initial and optimal states and controls. The convergence ..."

**Page 232, Exercise 7.3, 1st and 3rd equations**

$$\frac{\mathrm{d}y_1}{\mathrm{d}t} = -2y_1 - a_1 + (y_2 + a_1)\exp\left(\frac{a_2 y_1}{y_1 + 4a_1}\right) - (y_1 + 0.5a_1)u, \quad y_1(0) = y_{1,0}$$

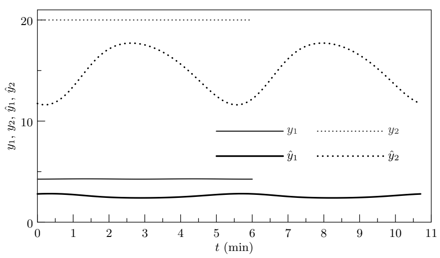
$$I = \int_0^{t_\mathrm{f}} (y_1^2 + y_2^2 + a_3 u^2)\,\mathrm{d}t$$

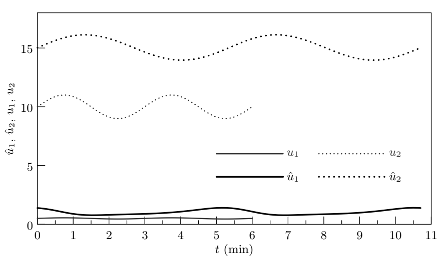
**Page 242, the equation before Equation (8.3)**

$$\begin{bmatrix} H_{u_1} \\ H_{u_2} \end{bmatrix} = \tau \begin{bmatrix} \lambda_1(y_\mathrm{f} - y_1) - \lambda_2 y_2 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

**Page 246, Table 8.1** $y_\mathrm{f}$ is 3.56 g/cm$^3$ and $k_0$ is $10^{-2}$ (cm$^3$/g)$^2$/min.

**Page 247, Figures 8.1 and 8.2**




**Page 247, 3rd line from bottom** "... production rate of $7.8\times10^{-2}$ g/(cm$^3$·min)."

**Page 247, last line to the 1st line of next page** "... maximized the average production rate to 0.17 g/(cm$^3$·min)."

**Page 248, Figure 8.3**